Quantum Field Theory - Friday Problems

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1.1 Consider a theory with the Hamiltonian $H = H_0 + H_{\rm int}$. Using the definition $U(t) = e^{iH_0 t}\, e^{-iHt}$, derive the evolution equation for $U(t)$:

$$i\frac{d}{dt}\,U(t) = H_{\rm int}(t)\,U(t),$$

where

$$H_{\rm int}(t) = e^{iH_0 t}\,H_{\rm int}\,e^{-iH_0 t}.$$

1.2 Use Wick's theorem to find an expression for $T[\phi(x_1)\phi(x_2)\phi(x_3)]$ in terms of $N[\phi(x_1)\phi(x_2)\phi(x_3)]$ and the Feynman propagators $D_F(x_i - x_j)$.

1.3 Given that $\phi_{\rm in}$ is a free field, obeying the Heisenberg equation of motion

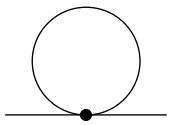
$$\dot{\phi}_{\rm in} = i\,[H_0(\phi_{\rm in}, \pi_{\rm in}),\, \phi_{\rm in}]\,,$$

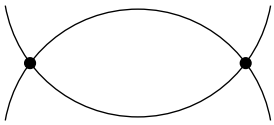
show that $\phi_{\rm out}$ is also a free field, which obeys

$$\dot{\phi}_{\rm out} = i\,[H_0(\phi_{\rm out}, \pi_{\rm out}),\, \phi_{\rm out}]\,.$$

[**Hint:** use $\phi_{\rm out} = S^\dagger \phi_{\rm in} S$ and $\pi_{\rm out} = S^\dagger \pi_{\rm in} S$. Keep in mind that the S-matrix has no explicit time dependence.]

1.4 (Harder) Find the expressions corresponding to the following *momentum space* Feynman diagrams for the scattering amplitude (i.e. the truncated Green's function)





Integrate out all the δ-functions but do not perform the remaining integrals. Argue from the behaviour of the integrands as the loop momenta diverge that both of these Feynman diagrams give infinite results. Which one is more divergent?