Non-local form factors in $b \to s\ell\ell$

Beyond the Flavour Anomalies III – 27/04/2022

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In collaboration with:
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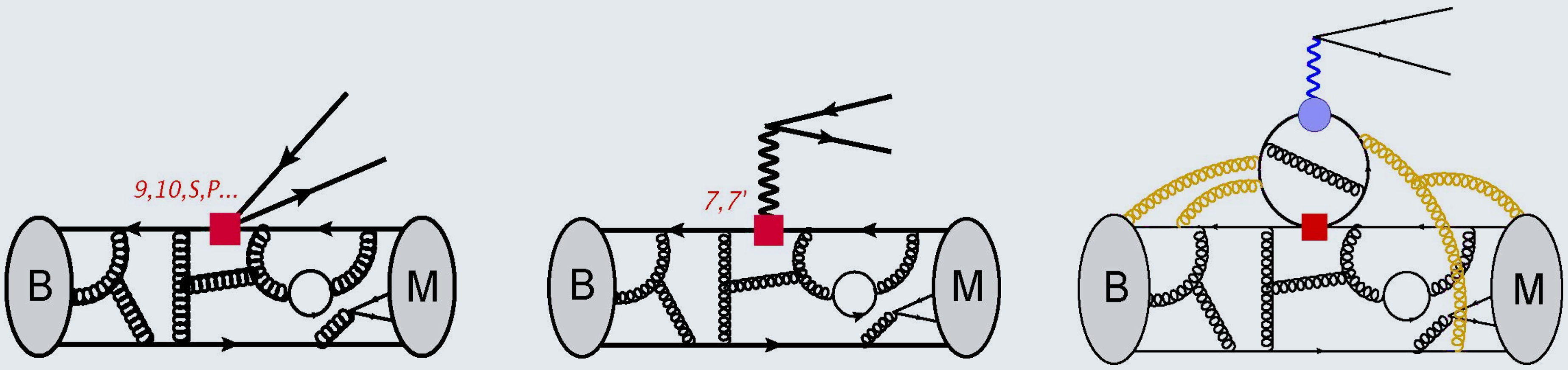
Form-factors in $b \to s\ell\ell$



$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2}\left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

Non-local form-factors

$$\mathcal{H}_\lambda(q^2) = i\mathcal{P}_\mu^\lambda \int d^4x\, e^{iq\cdot x} \langle \bar{M}_\lambda(k) | T\{\mathcal{J}^\mu_{\text{em}}(x), \mathcal{C}_i \mathcal{O}_i(0)\} | \bar{B}(q+k)\rangle$$

→ Main contributions: O_1^c, O_2^c the so-called **“charm-loops”**

A few remarks

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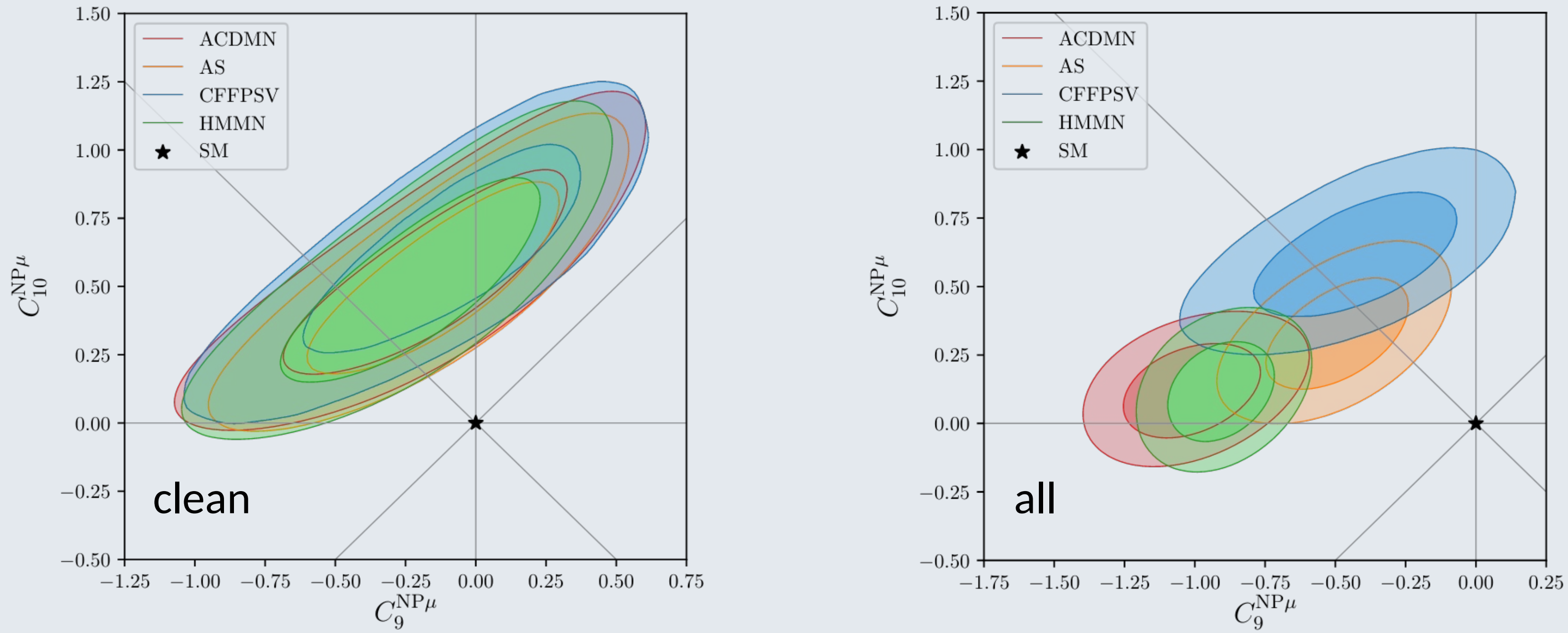
   Charm-loops effects cannot be very large!



[Capdevila, Fedele, Neshatpour, Stangl, '21; See Bernat's talk: here]

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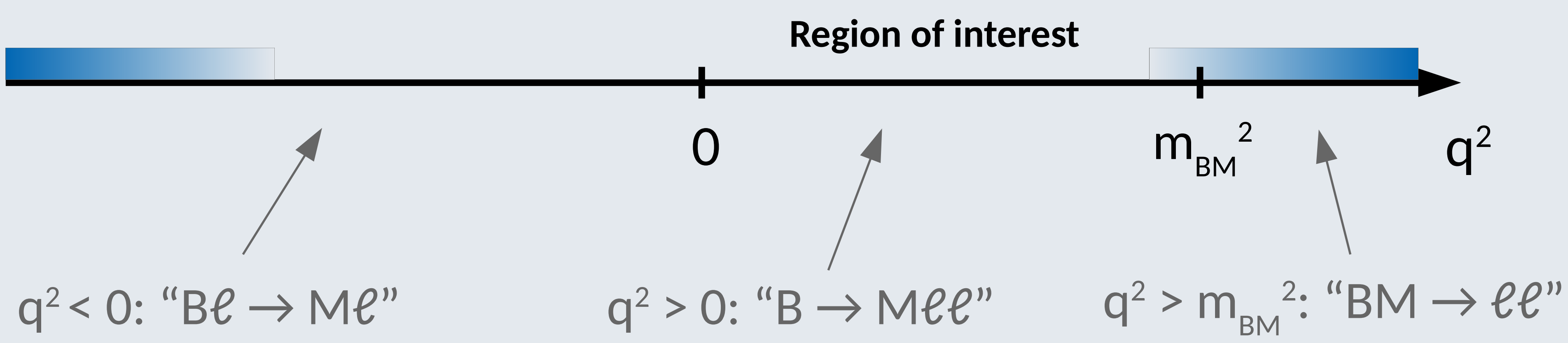
5. Theory **puzzles in $b \to s\bar{c}c$** [see e.g. Lyon, Zwicky, 2014]

   We need to be careful…

Constraints on H_λ

1. Two types of **OPE** can be used for H_λ:
   - **Local** OPE $|q|^2 \gtrsim m_b^2$ [Grinstein, Piryol 2004][Beylich, Buchalla, Feldmann 2011]

     → We will discuss it later



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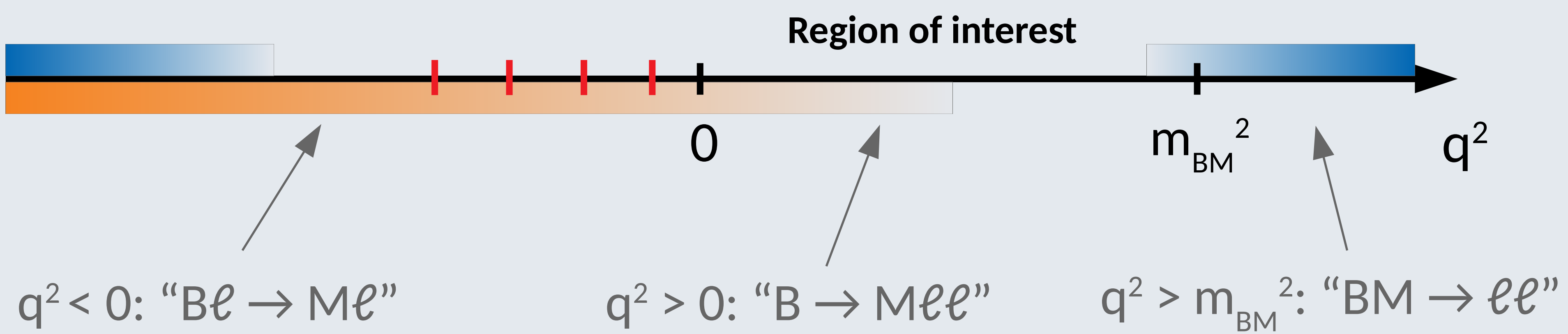
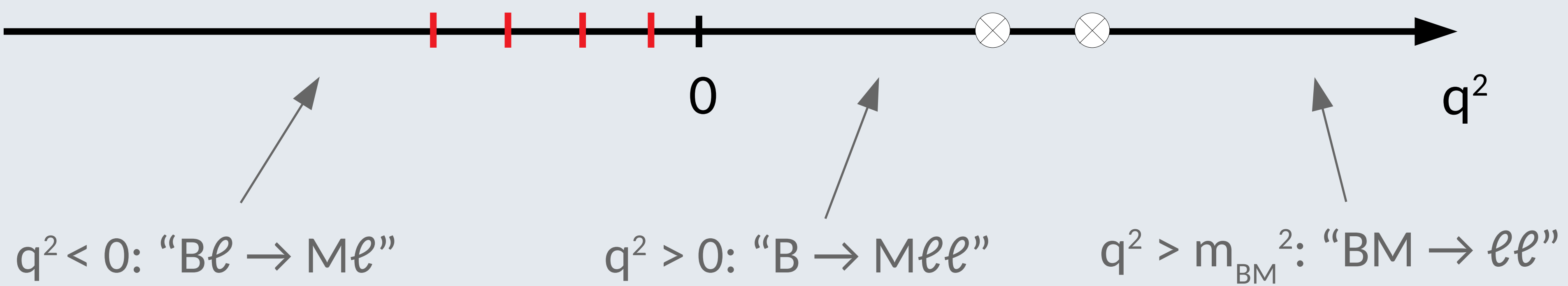
    - **Light Cone** OPE $q^2 \ll 4m_c^2$ [Khodjamirian, Mannel, Pivovarov, Wang 2010]

      → theory points at $q^2 < 0$ [Gubernari, van Dyk, Virto 2020]



Constraints on H_λ

2. Charmonium resonances [Bobeth, Chrzaszsz, van Dyk, Virto'17]:
   - H_λ presents **poles** at $q^2 = m_{J/\psi}^2$ and $m_{\psi(2S)}^2$
   - For this work we only use **B → M J/ψ** data



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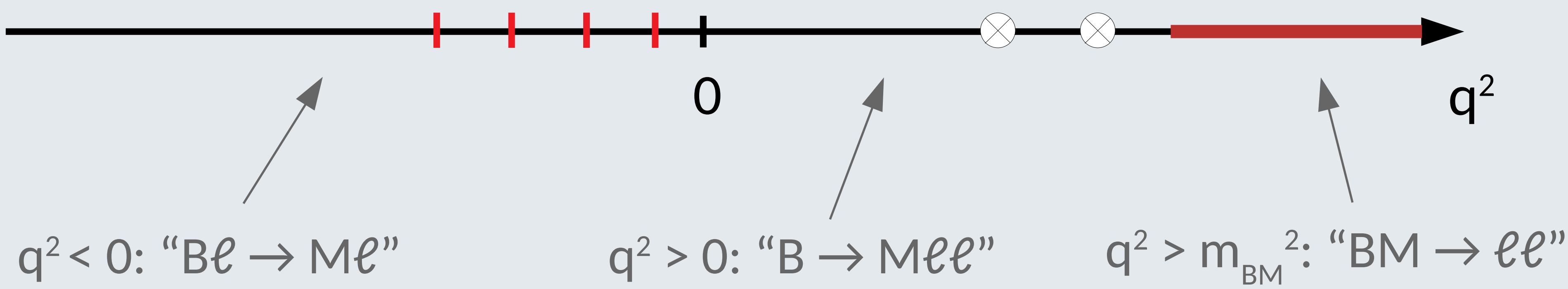
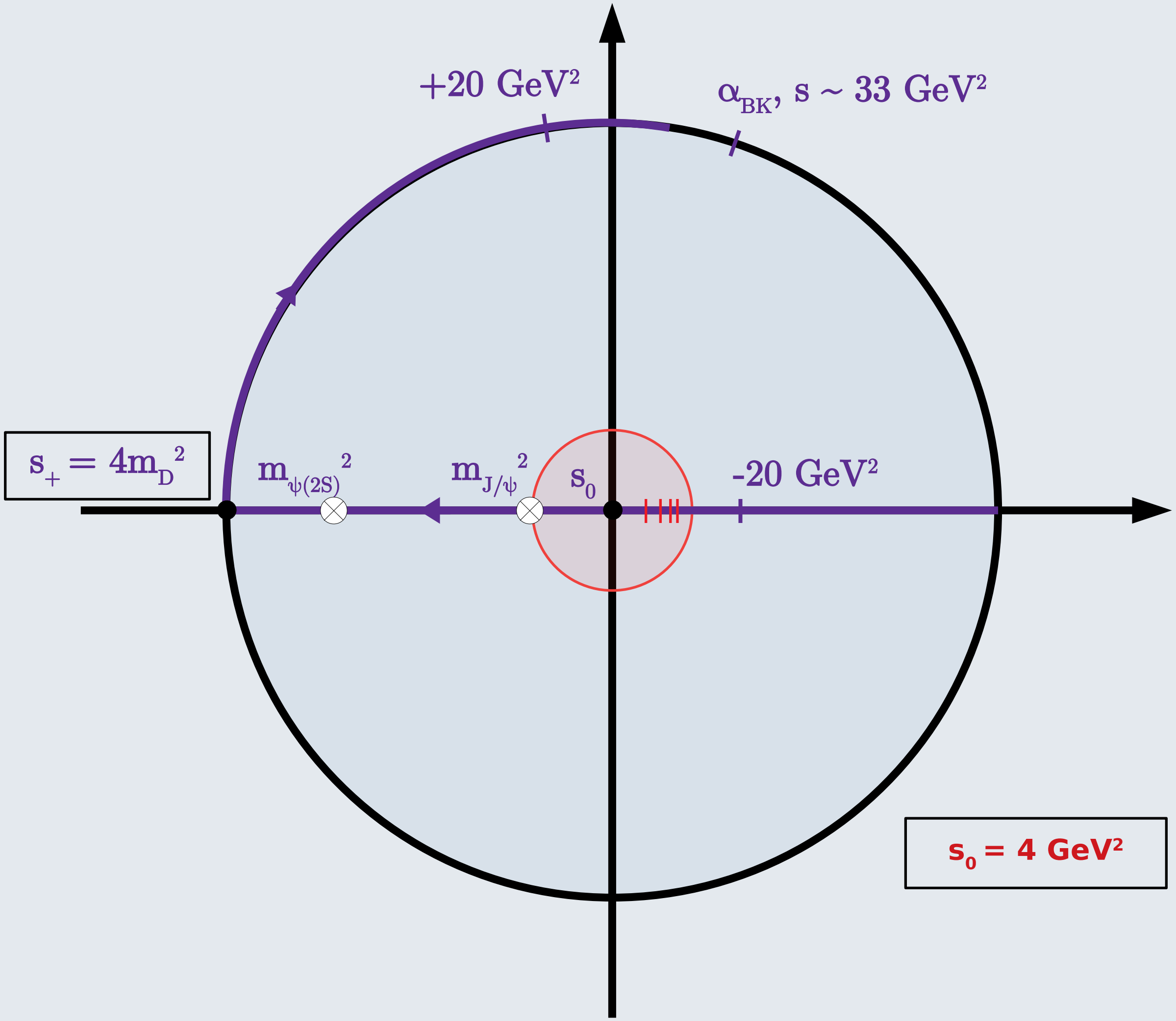
3. H_λ has a **branch cut** for $q^2 > 4m_D{}^2$



Parametrization of H_λ

- **z-mapping**

$$z(s) \equiv \frac{\sqrt{s_+ - s} - \sqrt{s_+ - s_0}}{\sqrt{s_+ - s} + \sqrt{s_+ - s_0}}$$



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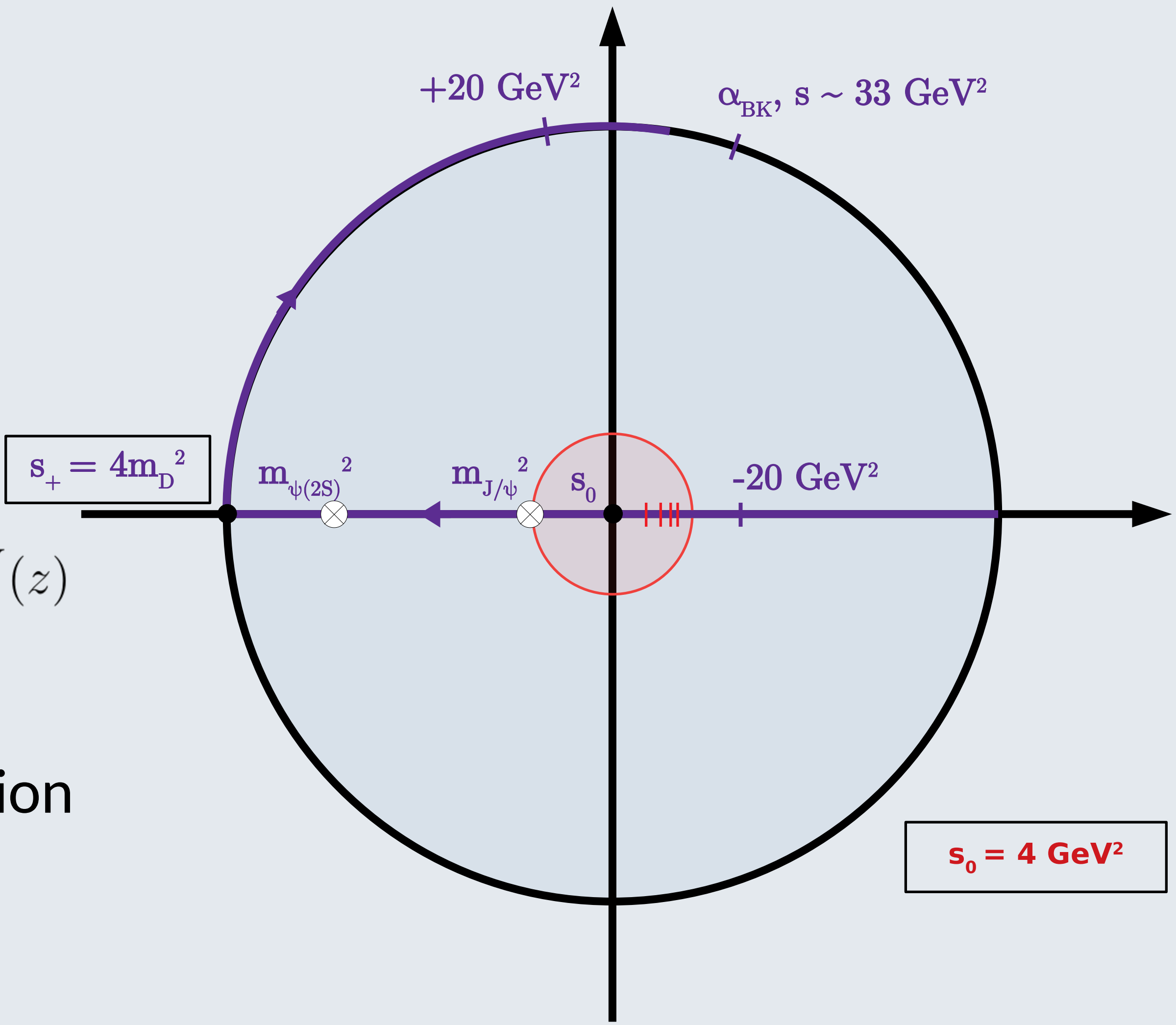
- **Analyticity**

$$\hat{\mathcal{H}}_\lambda^{B\to V}(z) \equiv \phi_\lambda^{B\to V}(z)\,\mathcal{P}(z)\,\mathcal{H}_\lambda^{B\to V}(z)$$

→ $\mathcal{P}(z)$ captures the poles

→ $\Phi(z)$ is a useful normalization



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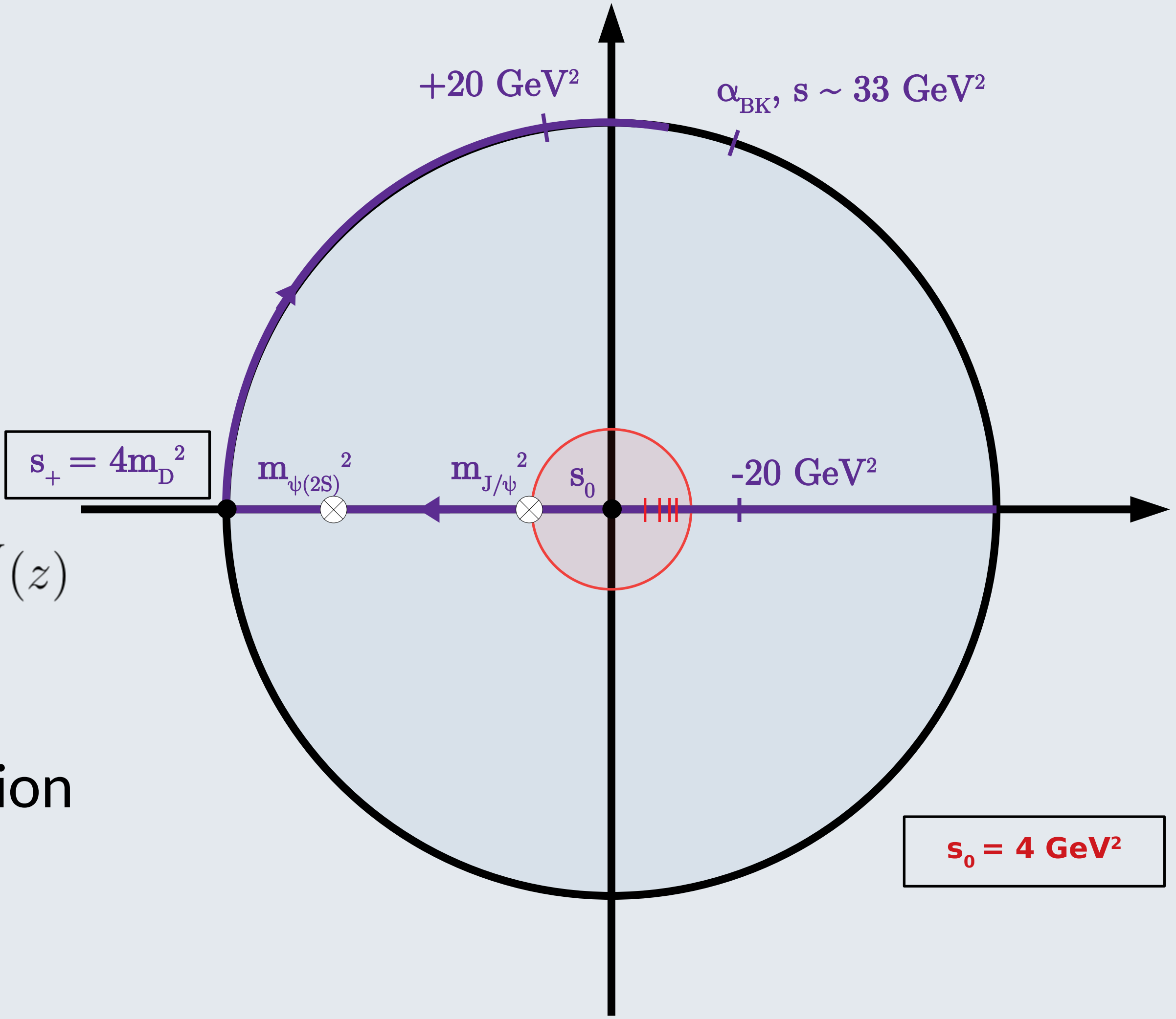
- **z-expansion**

$$\hat{\mathcal{H}}_\lambda^{B\to M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B\to M}\, z^n$$

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]



$$\hat{\mathcal{H}}_\lambda^{B\to M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B\to M}\, p_n^{B\to M}(z)$$

[Gubernari, van Dyk, Virto, 2020]

Dispersive bound

$$\hat{\mathcal{H}}_{\lambda}^{B\to M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B\to M}\, z^n$$

- In practice, $\mathrm{a}_{\lambda,\mathrm{n}} = 0$ for $\mathrm{n} > \mathrm{N}$. **What is the truncation error?**

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$$1 > 2\int_{-\alpha_{BK}}^{+\alpha_{BK}} d\alpha \left|\hat{\mathcal{H}}_0^{B\to K}(e^{i\alpha})\right|^2 + \sum_\lambda \left[2\int_{-\alpha_{BK^*}}^{+\alpha_{BK^*}} d\alpha \left|\hat{\mathcal{H}}_\lambda^{B\to K^*}(e^{i\alpha})\right|^2 + \int_{-\alpha_{B_s\phi}}^{+\alpha_{B_s\phi}} d\alpha \left|\hat{\mathcal{H}}_\lambda^{B_s\to\phi}(e^{i\alpha})\right|^2\right]$$

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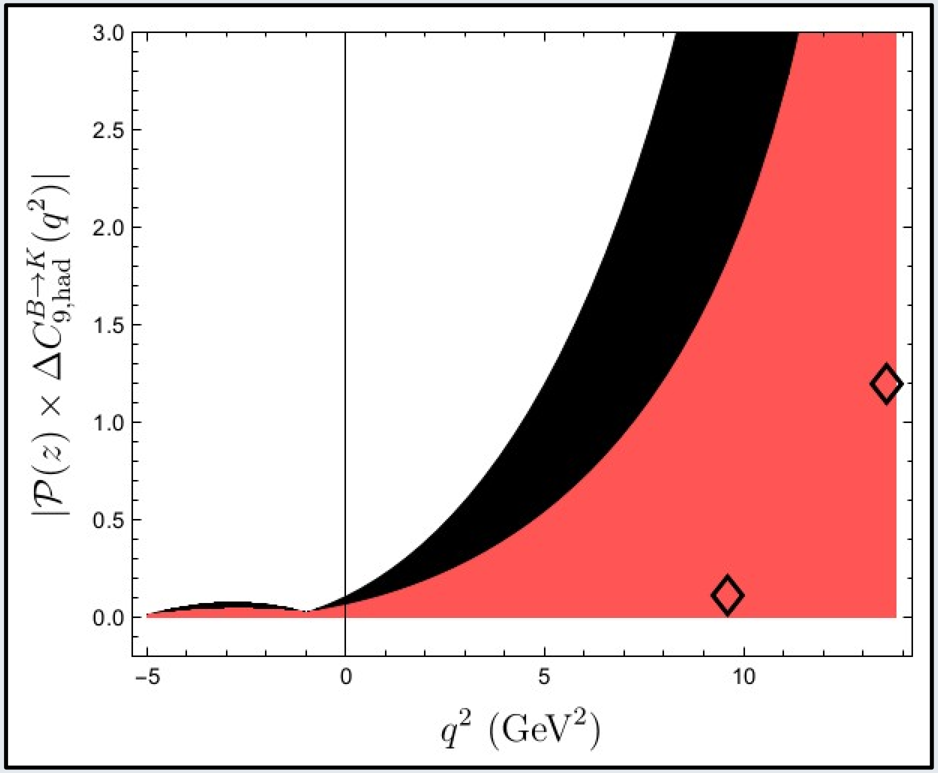
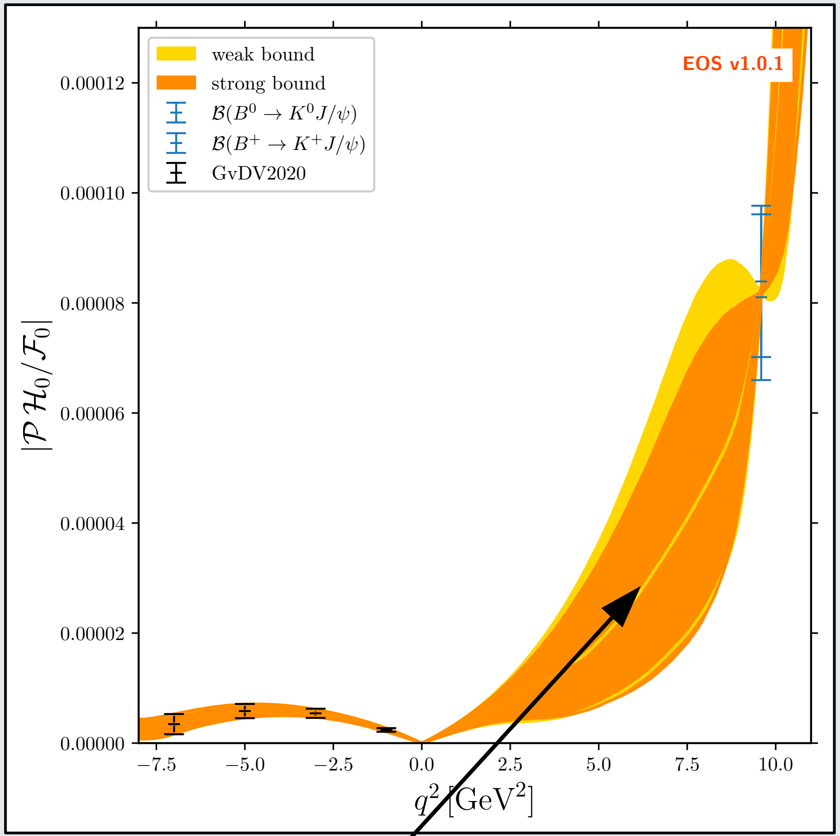
→ With **orthonormal polynomials:** $\hat{\mathcal{H}}_\lambda^{B\to M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B\to M} p_n^{B\to M}(z)$

$$\sum_{n=0}^{\infty} \left\{ 2\left|a_{0,n}^{B\to K}\right|^2 + \sum_{\lambda=\perp,\|,0} \left[2\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \left|a_{\lambda,n}^{B_s\to\phi}\right|^2\right]\right\} < 1$$

Anticipating on the results:



[Gubernari, van Dyk, Virto, 2020]



[Gubernari, Reboud, van Dyk, Virto, 2022]

1) Controlled uncertainty in the physical region

2) Adding an order in the expansion **doesn't increase this uncertainty!**

Putting everything together:

- The fit is performed in two steps…
  - Preliminary fits:
    - **Local** form factors:
      – BSZ parametrization (**8 + 19 + 19 parameters**)
      – LCSR + LQCD, **more in the backup**
    - **Non-local** form factors:
      – order 5 GvDV parametrization (**12 + 36 + 36 parameters**)
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- … using **EOS**:

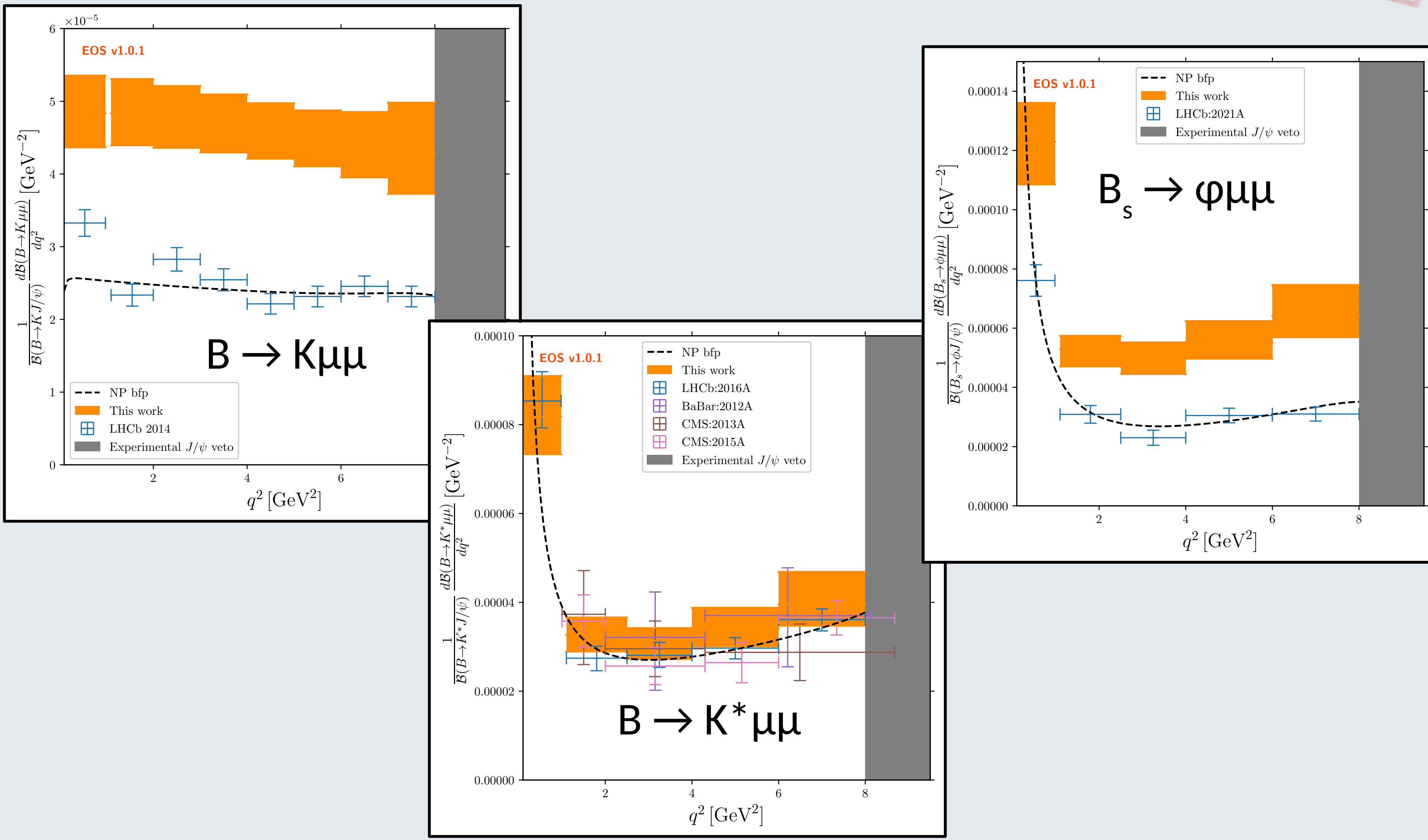
EOS is a software for a variety of applications in flavour physics. It is written in C++, but provides an interface to Python.

https://eos.github.io/

Putting everything together:

Preliminary

- New **semi data-driven SM predictions**



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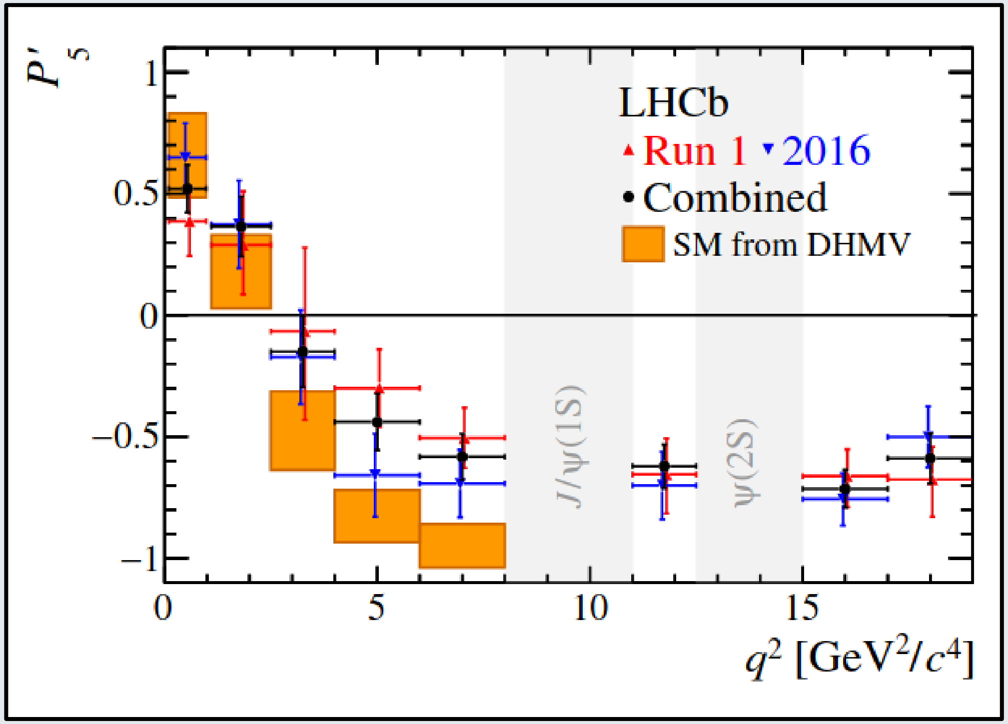

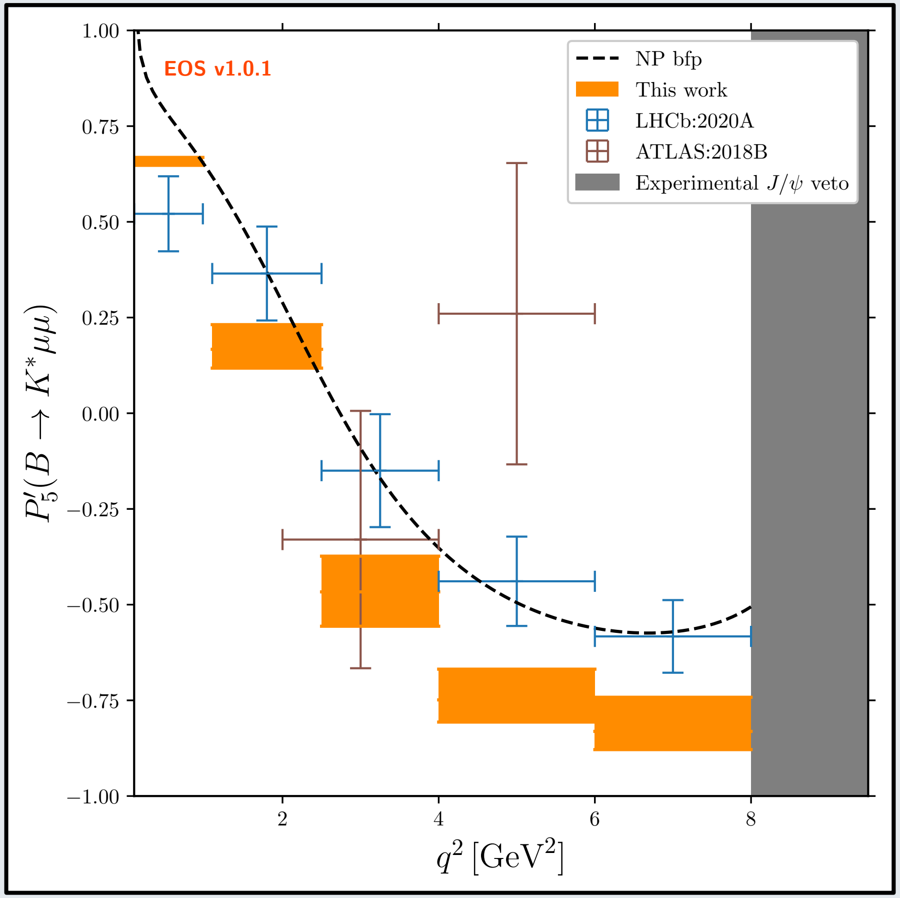

Simple NP analysis

Preliminary

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- Fit **separately** C_9 and C_{10} for the three channels:
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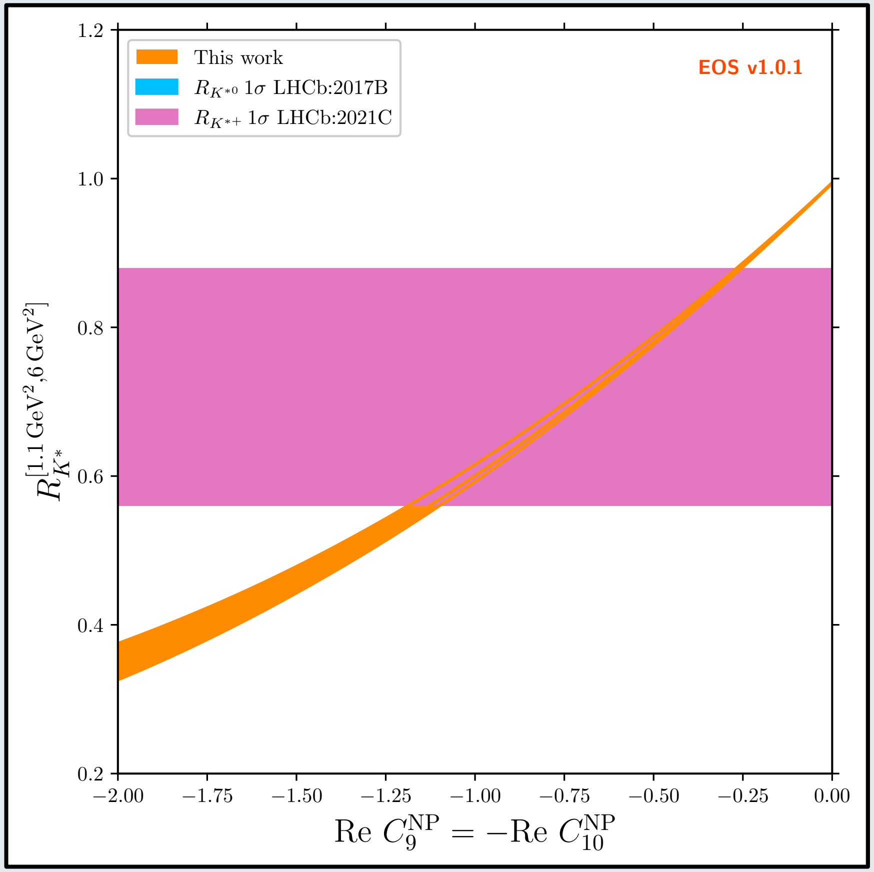

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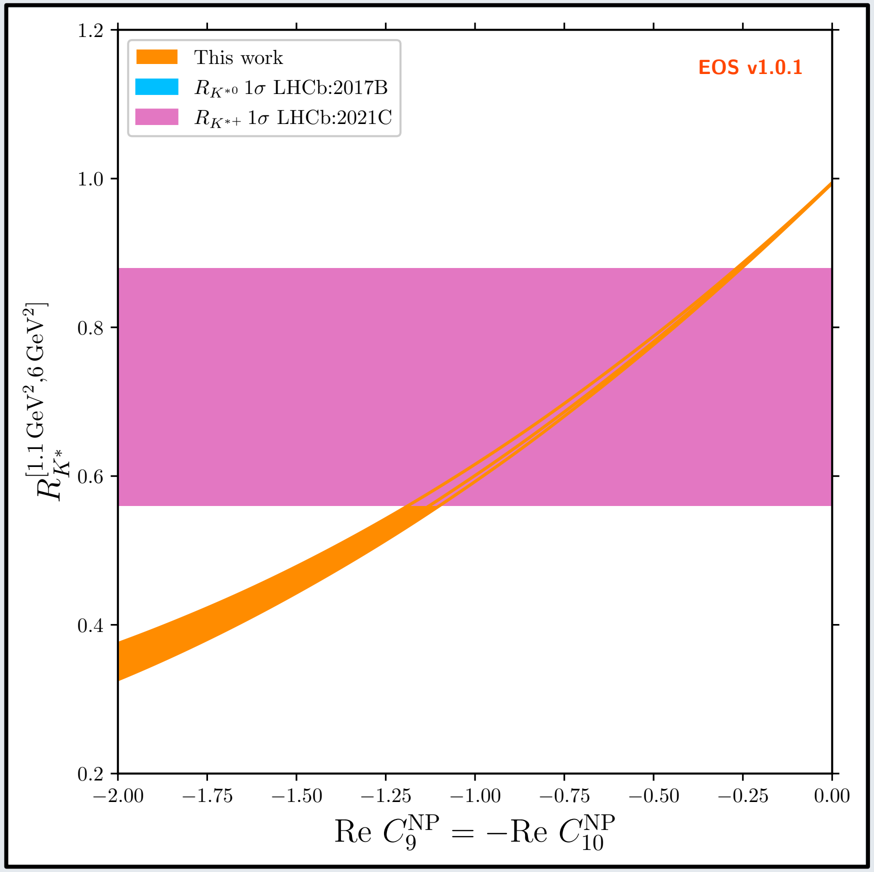
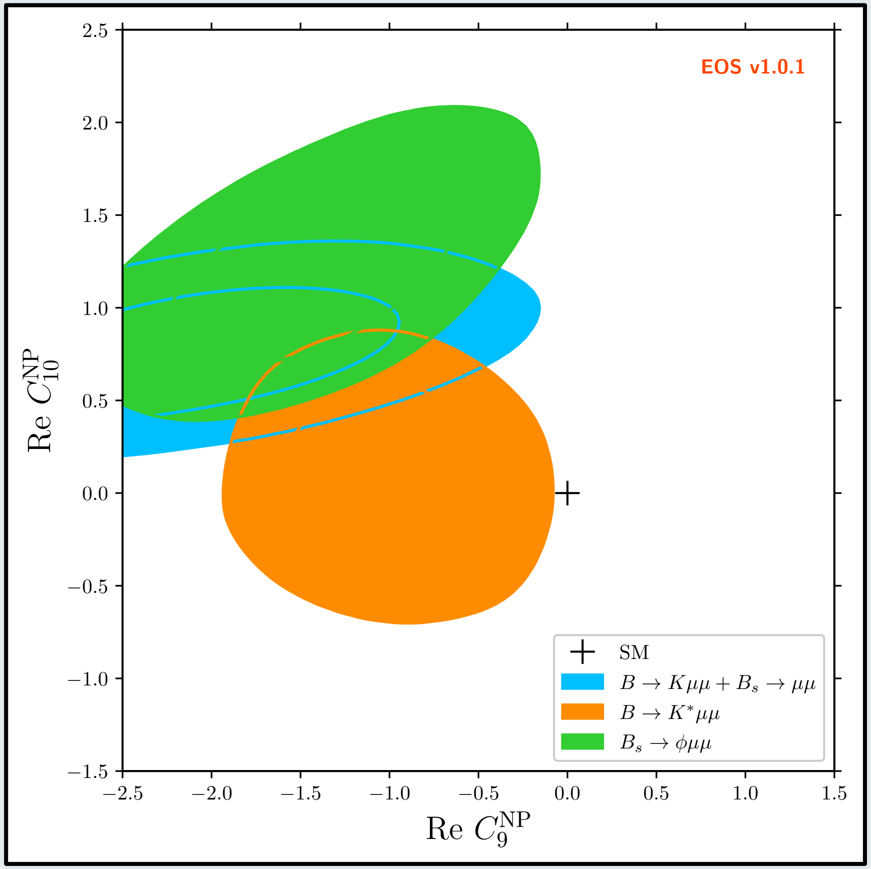


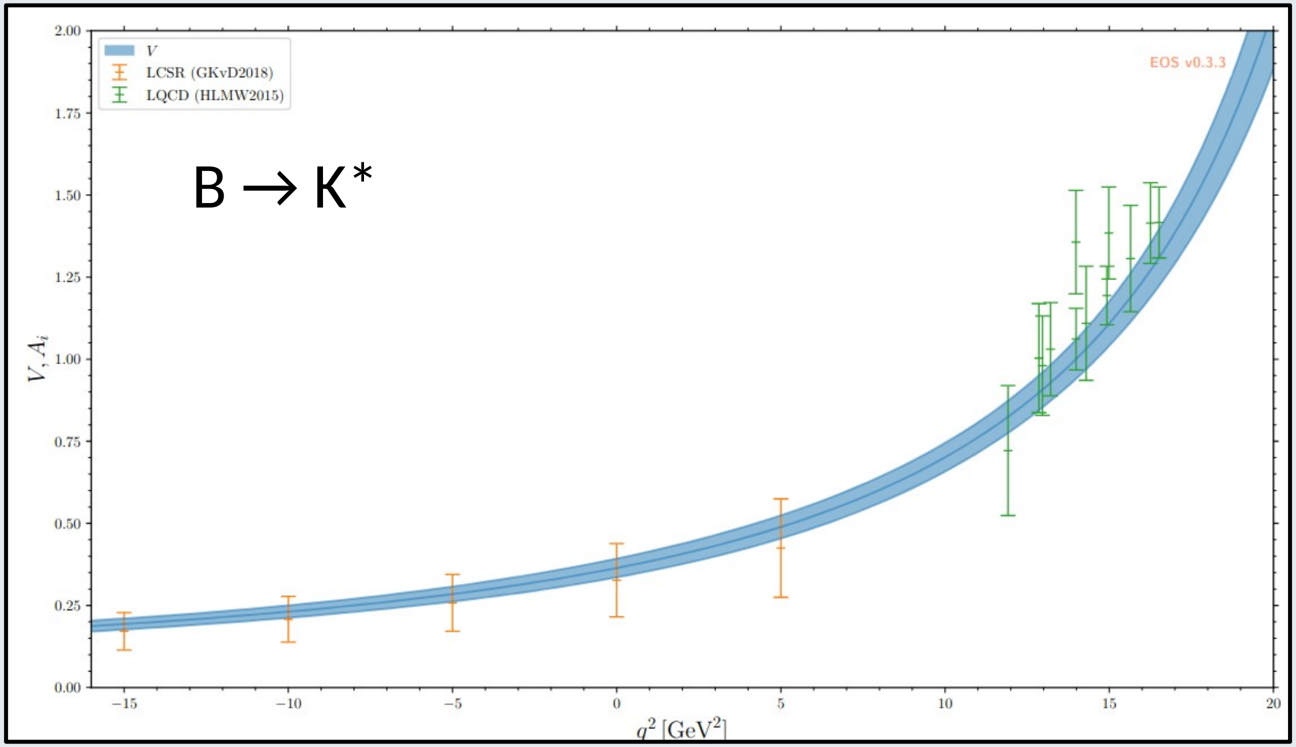


Back-up

Fit to local form factors

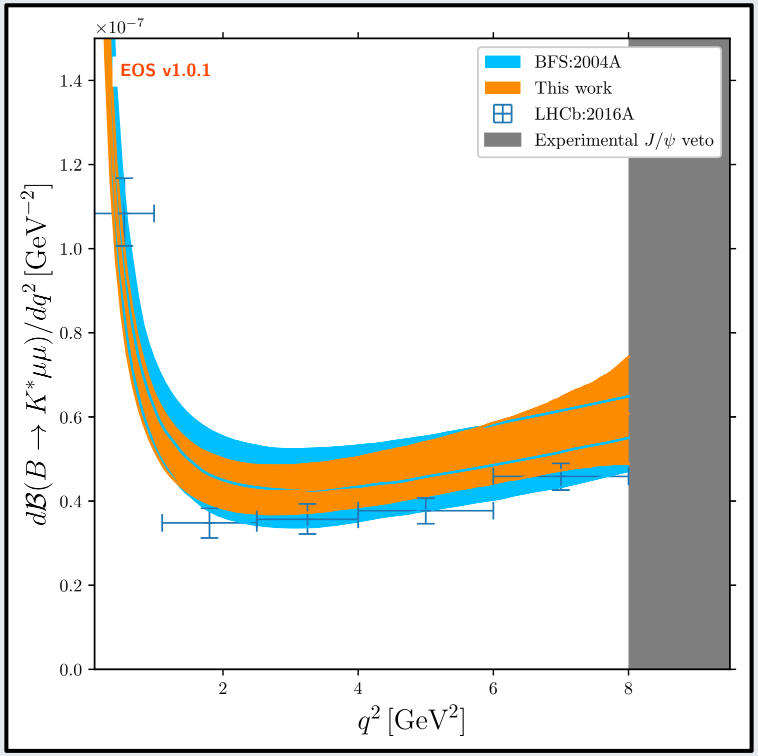
Combined fit to LCSR and lattice:

- $B \to K$:
  - HPQCD'17; FNAL/MILC'17
  - Khodjamiriam and Rusov'17
- $B \to K^*$:
  - Horgan, Liu, Meinel and Wingate'15
  - Gubernari, Kokulu and van Dyk'18
- $B_s \to \varphi$:
  - Horgan, Liu, Meinel and Wingate'15
  - Bharucha, Straub and Zwicky'15; Gubernari, van Dyk and Virto'20



 [Gubernari, Reboud, van Dyk, Virto, **imminent**]

Additional plots



- Comparison to [Beneke *et al.* ’01, ‘04]

- Weak ($|a_i| < 1$)
  vs.
  Strong ($\Sigma|a_i| < 1$) bounds