Cosmological and Quantum Gravity applications of QRG

On polygons

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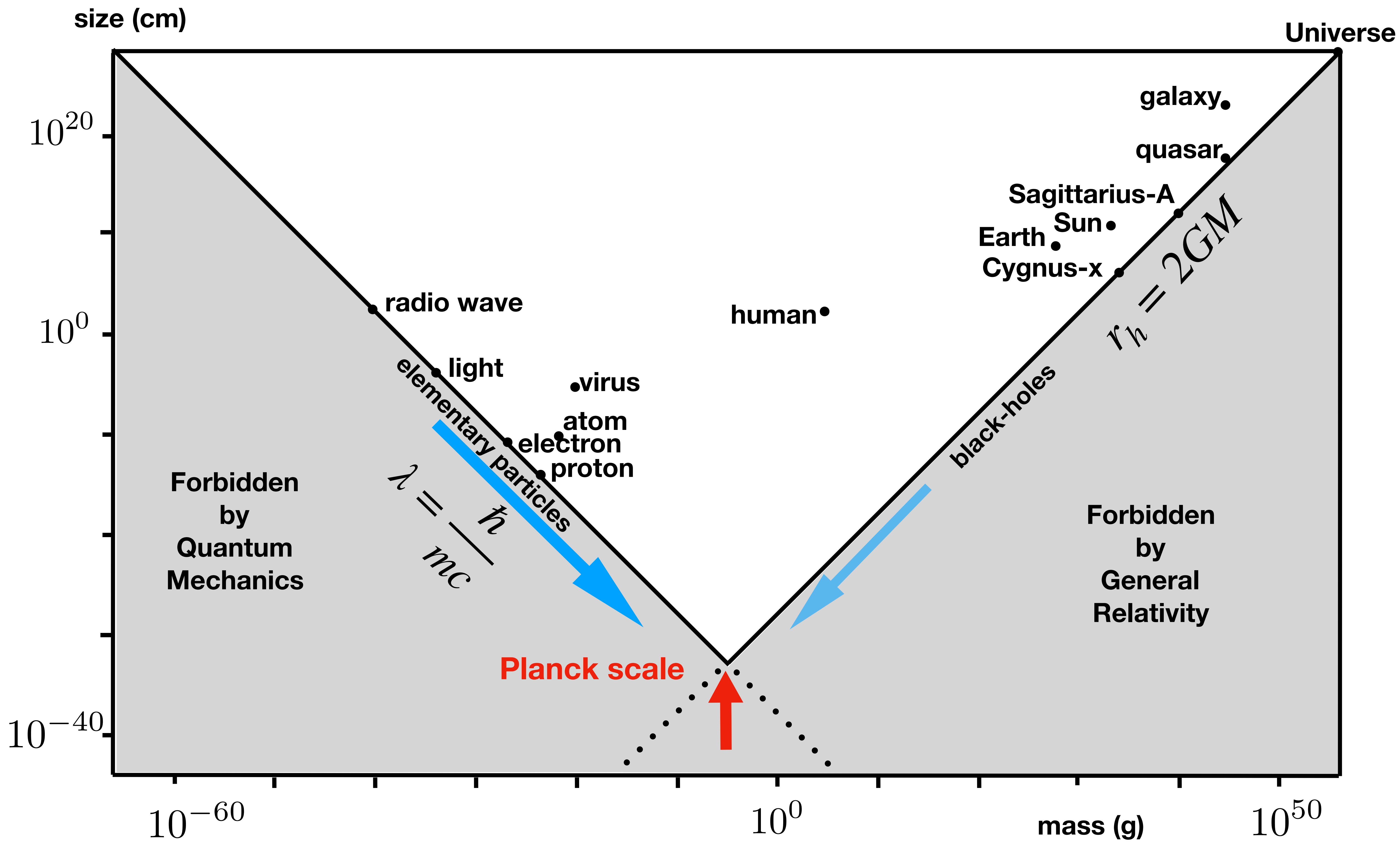
Spacetime at Plank scale



Quantum spacetime hypothesis: space-time coordinates better modelled as noncommutative due to quantum gravity effects

Quantum Riemannian Geometry

Differential calculus

- An algebra A over a field K
- An A-bimodule Ω^1 — $(a \cdot \omega) \cdot b = a \cdot (\omega \cdot b),\ a, b \in A,\ \omega \in \Omega^1$
- A linear map $d : A \to \Omega^1$ — $d(ab) = (da) \cdot b + a \cdot (db)$
- $\Omega^1 = \text{span}\{a \cdot db \mid a, b \in A\}$ — Surjectivity
- $\text{Ker}\, d = k \cdot 1_A,\ k \in K,\ 1_A \in A$ — Connectedness (Optional)
- $d : \Omega^n \to \Omega^{n+1}$ — $d^2 = 0$

Geometrical Structures on a differential calculus (A, Ω^1, d)

$g \in \Omega^1 \otimes_A \Omega^1$ Metric

$(,) : \Omega^1 \otimes_A \Omega^1 \to A$ Inverse metric

$\nabla : \Omega^1 \to \Omega^1 \otimes_A \Omega^1$ Connection

$\sigma : \Omega^1 \otimes_A \Omega^1 \to \Omega^1 \otimes_A \Omega^1$ Braiding map

$\mathrm{Ricci} \in \Omega^1 \otimes_A \Omega^1$ $\quad S = (\ ,\ )\mathrm{Ricci}$ $\quad i : \Omega^2 \to \Omega^1 \otimes \Omega^1$

$$\nabla(f\omega) = \mathrm{d}f \otimes \omega + f\nabla\omega \qquad \nabla(\omega f) = \sigma(\omega \otimes \mathrm{d}f) + (\nabla\omega)f$$

$$\nabla(\omega \otimes \eta) = \nabla\omega \otimes \eta + (\sigma \otimes \mathrm{id})(\omega \otimes \nabla\eta)$$

QLC $\quad \nabla g = 0 \qquad T_\nabla : \Omega^1 \to \Omega^2 \qquad T_\nabla = \wedge\nabla - \mathrm{d}$

Quantization of $\mathbb{Z}_n$

J. N. Argota-Quiroz and S. Majid, Quantum gravity on polygons and $\mathbb{R} \times \mathbb{Z}_n$ FLRW model, *Class. Quantum Grav. (2020) 245001 (43pp)*

Euclideanized Quantum Gravity

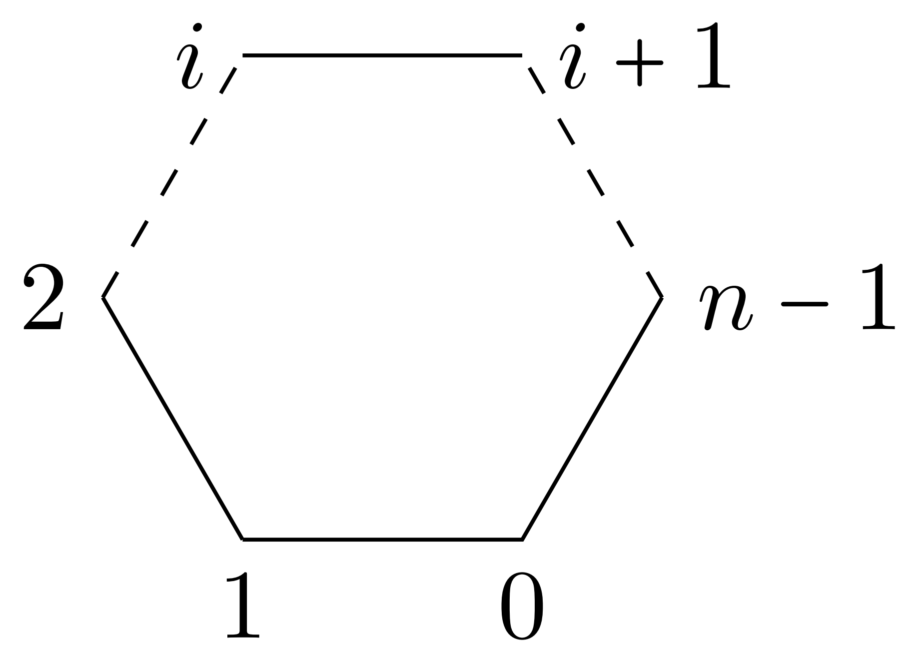
Algebra $A = \mathbb{C}(\mathbb{Z}_n)$

Calculus Ω

N-gon

$$e^{\pm} f = R_{\pm} f e^{\pm}, \quad \mathrm{d}f = \sum_{\pm} \partial_{\pm} f e^{\pm}; \quad (R_{\pm} f)(i) = f(i \pm 1), \quad \partial_{\pm} = R_{\pm} - \mathrm{id}$$

$a(i) = g_{i \to i+1}$
$b(i+1) = g_{i+1 \to i}$



$a(0) = g_{0 \to 1}$
$b(1) = g_{1 \to 0}$

Metric

$$g = ae^{+} \otimes e^{-} + be^{-} \otimes e^{+}$$

Edge symmetry condition

$$b = R\_(a)$$

Inverse metric

$$(e^{+}, e^{-}) = 1/R_{+}(b) \quad (e^{-}, e^{+}) = 1/R\_(a) \quad (e^{\pm}, e^{\pm}) = 0$$

Ω^2 calculus

$$(e^{\pm})^2 = 0, \quad e^{+} \wedge e^{-} + e^{-} \wedge e^{+} = 0, \quad \mathrm{d}e^{\pm} = 0$$

Geometrical structures for $\mathbb{Z}_n$

- $g = ae^+ \otimes e^- + R_-(a)e^- \otimes e^+$ Metric
- $\nabla e^+ = (1-\rho)e^+ \otimes e^+, \qquad \nabla e^- = (1 - R_-^2\rho^{-1})e^- \otimes e^-$ QLC $\qquad \rho = \dfrac{R_+(a)}{a}$
- $R_\nabla e^+ = \partial_-\rho e^+ \wedge e^- \otimes e^+, \qquad R_\nabla e^- = -\partial_+(R_-^2\rho^{-1})e^+ \wedge e^- \otimes e^-$ Curvature
- $\mathrm{Ricci} = \frac{1}{2}\left(\partial_-(R_-\rho)e^- \otimes e^+ - \partial_-\rho^{-1}e^+ \otimes e^-\right),$ Ricci tensor
- $S = \frac{1}{2}\left(-\frac{\partial_-\rho^{-1}}{a} + \frac{\partial_-(R_-\rho)}{R_-a}\right),$ Ricci scalar $\qquad \partial_\pm f(i) = f(i+1) \pm f(i)$
- $\Delta f = -\frac{R_-(\rho)+1}{a}(\partial_+ + \partial_-)f$ Laplacian

$$\int S = \sum_{\mathbb{Z}_n} aS = \frac{1}{4}\sum_{\mathbb{Z}_n} \rho(\partial_+ + \partial_-)\rho$$ Hilbert-Einstein Action

Quantum Gravity Applications

Euclideanized Quantum Gravity

Closed boundary condition $\rho_0\rho_1\cdots\rho_{n-1} = 1$ constraint manifold with metric $\mathfrak{g}_\rho$

Measure for integration over ρ

$$\mathcal{D}\rho = \left(\prod_{i=0}^{n-2} \mathrm{d}\rho_i\right)\sqrt{\det(\mathfrak{g}_\rho)}$$

Taking n = 3, and ρ_0, ρ_1 as coordinates

$$\det(\mathfrak{g}_\rho) = 1 + \frac{1}{\rho_0^4\rho_1^2} + \frac{1}{\rho_0^2\rho_1^4}$$

$$Z = \int_0^\infty \mathrm{d}\rho_0 \int_0^\infty \mathrm{d}\rho_1 \sqrt{\det(\mathfrak{g}_\rho)}\, e^{-\frac{1}{2G}(\rho_0^2+\rho_1^2+\rho_2^2-\rho_0\rho_1-\rho_1\rho_2-\rho_2\rho_0)}$$

For large G

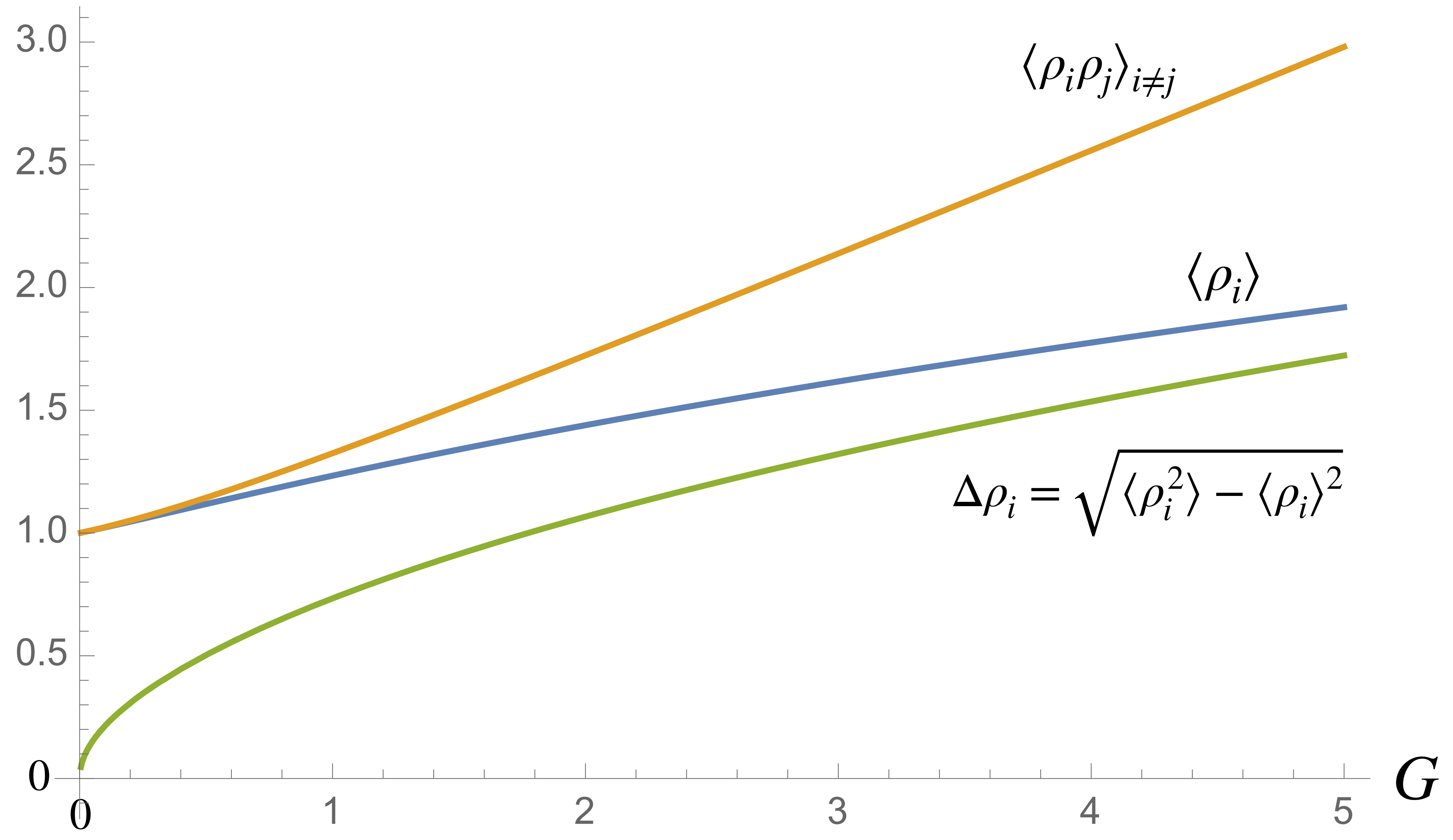
$$\frac{\Delta\rho_i}{\langle\rho_i\rangle} \sim 1.1$$

$$\frac{\langle\rho_i\rho_j\rangle}{\langle\rho_i\rangle\langle\rho_j\rangle} \geq .808$$

Uniform uncertainty in metric

For small G

$$\Delta\rho_i \to 0$$



Quantize fluctuations relative to the geometrical mean for n = 3

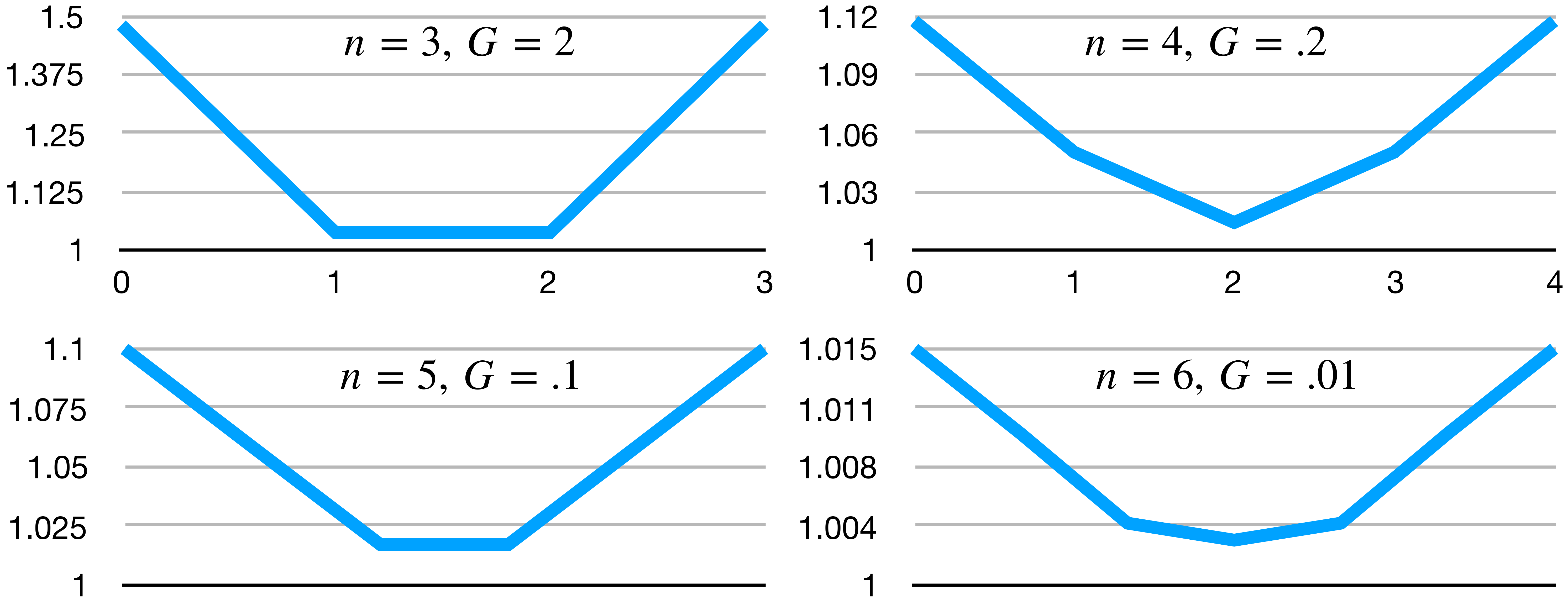
$$A = \left(\prod_i a_i\right)^{\frac{1}{n}}$$

With the variables $b_i = a_i/A$ And the restriction $b_0 b_1 \ldots b_n = 1$

Mesure $\mathrm{d}a_0\,\mathrm{d}a_1\,\mathrm{d}a_2 = \dfrac{3A^2}{b_0 b_1}\mathrm{d}b_0\,\mathrm{d}b_1\,\mathrm{d}A.$ Action $S_g = \dfrac{1}{2}\left(\dfrac{b_0}{b_1} + \dfrac{b_1}{b_2} + \dfrac{b_0}{b_2} - \left(\dfrac{b_1}{b_0}\right)^2 - \left(\dfrac{b_2}{b_1}\right)^2 - \left(\dfrac{b_0}{b_2}\right)^2\right)$

Partition function

$$Z = \int_0^\infty \mathrm{d}b_0 \int_0^\infty \mathrm{d}b_1 \frac{1}{b_0 b_1} e^{\frac{1}{2Gb_0^2 b_1^4}(-1+(1+b_0^3)b_1^3+(-1+b_0^3-b_0^6)b_1^6)}$$



Cosmological applications

Geometrical structures of FLRW model

$$g=-\mathrm{d}t\otimes\mathrm{d}t-\mathrm{R}^2(t)e^+\otimes_s e^-$$

Metric

$$\nabla\mathrm{d}t=R\dot{R}e^+\otimes_s e^-,\quad \nabla e^\pm=-\frac{\dot{R}}{R}e^\pm\otimes_s\mathrm{d}t,$$

QLC

$$R_\nabla e^\pm=-\frac{\ddot{R}}{R}\mathrm{d}t\wedge e^\pm\otimes\mathrm{d}t\pm\left(\frac{\dot{R}}{R}\right)^2R^2e^+\wedge e^-\otimes e^\pm,\quad R_\nabla\mathrm{d}t=\ddot{R}R\mathrm{d}t\wedge e^+\otimes_s e^-,$$

Curvature

$$\mathrm{Ricci}=\frac{\ddot{R}}{R}\mathrm{d}t\otimes\mathrm{d}t+\frac{1}{2}\left(\frac{\dot{R}^2}{R^2}+\frac{\ddot{R}}{R}\right)R^2e^+\otimes_s e^-,\quad S=-2\frac{\ddot{R}}{R}-\left(\frac{\dot{R}}{R}\right)^2$$

Ricci

$$\mathrm{Eins}=\mathrm{Ricci}-\frac{1}{2}Sg=-\frac{1}{2}\left(\frac{\dot{R}}{R}\right)^2\mathrm{d}t\otimes\mathrm{d}t-\frac{R\ddot{R}}{2}e^+\otimes_s e^-\qquad\nabla\cdot\mathrm{Eins}=0$$

Naive Einstein Tensor

$$T=f\mathrm{d}t\otimes\mathrm{d}t-pR^2e^+\otimes_s e^-\qquad p=-\frac{1}{8\pi G}\left(\frac{\ddot{R}}{R}\right),\quad f=\frac{1}{8\pi G}\left(\frac{\dot{R}}{R}\right)^2.$$

Fluid tensor for a pressure p and density f

$$R(t)=R_0\left(1+\sqrt{8\pi Gf_0}(1+w)t\right)^{\frac{1}{1+w}}$$

1+ 2 rate of expansion

Results of FLRW model

With R being constantan (QFT)

$$< 0 \mid T[\phi_i(t_a)\phi_j(t_b)] \mid 0 > = \sum_{k=0}^{n-1} \frac{1}{w_k} \cos\left(\frac{2\pi}{n}k(i-j)\right) e^{-\imath w_k |t_a - t_b|}.$$

Particle creation in the adiabatic limit $\dot{R}/R \to 0,\ \ddot{R}/R \to 0$

$$w_k(t) = \sqrt{m^2 + \frac{8}{R^2(t)}\sin^2\left(\frac{\pi}{n}k\right)}$$

$$\Delta = -\partial_t^2 - 2\frac{\dot{R}}{R}\partial_t + \frac{2}{R^2}(\partial_+ + \partial_-) \qquad (-\Delta + m^2)\phi = 0$$

No particle creation

$\mathbb{R} \times S^1$

$R \propto t^2$

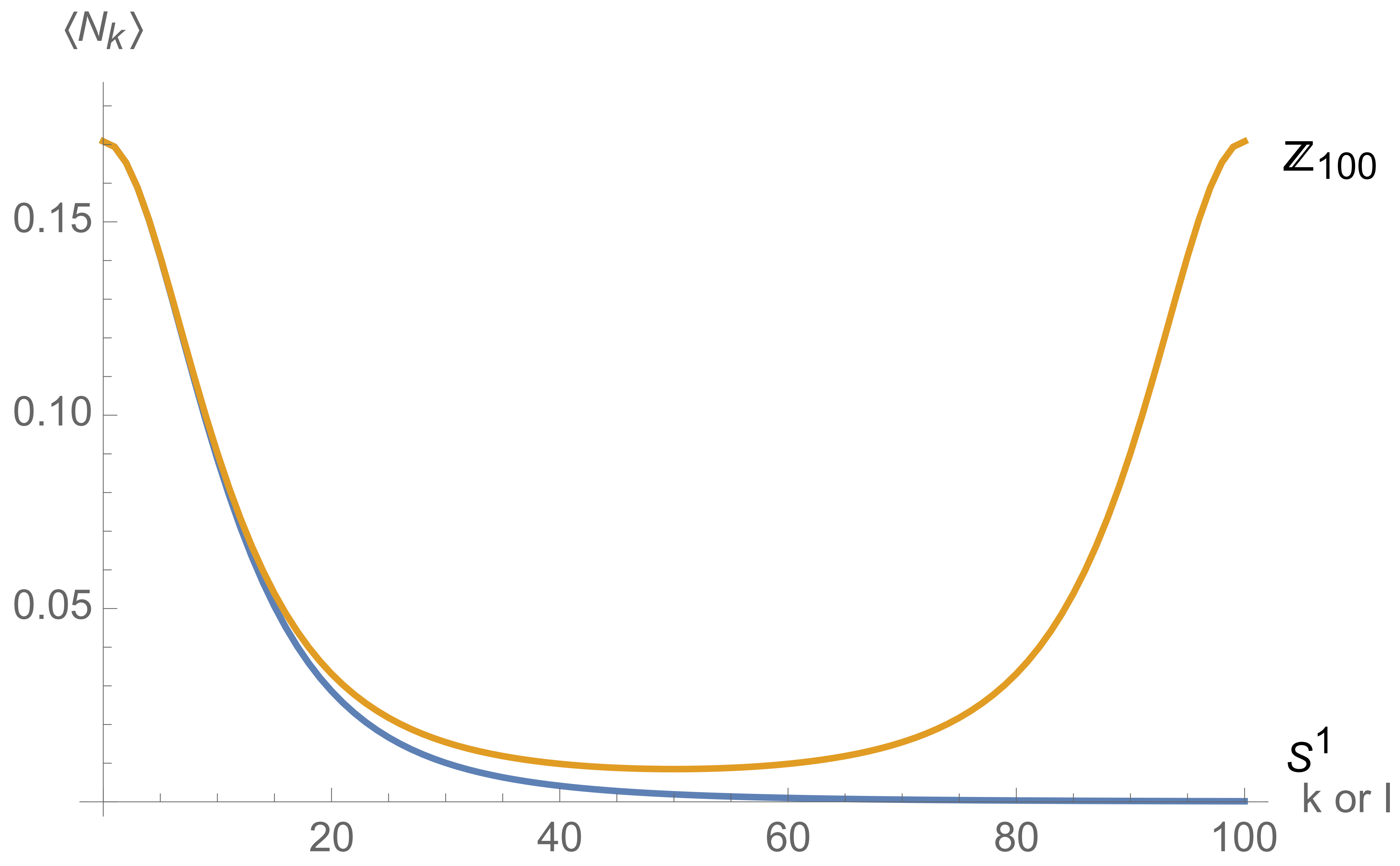
$$\frac{1}{2}\frac{\ddot{R}}{R} = \frac{1}{4}\left(\frac{\dot{R}}{R}\right)^2 \quad (m \to \infty)$$

$\mathbb{R} \times \mathbb{Z}_n$

$\ddot{R} = 0\ (m \to \infty) \qquad R \propto t$

$$\frac{\ddot{R}}{R} = -\frac{1}{2}\left(\frac{\dot{R}}{R}\right)^2,\ (m \to 0)$$

$R \propto t^{\frac{2}{3}}$



Thank you for your attention