Journeys through the moduli space using generalized geometry

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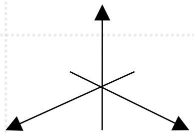
Consider a base manifold B as our spacetime

Let E be a vector bundle over B

A frame bundle $\pi : F(E) \to B$ is a principal $GL^{+}(d;\mathbb{R})$-bundle



$$e_a = \begin{pmatrix} e_1 \\ \vdots \\ e_d \end{pmatrix}$$

Typical fibre $F(E)|_x$ is the space of all frames $\{e_a(x)\}$ on $E|_x$ for $x \in B$

$e_a \in F(E) : e_a \lhd GL^{+}(d;\mathbb{R})$ $\forall\, A \in GL^{+}(d;\mathbb{R}) \colon e_a \lhd A = e_b A^{b}{}_{a}$

Given a tangent bundle TB

$e_a \lhd GL^{+}(d;\mathbb{R})$ induces $GL^{+}(d;\mathbb{R}) \rhd v^a$, $\forall\, v^a \in TB$ via an inner product $e_a v^a$

Choose a **metric** g_{ab} on B : $g_{ab} = e^{\alpha}{}_{a} e^{\beta}{}_{b} \eta_{\alpha\beta}$

$e_a \lhd GL^+(d;\mathbb{R})$ induces $GL^+(d;\mathbb{R}) \rhd g_{ab}$

$$\forall\, A \in GL^+(d;\mathbb{R}) : A \rhd g = A^T g A$$

$\boldsymbol{\eta_{\alpha\beta} = diag(1, \dots, 1)}$ **is the** $\boldsymbol{SO(d;\mathbb{R})}$ **invariant** $\Rightarrow$ induced $SO(d;\mathbb{R}) \rhd e^{\alpha}{}_{a}$

Naturally, $\exists\, N \in GL^+(d;\mathbb{R})$: $\boldsymbol{e_a \lhd N = v \rhd e_a}$ for some $v \in SO(d;\mathbb{R})$

$\Longrightarrow \boldsymbol{N \rhd g_{ab} = g_{ab}}$

$\Longrightarrow$ metrics on B form $\boldsymbol{SO(d;\mathbb{R})}$ **cosets**

A choice of metric g_{ab} on B is a choice of coset $[g_{ab}] = g_{ab}(SO(d;\mathbb{R}))$

$\Longleftrightarrow$ **Reduction of the structure group on** $\boldsymbol{TB}$ **from** $\boldsymbol{GL^+(d;\mathbb{R})}$ **to** $\boldsymbol{SO(d;\mathbb{R})}$.

Moduli space $\mathcal{M}$: space of all metrics $\Longleftrightarrow$ **space of degrees of freedom of the metric which you can tune in the theory to fix your spacetime geometry**

Consider a bundle $L \to B$ with $\forall x \in B$: $L|_x \cong \frac{GL^+(d;\mathbb{R})}{SO(d;\mathbb{R})}$ $\Longrightarrow$ $\boldsymbol{\mathcal{M} \cong C^\infty(L)}$

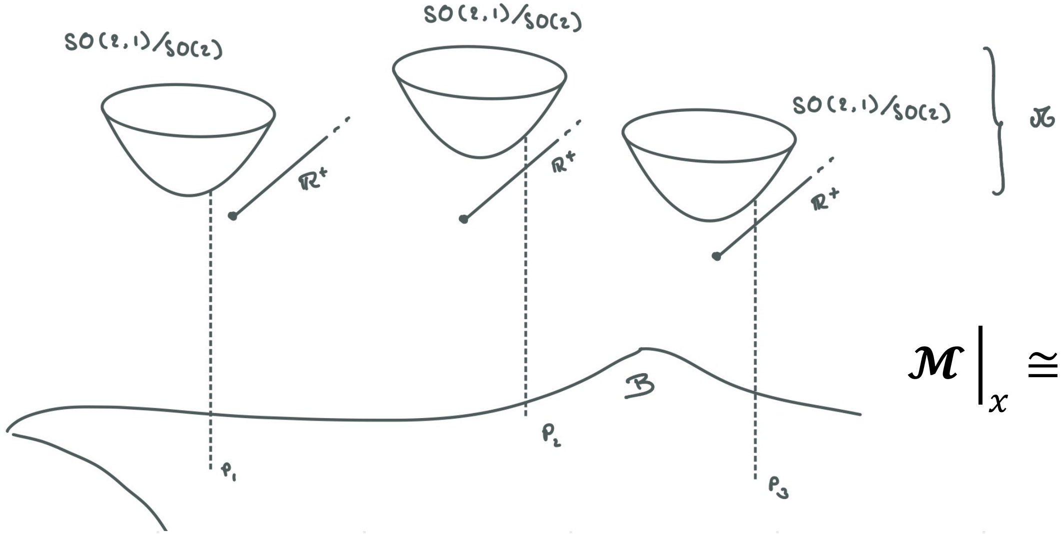
$\Longrightarrow$ use group theory to study geometry pointwise:

$\Longrightarrow$ a metric on $\boldsymbol{\mathcal{M}}$

$\Longrightarrow$ **geodesics**

$\Longrightarrow$ notions of **distance** on the moduli space

Example: $d = 2$:



$$\mathcal{M}\Big|_x \cong \frac{GL^+(2;\mathbb{R})}{SO(2;\mathbb{R})} \cong \mathbb{R}^+ \times \frac{SL(2;\mathbb{R})}{SO(2;\mathbb{R})}$$

Generalized Geometry : instead of taking TB and T^*B separately, we propose

E ($\cong_{Kinda}$ $TB \bigoplus T^*B$) as a generalized tangent bundle

$\forall\, V^M \in E$: $V^M = \begin{pmatrix} v^m \\ \lambda_m \end{pmatrix}$ where morally $v^m \in TB$ and $\lambda_m \in T^*B$

In supergravities, the objects are fields and forms so why not bunch them all together. This allows us to make more of these objects geometric…

Possible structure groups : $GL(2d;\mathbb{R}) \supset O(d,d;\mathbb{R}) \supset O(d;\mathbb{R}) \times O(d;\mathbb{R})$

$$O(d;\mathbb{R}) \hookrightarrow O(d,d;\mathbb{R}) : A \to \begin{pmatrix} 0 & A \\ A & 0 \end{pmatrix}$$

$$O(d;\mathbb{R}) \hookrightarrow O(d,d;\mathbb{R}) : A \to \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix}$$

$$V^M = \begin{pmatrix} v^m \\ \lambda_m \end{pmatrix} \to \begin{pmatrix} A^m{}_n v^n \\ A^n{}_m \lambda_n \end{pmatrix}$$

To describe a supergravity, we want a reduction of the structure group to $\boldsymbol{O(d;\mathbb{R})\times O(d;\mathbb{R})}$ acting on $\boldsymbol{E(\cong_{Kinda} TB \oplus T^*B)}$

FYI : $GL(2d;\mathbb{R}) \supset O(d,d;\mathbb{R}) \supset O(d;\mathbb{R})\times O(d;\mathbb{R})$

The $O(d,d;\mathbb{R})$ invariant : $J = \begin{Bmatrix} 0 & \mathbb{I}_d \\ \mathbb{I}_d & 0 \end{Bmatrix}$

Use J to define an $O(d,d;\mathbb{R})$-structure (E,J) by a choice of metric

$$\mathcal{N}_{IJ} = \mathrm{e}^{A}{}_{\mathrm{I}}\mathrm{e}^{B}{}_{\mathrm{J}}\, J_{AB} \in \left[\mathcal{N}_{IJ}\right] \approx O(d,d;\mathbb{R})$$

The $O(d;\mathbb{R})\times O(d;\mathbb{R})$ invariant : $O(d;\mathbb{R}) \hookrightarrow O(d,d;\mathbb{R}) \Rightarrow \mathbb{I}_d \hookrightarrow \delta_{AB} = \begin{pmatrix} \mathbb{I}_d & 0 \\ 0 & \mathbb{I}_d \end{pmatrix}$

Use δ to define an $\boldsymbol{O(d;\mathbb{R})\times O(d;\mathbb{R})}$**-structure** $\boldsymbol{(E,J,\delta)}$ by a choice of metric

$$\mathcal{H}_{IJ} = \mathcal{E}^{A}{}_{\mathrm{I}}\mathcal{E}^{B}{}_{\mathrm{J}}\, \delta_{AB} \in [\mathcal{H}_{IJ}]$$

Moduli space $\mathcal{M} = C^{\infty}(L)$ **:** $\forall x \in B\colon\ \mathcal{M}|_x \cong \frac{O(d,d;\mathbb{R})}{O(d;\mathbb{R})\times O(d;\mathbb{R})}$

Why do we care?

We use generalized geometry to describe supergravities where the objects in the SUGRA map to geometric object in the generalized geometry.

$$\mathcal{H}_{IJ} \text{ has } d^2 = \frac{1}{2}d(d+1) + \frac{1}{2}d(d-1) \text{ degrees of freedom}$$

$$g \in GL(d;\mathbb{R})\colon g^T = g \qquad\qquad B \in GL(d;\mathbb{R})\colon B^T = -B$$

If anyone tells you this in a seminar they are lying

Without too much effort, add a scalar ϕ (conformal factor) "size" of $\mathcal{H}_{IJ}$ (determinant).

$\Rightarrow \mathcal{H}_{IJ}\ (g, B, \phi)(x)$ and $\mathcal{M}$ is spanned by the $NS-NS$ bosonic field (g, B, ϕ) of the SUGRA

$$\forall x \in B\colon\ \mathcal{M}\Big|_x \cong \mathbb{R}^+ \times \frac{O(d,d;\mathbb{R})}{O(d;\mathbb{R}) \times O(d;\mathbb{R})}$$

Simple application

The generalized distance conjecture of the swampland program:

Dieter Lüst, Eran Palti, and Cumrun Vafa , AdS and the Swampland, Physics Letters B 797 (2019) 134867

We are now ready to state the generalized distance conjecture: Consider the non-compact space to be an Einstein space, i.e. AdS, Minkowski, or dS (if it exists). Then for large distance variation in fields, we get a light tower of states in the Einstein frame of the external effective field theory, whose mass scale in Planck units is given by

$$m \sim e^{-\alpha\Delta} \, ,$$

where $\alpha \sim \mathcal{O}(1)$.

Δ is a "distance" accounting for the distance travelled on the metric moduli space and a change in **all** non-trivial fluxes of the SUGRA. **This is a hard computation!**

*But these can **all be** metric moduli of the generalized metric in generalized geometry. **Just need to find geodesics on $\mathcal{M}$**.*

Much easier!

Let’s see what happens…