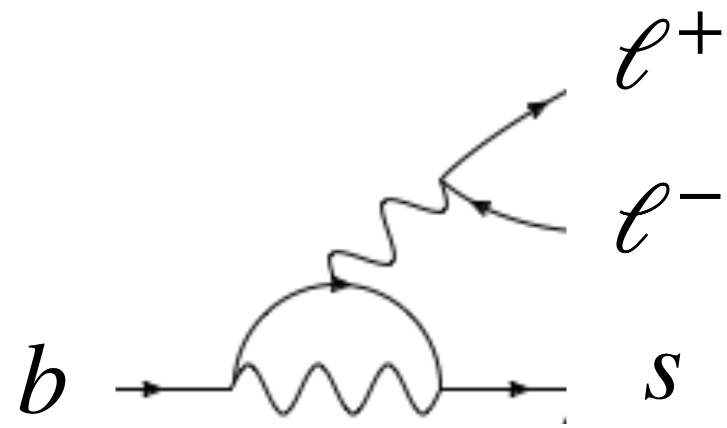
What are the future $b \to s$ measurements?

Tom Hadavizadeh and Rafael Silva Coutinho

Beyond the Flavour anomalies workshop
26th-28th April 2022

$b \to sl^+l^-$ outline

The prospects for $b \to s$ measurements is quite rich and includes several processes



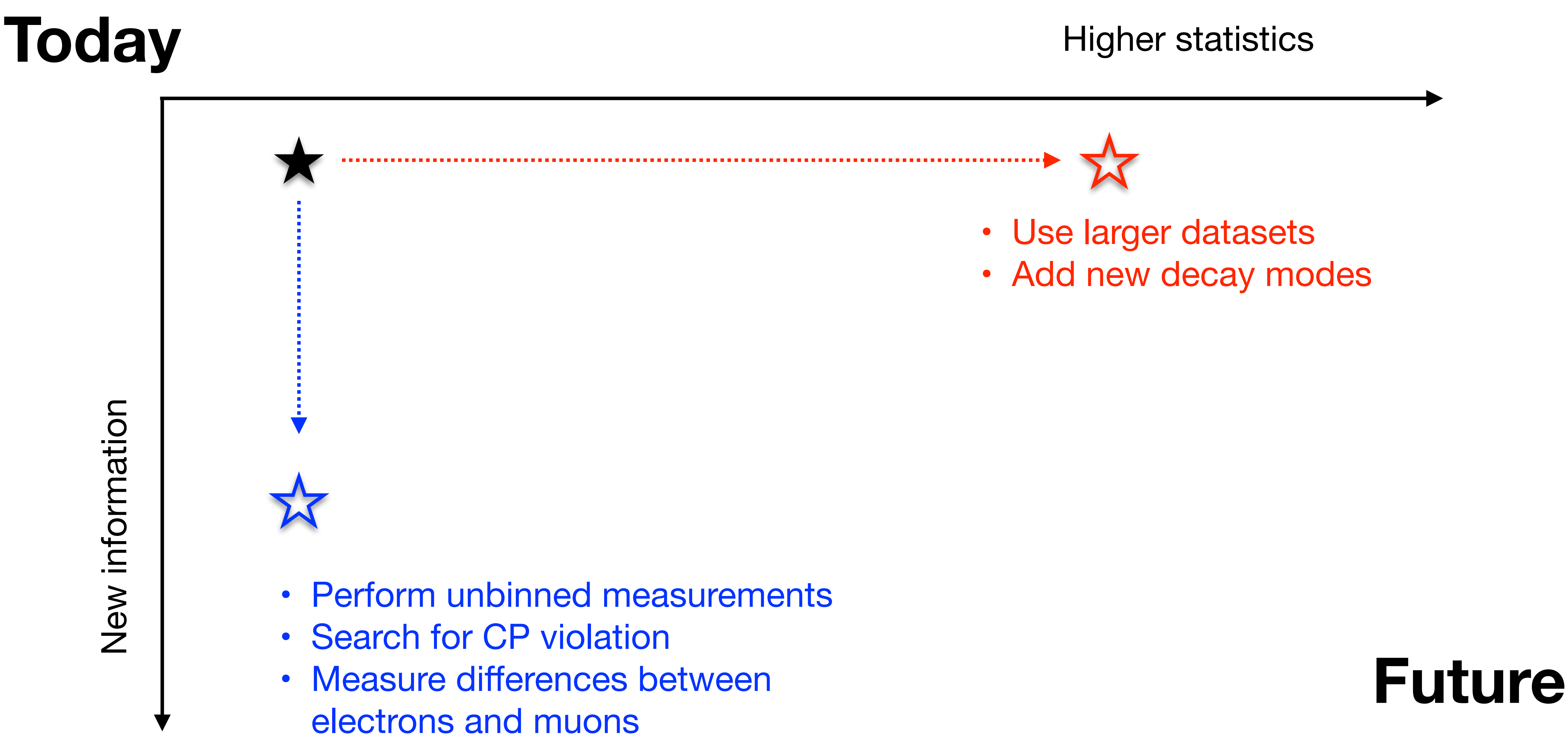
In order to concentrate in some key measurements we will discuss mainly ***angular*** and ***amplitude analyses*** in $b \to sl^+l^-$ ($l = \mu, e$) in transitions

[Lots of BF results are also foreseen, *e.g.* with b-baryon and searches with tau's]

We will hear updates soon about the GPDs, Belle 2 and LHCb Run3, so we'll concentrate on the ***near future*** at **LHCb**

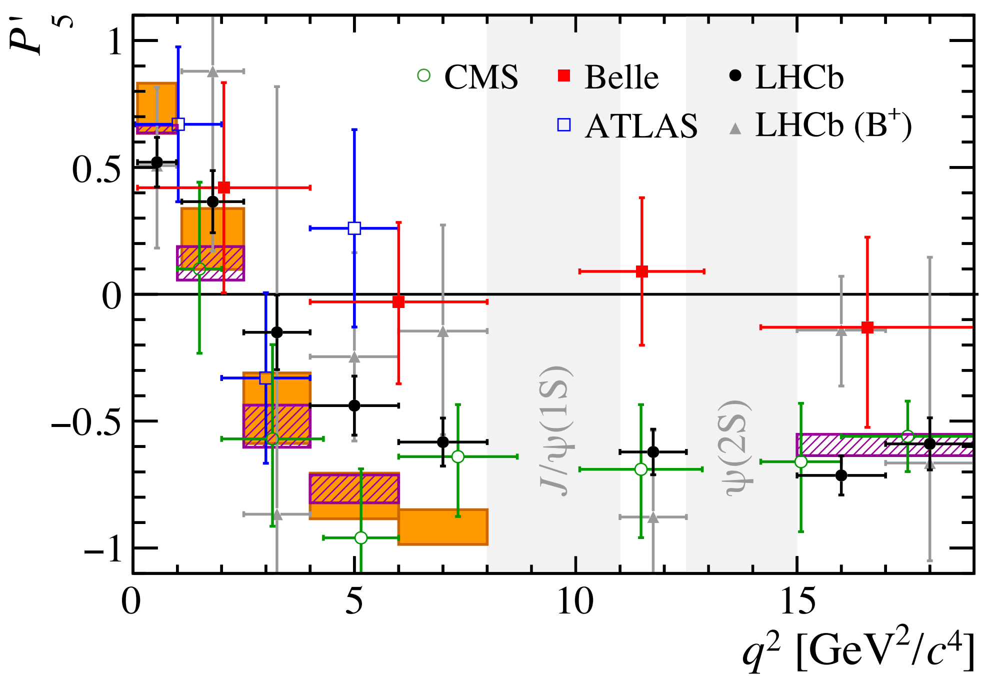
Looking to the future

- Future measurements will allow us to add ***more data*** to existing measurements and add ***new information*** by using different techniques

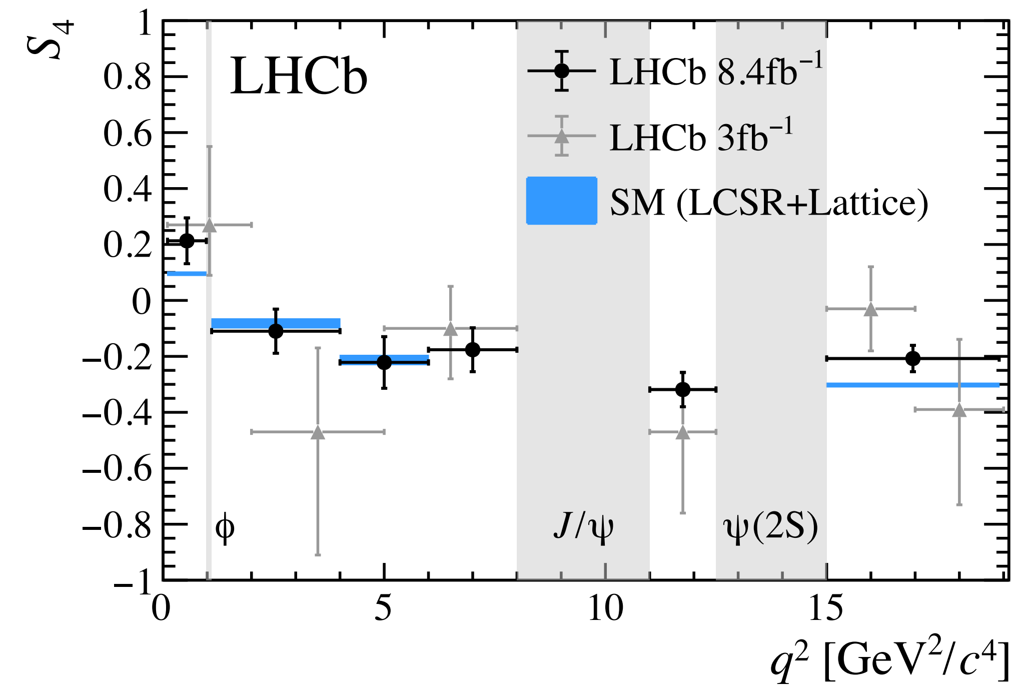


"Binned" angular analyses

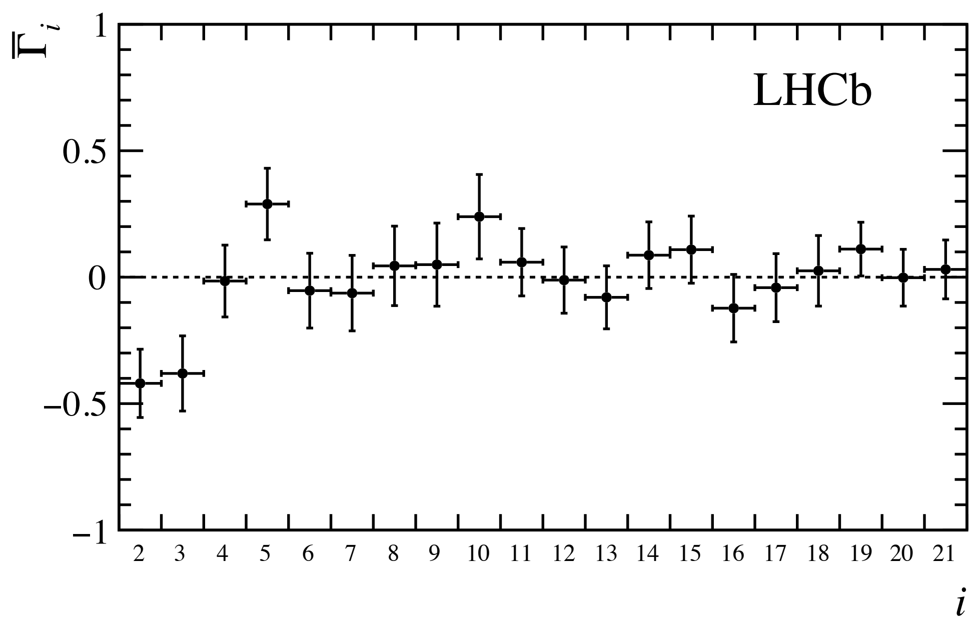
$[B \to K^* \mu^+ \mu^-]$



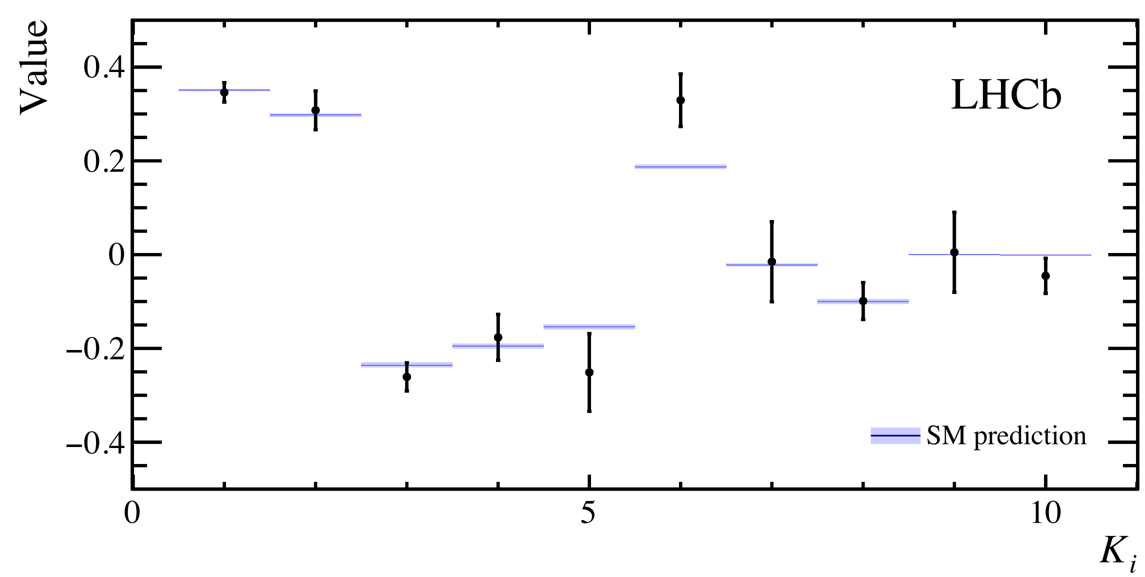
$[B_s^0 \to \phi \mu^+ \mu^-]$



$[B^0 \to (K\pi)^0_{[1330,1530]} \mu^+ \mu^-]$



$[\Lambda_b^0 \to \Lambda \mu^+ \mu^-]$



[LHCb, PRL 125 (2020) 011802, PRL 126 (2021) 161802, arXiv:2107.13428, JHEP 12 (2016) 065, JHEP 09 (2018) 146]

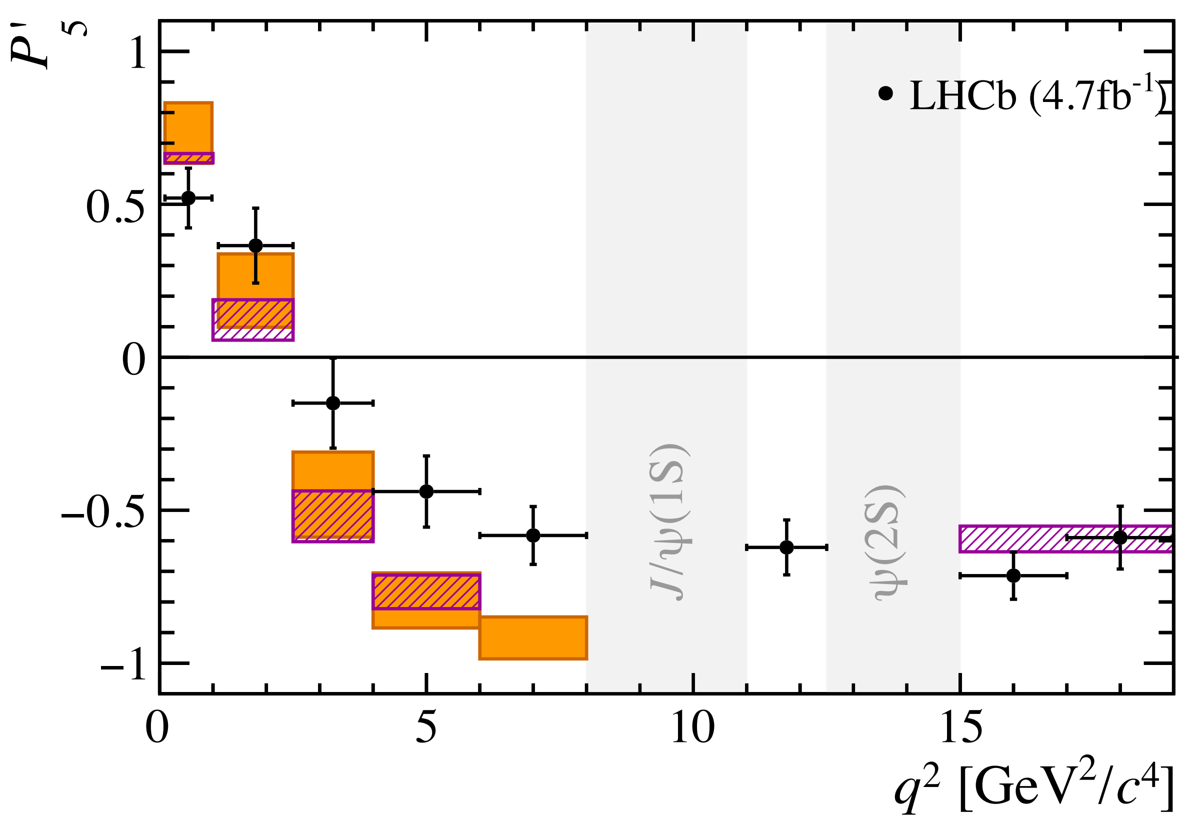
$B^0 \to K^{*0}\mu^+\mu^-$ binned analyses

[How to extract the most of the LHCb available data?]

Increase in statistics

More information

[LHCb, PRL 125 (2020) 011802]



Strategy of previous analysis (Run1+2016):

- Perform angular analysis (θ_K, θ_l, φ) in bins of q^2 to extract P-wave angular obs
- Simultaneously fit $m_{K\pi}$ spectrum (4D+1D PDF) to better constrain S-wave contribution
- CP-asymmetries were not measured

$B^0 \to K^{*0}\mu^+\mu^-$ binned analyses

[How to extract the most of the LHCb available data?]

Increase in statistics

More information

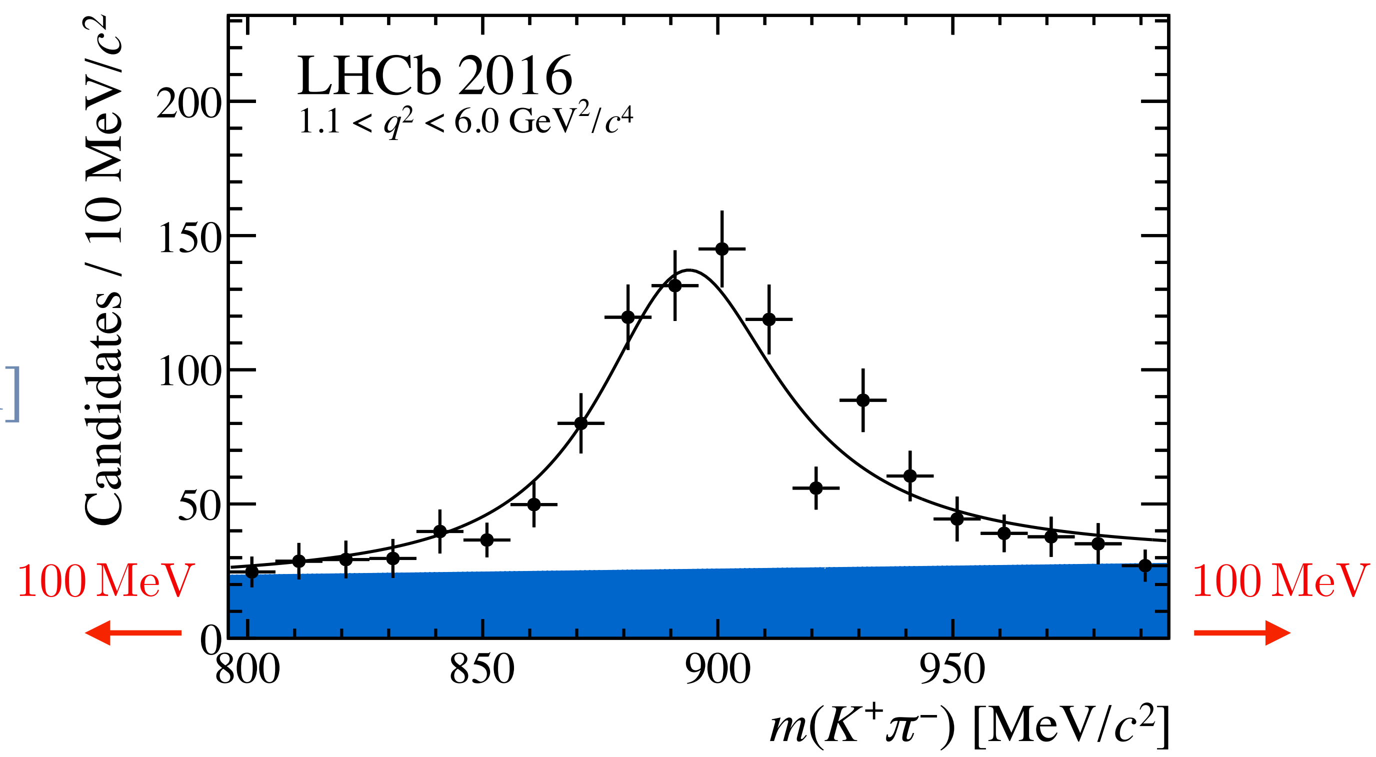
- Additional 2017/2018 data
- Improved selection
- Wider $m_{K\pi}$ window

[possible due to better peaking bkg selection]

- Improved per-event sensitivity in fitter (4D+1D → 5D PDF)

[$m_{K\pi}$ dependence now directly included into PDF]

[LHCb, PRL 125 (2020) 011802]



[795.9, 995.9] → [745.9, 1095.9] MeV

[How to extract the most of the LHCb available data?]

Increase in statistics

More information

- Include both CP-averages and CP-asymmetries in the fit

[M. Algueró *et al*, JHEP 12 (2021) 085]

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\bigg|_{\mathrm{P+S}} \frac{\mathrm{d}^5\overset{\leftrightarrow}{\Gamma}}{\mathrm{d}q^2\,\mathrm{d}m_{K\pi}\,\mathrm{dcos}\theta_\ell\,\mathrm{dcos}\theta_K\,\mathrm{d}\phi} = (1-F_\mathrm{S})\frac{9}{32\pi}\sum_{i\,\in\,\text{P-wave}}\tfrac{1}{2}(S_i \pm A_i) f_i(\cos\theta_\ell,\cos\theta_K,\phi)|\mathcal{BW}_\mathrm{P}(m_{K\pi})|^2$$

$$= \frac{3}{16\pi}\Big[\Big(\tfrac{1}{2}(S_{10}\pm A_{10}) + \tfrac{1}{2}(S_{12}\pm A_{12})\cos 2\theta_\ell\Big)\times\big|\mathcal{BW}_\mathrm{S}(m_{K\pi})\big|^2$$

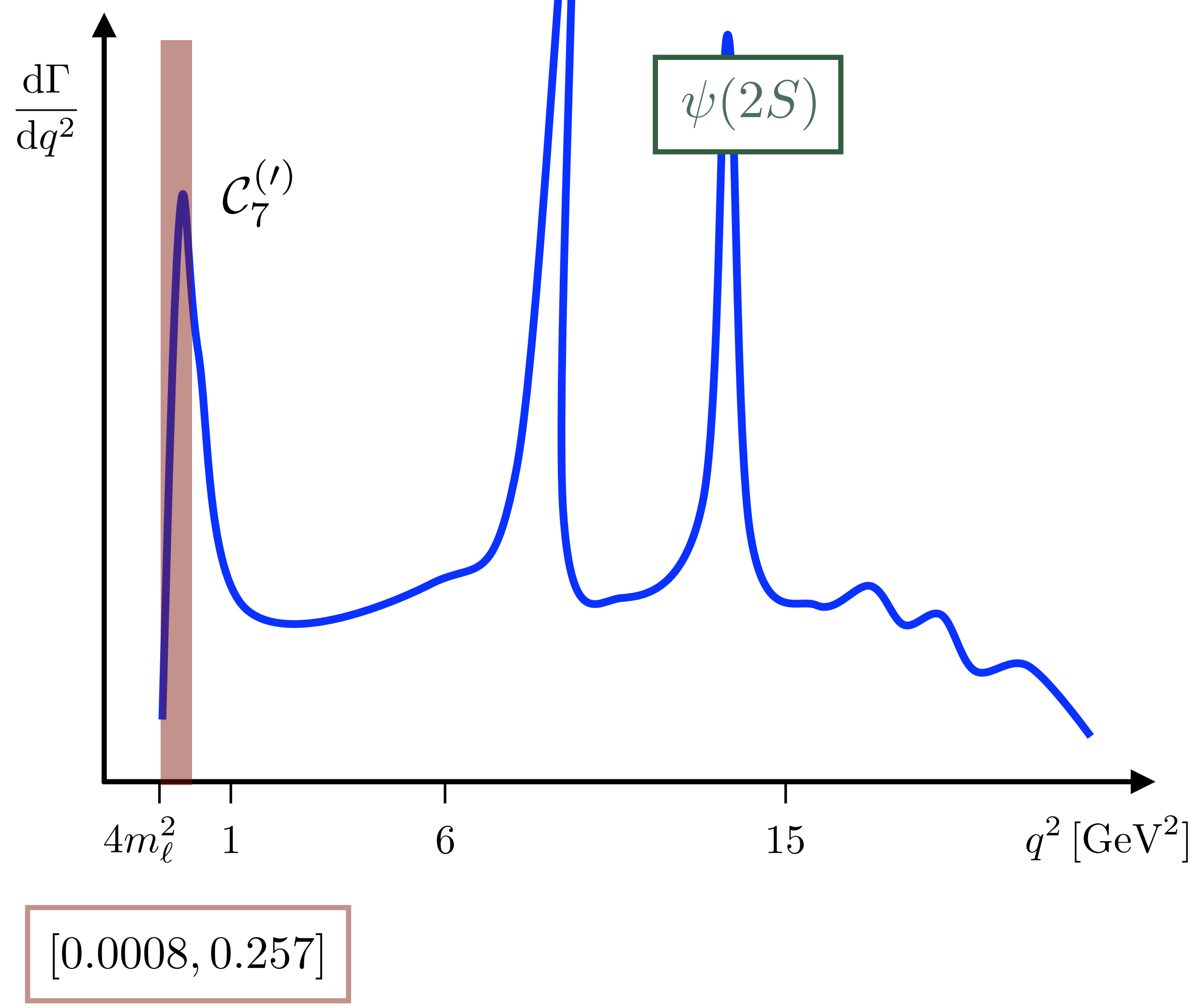
$$+\tfrac{1}{2}\cos\theta_K\Big((S_{11}^{\mathrm{re}}\pm A_{11}^{\mathrm{re}})\times\mathrm{Re}[\mathcal{BW}_\mathrm{S}(m_{K\pi})\mathcal{BW}^*_\mathrm{P}(m_{K\pi})] - (S_{11}^{\mathrm{im}}\pm A_{11}^{\mathrm{im}})\times\mathrm{Im}[\mathcal{BW}_\mathrm{S}(m_{K\pi})\mathcal{BW}^*_\mathrm{P}(m_{K\pi})]\Big)+\ldots$$

- Drop assumption of massless leptons in the [0.1, 0.98] GeV q^2 bin
  - 6 (2) more P(S)-wave terms: S_{2s}, S_{1c}, S_{6c}, A_{2s}, A_{1c}, A_{6c} (S_{12}, A_{12})
  - 4 more P/S-wave interference terms: S_{13}^{re}, A_{13}^{re}, S_{13}^{im}, A_{13}^{im}

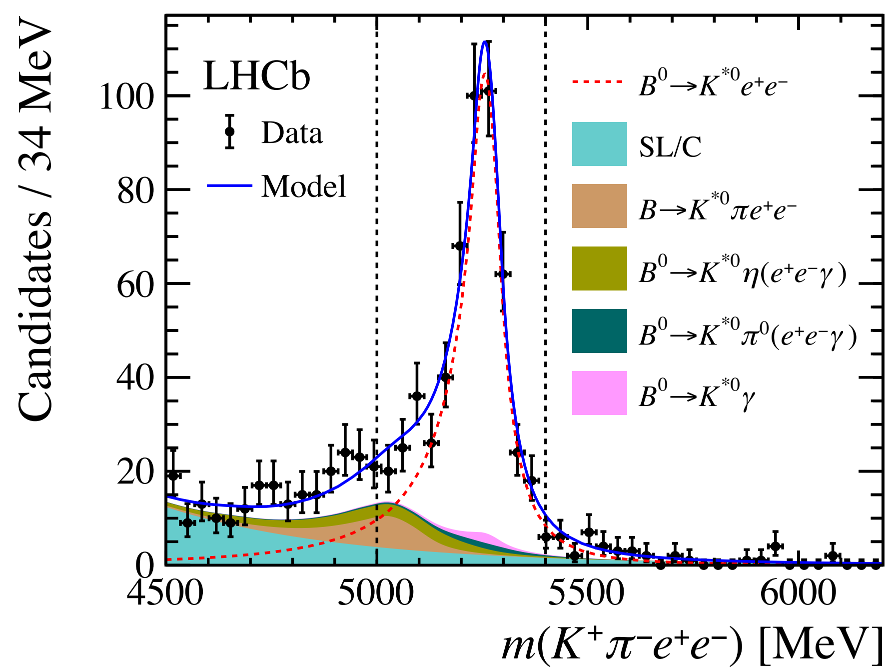
- Finer q^2 granularity (*e.g.* half-width)?

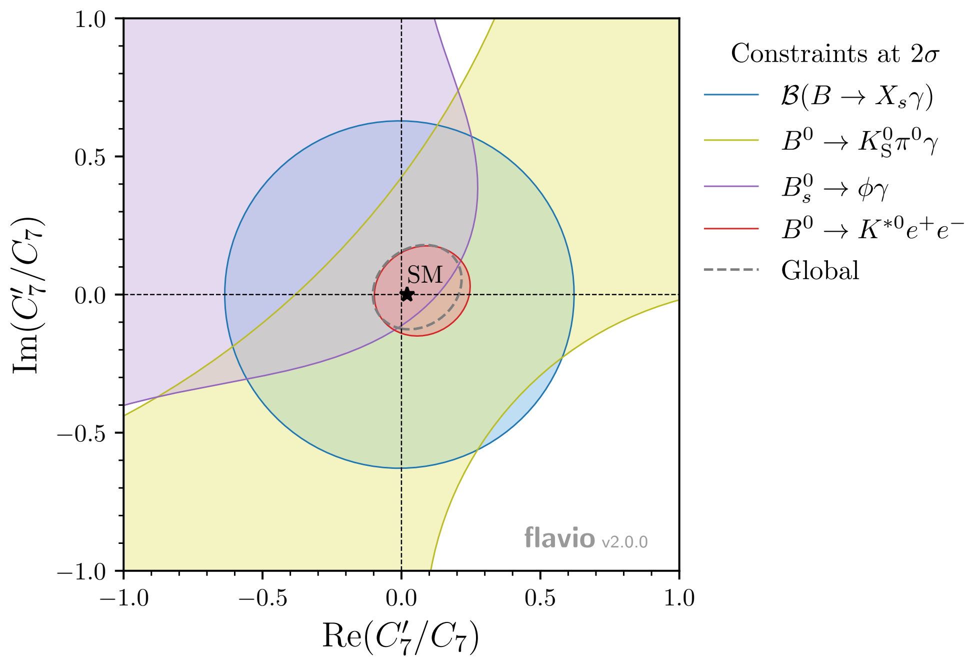
What about analyses with electrons?

Guinea pig: $B^0 \to K^{*0}e^+e^-$



[LHCb, JHEP 12 (2020) 081]



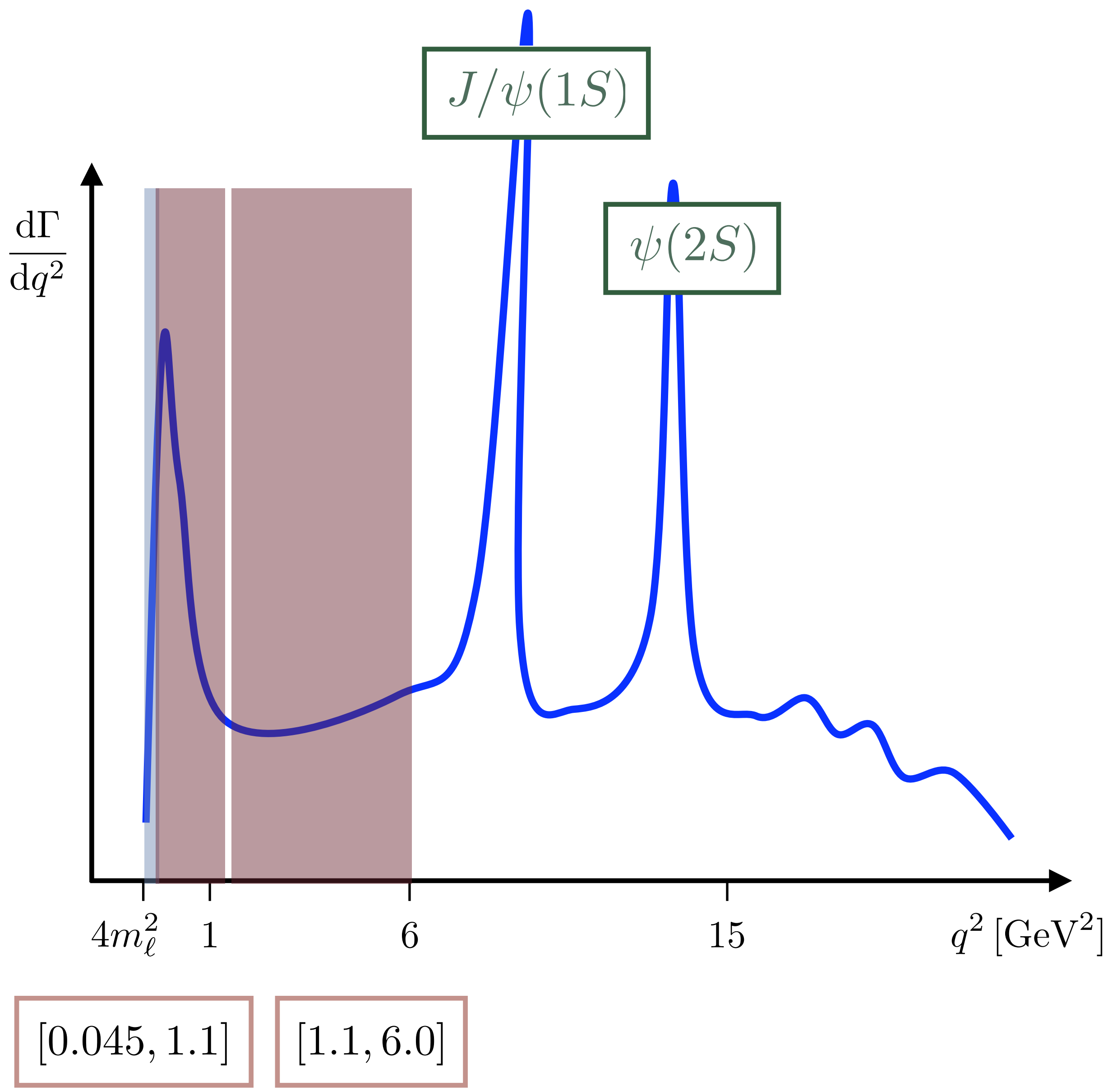


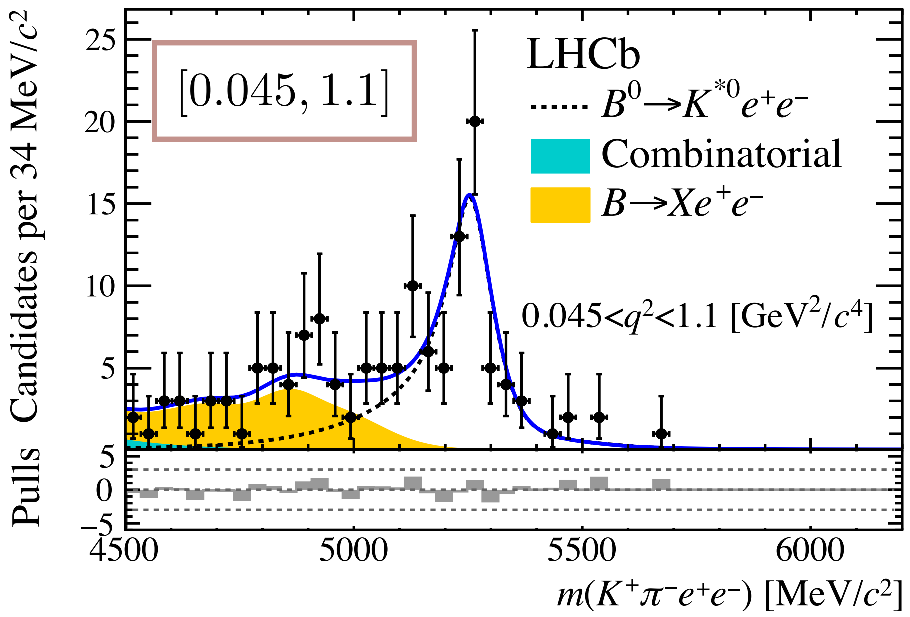
* see more details about electrons in Marie-Hélène's talk

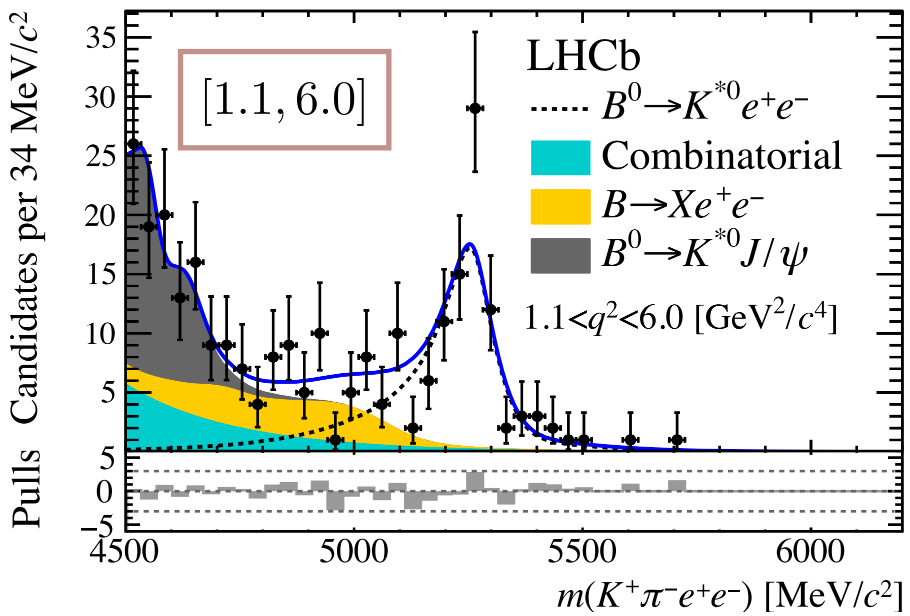
What about analyses with electrons?

Guinea pig: $B^0 \to K^{*0}e^+e^-$

[LHCb, JHEP 08 (2017) 055]





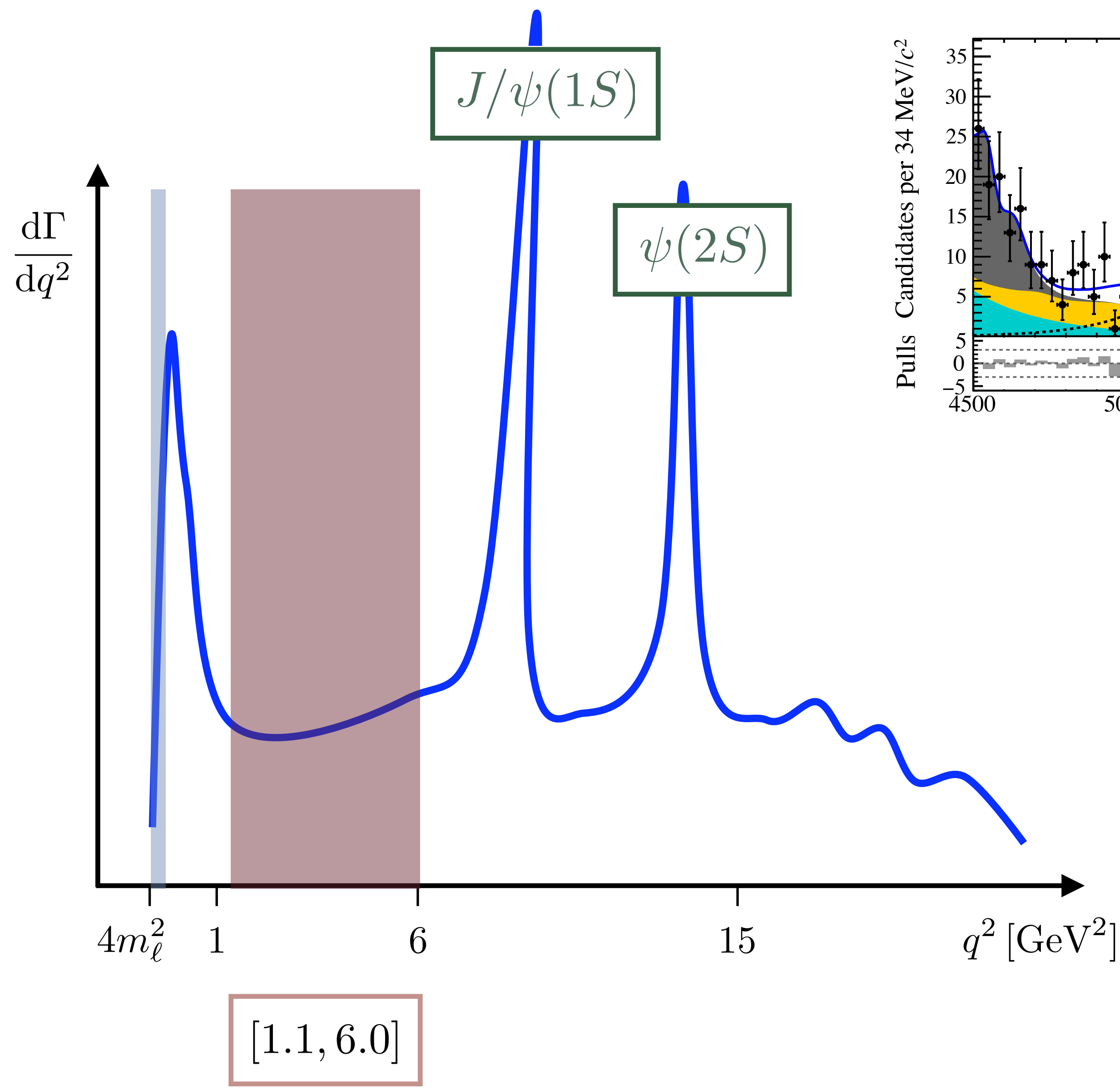


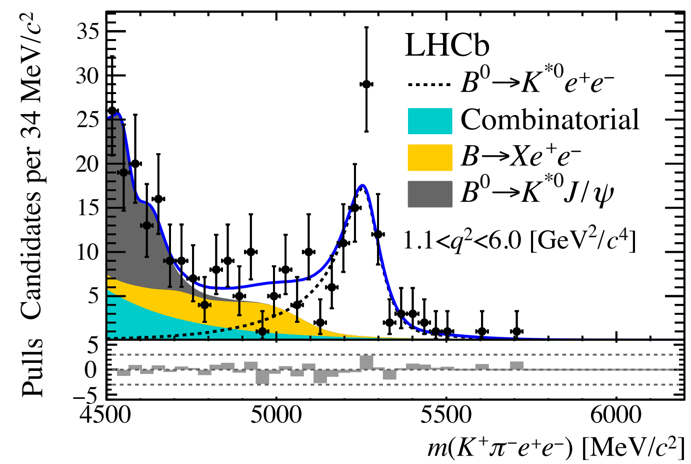
What about analyses with electrons?

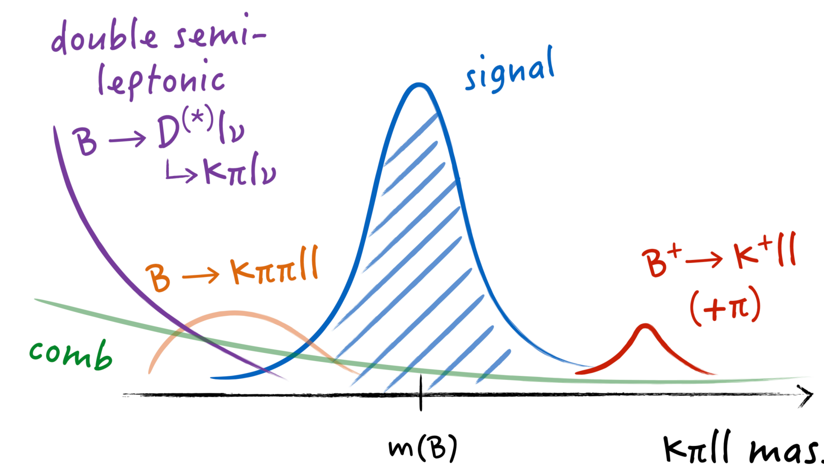
MONASH University

Guinea pig: $B^0 \rightarrow K^{*0}e^+e^-$

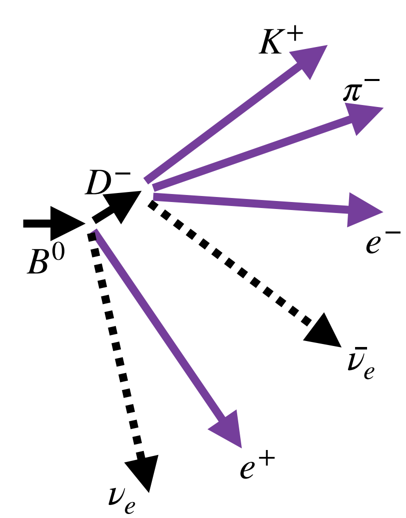
[LHCb, ongoing]

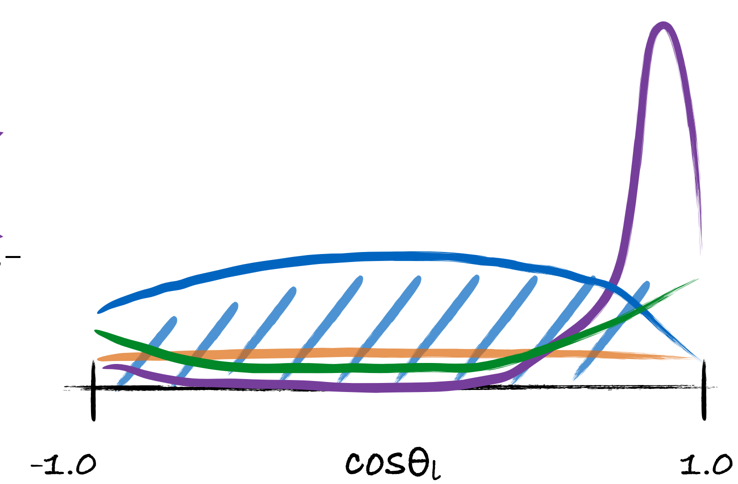






Good control over q^2 and $\vec{\Omega}(\cos\theta_l, \cos\theta_K, \phi)$:



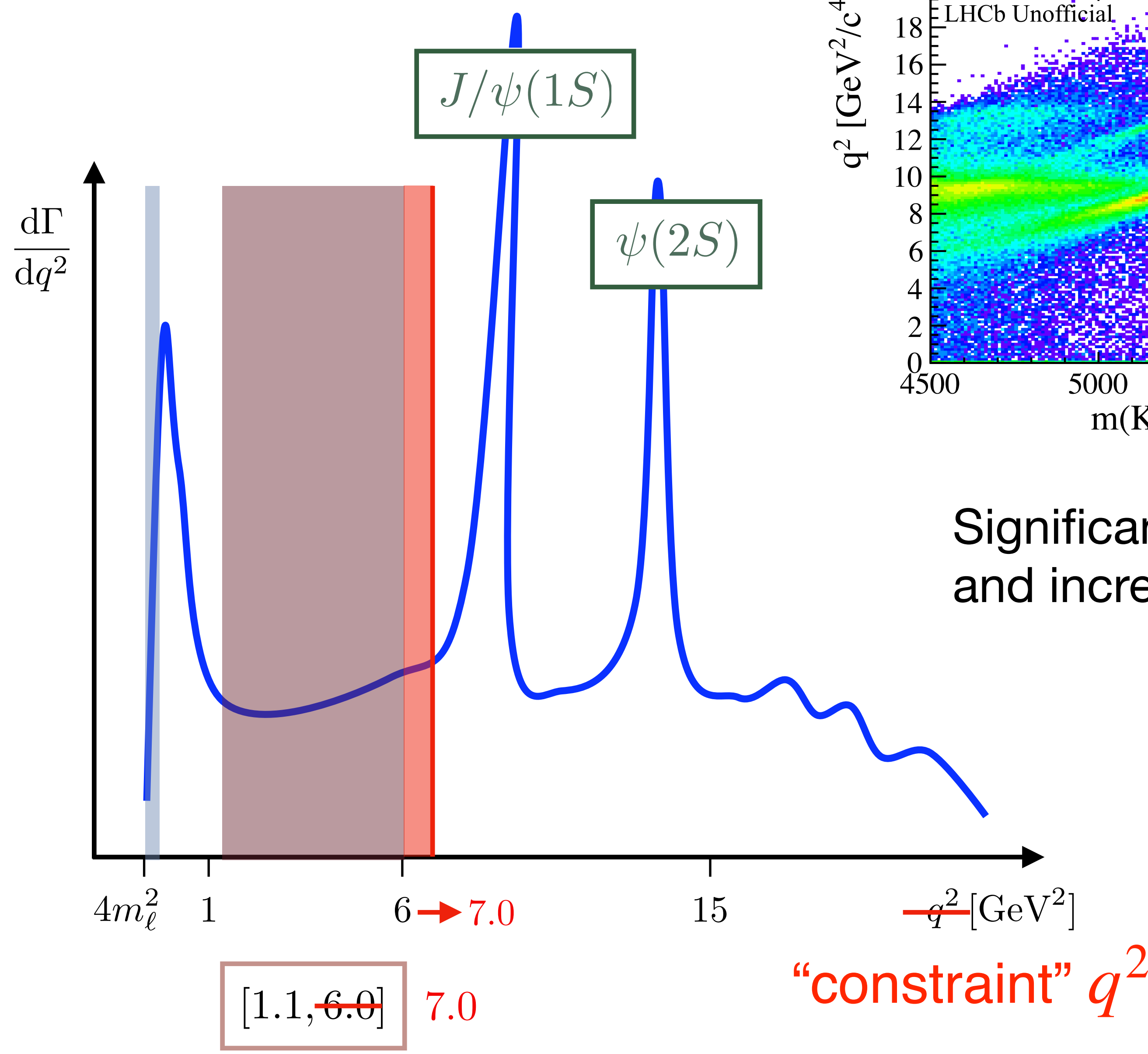


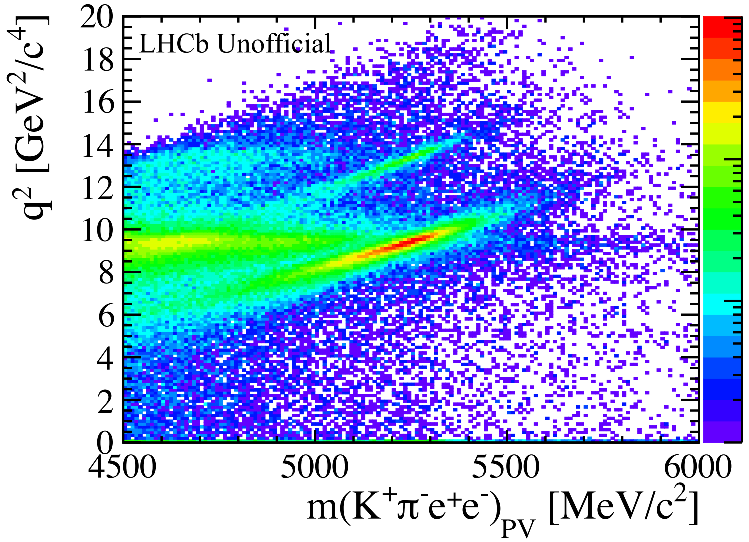
What about analyses with electrons?

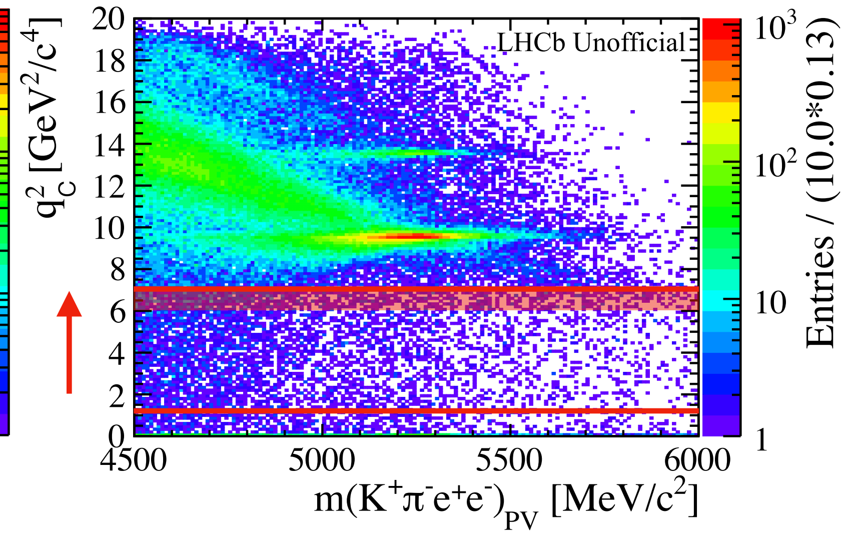
Guinea pig: $B^0 \to K^{*0}e^+e^-$

[LHCb, ongoing]

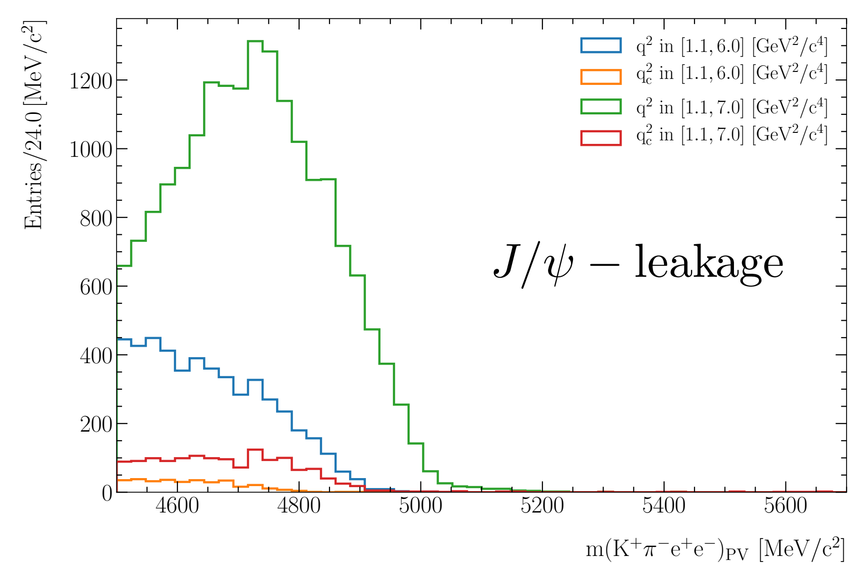
[F. Lionetto PhD Thesis]







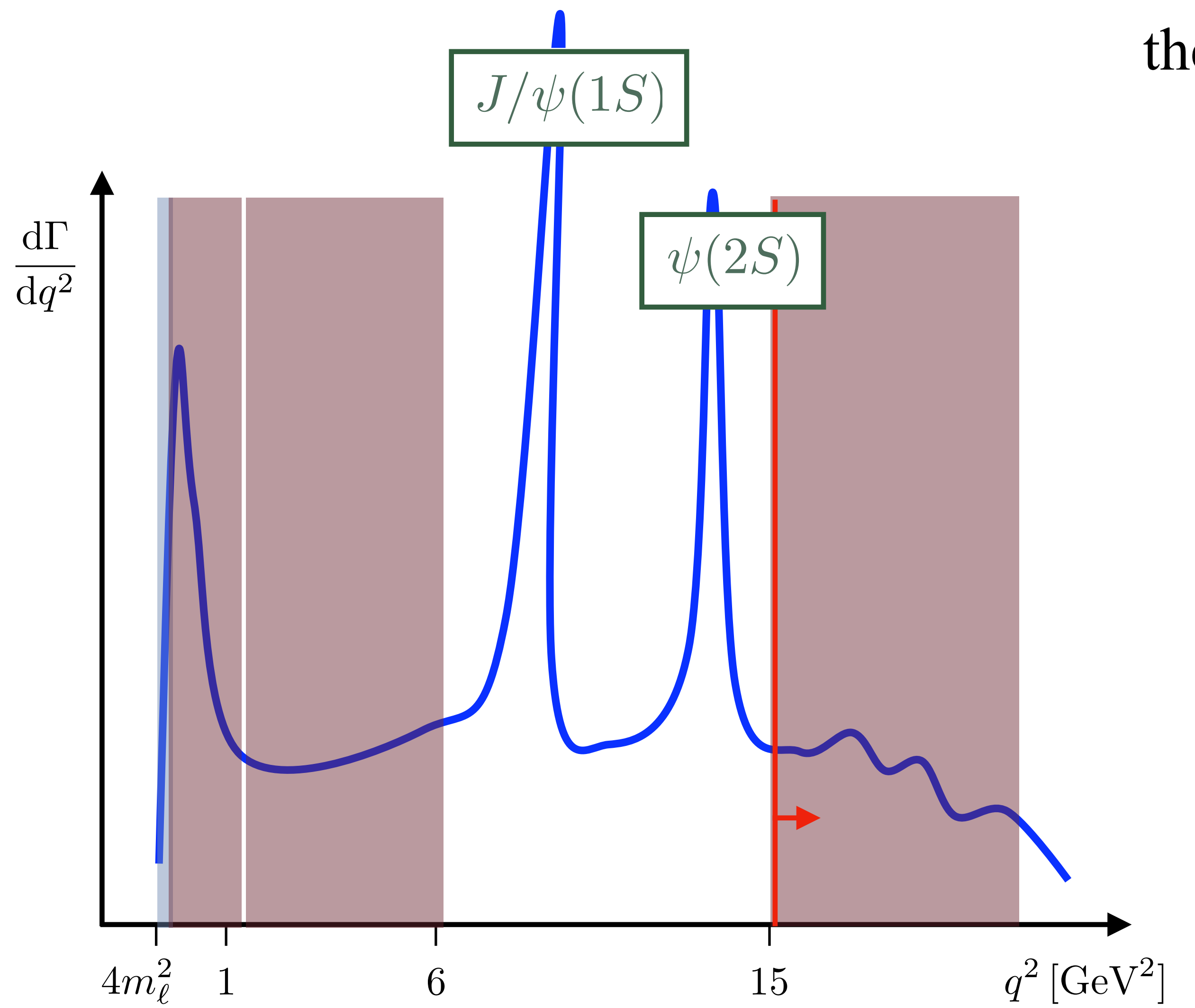
Significantly improve resolution (reduce bin migration) and increase statistics ~ 15-20%



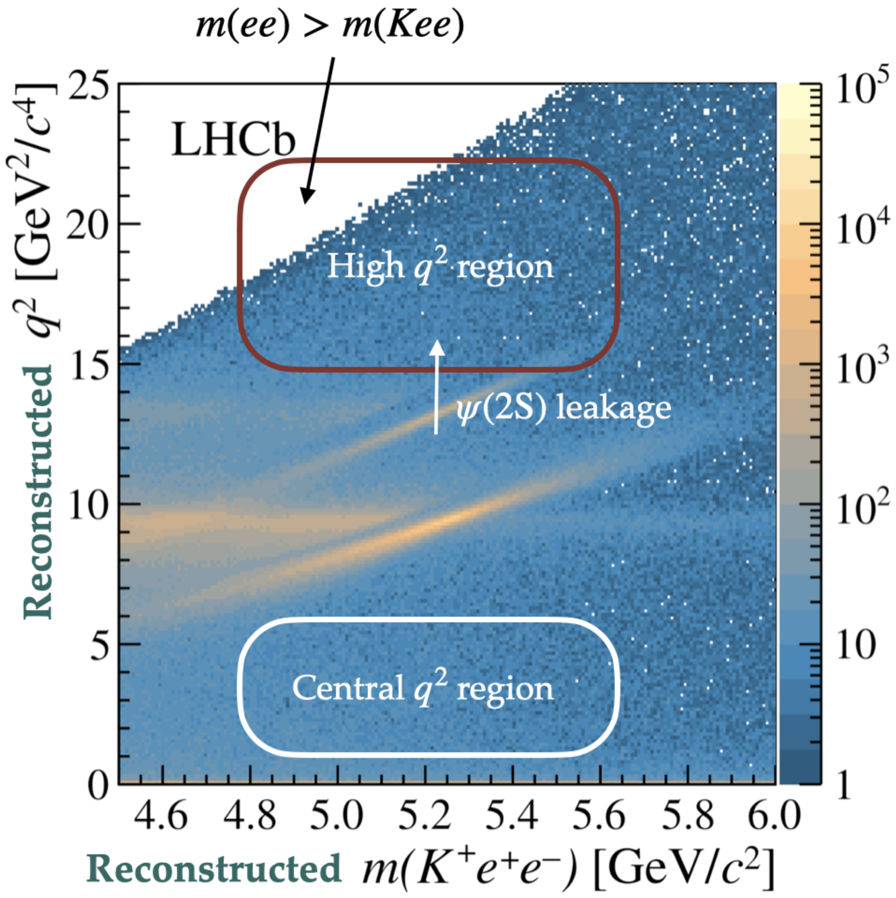
What about analyses with electrons?

Guinea pig: $B^0 \to K^{*0}e^+e^-$

[LHCb, ongoing]



While the low-q² region has similar features as the central-q², high-q² is a challenge regime:



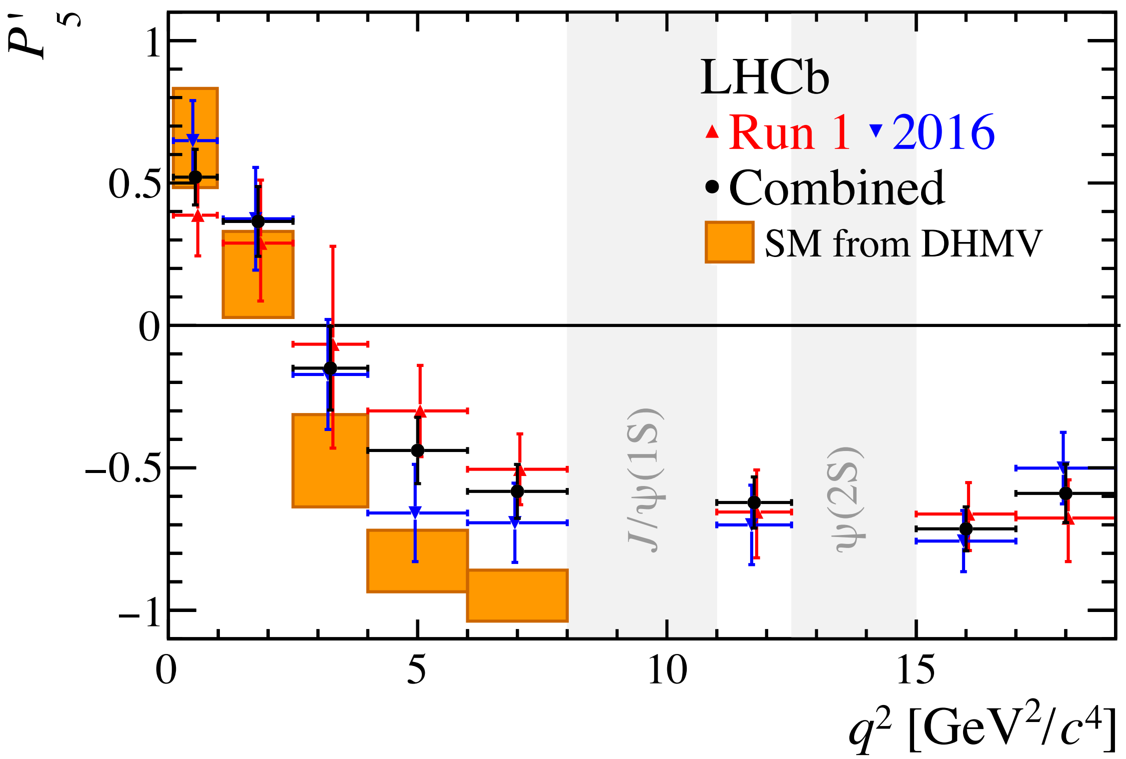
Different ideas under study, *e.g.* $q^2_{\text{no Brem}}$ or MVA based on $q^2_{\text{no Brem}}$, q^2_{PV} and Brem info

Going unbinned

- Current amplitude analyses show tension with the SM

LHCb Run1+2016 $B^0 \to K^{*0}\mu^+\mu^-$ **update**

LHCb-PAPER-2020-002 [PhysRevLett.125.011802]



- Updates with higher statistics will help identify if this is a genuine effect or a fluctuation

- However, performing measurements that are unbinned in q^2 can help to untangle New Physics and SM hadronic effects
  - e.g. shifts resulting from $c\bar{c}$ loops may vary as a function of q^2

Decay rates

$$\frac{d^4\Gamma(B^0 \to K^{*0}\mu^+\mu^-)}{dq^2 d\vec{\Omega}} = \frac{9}{32\pi}\sum_i J_i(q^2) f_i(\cos\theta_\ell, \cos\theta_K, \phi)$$

Penguin decay rate is written in terms of 6 P-wave and 2 S-wave amplitudes

P-wave	**S-wave**	**Time-like**
$A_\lambda^{L,R}(q^2)$	$A_{00}^{L,R}(q^2)$	$A_{time}(q^2)$

- These are written in terms of effective Wilson coefficients and Form factors, e.g.

$$A_\lambda^{L,R}(q^2) = N_\lambda\left(\left(\left[C_9^{\text{eff},\lambda}(q^2) \pm C_9'\right] \mp \left[C_{10} \pm C_{10}'\right]\right) F_V^\lambda(q^2) + \frac{2m_b}{q^2} C_7^{\text{eff},\lambda}(q^2) F_T^\lambda(q^2)\right)$$

- Non-local effects can be included by adding addition terms to the amplitudes or modifying the Wilson coefficients to have additional q^2-dependent terms

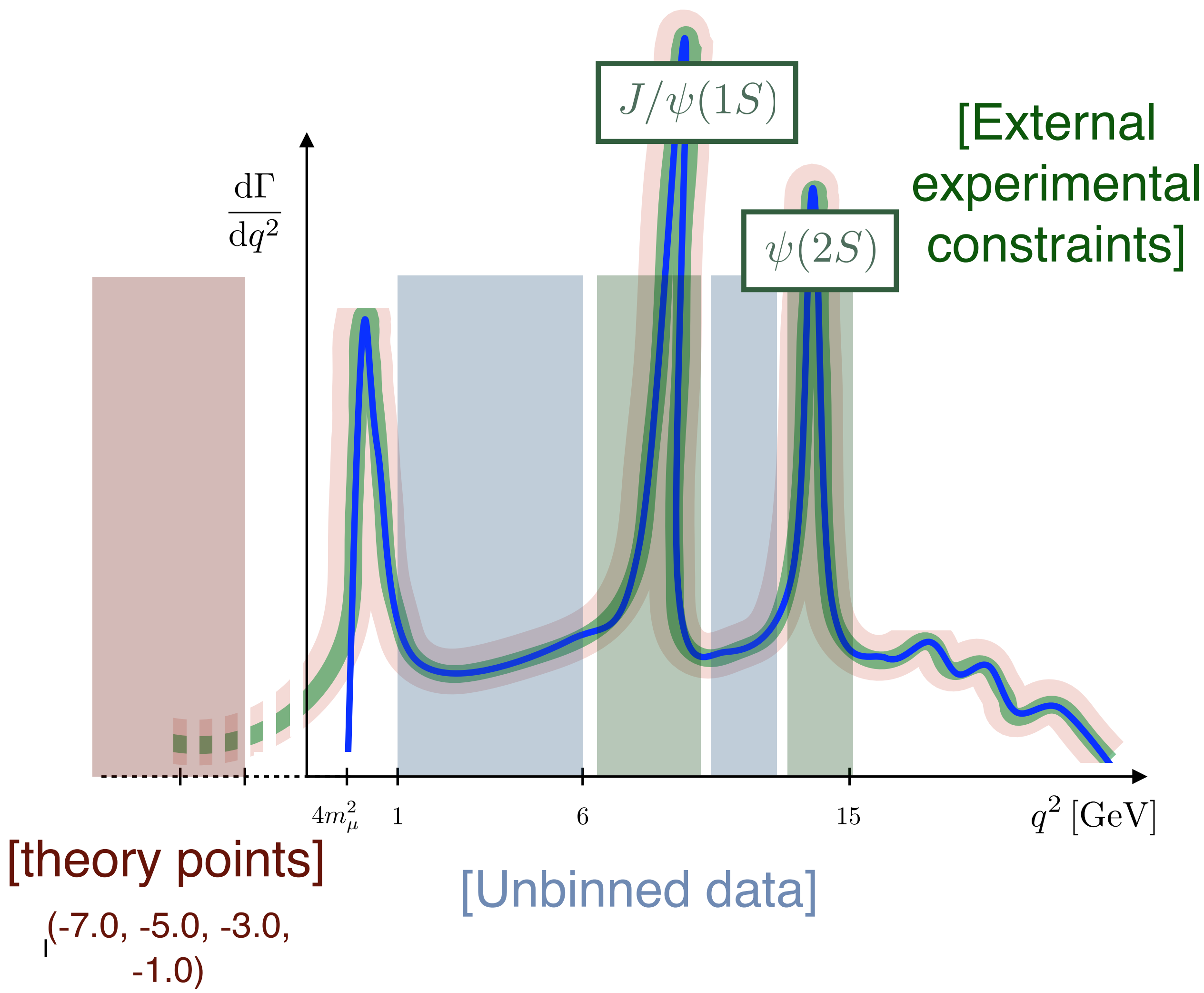
$$A_\lambda^{L,R} = A_\lambda^{L,R,\text{Local}} + H_\lambda(q^2) \qquad\qquad C_9^{\text{eff},\lambda}(q^2) = C_9^\mu + G_\lambda(q^2)$$

z-expansion

Method 1

Hybrid theory-experimental approach to perform direct fits to WCs

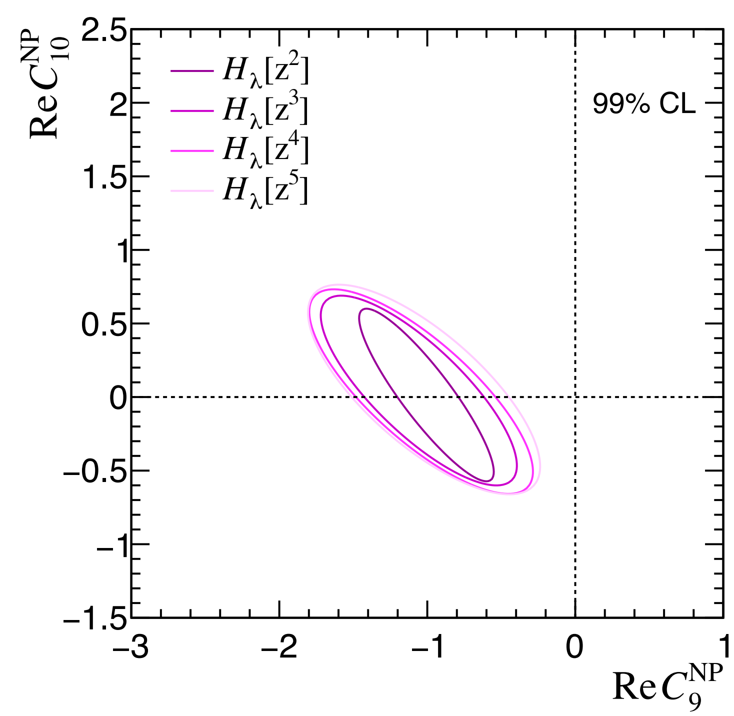
[EPJC 78 (2018) 6, 451, JHEP 02 (2021) 088, JHEP 10 (2019) 236]



- Constrain from theory ($q^2 < 0$) with*

$$\mathcal{H}_\lambda(z) = \frac{1 - zz^*_{J/\psi(1S)}}{z - z_{J/\psi(1S)}} \frac{1 - zz^*_{\psi(2S)}}{z - z_{\psi(2S)}} \times \hat{\mathcal{H}}_\lambda(z)$$

$$\hat{\mathcal{H}}_\lambda(z) = \phi_\lambda^{-1}(z) \times \sum_n \alpha_{\lambda,n} p_n(z)$$



- Constraints from $B^0 \to \psi(n)K^{*0}$

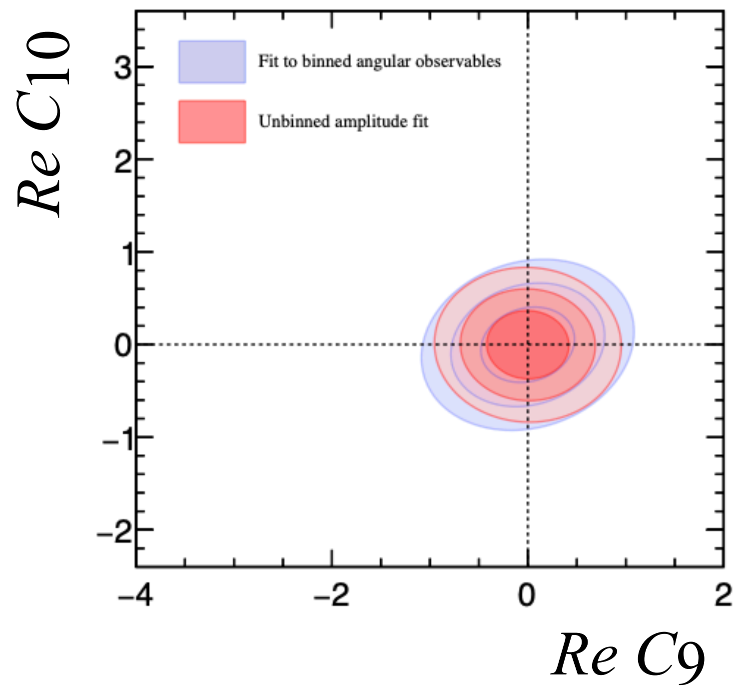
$$\mathrm{Res}_{q^2 \to m_\psi} \mathcal{H}_\lambda = \frac{m_\psi f^*_\psi \mathcal{A}^\psi_\lambda}{m_B^2}$$

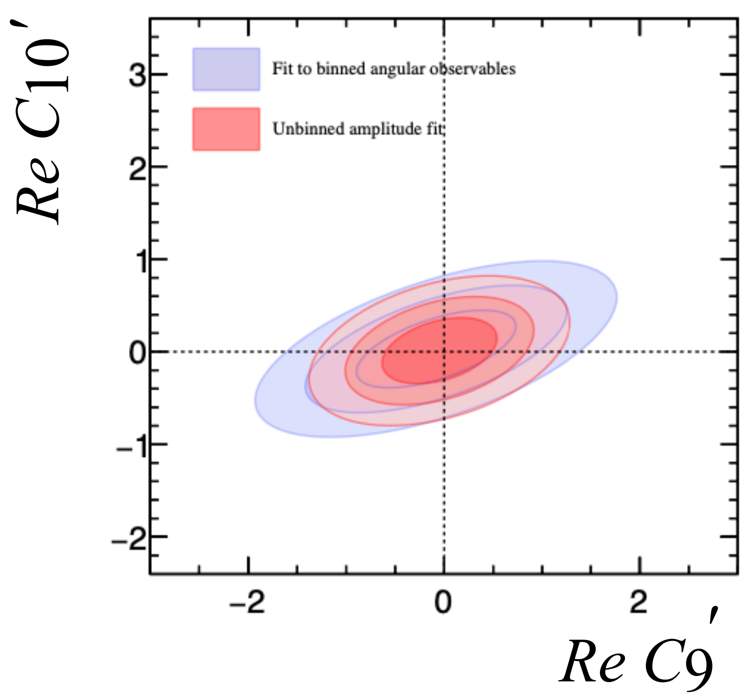
What is the compatibility when removing theory point?

* see more details about the non-local parameterisation in M. Rebound's talk

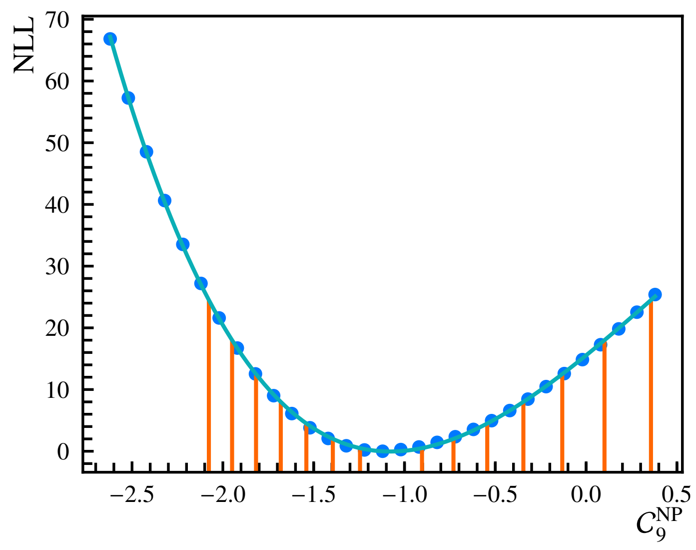
z-expansion

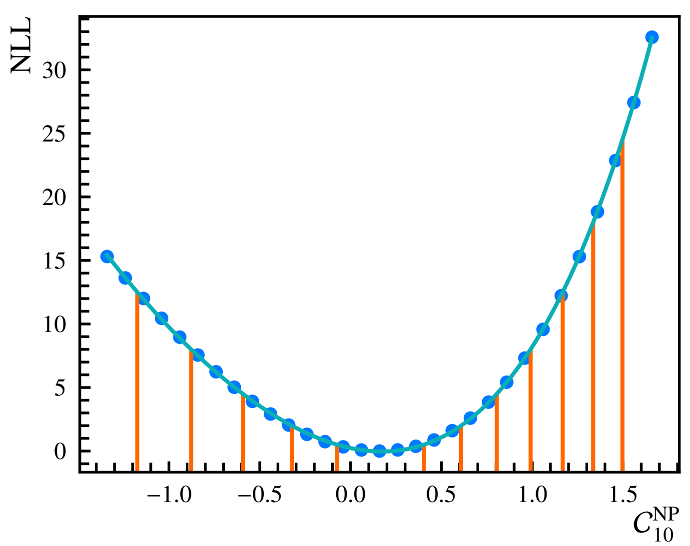
- Improved sensitivity wrt standard binned analysis (including BR info in both):

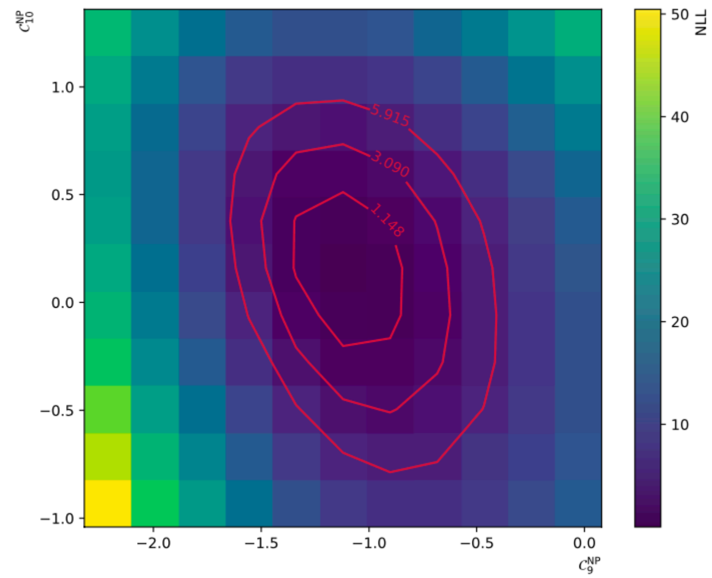




- Outputs: Analysis aims to provide WCs likelihoods fit results (*e.g.*):





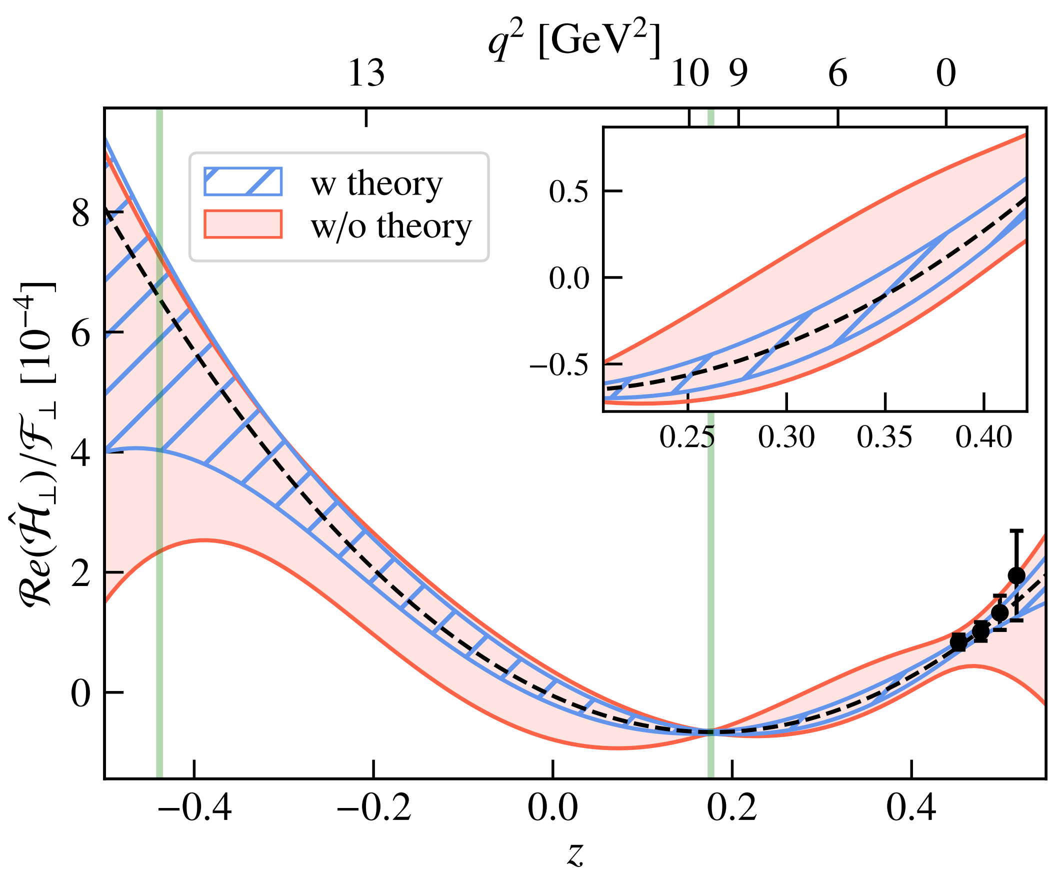


z-expansion

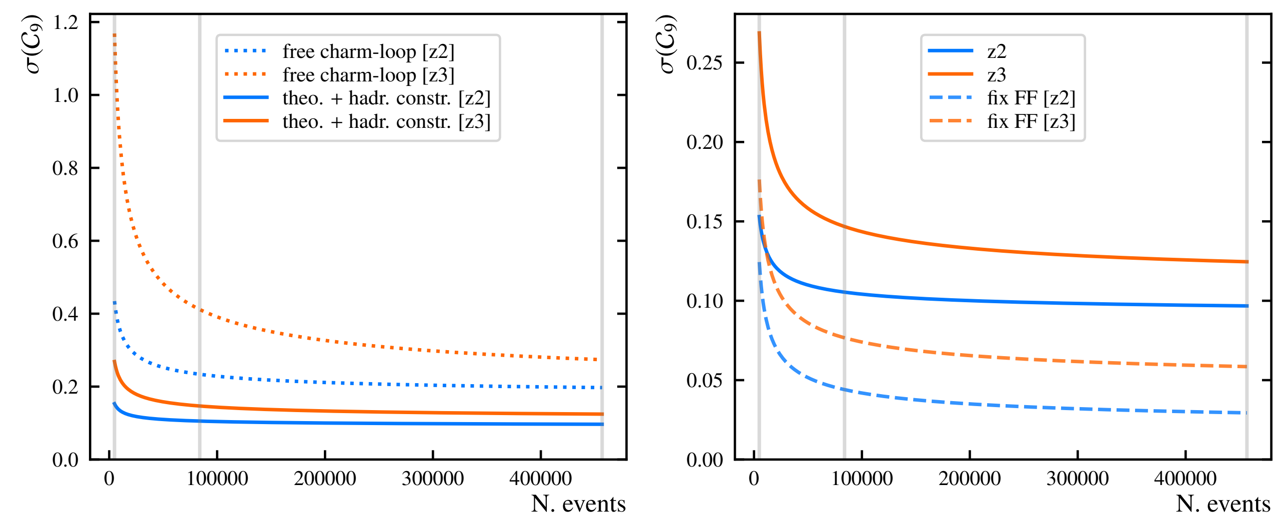
Plan to provide the “anatomy” of the impact of the theory constraints

Compatibility w vs w/o theory inputs

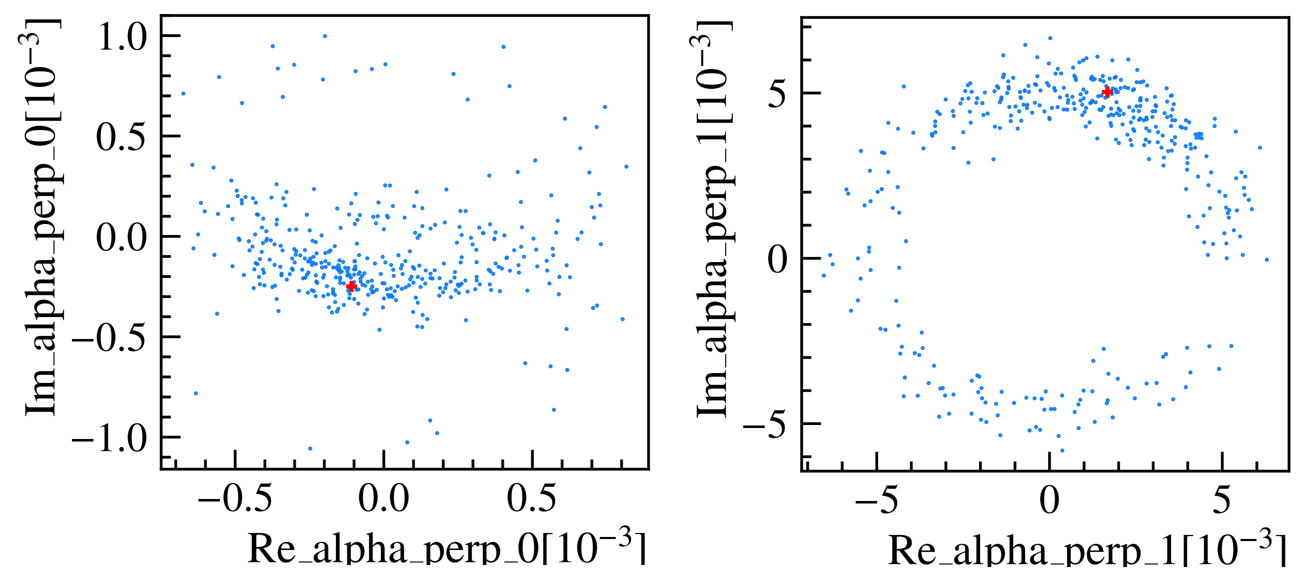
[A. Mauri *et al*, JHEP 10 (2019) 236]



Also in terms of e.g. ΔC_9



[toy simulation]



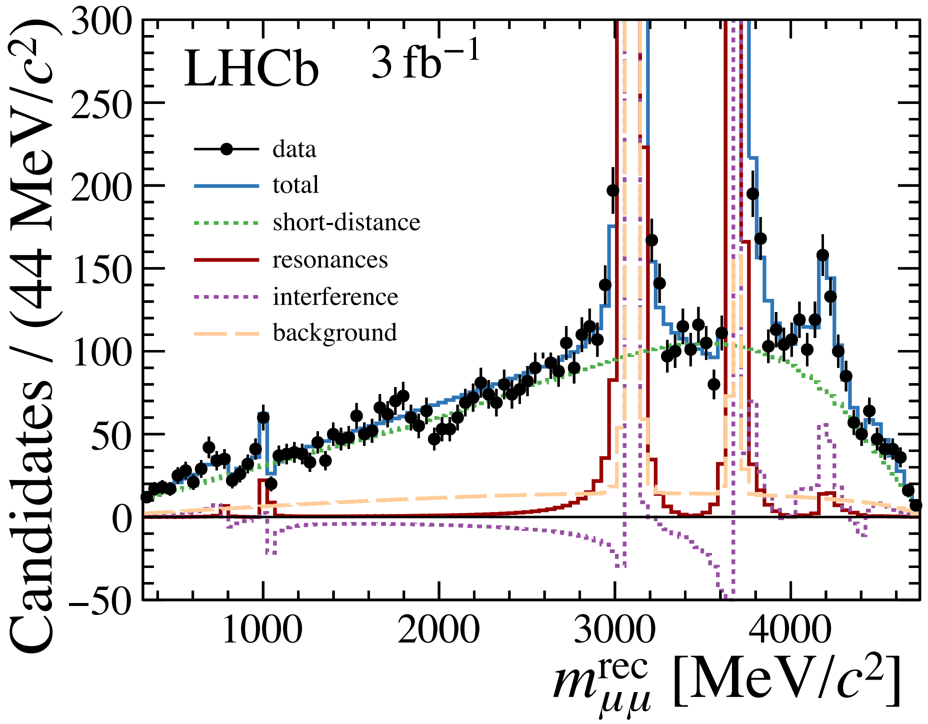
Provide full set of a posteriori non-local parameters (and FFs) bootstrapped from fit results

Non-local models

- Another method involves using explicit models for the vector resonates
  - The full q^2 range can be modelled accounting for J/ψ, $\psi(2S)$ *etc.*
- Non-local models could be ***isobar*** or ***dispersion relation*** models

T. Blake, U. Egede, P. Owen, K. A. Petridis & G. Pomery [Eur. Phys. J. C 78, 453 (2018)]

Previous measurements of $B^+ \to K^+\mu^+\mu^-$ used an isobar model



LHCb-PAPER-2016-045 [Eur.Phys.J.C 77 (2017) 3, 161]

$$C_7^{\text{eff},\lambda}(q^2) = C_7^{\mu} + \zeta^{\lambda} e^{i\omega^{\lambda}}$$

$$C_9^{\text{eff},\lambda}(q^2) = C_9^{\mu} + \sum_j \eta_j^{\lambda} e^{i\delta_j^{\lambda}} A_j^{\text{res}}(q^2)$$

Magnitude and phase of Relativistic Breit-Wigners free to vary relative to C_9

Dispersion model

- In the dispersion relation model these effective Wilson coefficients are written in terms of local and non-local contributions

Local — Non-local contributions

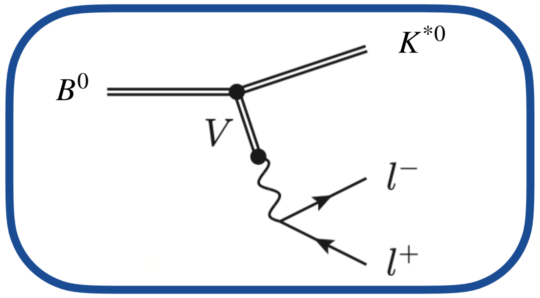
$$C_7^{\mathrm{eff},\lambda}(q^2) = C_7^{\mu} + \epsilon^{\lambda} e^{i\omega^0}$$

$$C_9^{\mathrm{eff},\lambda}(q^2) = C_9^{\mu} + Y_{c\bar{c}}^{(0),\lambda} + Y_{c\bar{c}}^{1P,\lambda}(q^2) + Y_{\mathrm{light}}^{1P,\lambda}(q^2) + Y_{c\bar{c}}^{2P,\lambda}(q^2) + Y_{\tau\bar{\tau}}(q^2)$$

This is determined theoretically at negative q^2 values

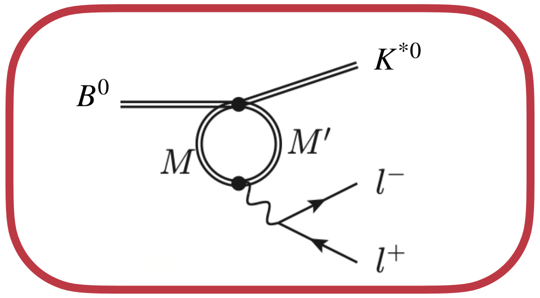
Constant offset

Asatrian, Greub, Virto [JHEP 04 (2020) 012]



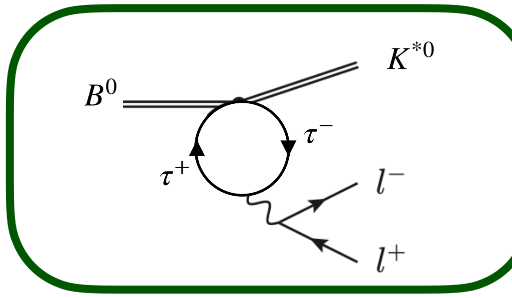
1-particle contributions

Includes:
$\rho(770)$, $\phi(1020)$, J/ψ, $\psi(2S)$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$



2-particle contributions

Includes:
$D\overline{D}$, $D^*\overline{D}$, $D^*\overline{D}^*$



Tau loop contribution

C. Cornella, G. Isidori, M. König, S. Liechti, P. Owen, N. Serra [Eur.Phys.J.C 80 (2020) 12, 1095]

Method 2

Dispersion model

- In the dispersion relation model these effective Wilson coefficients are written in terms of local and non-local contributions

Local — Non-local contributions

$$C_7^{\text{eff},\lambda}(q^2) = C_7^\mu + \epsilon^\lambda e^{i\omega^0}$$

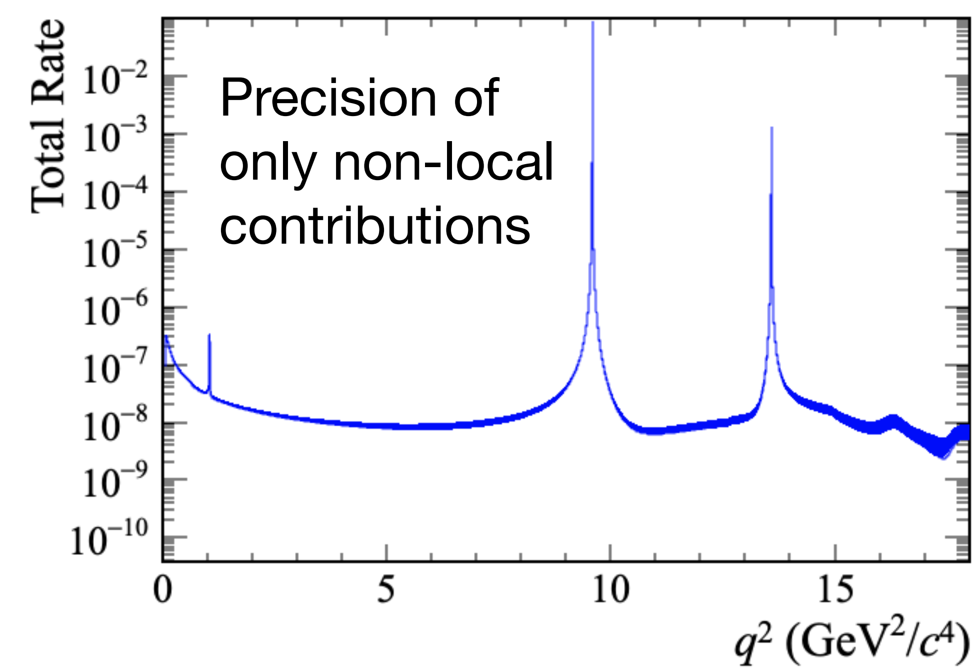
$$C_9^{\text{eff},\lambda}(q^2) = C_9^\mu + Y_{c\bar{c}}^{(0),\lambda} + Y_{c\bar{c}}^{1P,\lambda}(q^2) + Y_{\text{light}}^{1P,\lambda}(q^2) + Y_{c\bar{c}}^{2P,\lambda}(q^2) + Y_{\tau\bar{\tau}}(q^2)$$

Outputs:

- $|C_9^\mu|$, $|C_{10}^\mu|$, C_9' and C_{10}'
- Magnitude and phase of 1P resonances for each helicity λ
- $\Re(C_9^\tau)$
- $D^{(*)}\overline{D}^{(*)}$ amplitudes per helicity λ
- Non-local ΔC_7 contribution per helicity λ

Expected statistical sensitivity:

$$|C_9^\mu|, |C_{10}^\mu| \sim 0.2$$



Amplitude Ansatz

Method 3

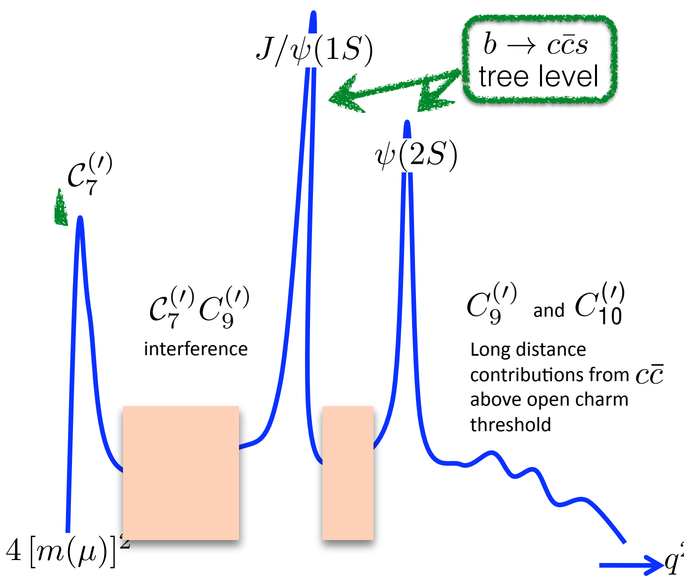
U. Egede, M. Patel, K. Petridis [JHEP06(2015)084]

- Rather than determining observables, the amplitudes can be extracted directly
- Use a parametric q^2 ansatz to avoid model dependence

$$A_\lambda^{L,R}(q^2) = \sum_i a_{\lambda,i} f_i(q^2)$$

e.g. Legendre polynominal

- Can be performed in regions of q^2 where the ansatz is a good approximation
  - *i.e.* low q^2 or between J/ψ and $\psi(2S)$ resonances
- Parametric distributions determined for P-wave and S-wave and/or B^0 and $\overline{B}^0$ separately



Method 3

Amplitude Ansatz

MONASH University

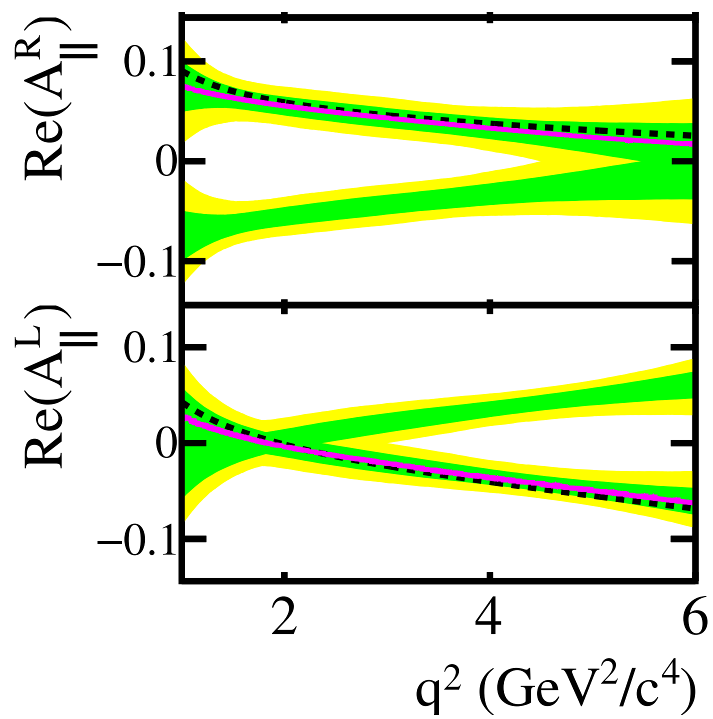
U. Egede, M. Patel, K. Petridis
[JHEP06(2015)084]

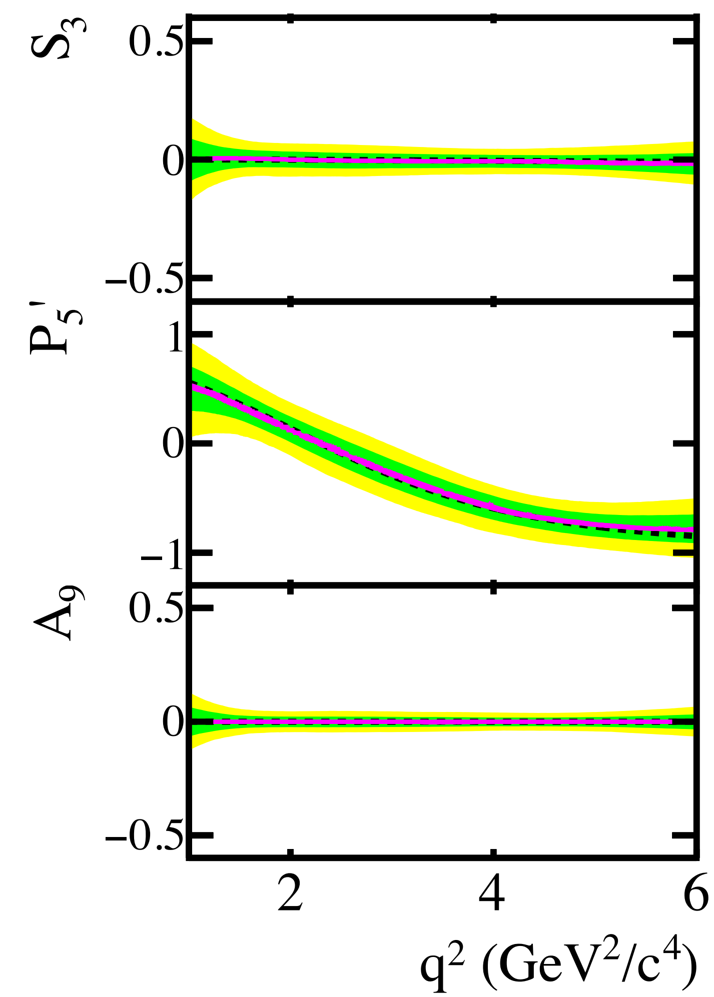
- Information is gained by exploiting the shape in q^2 of amplitudes

- Doesn't introduce specific model independence, but choice of ansatz is necessary

Outputs:

- Parametric form of the P-wave and S-wave amplitudes within the q^2 windows for each helicity





- Results can be used to determine C_9 and C_{10} and observables with improved precision

Complications

- With each unbinned model approach comes additional complications

	Z- expansion	Isobar/dispersion	Amplitude ansatz
$B \to K$ form factors	External inputs + constrained	External inputs + constrained	N/A
$m(K\pi)$ line shape	Parameterised	Integrated over	Parameterised
Exotic contributions e.g. $B^0 \to Z(4430)K$	Ignored (systematic)	Ignored (systematic)	N/A

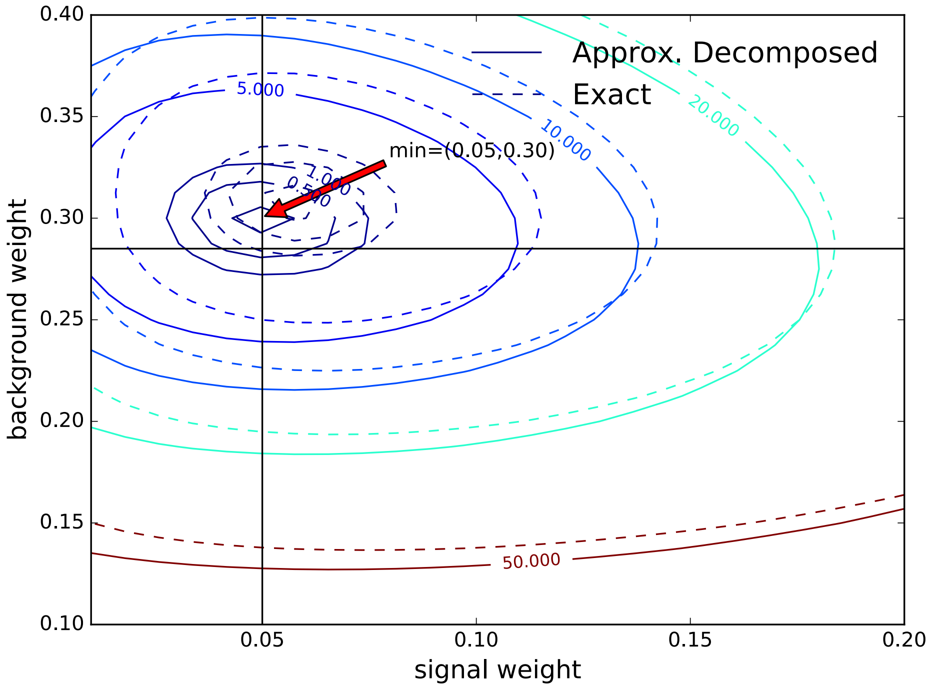
Combining unbinned measurements

- These future unbinned measurements will provide valuable insights to the impact of non-local effects
- Analyses performed on the ***same dataset***, however there will be some necessary differences
    - e.g. the dispersion relation model uses the whole q^2, where as other are restricted to regions
- Care must be taken when interpreting the results
    - Obviously they cannot be combined
    - They also shouldn't be combined with the binned measurements
- The observables can be compared to determine the consistency between the methods
    - Inconsistencies between models may give hints

Extracting likelihoods

- ***Portable*** likelihood functions
    - Large unbinned measurements could have complex high-dimensional likelihood functions with many nuisance parameters
    - ML algorithms can be used to provide portable ***approximations*** of the surfaces for combinations or inference

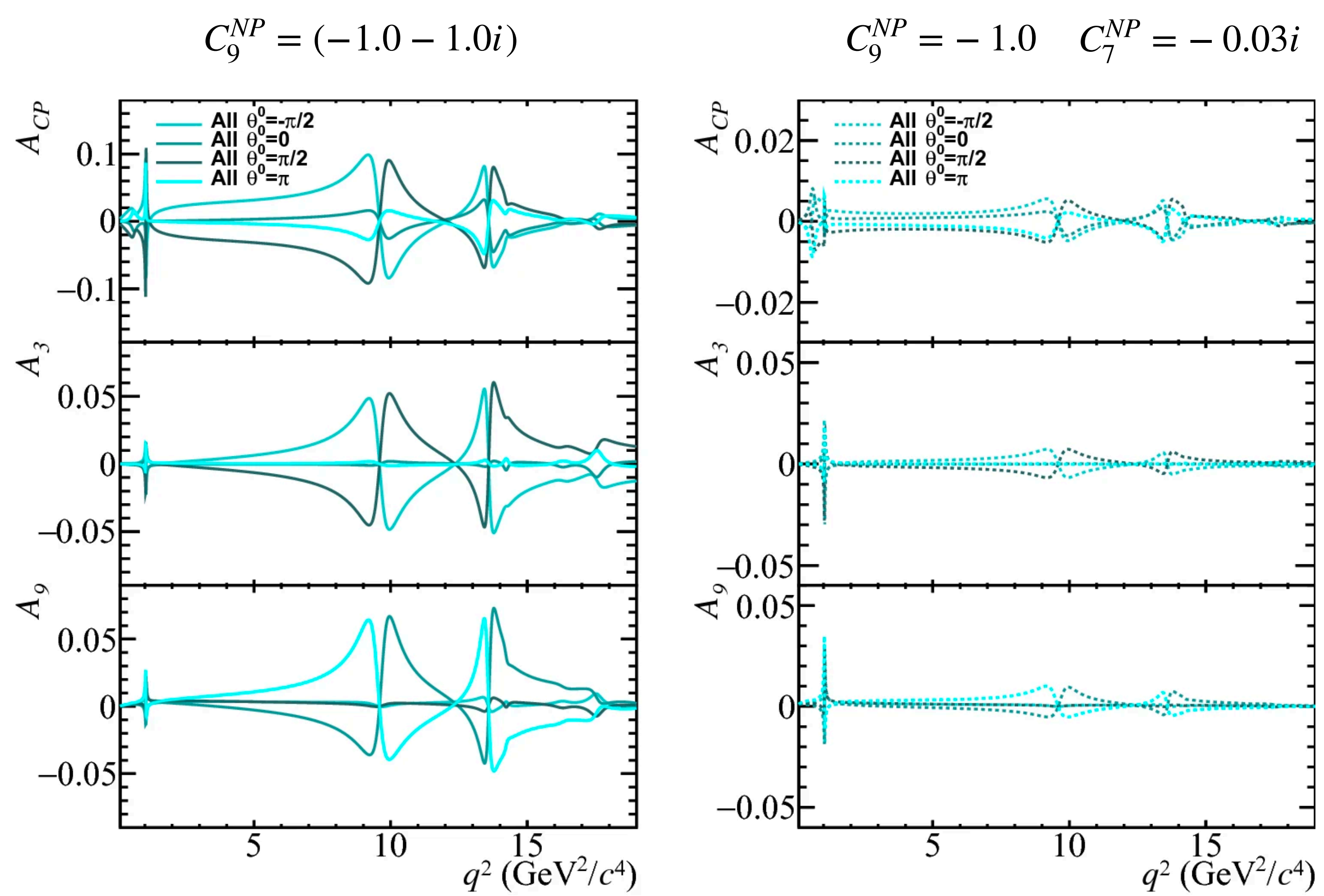
- Future $b \to s$ measurements could provide such information
    - Measurements that use ***external form factors*** could then provide correlations with these inputs



K Cranmer, J Pavez, G Louppe and W K Brooks
[J.Phys.Conf.Ser. 762 (2016) 1, 012034]

CP violation measurements

- CP violation requires different strong and weak phases
- Unbinned measurements of $B \to K^{*0}\mu^+\mu^-$ are a system in which the resonances provide the varying strong phases
- Direct CP asymmetry A_{CP} and CP-odd observables A_i are sensitive to NP with a different weak phases
- This gives sensitivity to phase of C_9



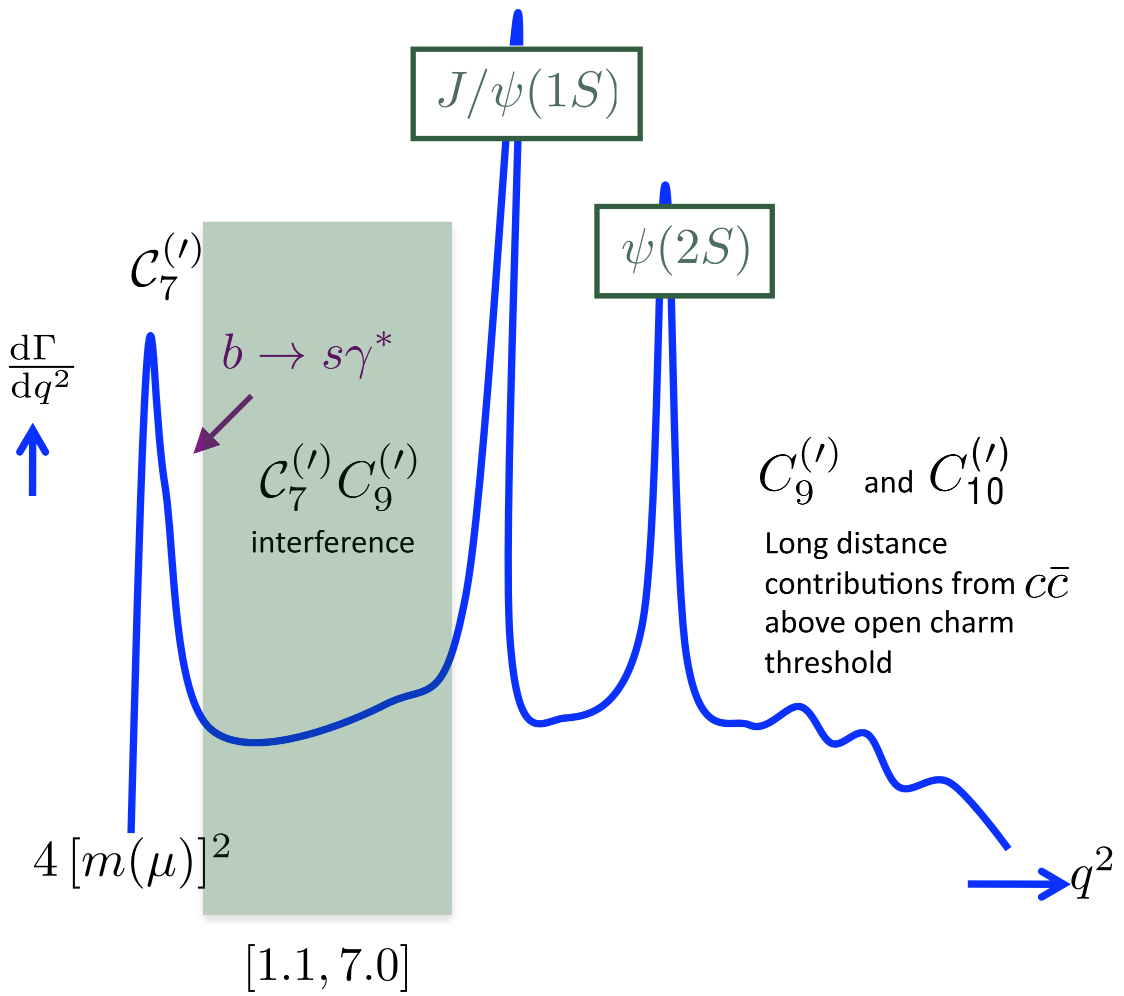
T. Blake, U. Egede, P. Owen, K. A. Petridis & G. Pomery [Eur. Phys. J. C 78, 453 (2018)]

Unbinned LFU WCs fits

Simultaneous unbinned analysis of $B^0 \to K^{*0}\mu^+\mu^-$ and $B^0 \to K^{*0}e^+e^-$

[A. Mauri *et al*, JHEP 10 (2019) 236]

$$\mathcal{A}_\lambda^{(\ell)\,L,R} = \mathcal{N}_\lambda^{(\ell)} \left\{ (C_9^{(\ell)} \mp C_{10}^{(\ell)}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7^{(\ell)} \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$



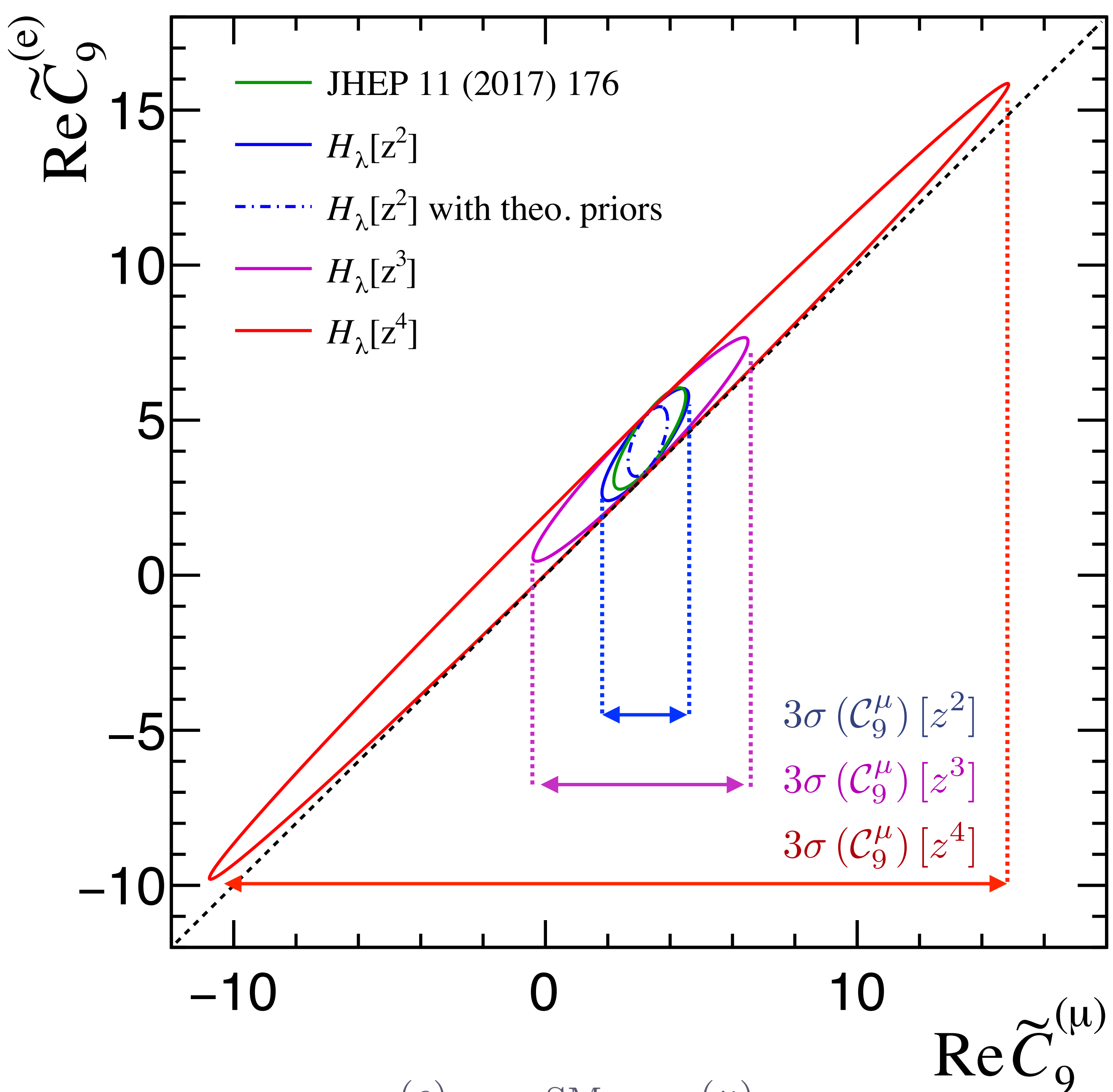
Same interval for both leptons

- All nuisance parameters are shared between electron and muons, *i.e.* CKM and (non) local hadronic
- Extended maximum-likelihood fit, ***i.e.* includes P'_5, R_{K^*} information**

* see more details about LFU in Inguglia and Amhis's talk

Unbinned LFU WCs fits

[A. Mauri *et al*, JHEP 10 (2019) 236]



$$\mathcal{C}_9^{(e)} = \mathcal{C}_9^{\rm SM} = \mathcal{C}_9^{(\mu)} + 1$$

$C_i^{(\ell)}$: strongly dependent on the model assumption (renamed for simplicity)

Key feature: *model-independent* determination of the difference between electron and muons WCs

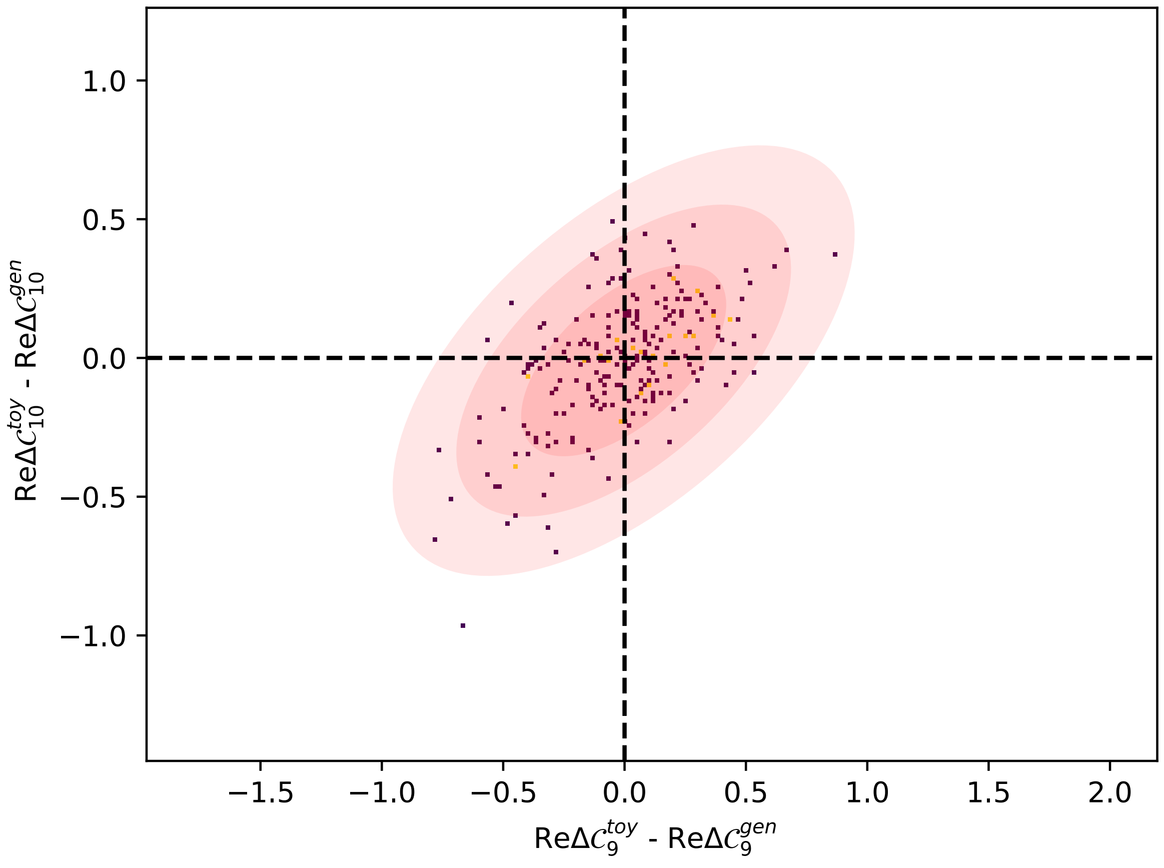
$$\Delta\mathcal{C}_i = \widetilde{\mathcal{C}}_i^{(\mu)} - \widetilde{\mathcal{C}}_i^{(e)}$$

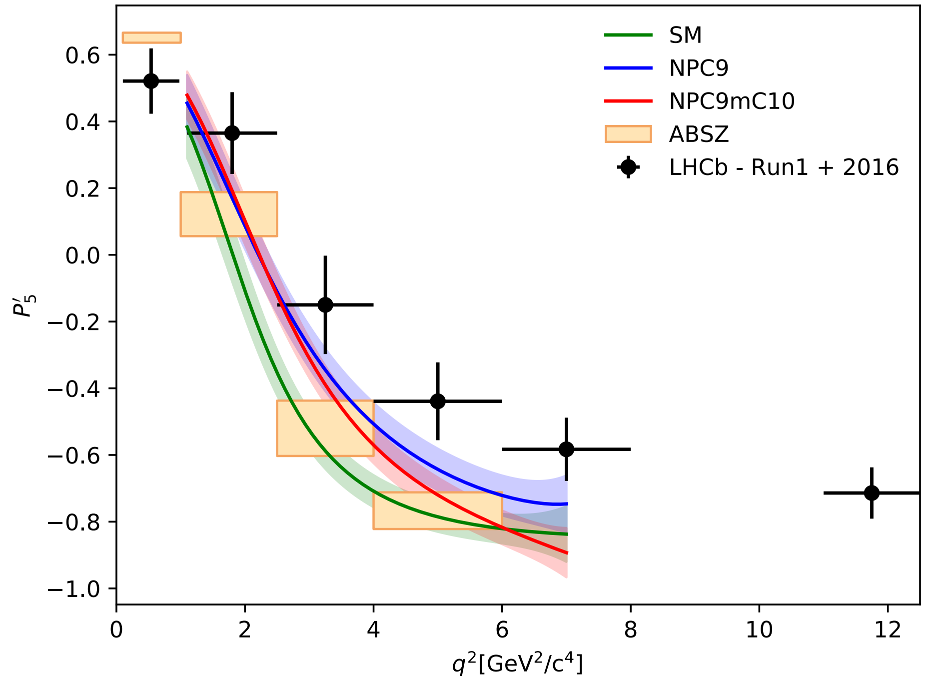
- Insensitive to the parametrisation of the non-local contributions
- Significance wrt LFU hypothesis is unbiased

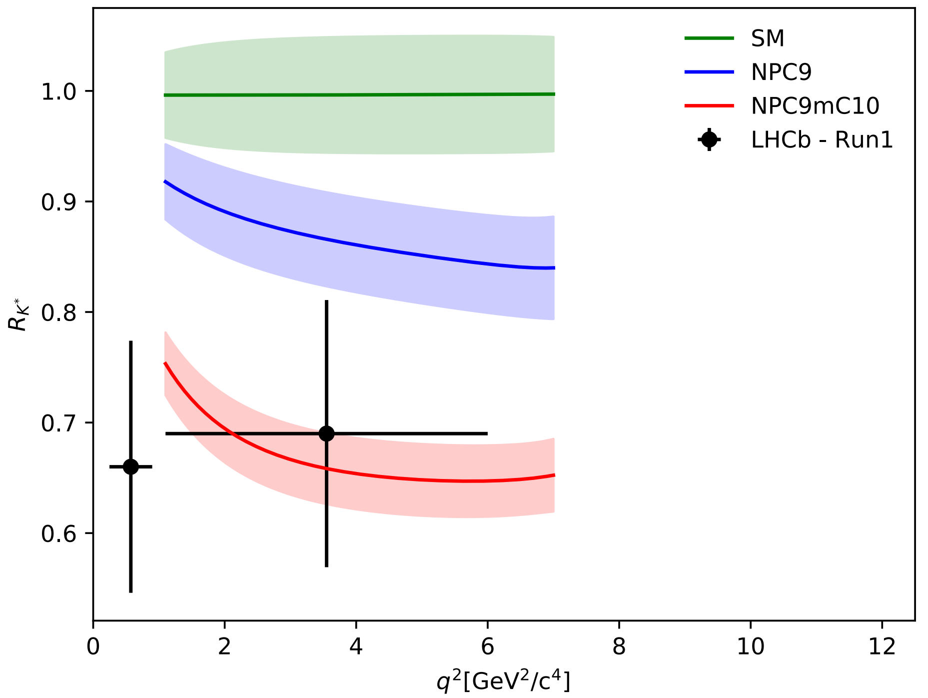
Unbinned LFU WCs fits

[toy simulation]

Difference in WCs, i.e. ΔC9,10 and primed observables, 2D likelihood scans and 1D projections of standard observables







Looking further ahead

- To make the most of the available data we want to update existing measurements with higher statistics *and* perform new measurements that provide complementary information

- What further information could be extracted from binned measurements?
  - Is there anything beyond moments analysis that could be useful?

- Where else could benefit from unbinned measurements?
  - **Unbinned measurements in baryonic systems** e.g. $\Lambda_b \to \Lambda\mu^+\mu^-$: would we expect hadronic effects to differ between baryons vs. mesons? If so these measurements could be very useful crosschecks
  - **Higher Lorentz structures:** what about Wilson coefficients for tensor Lorentz structures? Spin-2 K* states could provide insight

Back up

Dispersion model

- In the dispersion relation model these effective Wilson coefficients are written in terms of local and non-local contributions

Local — Non-local contributions

$$C_7^{\text{eff},\lambda}(q^2) = C_7^{\mu} + \epsilon^{\lambda} e^{i\omega^0}$$

$$C_9^{\text{eff},\lambda}(q^2) = C_9^{\mu} + Y_{c\bar{c}}^{(0),\lambda} + Y_{c\bar{c}}^{1P,\lambda}(q^2) + Y_{\text{light}}^{1P,\lambda}(q^2) + Y_{c\bar{c}}^{2P,\lambda}(q^2) + Y_{\tau\bar{\tau}}(q^2)$$

Inputs:

- P-wave form factors available from Light Cone Sum Rules and Lattice QCD

Bharucha, Straub, Zwicky [JHEP 08 (2016) 098]

- The S-wave form factors not well known, so it is possible to decouple the S-wave and P-wave C_7, C_9 and C_{10} values

Doring, Meißner, Wang [JHEP 10 (2013) 011]