Lattice Simulations for first order deconfinement phase transitions

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Overview

Why are we interested in models with first order deconfinement transitions?

Properties of the deconfinement transition

Metastability problems with standard lattice methods

The linear logarithmic relaxation method (LLR).

Deconfinement in the standard model

- Current Universe: Quarks and gluons confined in Hadrons.
- Early Universe: Plasma of free quarks and gluons.

Between these phases we may have a phase transition

Extensions to the standard the model may also contain additional strong sectors that will undergo this type of phase transition, e.g.

- Stealth dark matter models.
- Composite Higgs models.

Why first order?

First order transitions have coexisting phases separated by a potential barrier.

Once one point in space transitions, a bubble of the low energy solution forms and expands.

These bubbles give rise to gravitational waves from[1]:

- Compression waves due to the expansion
- Bubble collisions

The signature of the waves depend on thermodynamic observables, e.g. the latent heat, of the transitions which are calculable on lattice simulations.

[1]:Hindmarsh, Mark, et al. "Gravitational waves from the sound of a first order phase transition." *Physical review letters* 112.4 (2014): 041301.

The Model

Model: SU(3) pure gauge field

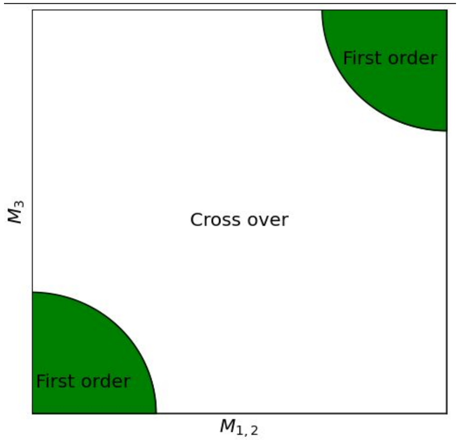
Columbia plot shows the regions of QCD we expect to have first order transitions

Lattice simulations suggest Quantum Chromodynamics this is a cross-over[1]

First order phase transition for very light or very heavy fermions.

Pure gauge limit corresponds to limit of infinite quark mass.

All graphs shown will be for SU(3) pure gauge theory on a lattice of size 4x20x20x20.



Columbia plot of QCD. M_i correspond to the mass of the quarks.

[1]: Aoki, Yasumichi, et al. "The order of the quantum chromodynamics transition predicted by the standard model of particle physics." *Nature* 443.7112 (2006): 675-678.

Simulating on the lattice

$$\int \mathcal{D}A \exp(-\frac{1}{4}\int d^4x Tr(F_{\mu\nu}^2)) \longrightarrow \int \mathcal{D}U \exp(-\beta \sum_{j,\mu>\nu}[1-\frac{1}{N}\Re(Tr(U_{\mu\nu}(j)))])$$

Discretise spacetime onto hypercubic lattice.

Replace gauge field with link variables U_μ

Temperature set by changing β

Generate configurations by using MC methods to update links with a probability distribution ∝exp(-ΔS)), ΔS is the change in the action due to the change in a link

Calculate expectation values by averaging over observables measured on configurations.

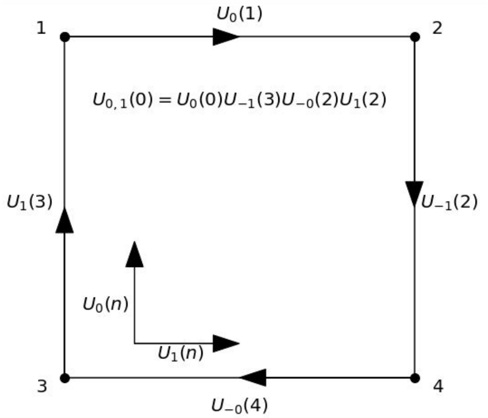


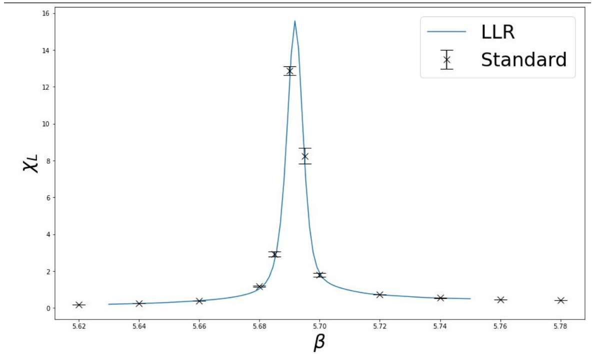
Diagram of Plaquette .

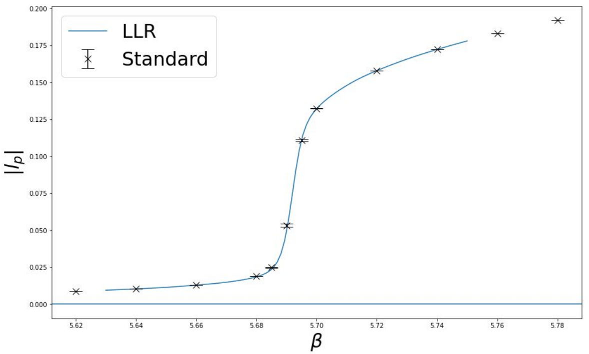
Deconfinement transition

$$\langle |l_p| \rangle = \langle |Tr(\prod_{n_0=0}^{N_0-1} U_0(n_0, \vec{n_s}))| \rangle \propto \exp(-F\beta) = \begin{cases} 0 \text{ confined} \\ \neq 0 \text{ deconfined} \end{cases}$$

F - free energy required to create an additional free colour source[1]

Associated with the phase transition is a spontaneous symmetry breaking of the $\mathbb{Z}_N$ centre symmetry with 1 confined vacua and N deconfined vacua





Top diagram: Polyakov loop susceptibility against the inverse temperature.
Bottom diagram : Polyakov loop against inverse temperature .

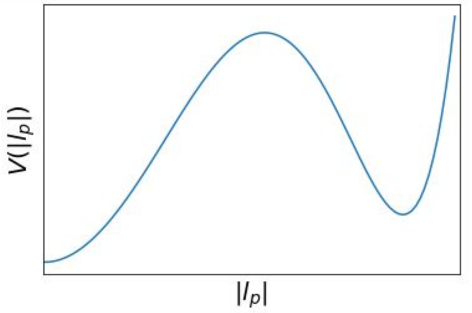
[1]: McLerran, Larry D., and Benjamin Svetitsky. "Quark liberation at high temperature: A Monte Carlo study of SU (2) gauge theory." *Physical Review D* 24.2 (1981): 450.

Simulating first order phase transitions

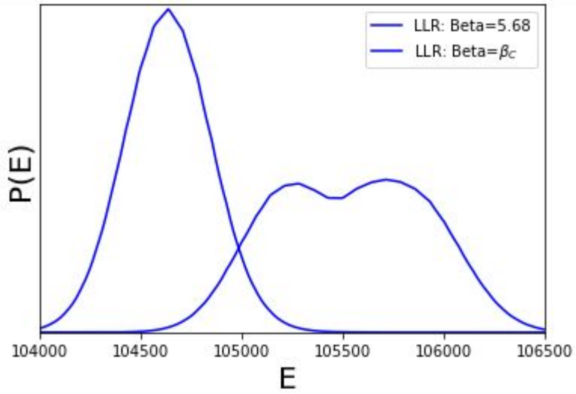
Around critical point there exists multiple coexisting phases separated by potential barriers

To get accurate expectation values we require the system to tunnel into all of the phases multiple times.

In the large volume limit the potential barriers increase, increasing the tunnelling times



Heuristic plot of the shape of the Polyakov loop potential.



Energy distribution for different β values

LLR method

Break down system into subsystems with energies confined to an interval of size δ_E

Extract the density of states for each interval ρ(E)

Reconstruct observables

$$Z_\beta = \int dE \rho(E) e^{-\beta E}$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z_\beta} \int dE \rho(E) \mathcal{O}(E) e^{-\beta E} = \int dE P_\beta(E) \mathcal{O}(E)$$

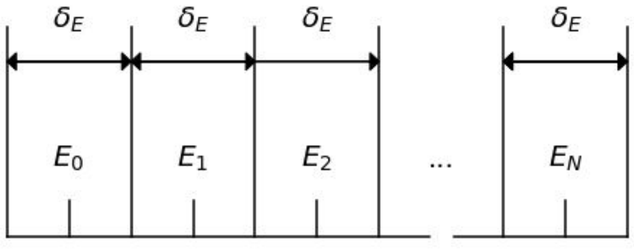
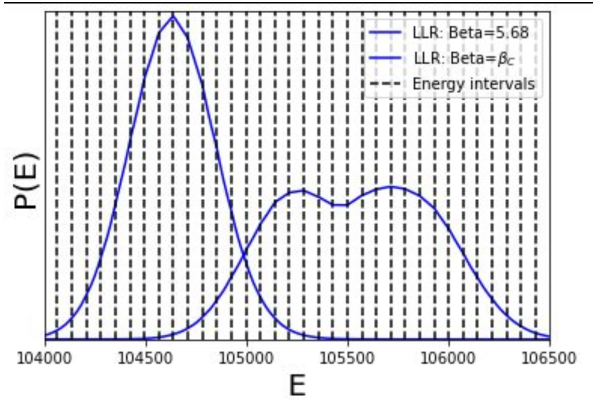


Diagram showing energy intervals for range E_0-δ_E/2 to E_N+δ_E/2, with interval size δ_E



Energy distribution for different β values

Finding the density of states

To find the density of states introduce the microcanonical temperature a_k.

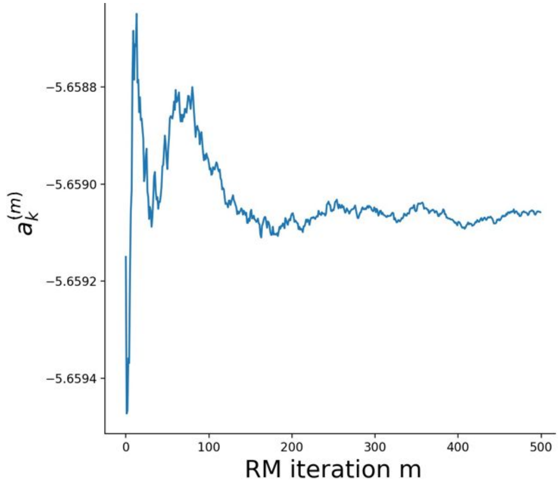
Use Robbins Monroe to iteratively solve the equation

$$\langle\!\langle \Delta E \rangle\!\rangle_{a_k} = \langle\!\langle E - E_k \rangle\!\rangle_{a_k} = 0, \qquad a_k^{(m+1)} = a_k^{m} - \frac{12 \langle\!\langle \Delta E \rangle\!\rangle_{a_k^{(m)}}}{\delta_E^2 (m+1)}, \qquad \lim_{m \to \infty} a_k^{(m)} = a_k$$

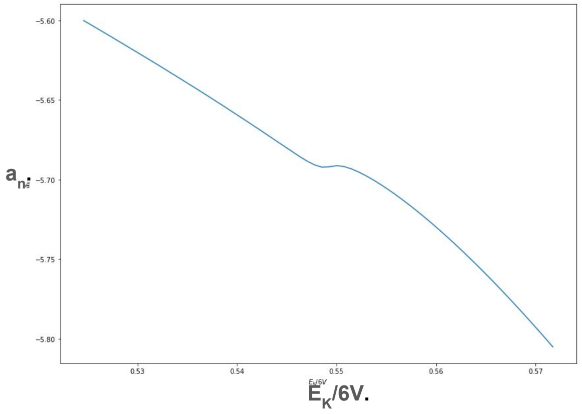
$\langle\!\langle ... \rangle\!\rangle$ is expectation value with configurations restricted to the interval $E_k - \frac{\delta_E}{2} \leq E \leq E_k + \frac{\delta_E}{2}$

Repeat this for all intervals, then estimate the density of states using

$$\rho(E) = \tilde{\rho}(E) \exp\left(\mathcal{O}(\delta_E)\right) \approx \rho_0 \exp\left(\sum_{n=1}^{k-1} (a_n \delta_E) + a_k (E - E_k + \delta_E/2) \right)$$



Example RM iteration for a_n.



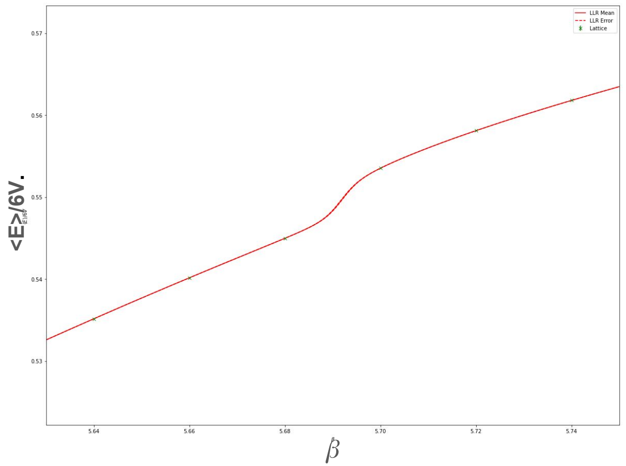
Centre energy per link against a_n.

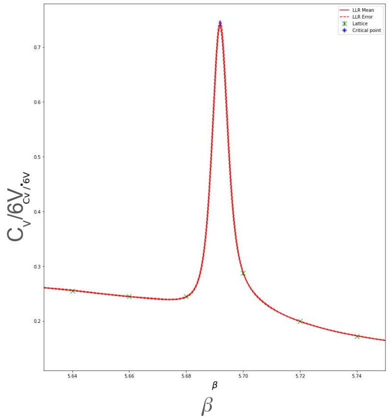
LLR observables

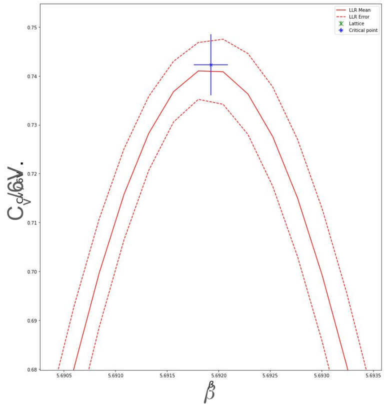
When the observables only depend on the energy we can simply analytically solve:

$$\langle \mathcal{O} \rangle = \frac{1}{Z_\beta} \int dE \rho(E) \mathcal{O}(E) e^{-\beta E}$$

Errors found by repeating LLR process multiple times.





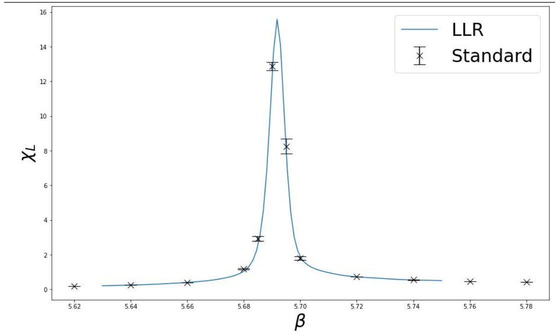


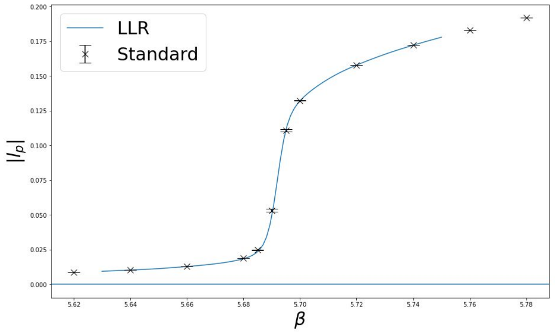
LLR observables

For more general observables, we require have to measure observables on the restricted lattice for fixed a and use:

$$\langle B[U] \rangle_\beta = \frac{1}{Z(\beta)} \sum_k \delta_E \rho(E_k) \langle\!\langle B[U] \exp(-\beta S[U] + a_k(S[U] - E_k)) \rangle\!\rangle$$

[1]





[1]: Langfeld, Kurt, et al. "An efficient algorithm for numerical computations of continuous densities of states." *The European Physical Journal C* 76.6 (2016): 1-18.

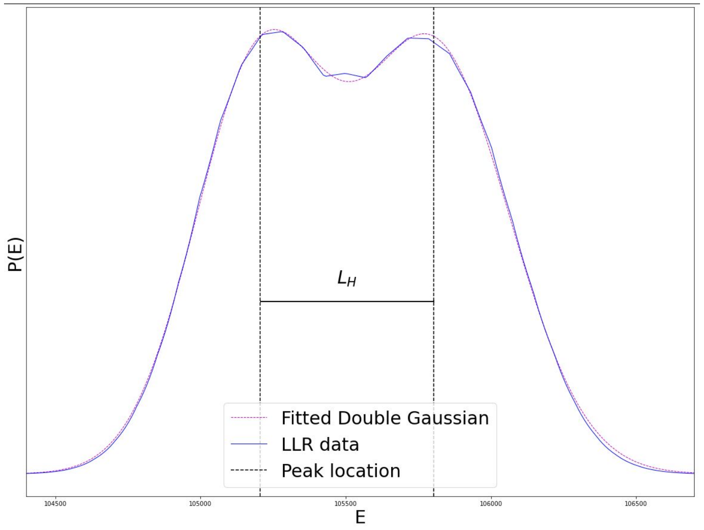
Latent heat

The energy probability distribution is given by

$$P_\beta(E) = \frac{1}{Z}\rho(E)e^{-\beta E}$$

Around the transitions we get a double Gaussian

The latent heat is the energy difference between the peaks of the double Gaussian when they are of equal height.



Conclusion and outlook.

First order deconfinement transitions in the early universe may lead to gravitational waves with signatures dependent on the thermodynamic properties of the transition.

Standard lattice methods have a strong metastability problems around first order transition particularly with large volumes.

The LLR method allows the reconstruction of thermodynamic observables without these metastability problems.

To do:

- More lattice sizes
- New gauge groups (Sp(2N))