Collider Phenomenology

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STFC school, Oxford
9-16/9/22

Plan for the lectures

- Basics of collider physics
- Basics of QCD
  - DIS and the Parton Model
  - Higher order corrections
  - Asymptotic freedom
  - QCD improved parton model
- State-of-the-art computations for the LHC
- Monte Carlo generators
- Higgs phenomenology
- Top phenomenology
- Searching for New Physics: EFT

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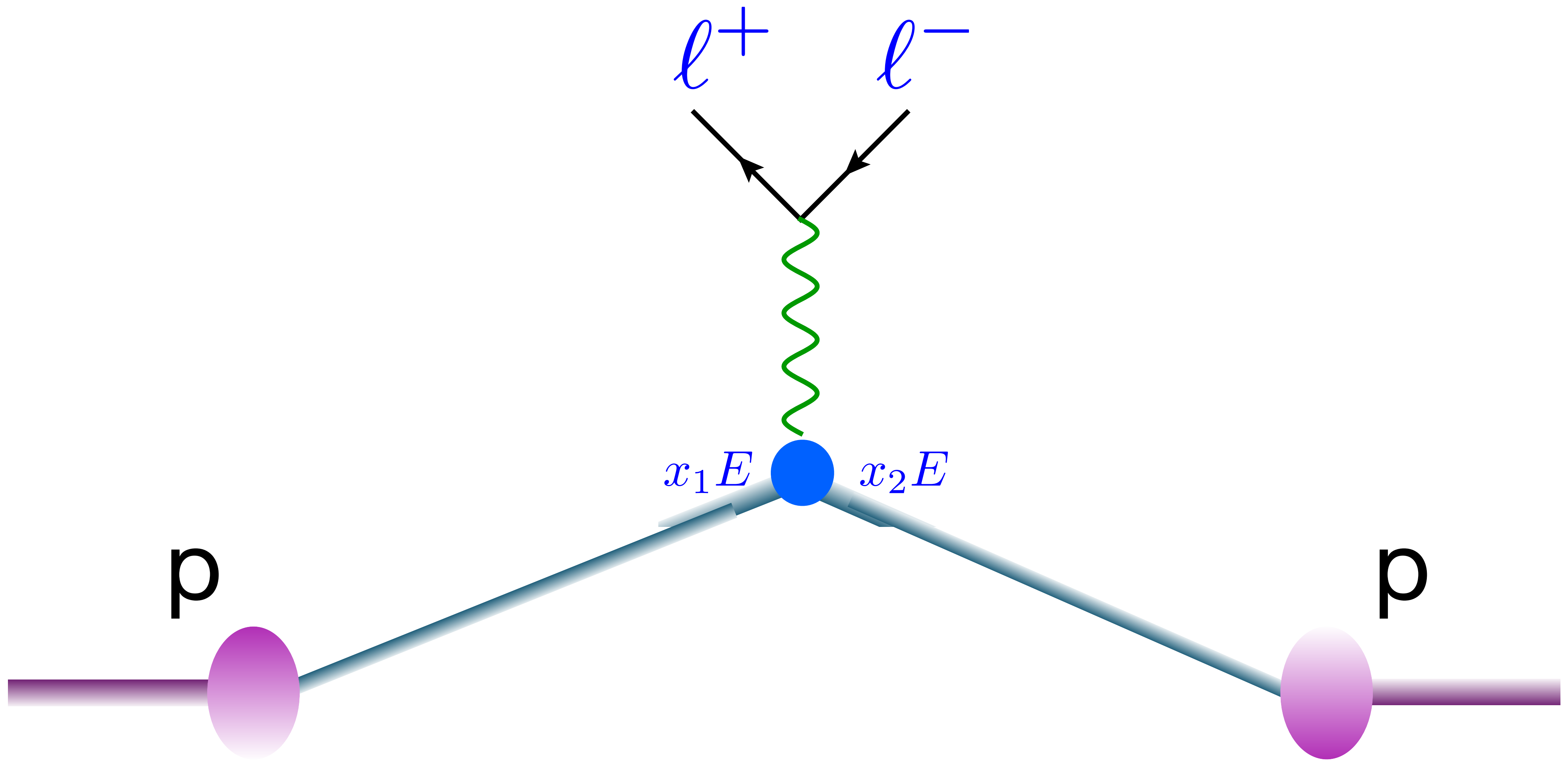
Fixed order computations

Going to higher orders



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$$\hat{\sigma} = \sigma^{\rm Born}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma^{(3)} + \dots\right)$$

LO NLO NNLO N3LO

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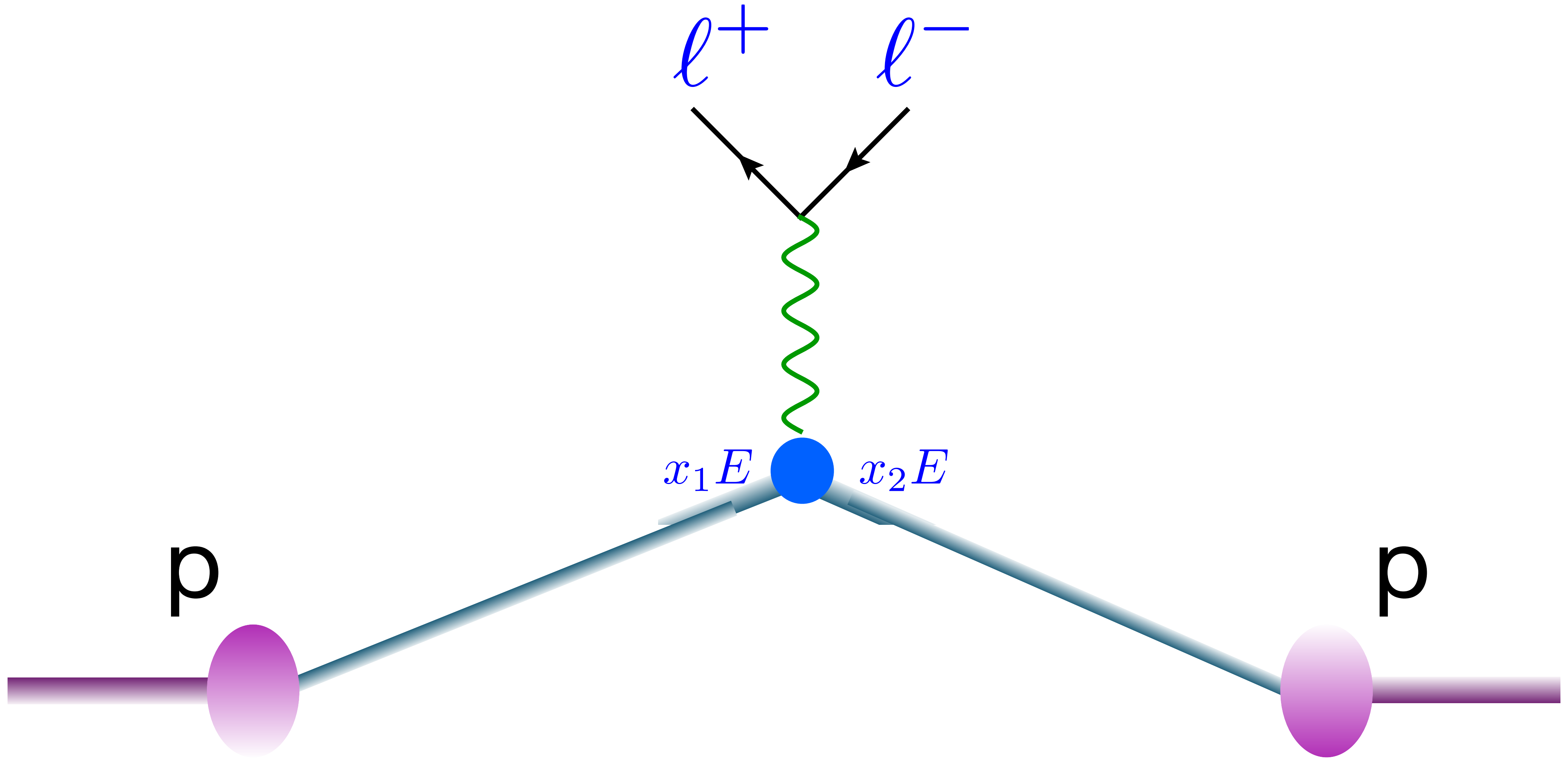

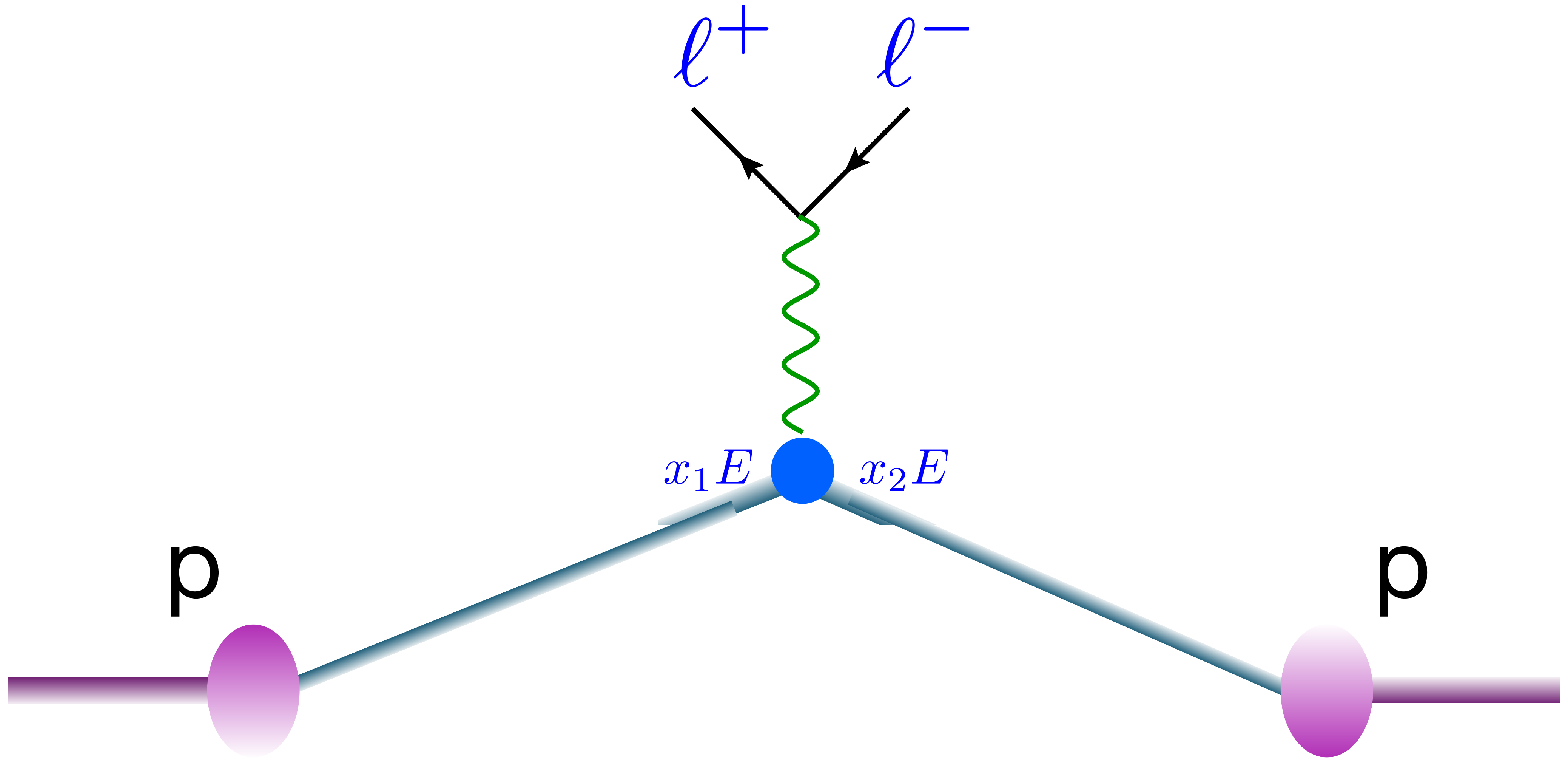

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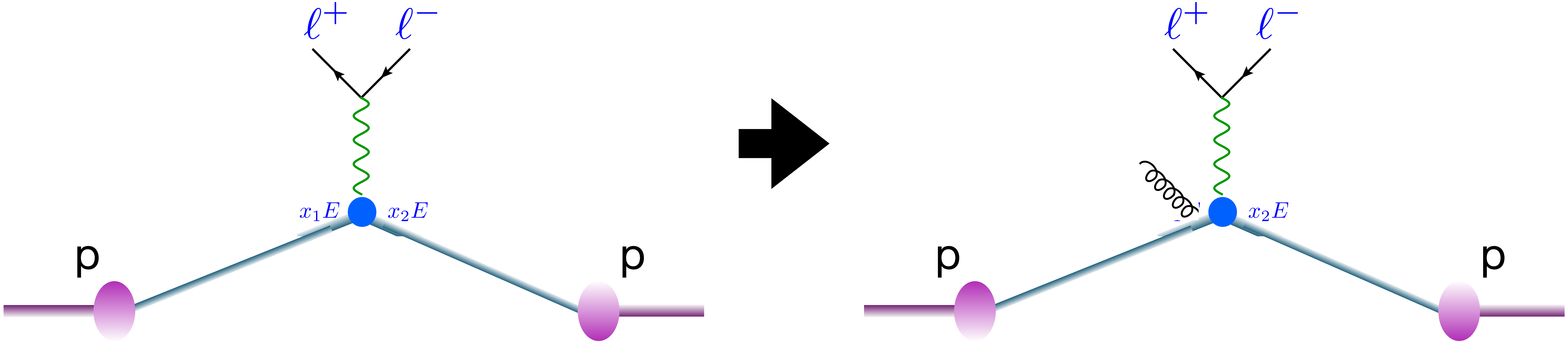

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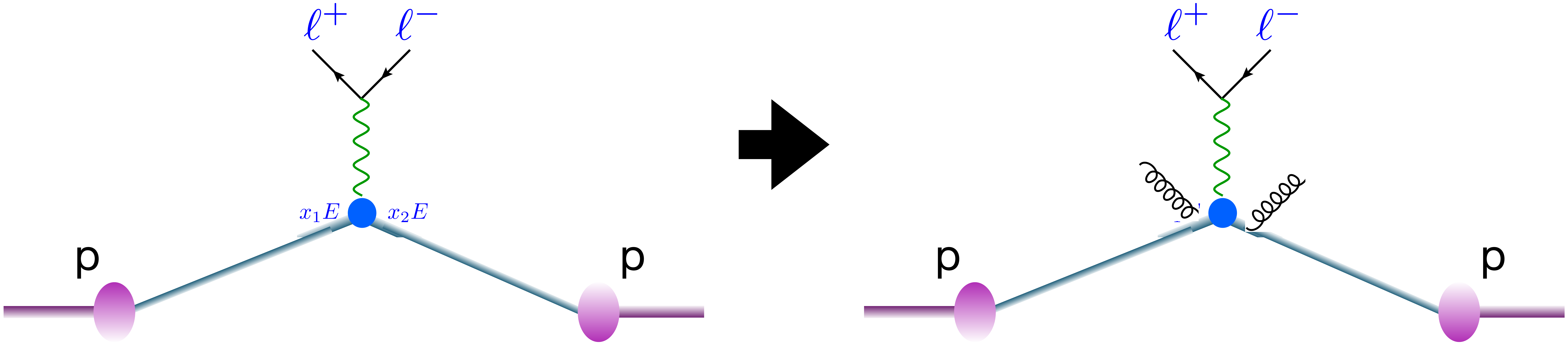

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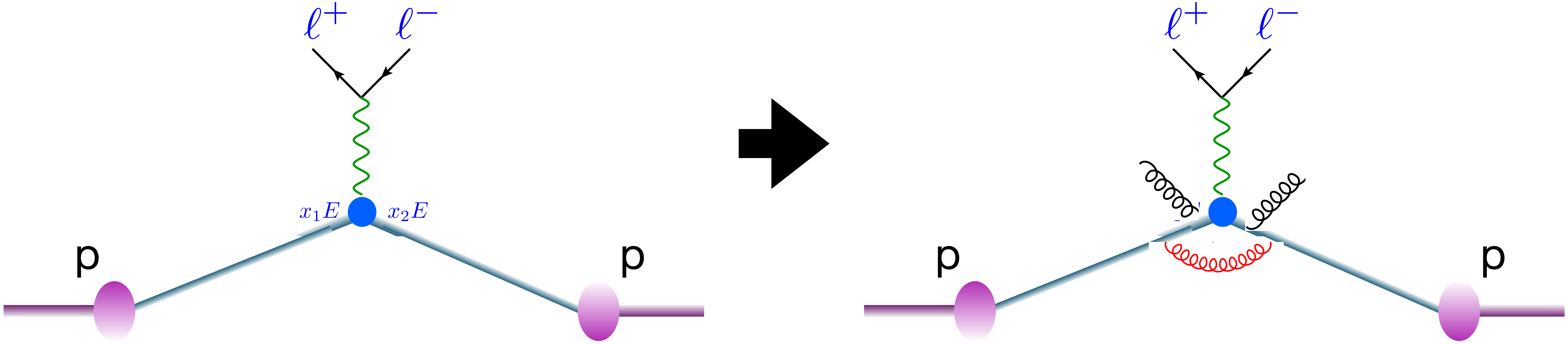

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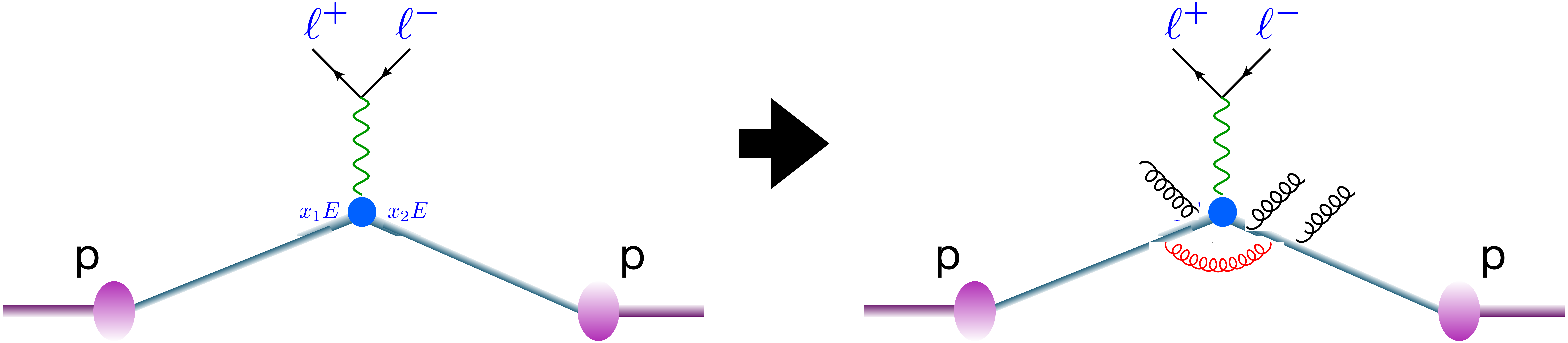

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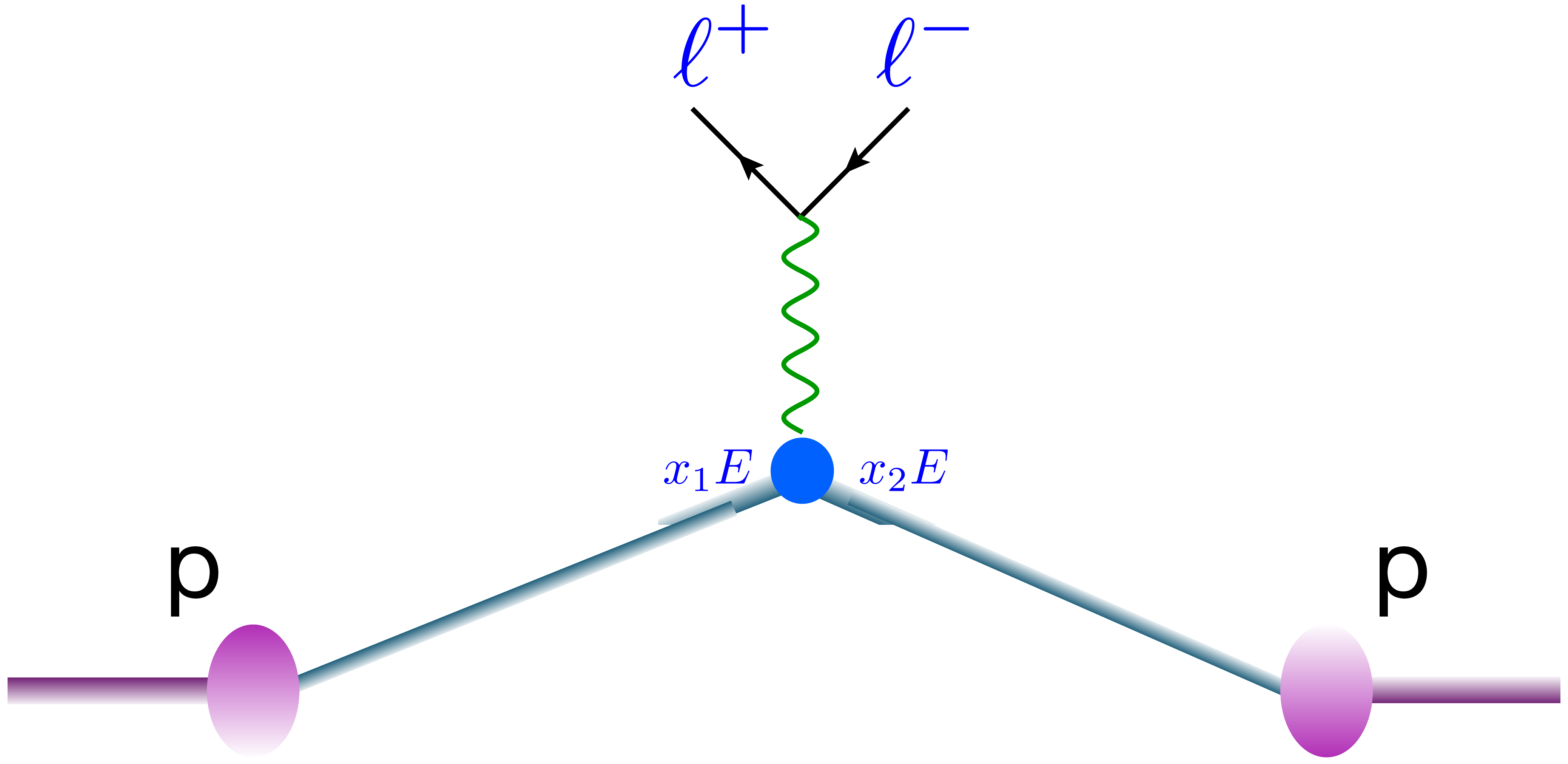

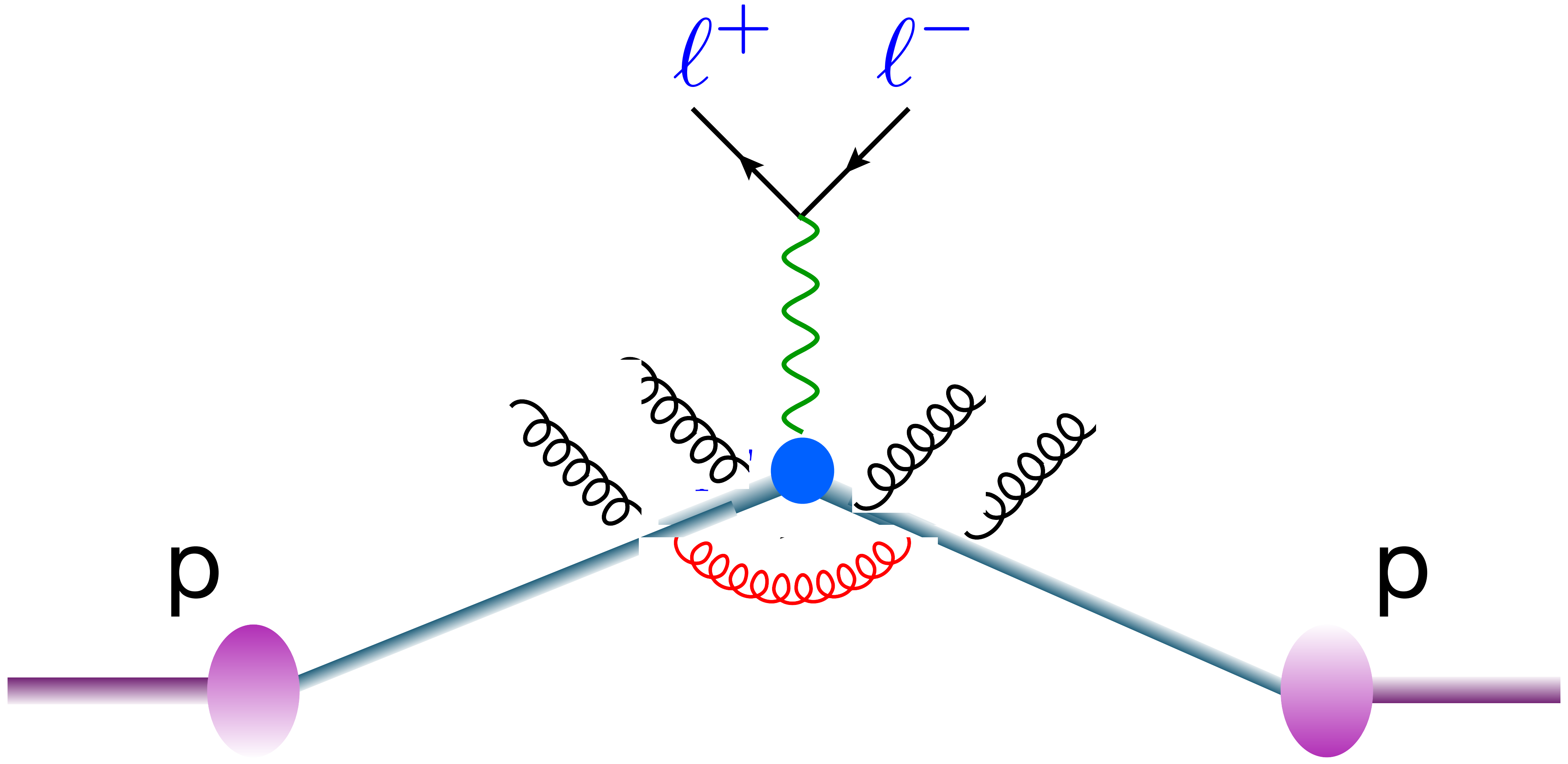

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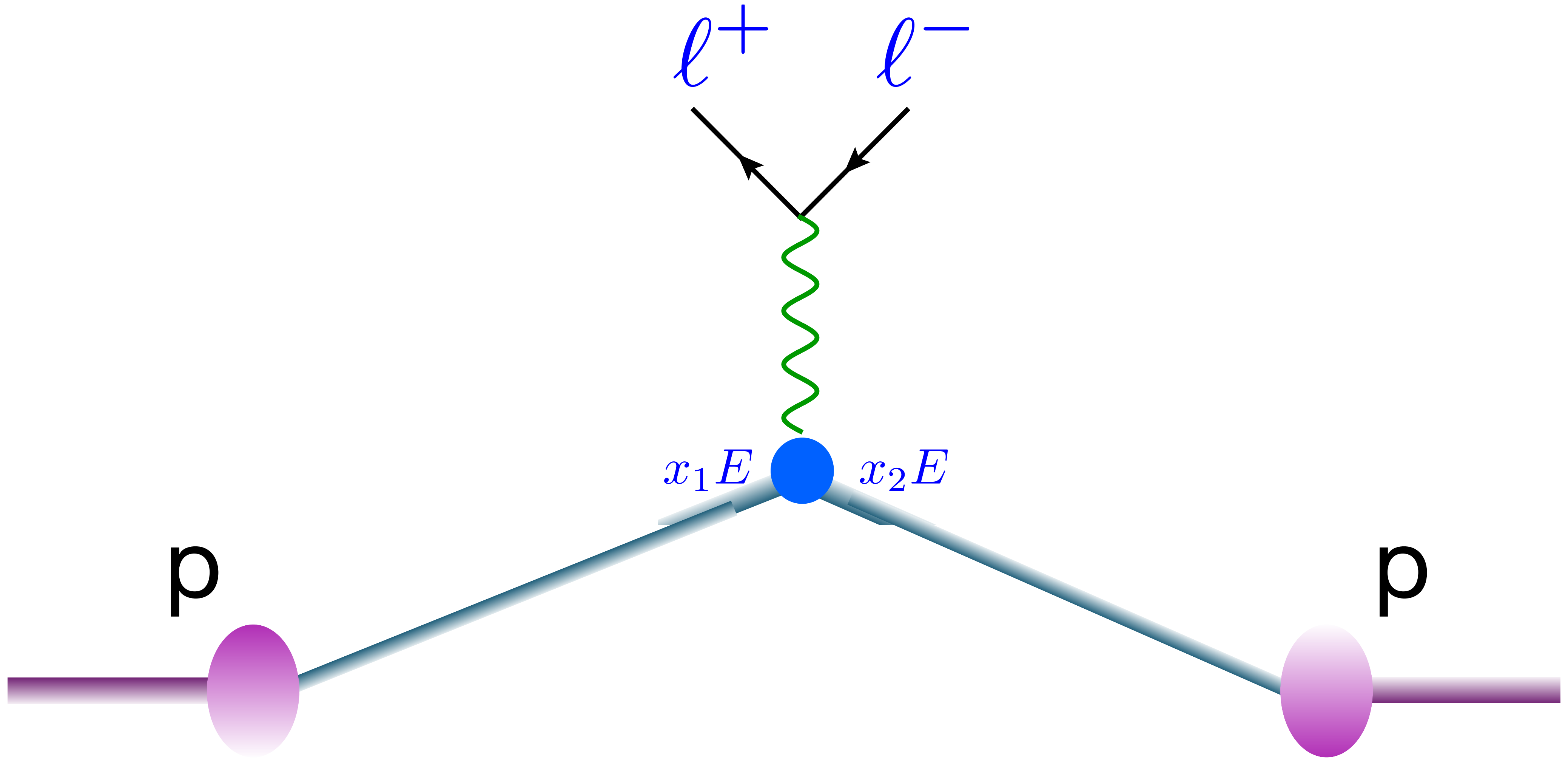

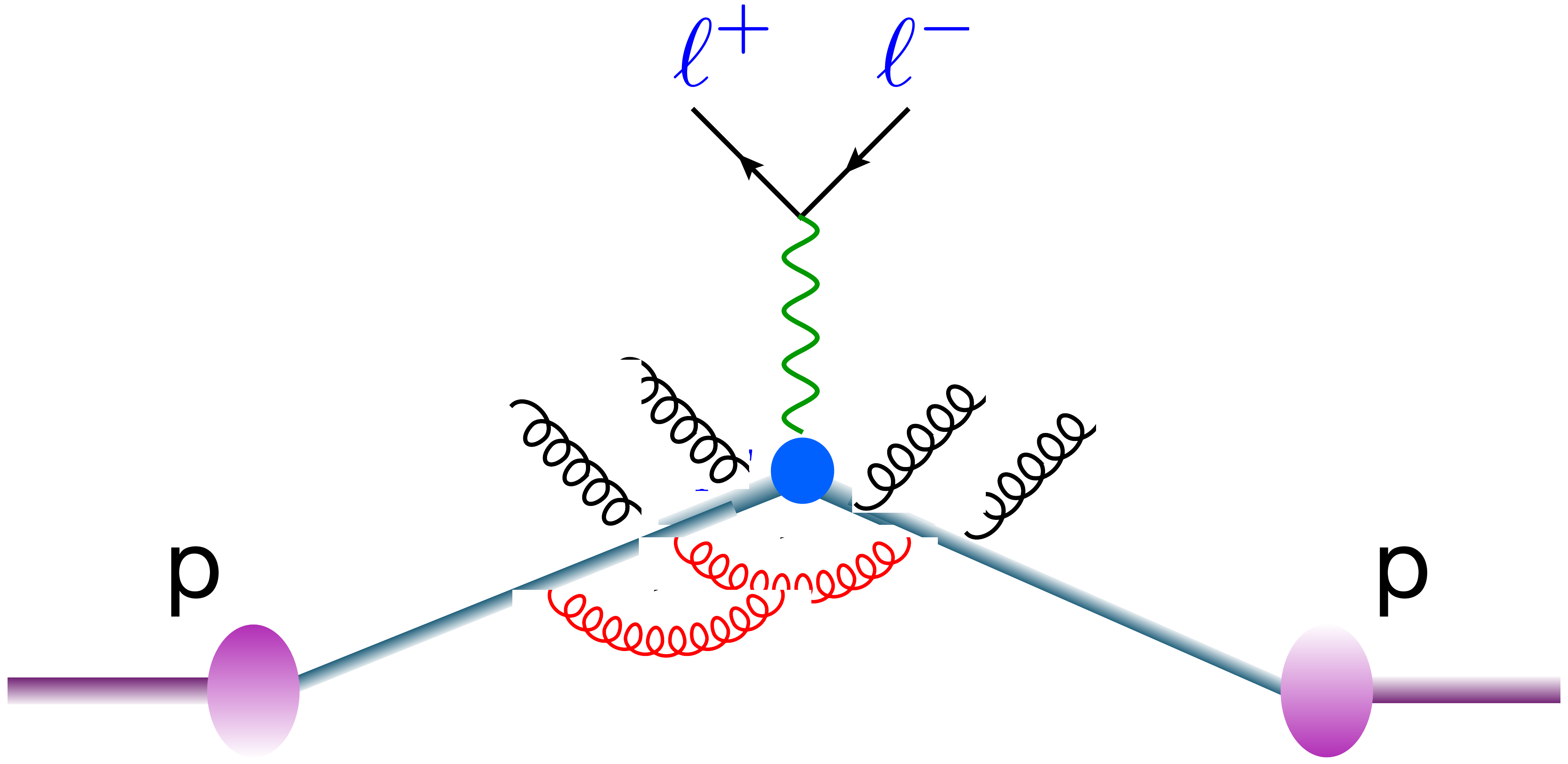

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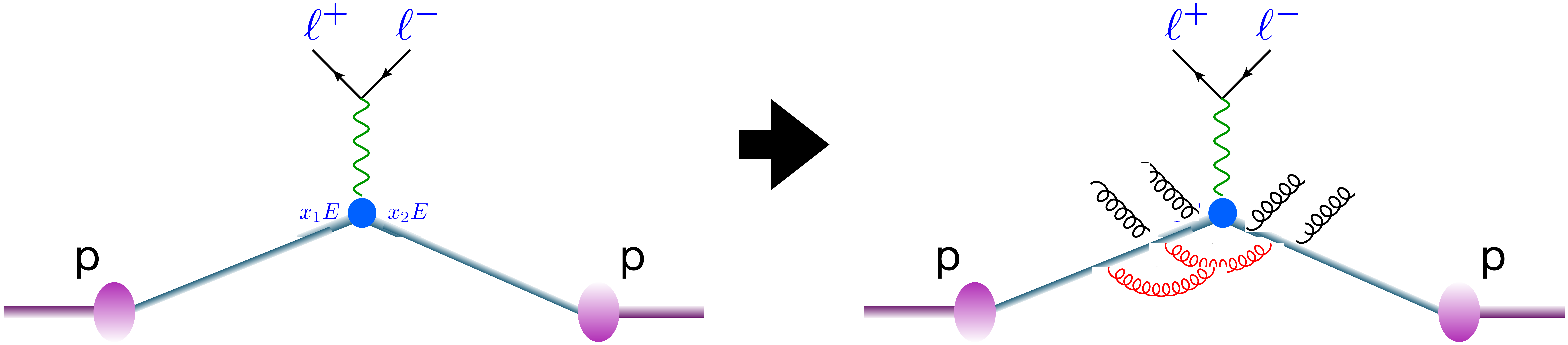
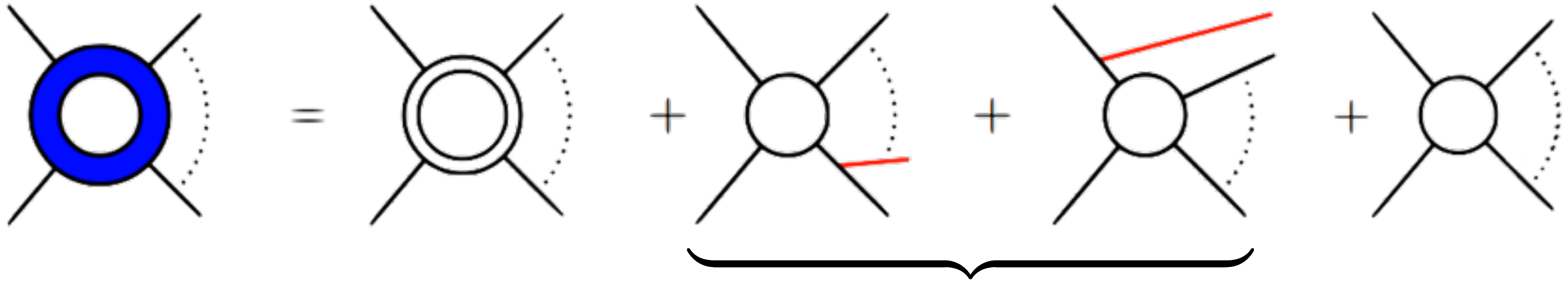


We need to add real and virtual corrections to the hard scattering dealing with singularities

Relatively straightforward at NLO (automated), complicated at NNLO (tens of processes), extremely hard at NNNLO (handful of processes known)

Structure of an NLO calculation



$$\sigma^{\rm NLO} = \int_m d^{(d)}\sigma^V + \int_{m+1} d^{(d)}\sigma^R + \int_m d^{(4)}\sigma^B$$

Virtual part — Real emission part — Born

Difficulties:

- Loop calculations: tough and time consuming
- Divergences: Both real and virtual corrections are divergent
- More channels, more phase space integrations

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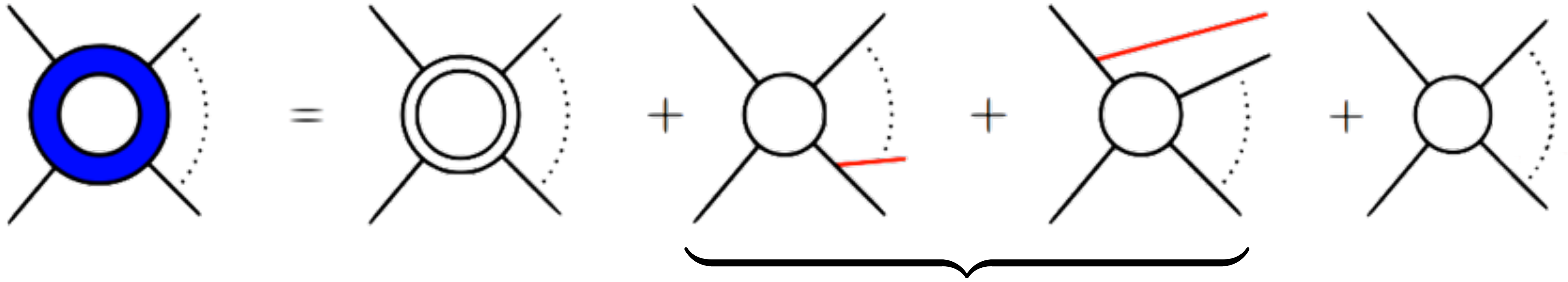

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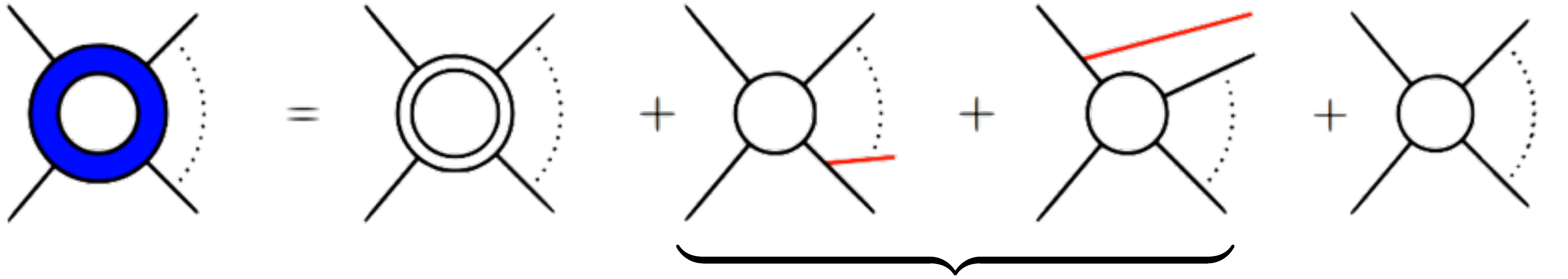

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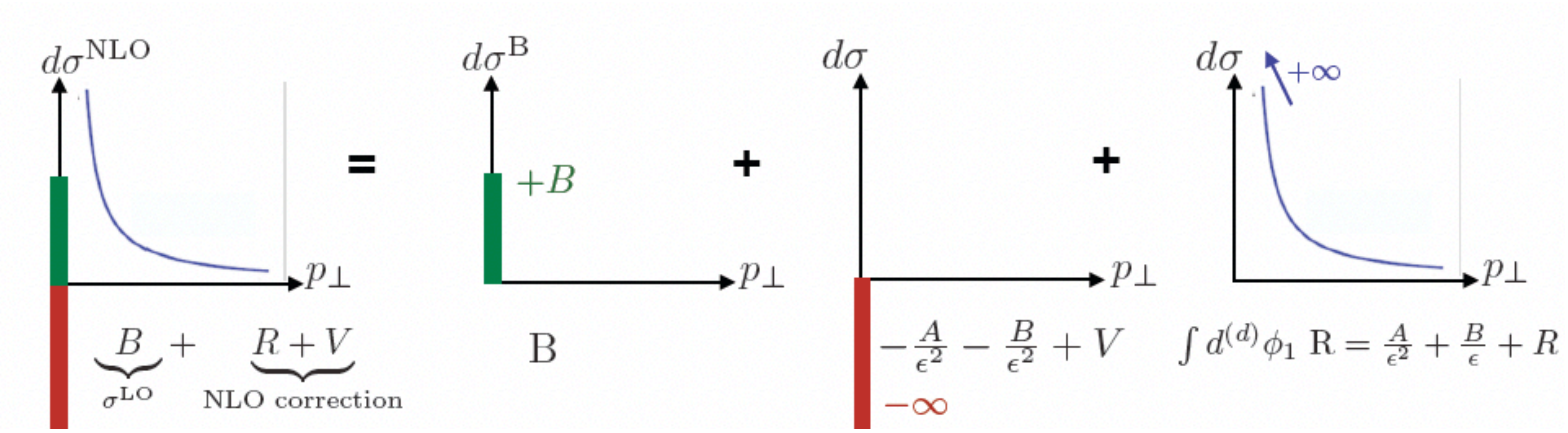
How to deal with NLO in practice?

NLO corrections involve divergences: Divergences are bad for numerical computations



$$\sigma_{\rm NLO} = \int d\Phi^{(n)} \mathcal{B} + \int d\Phi^{(n)} \mathcal{V} + \int d\Phi^{(n+1)} \mathcal{R}$$

Subtraction:

$$= \int d\Phi^{(n)} \mathcal{B} + \int d\Phi^{(n)} \left[\mathcal{V} + \int d\Phi^{(1)} S\right] + \int d\Phi^{(n+1)} \left[\mathcal{R} - S\right]$$

finite   finite

Subtraction techniques at NLO

Dipole subtraction

- Catani, Seymour hep-ph/9602277
- Automated in MadDipole, Sherpa, HELAC-NLO

FKS subtraction

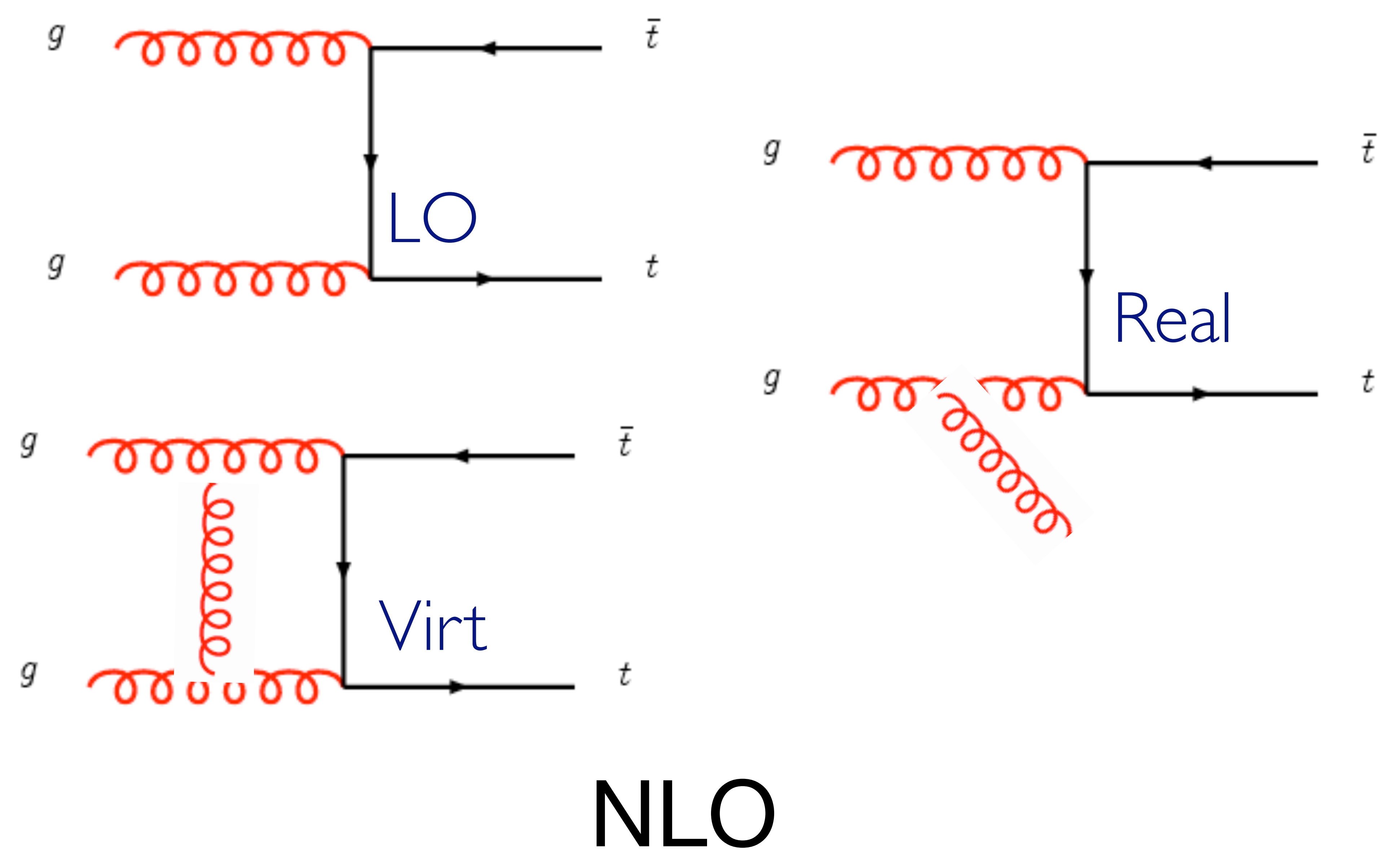
- Frixione, Kunszt, Signer hep-ph/9512328
- Automated in MadGraph5_aMC@NLO and Powheg/Powhel

Detailed discussion of these could be another lecture course!

A note about NLO

NLO is relative

Example: top pair production



Which observables do we compute at NLO?

Total cross-section

pT of a top quark

pT of top pair

pT of hardest jet

tt invariant mass

It is certain observables which are computed at NLO

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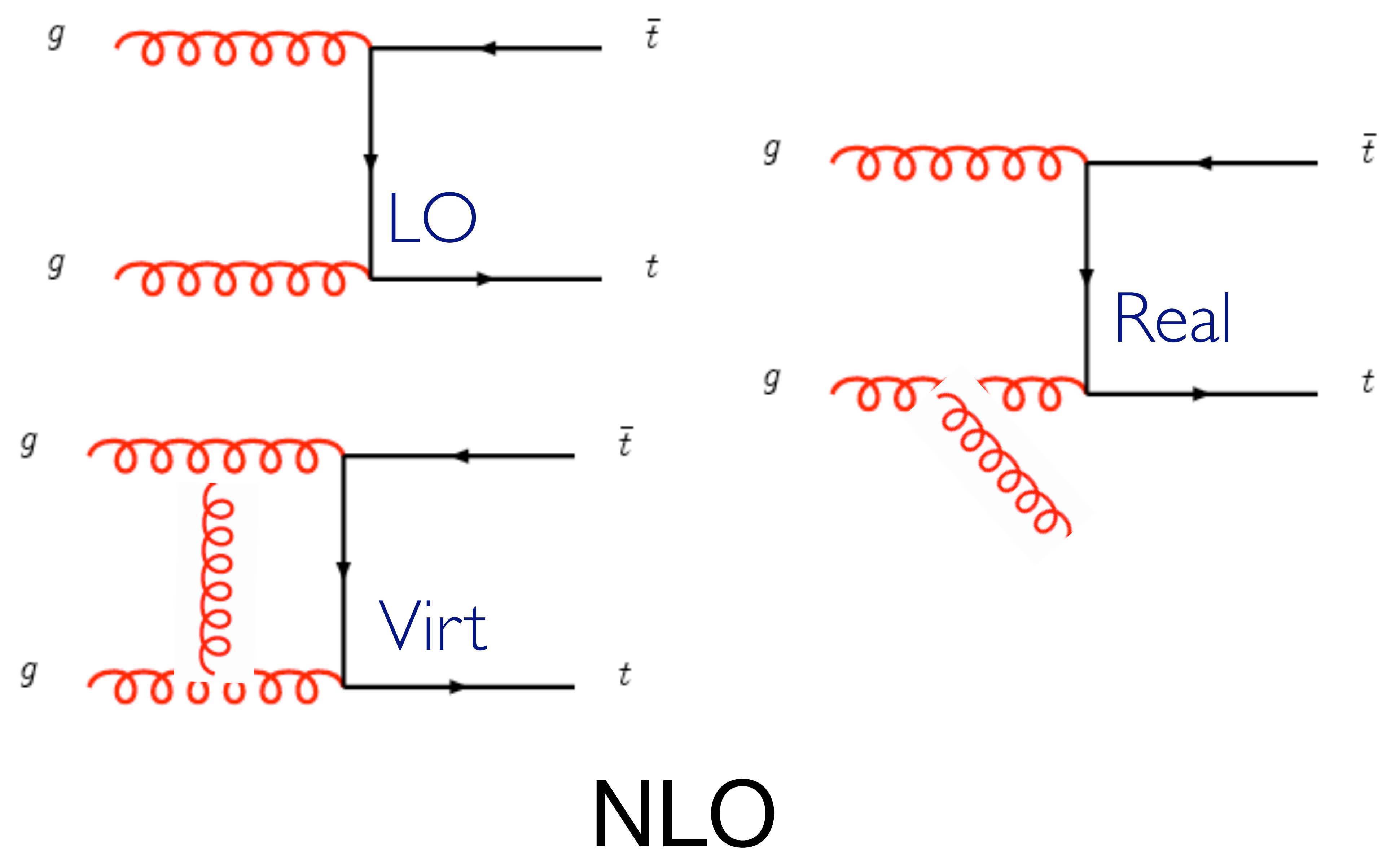

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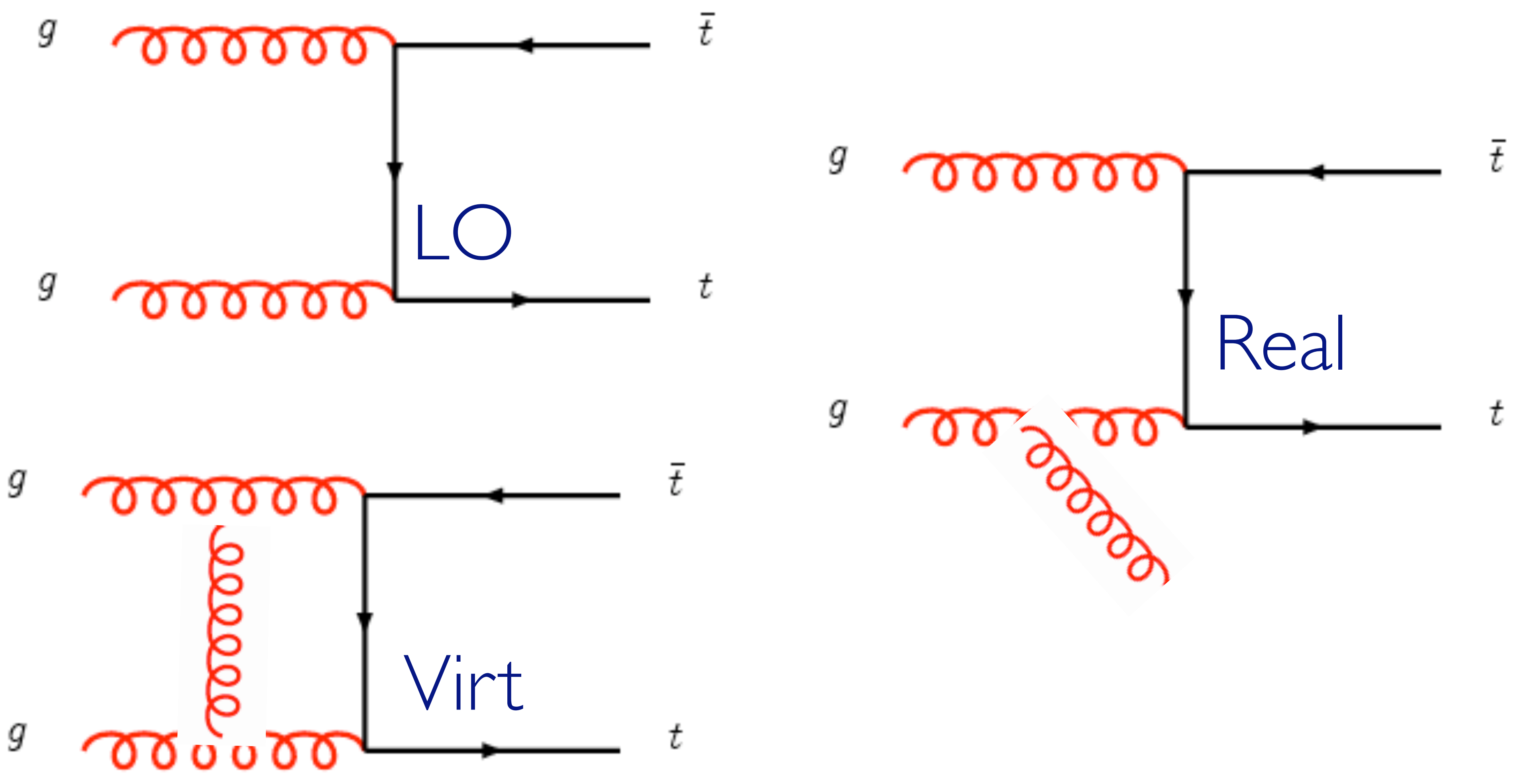

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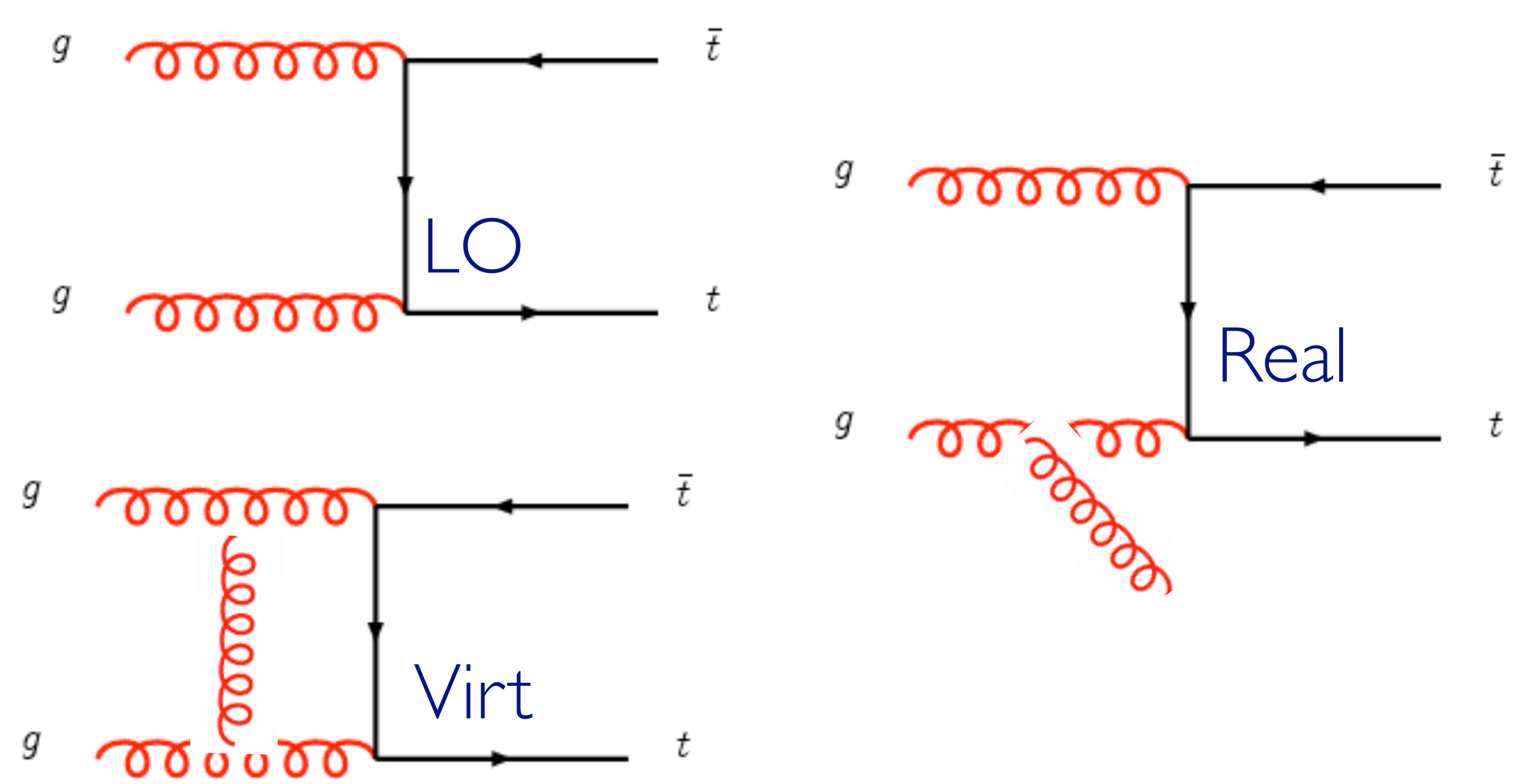

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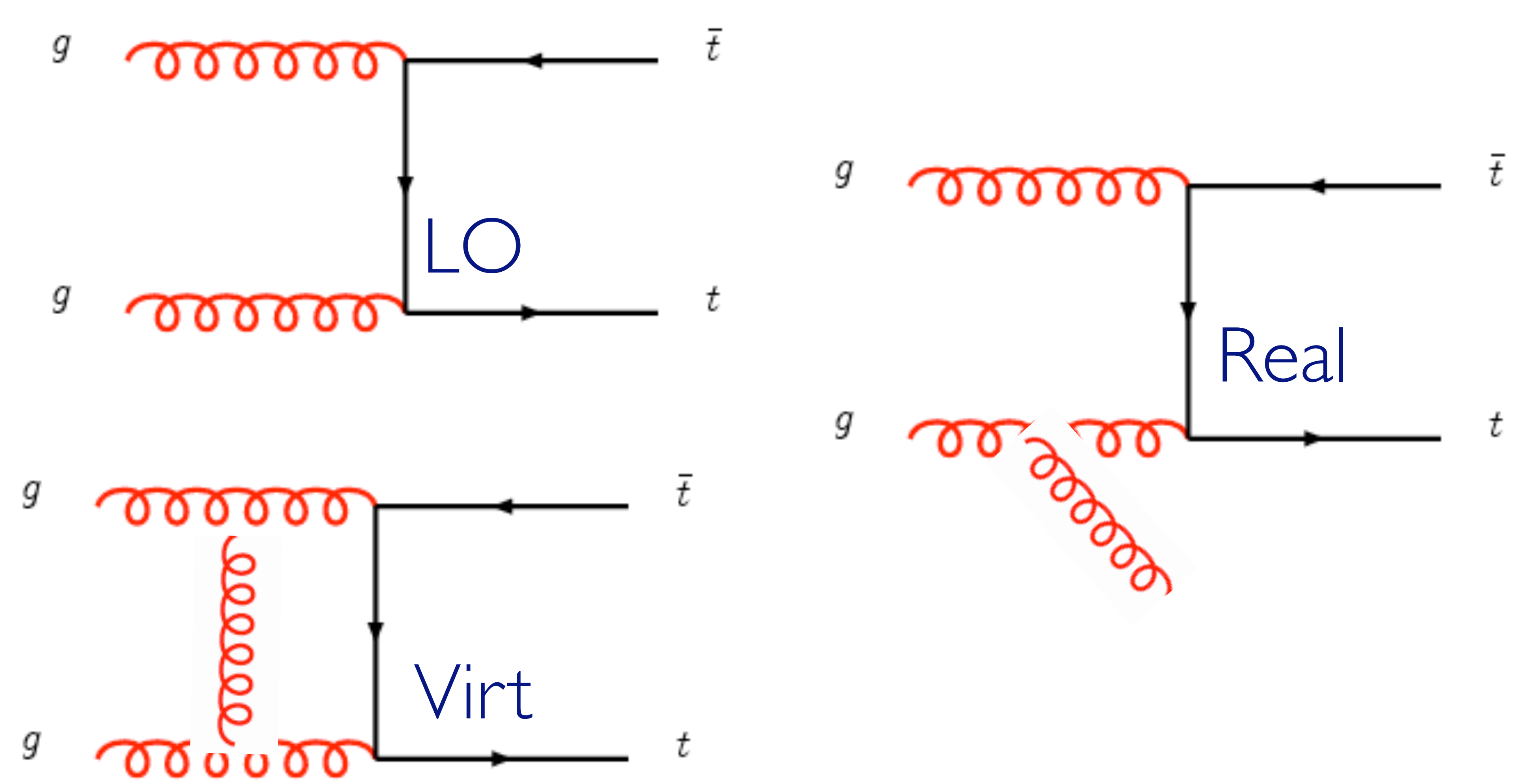

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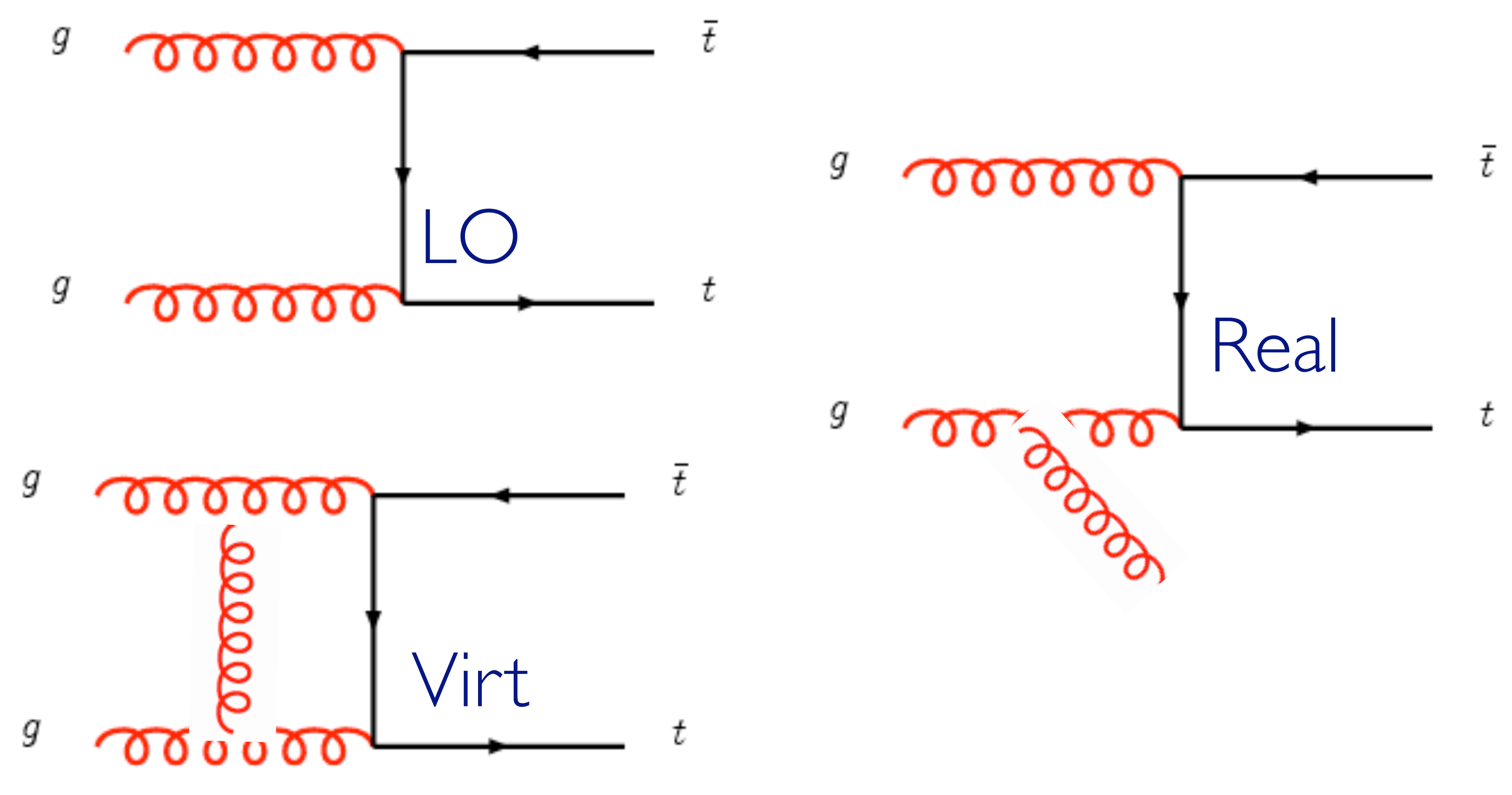


NLO

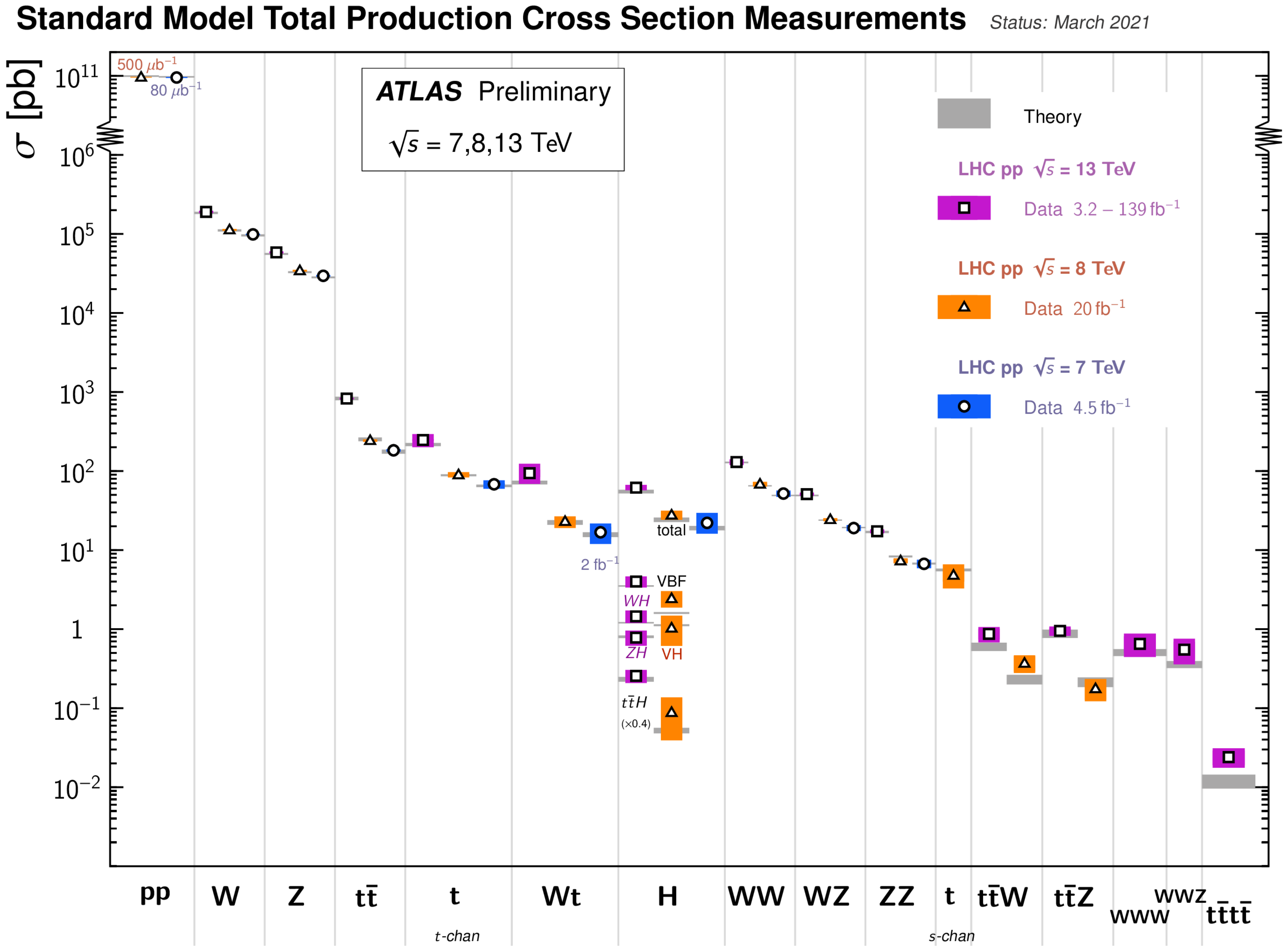
Which observables do we compute at NLO?

- Total cross-section ✓
- pT of a top quark ✓
- pT of top pair ✗
- pT of hardest jet ✗
- tt invariant mass ✓

It is certain observables which are computed at NLO

Need for higher-orders

Why is this so important?

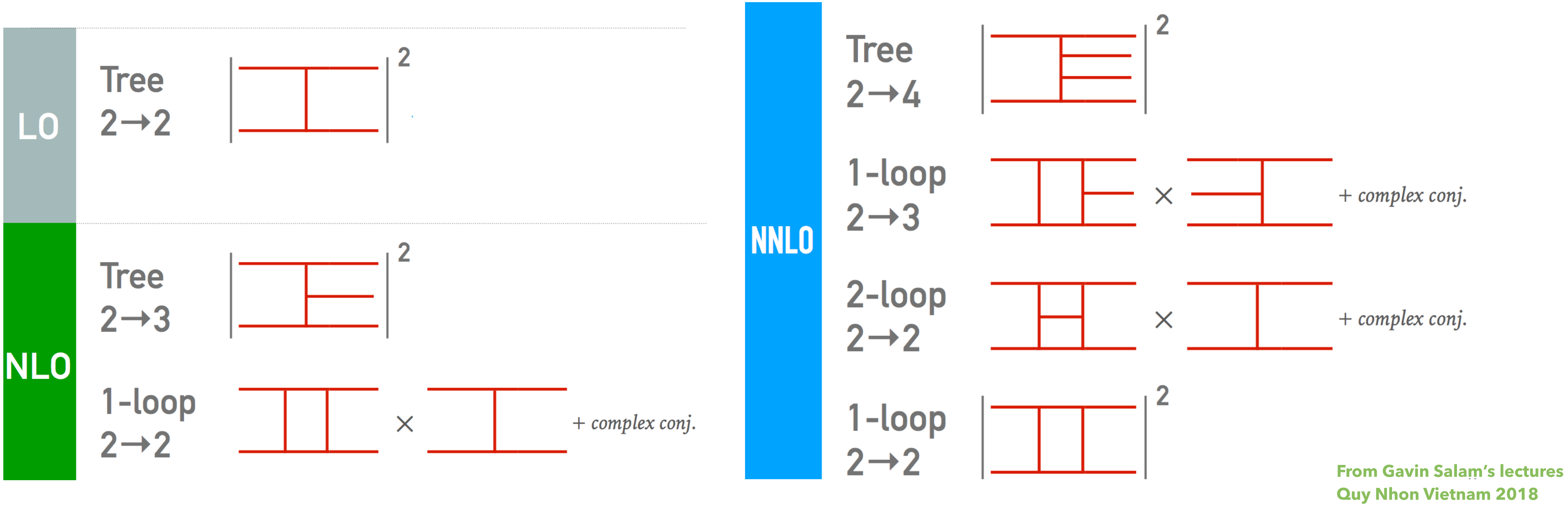


Reminder:

Level of experimental precision demands precise theoretical predictions

Theorists are not simply having fun!!!

Higher order computations



Complexity rises a lot with each N!

Status of hard scattering cross-sections

LO automated

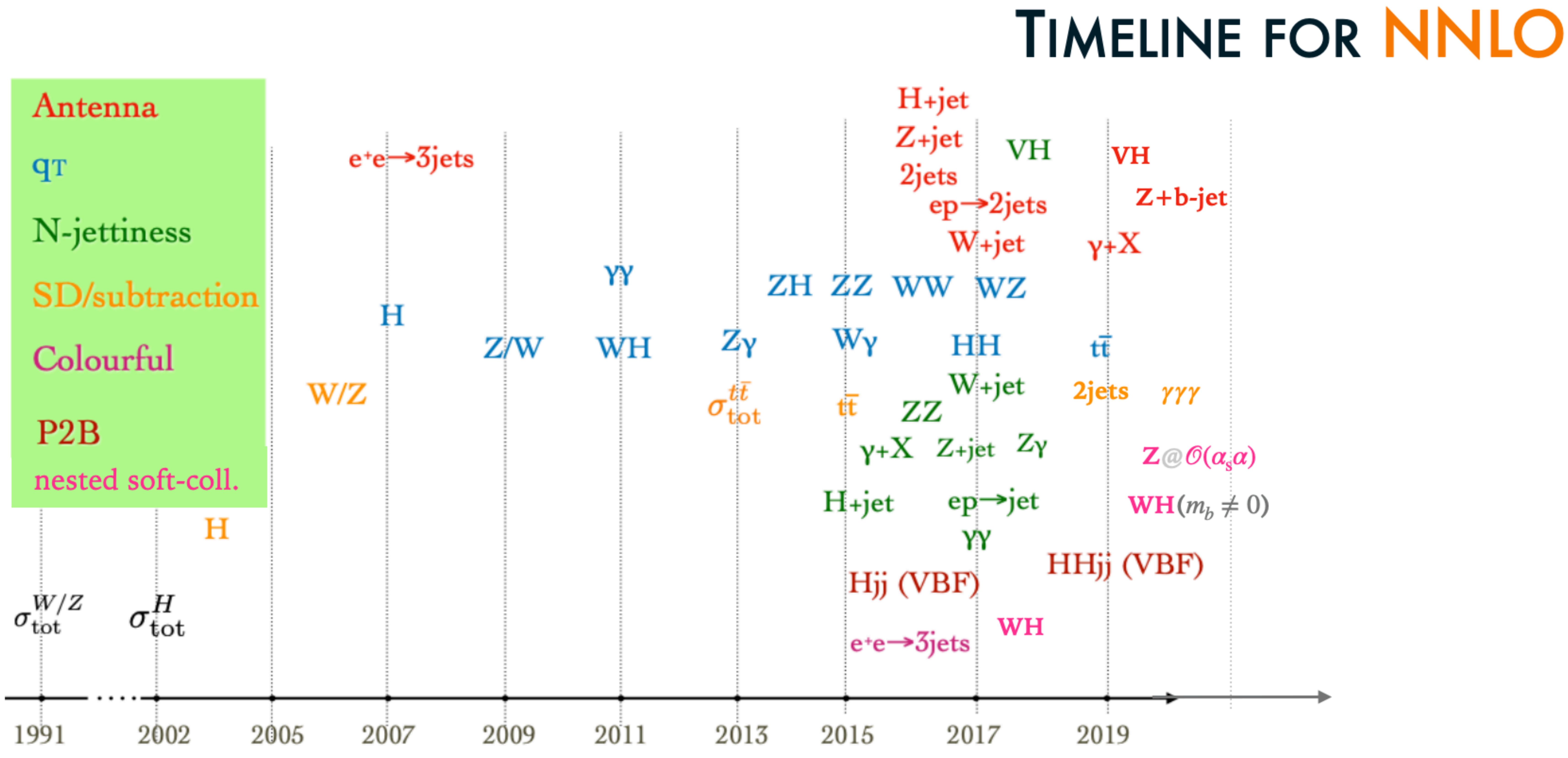
NLO automated

NNLO: Several processes known (VV production, top pair production, all $2 \to 1$ processes)

NNNLO: only a handful of processes!

- Higgs in gluon fusion (Anastasiou et al, arXiv:1602.00695)
- Higgs in VBF (Dreyer et al, arXiv:1811.07906)
- Higgs in bottom annihilation (Duhr et al, arXiv:1904.09990)
- Drell-Yan (Duhr et al, arXiv:2001.07717, 2007.13313)

Progress in higher-order computations



A. Huss, QCD@LHC-X 2020

Hard scattering cross-section

Perturbative expansion

$$\hat{\sigma} = \sigma^{\mathrm{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

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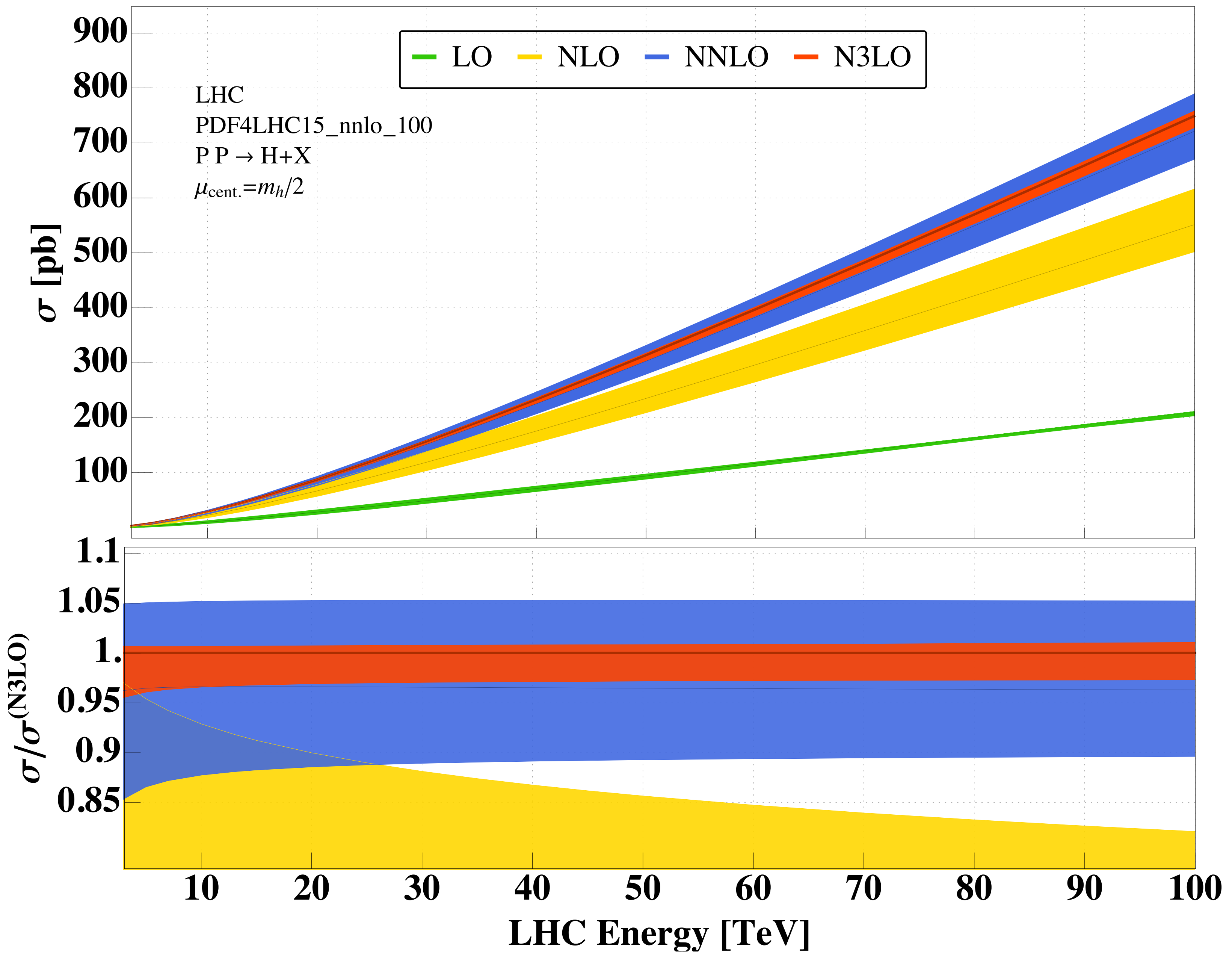
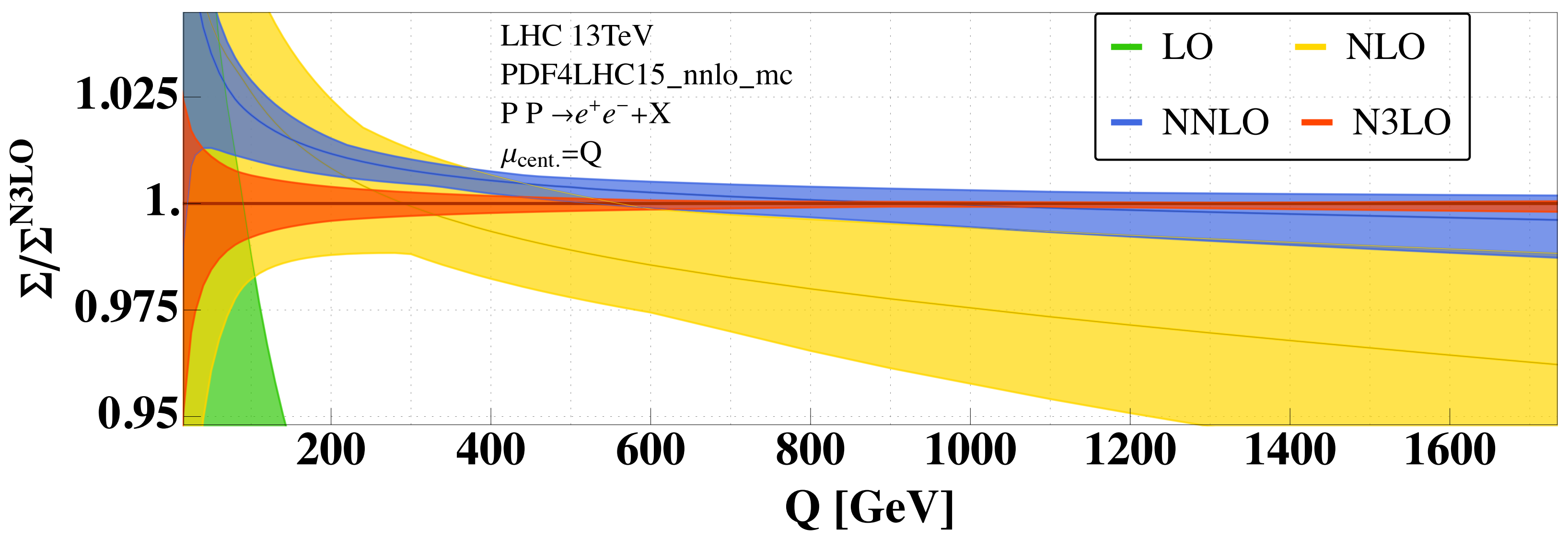


Improved accuracy and precision



Dilepton production

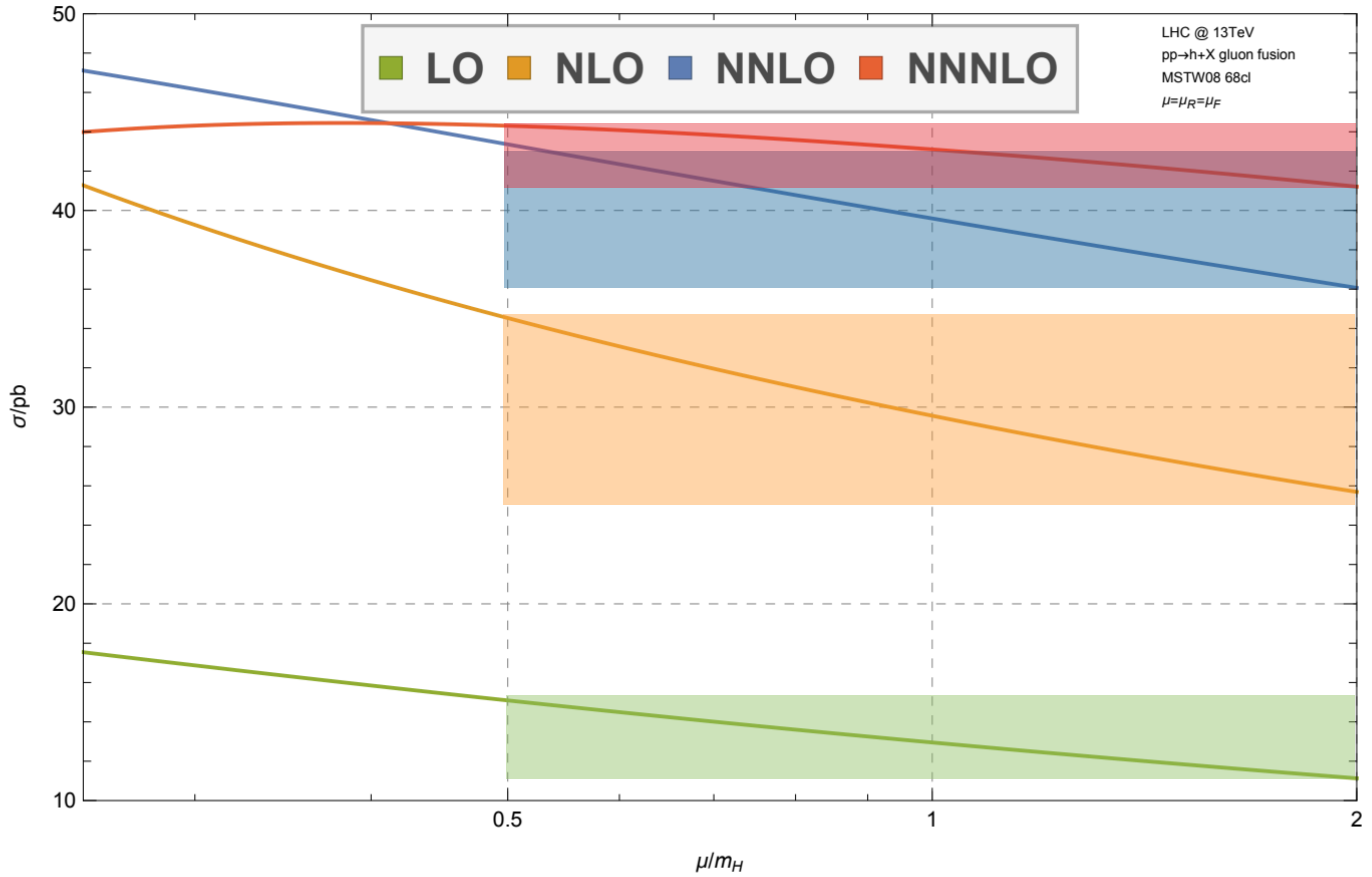
Higgs production arXiv:2203.06730

Uncertainties in theory predictions

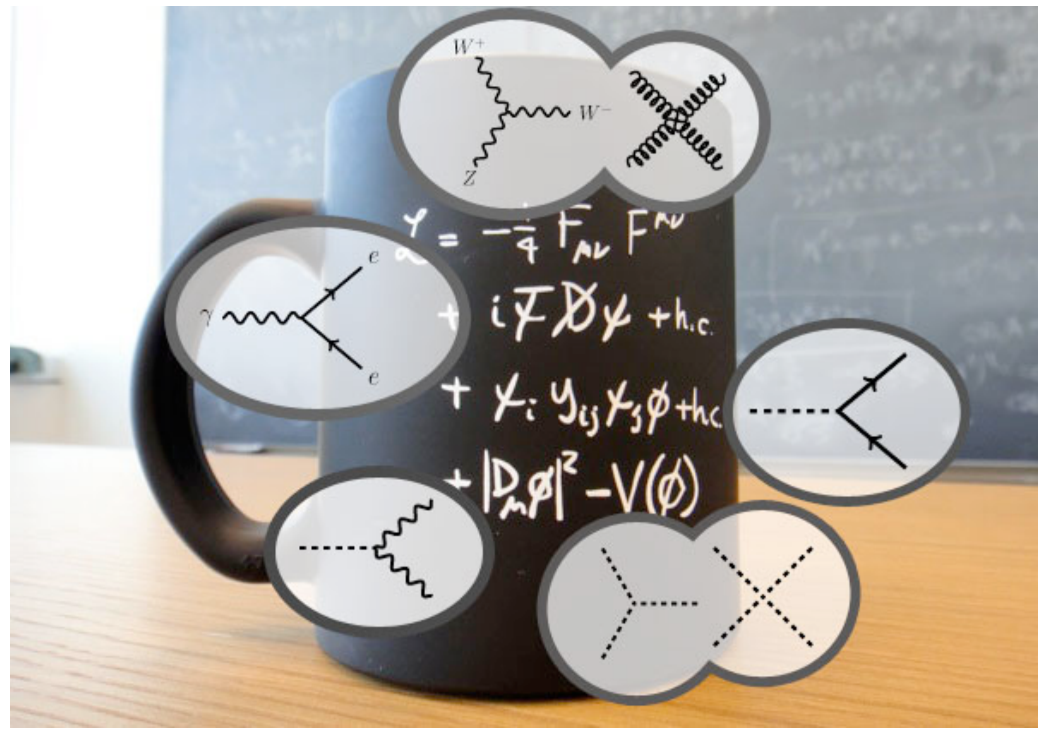
How do we estimate uncertainties?

Vary the renormalisation and factorisation scale

Typically pick some central scale μ_0 and vary the scale up and down by a factor of 2



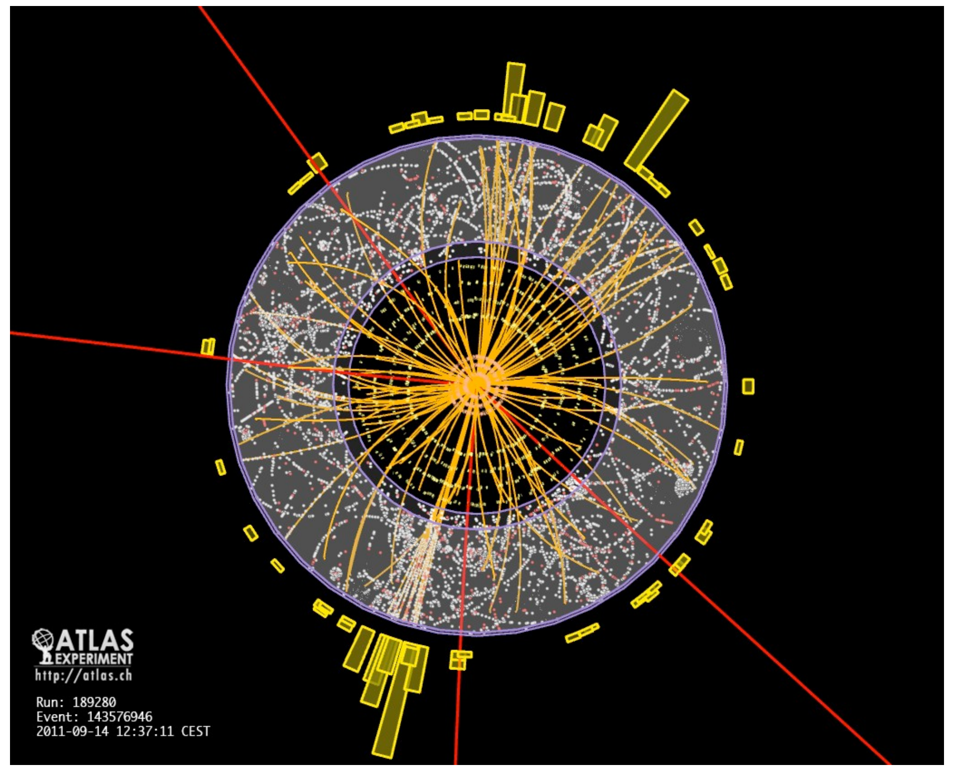
How do we actually compute all of these?



Theory

Monte Carlo

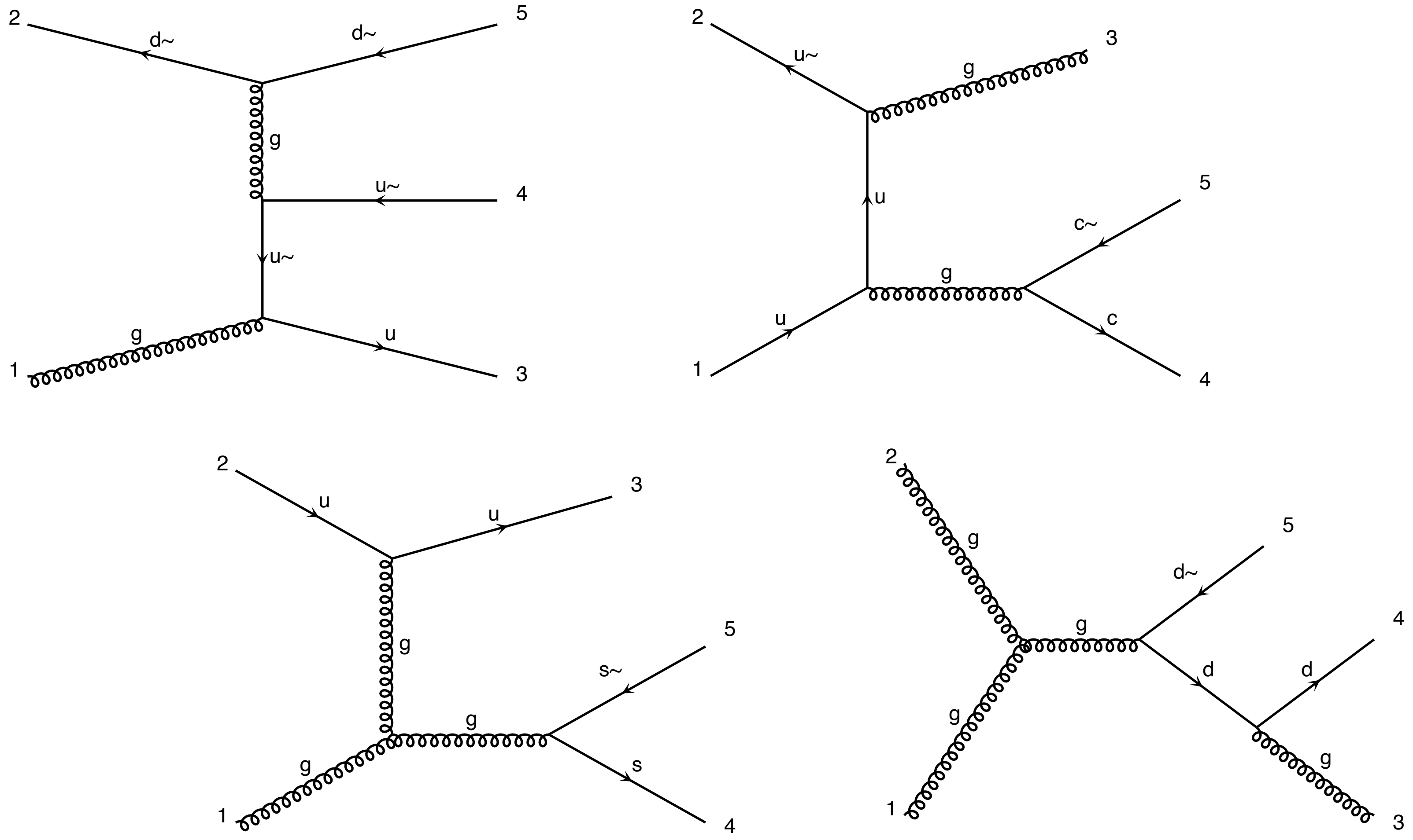
Experiment



Focusing on LO

How to compute a LO cross-section

Example: 3 jet production in pp collisions



1. Know the Feynman rules (SM or BSM)
2. Find all possible Subprocesses

97 processes with 781 diagrams generated in 2.994 s

Total: 97 processes with 781 diagrams

3. Compute the amplitude
4. Compute $|M|^2$ for each subprocess, sum over spin and colour
5. Integrate over the phase space

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

LO calculation of a cross-section

How many subprocesses?

Amplitude computation (Feynman diagrams)

Square the amplitude, sum over spin and colour

Integrate over the phase space

Difficulty

Complexity increases with

- number of particles in the final state
- number of Feynman diagrams for the process (typically organise these in terms of leading couplings: see tutorial)

Structure of an automated MC generator

I. Input Feynman rules
II. Define initial and final state
III. Automatically find all subprocesses
IV. Compute matrix element (including tricks like helicity amplitudes)
V. Integrate over the phase space by optimising the PS parametrisation and sampling
VI. Unweight and write events in the Les Houches format

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Next: Shower+Hadronisation
Detector simulation and reconstruction

Output of LO MC generators

Les houches events

Example: gg>ZZ

```
<event>
 4      0 +1.1211000e+00 1.89058500e+02 7.81859000e-03 1.15931300e-01
       21 -1    0    0  502  501 +0.0000000000e+00 +0.0000000000e+00 +4.6570159241e+01 4.6570159241e+01 0.0000000000e+00 0.0000e+00 1.0000e+00
       21 -1    0    0  501  502 -0.0000000000e+00 -0.0000000000e+00 -1.9187776299e+02 1.9187776299e+02 0.0000000000e+00 0.0000e+00 1.0000e+00
       23  1    1    2    0    0 +1.3441082214e+01 +1.3065682927e+01 -5.1959303141e+01 1.0661295577e+02 9.1187600000e+01 0.0000e+00 1.0000e+00
       23  1    1    2    0    0 -1.3441082214e+01 -1.3065682927e+01 -9.3348300610e+01 1.3183496646e+02 9.1187600000e+01 0.0000e+00 1.0000e+00
</event>
```

PDG — Momenta — Mass

All Information needed to pass to parton shower is included in the event

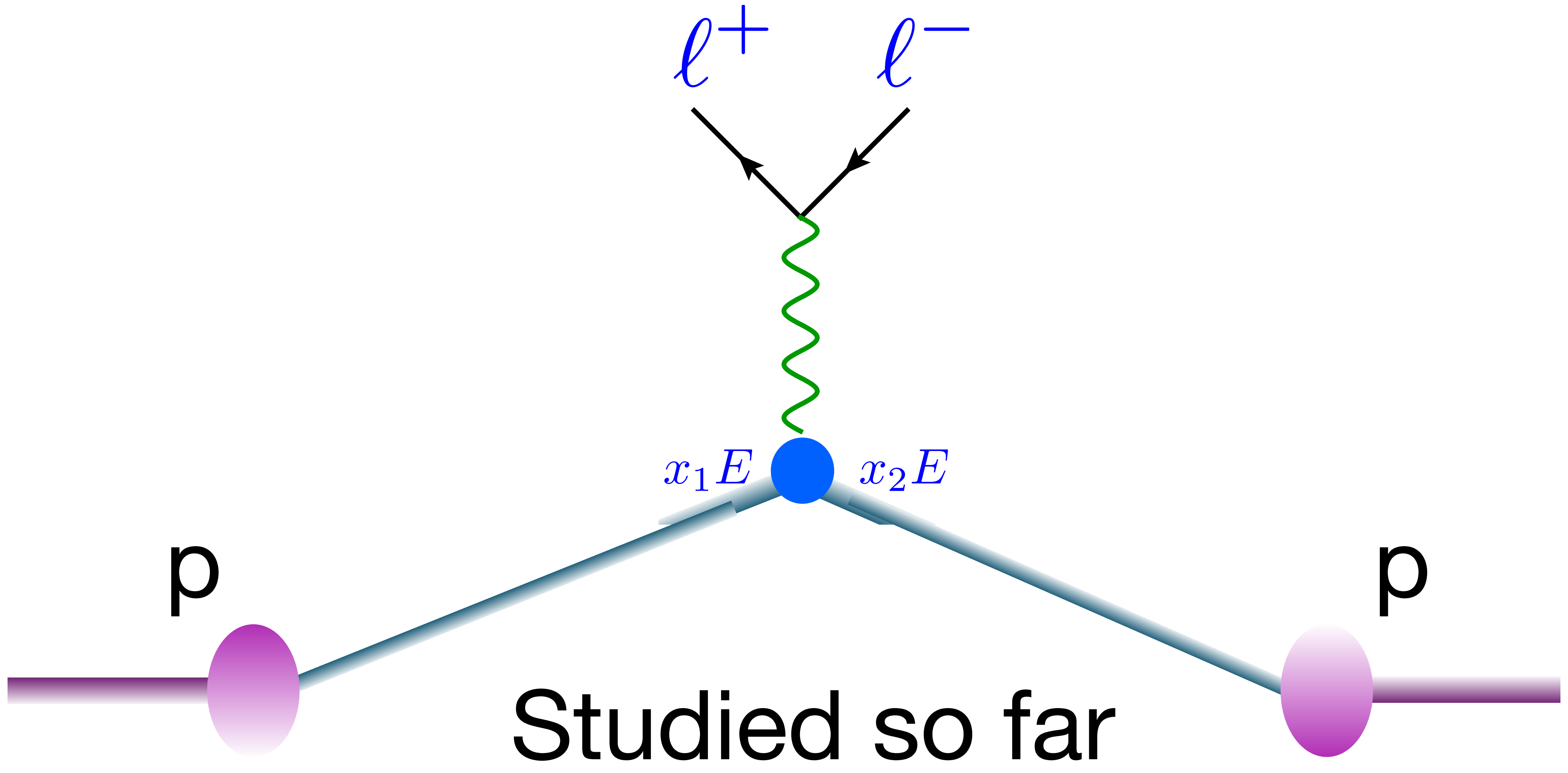
Available public MC generators

Matrix element generators (and integrators):

- MadGraph/MadEvent
- Comix/AMEGIC (part of Sherpa)
- HELAC/PHEGAS
- Whizard
- CalcHEP/CompHEP

Is Fixed Order enough?

Fixed order computations can't give us the full picture of what we see at the LHC



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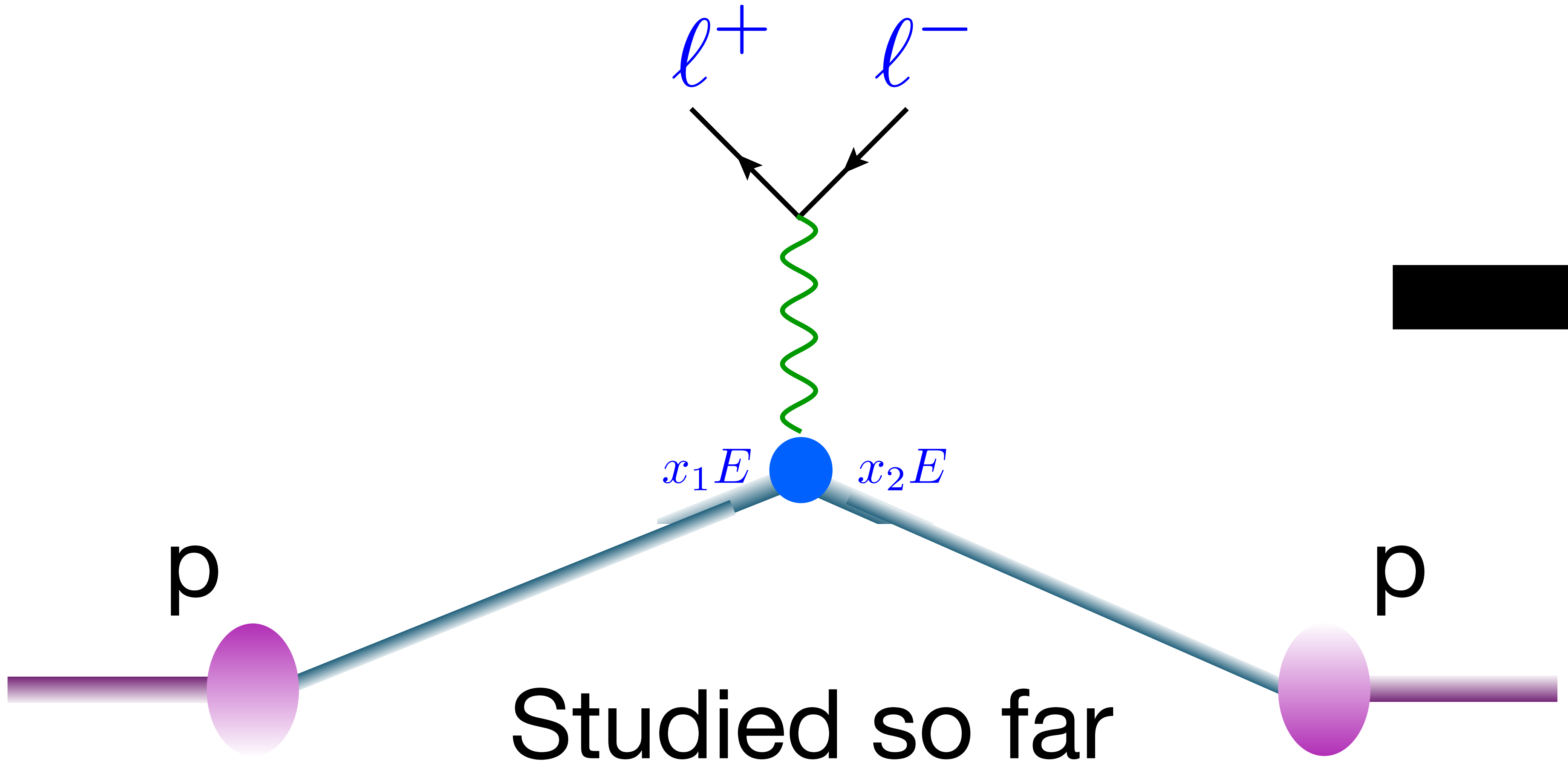
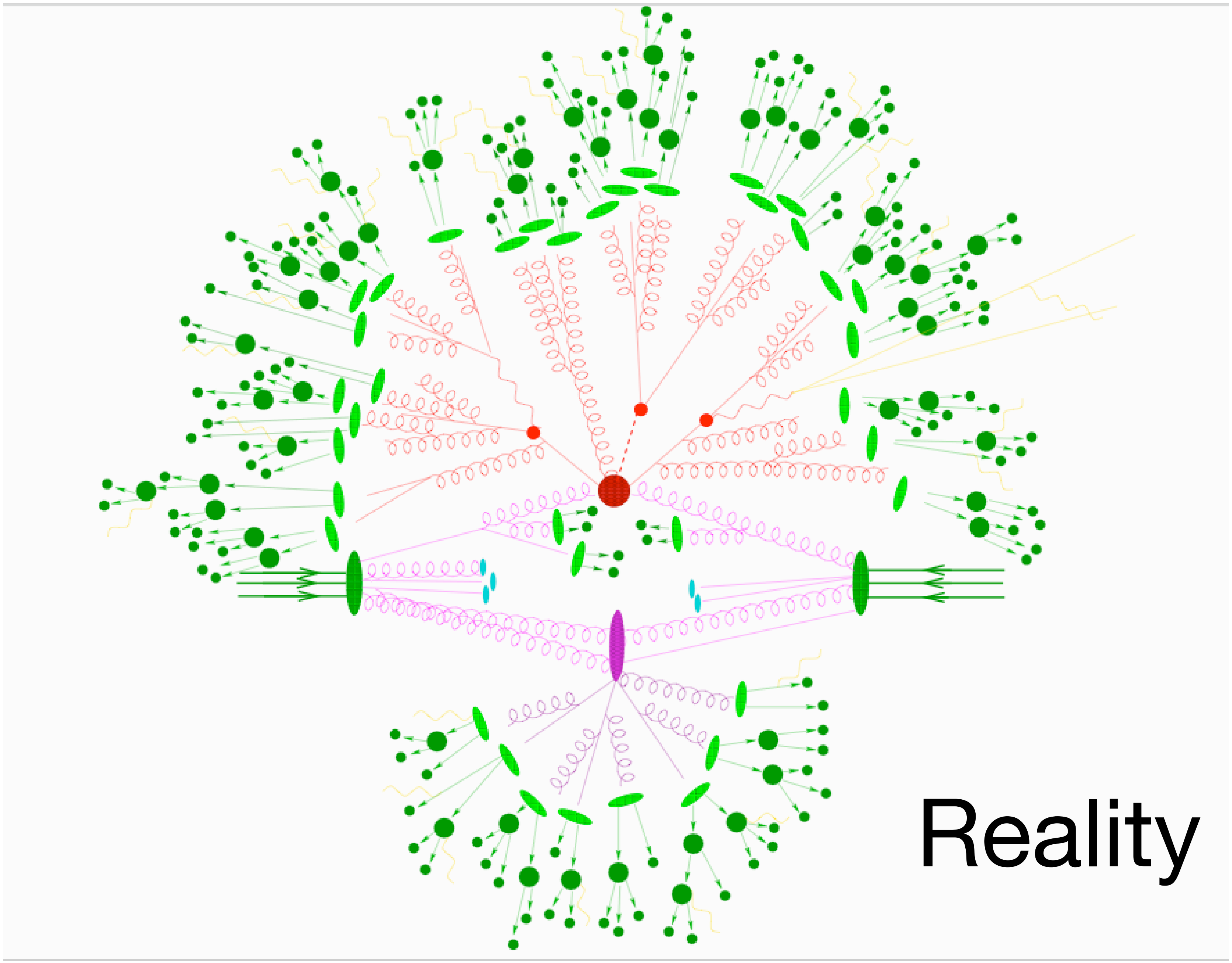
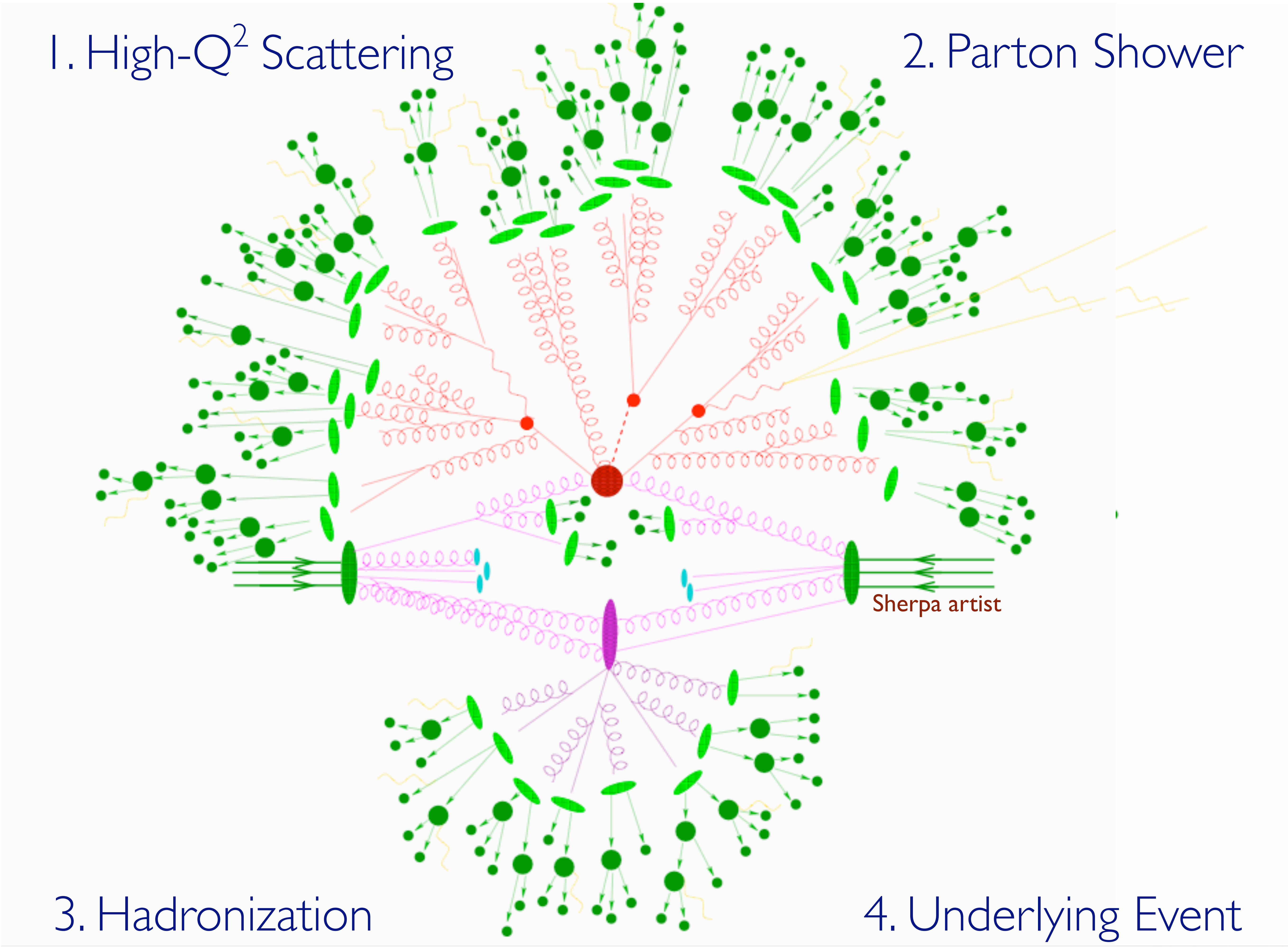




An LHC event


1. High-Q^2 Scattering
2. Parton Shower
Sherpa artist
3. Hadronization
4. Underlying Event

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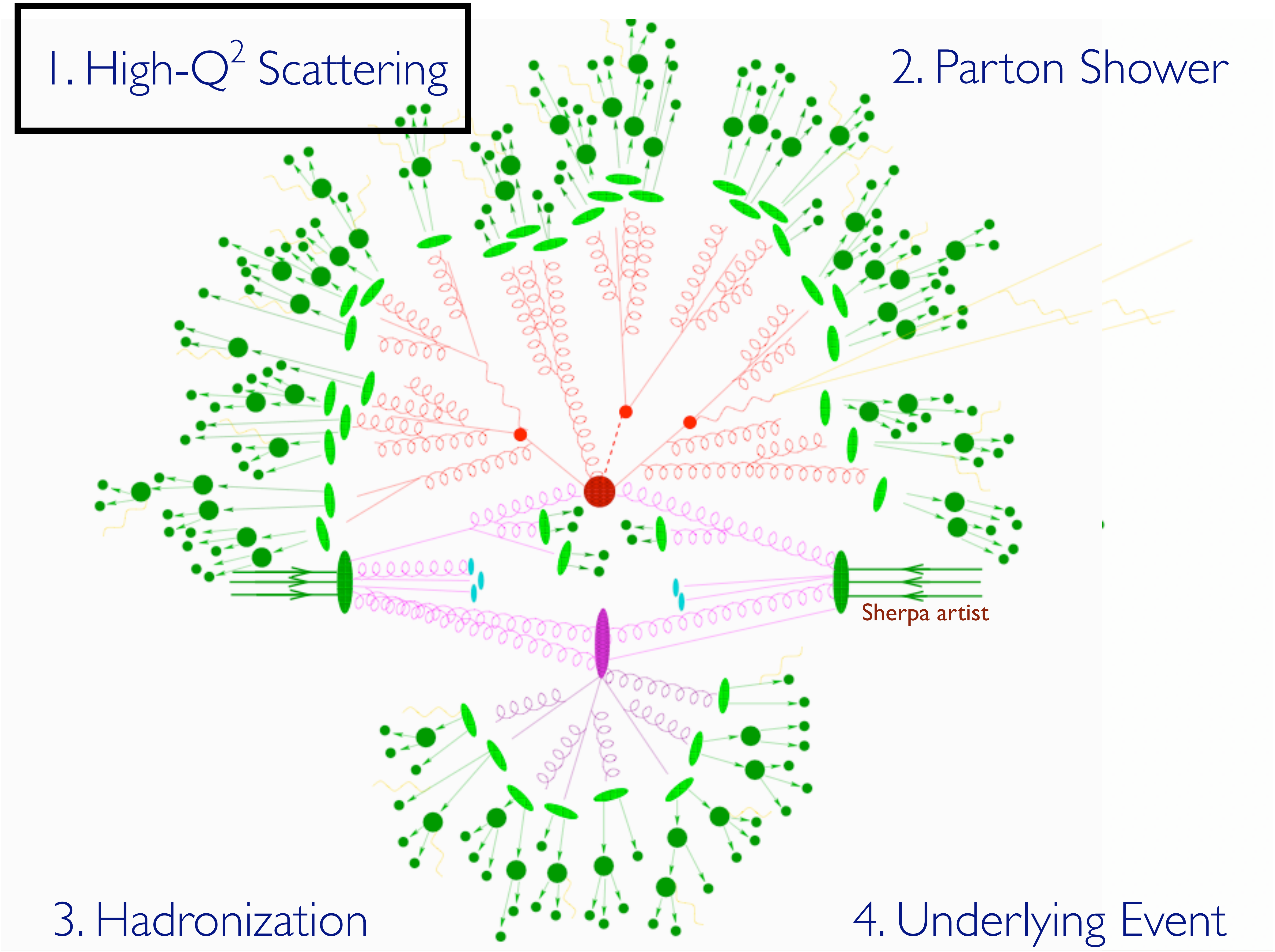
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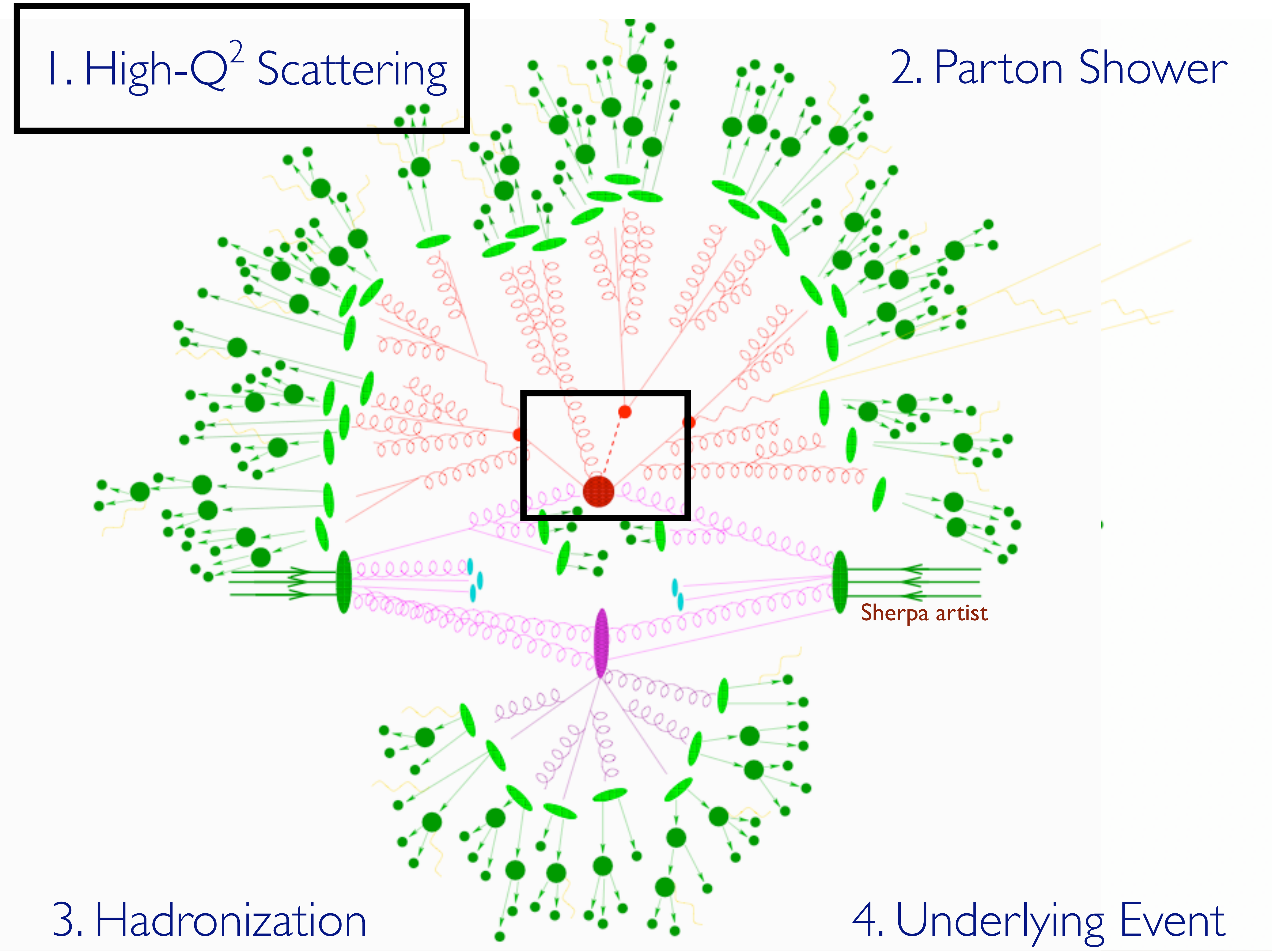

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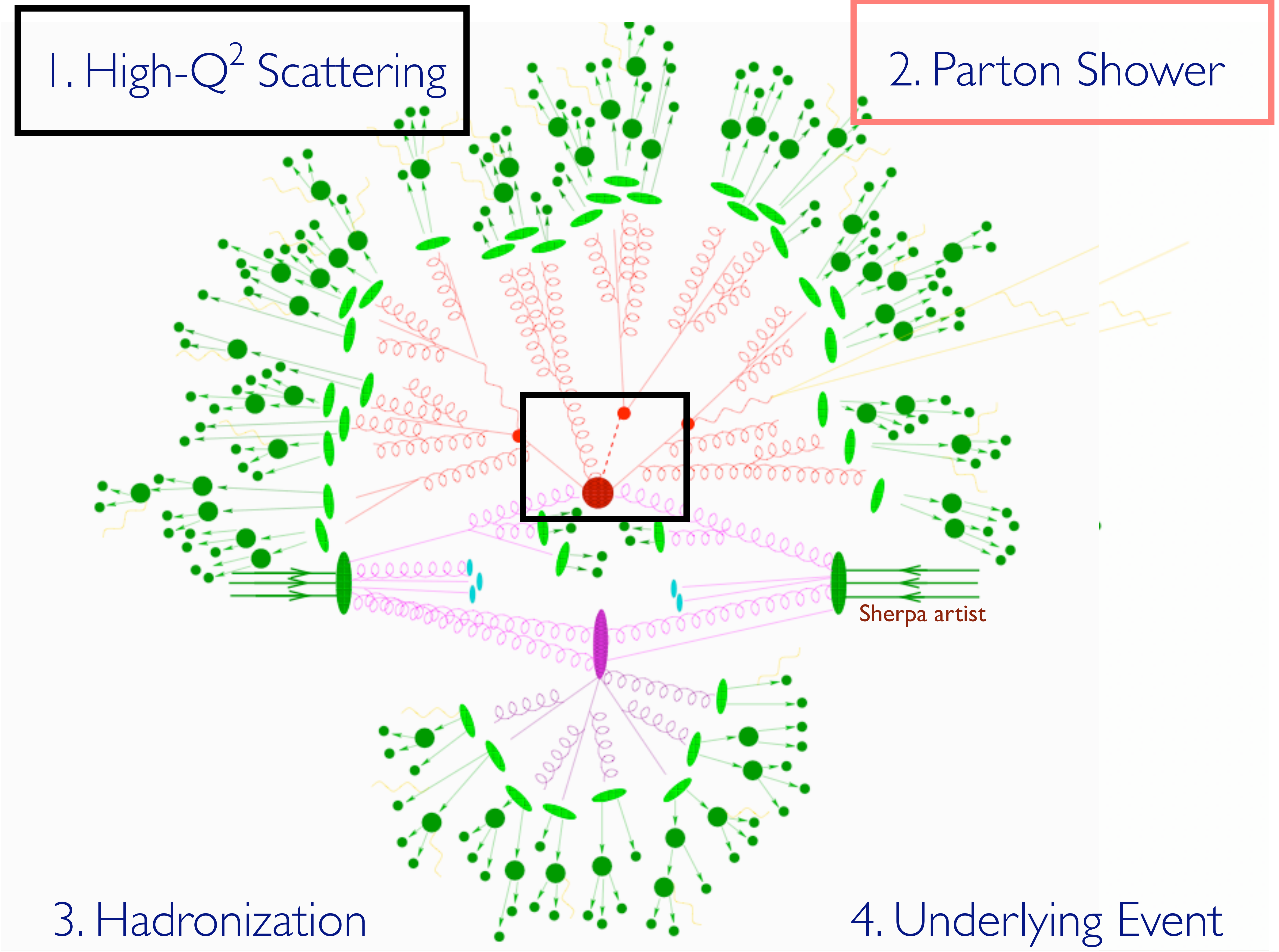
3. Hadronization
4. Underlying Event

An LHC event



1. High-Q2 Scattering
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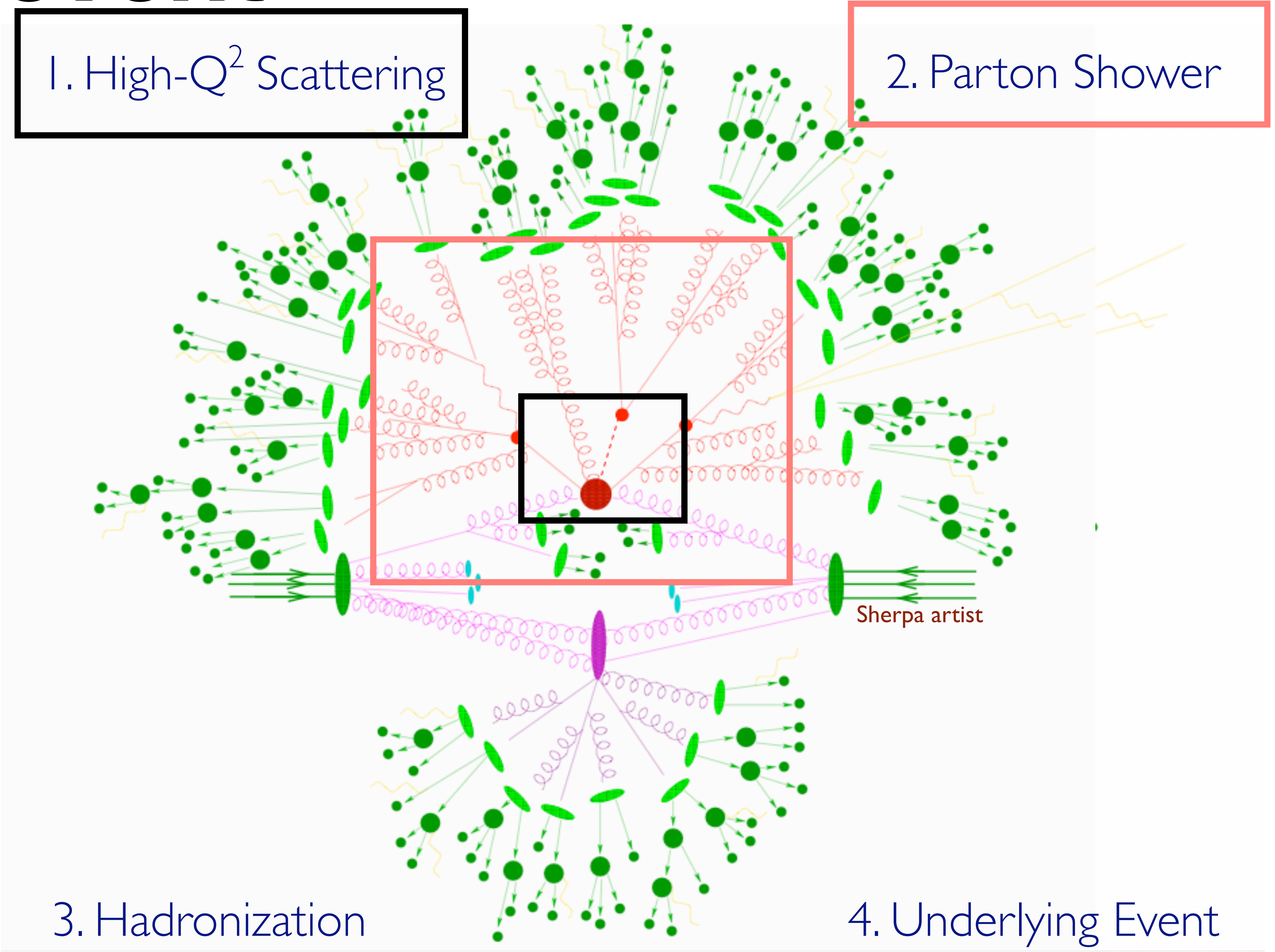
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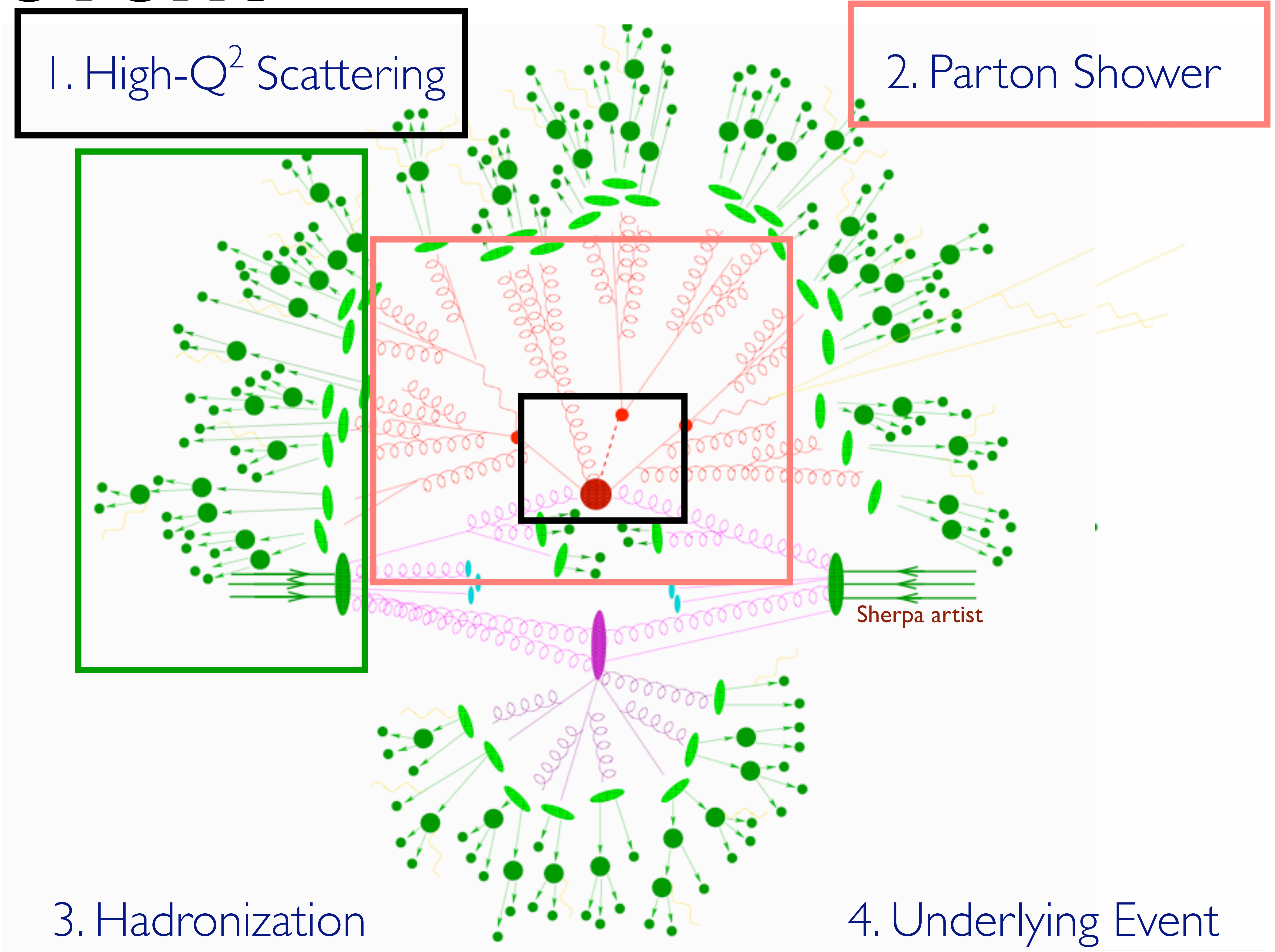
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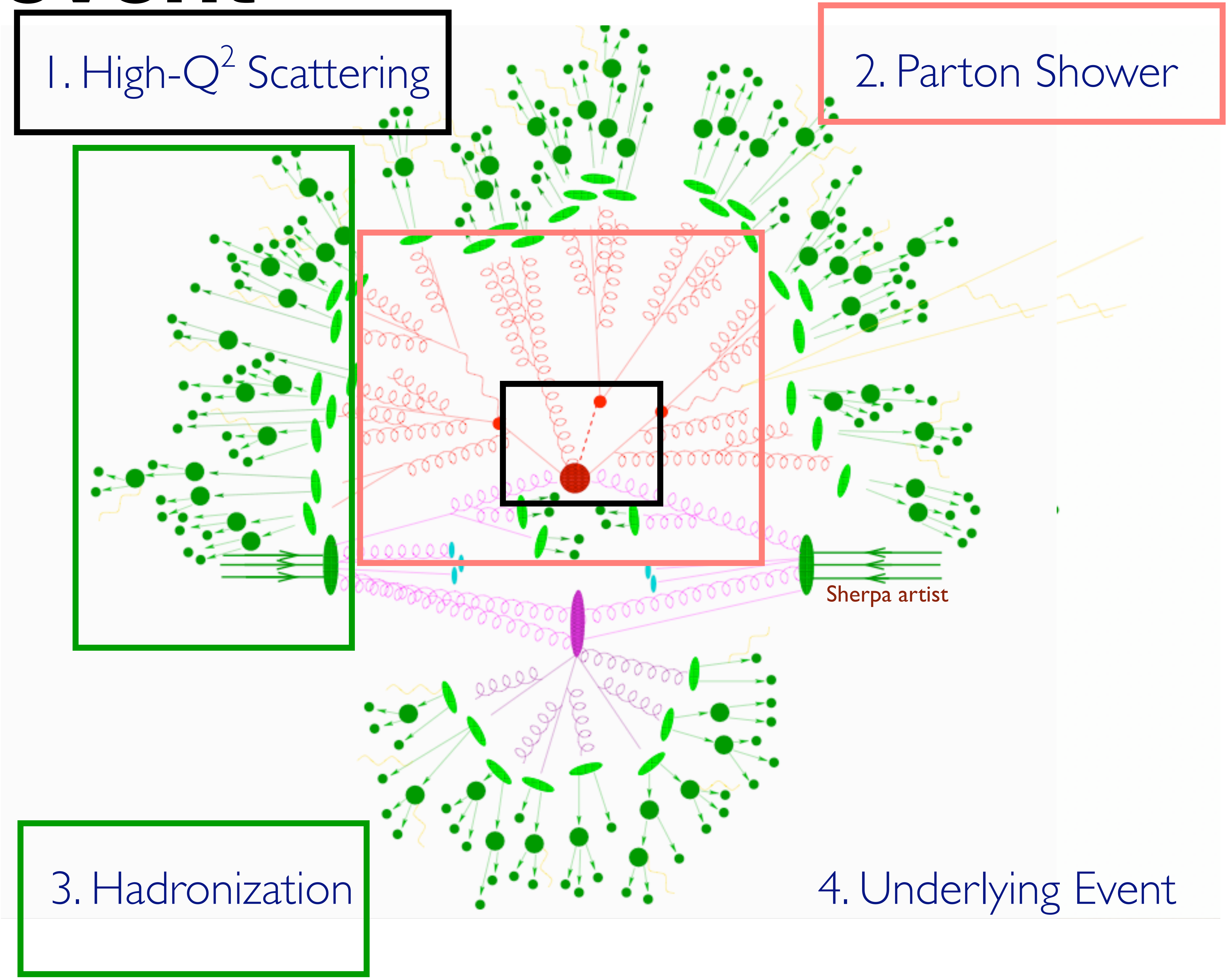

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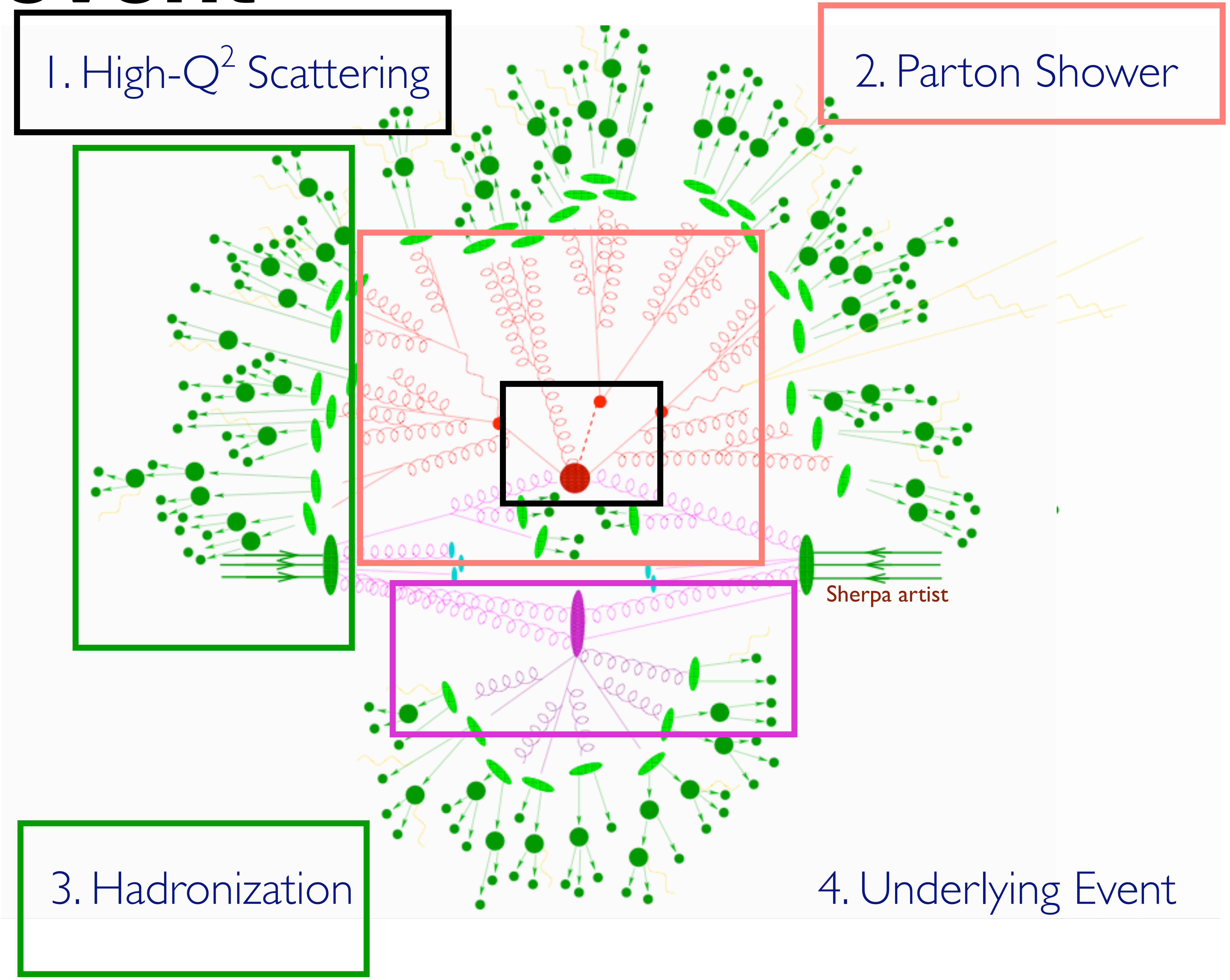
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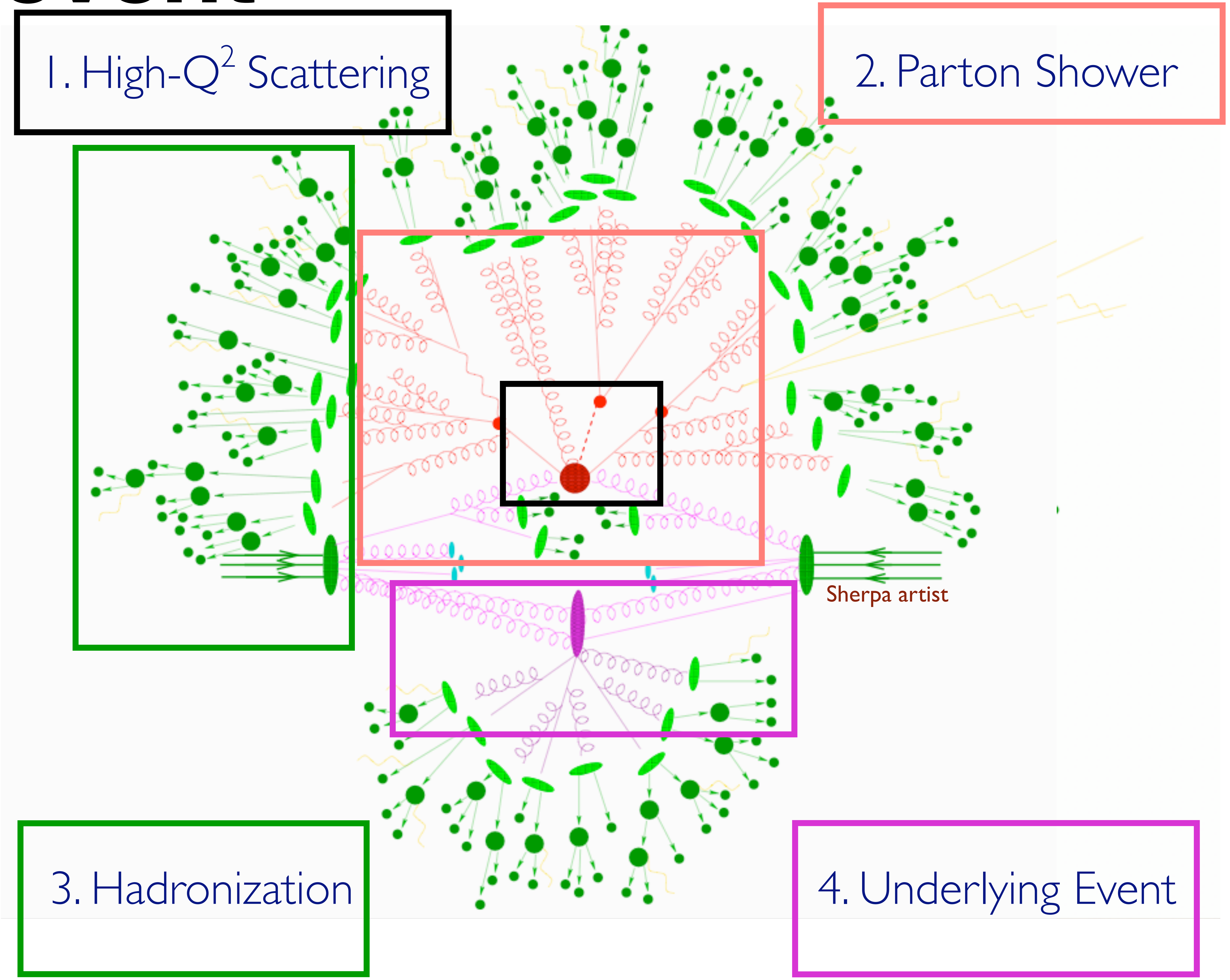

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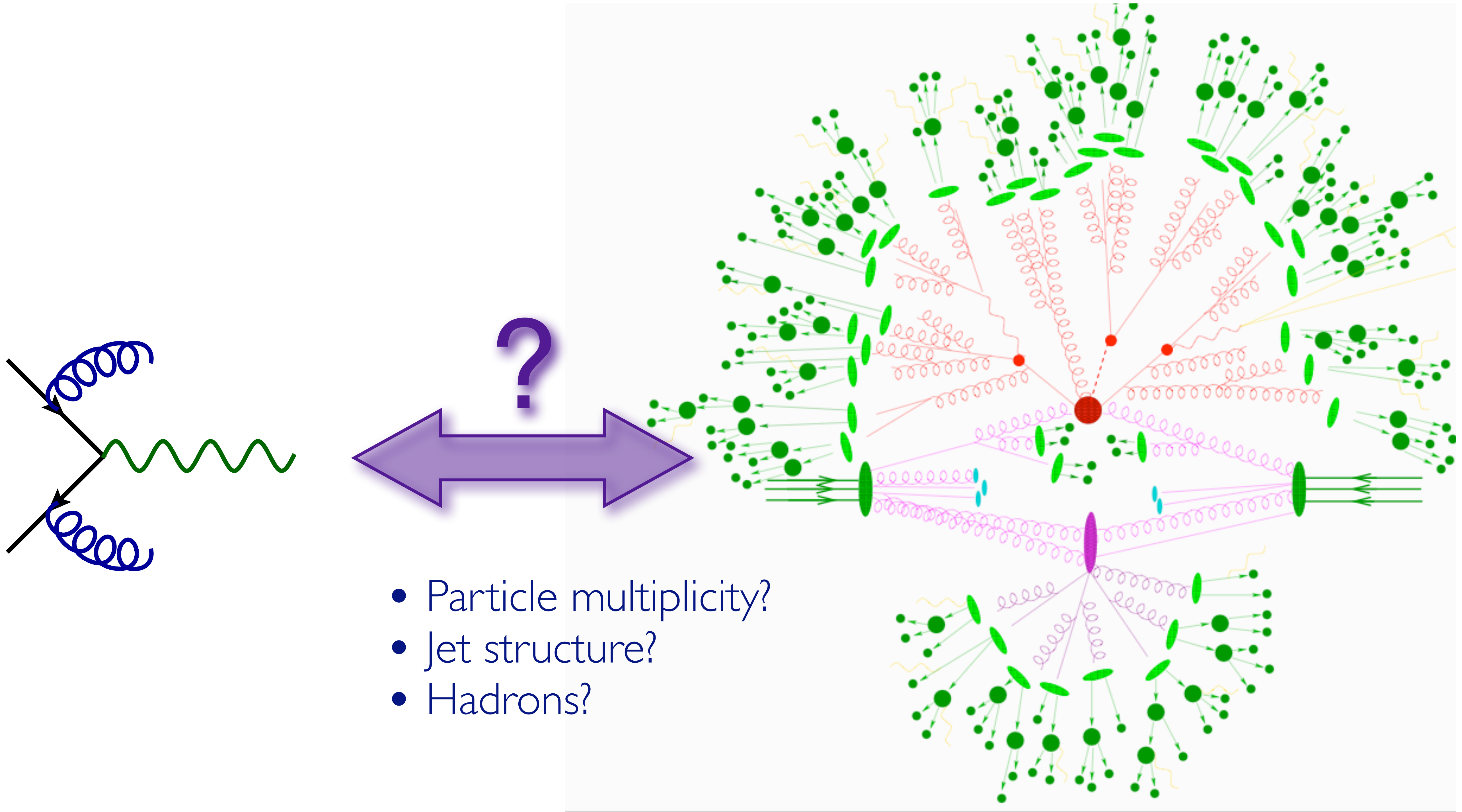
Is fixed order enough?



- Fixed order calculations involve only a few partons
- Not what we see in detectors
- Need Shower and Hadronisation

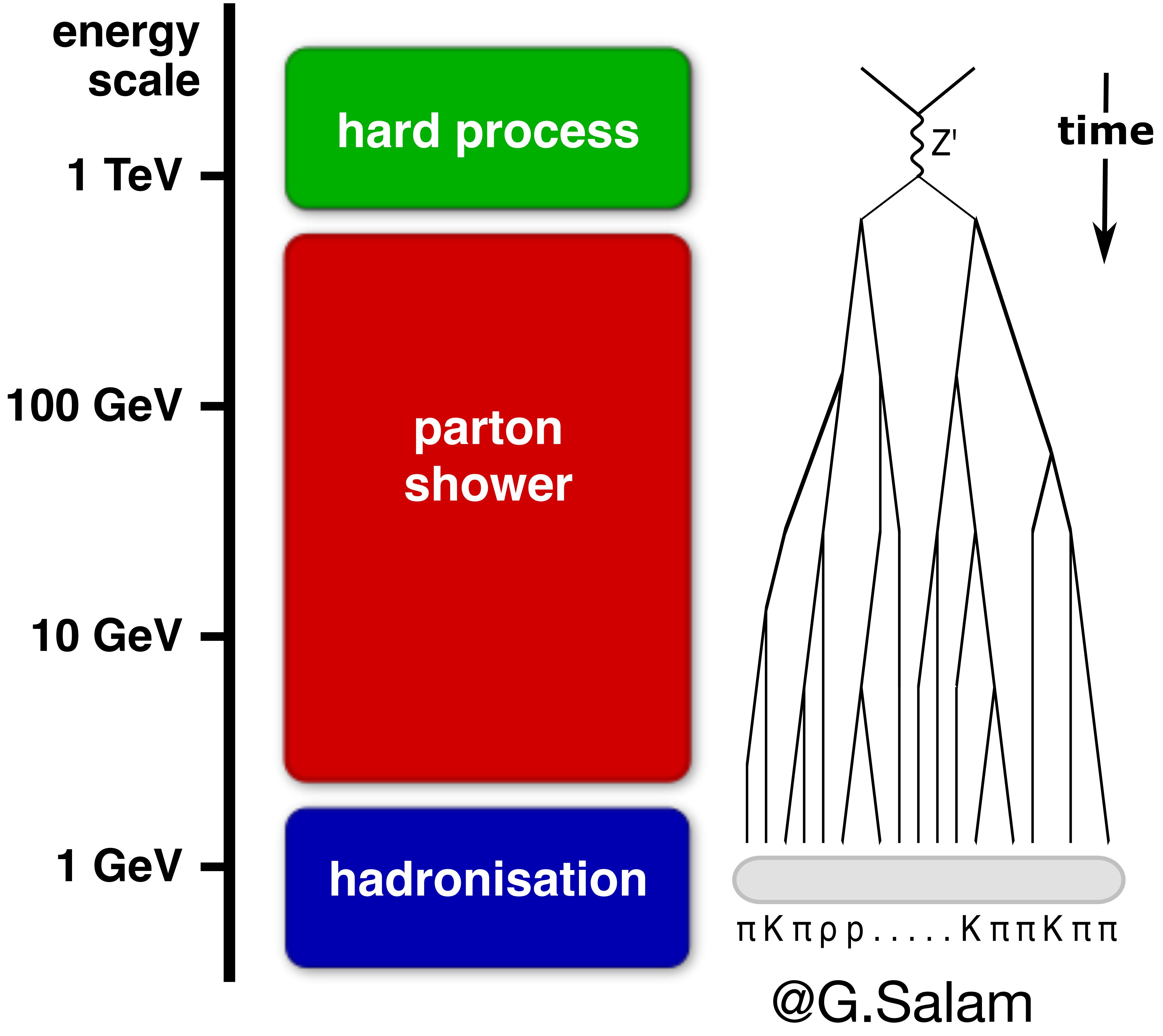
A multiscale story

High-Q^2 scattering: process dependent, systematically improvable with higher order corrections, where we expect new physics

Parton Shower: QCD, universal, soft and collinear physics

Hadronisation: low Q^2, universal, based on different models

Underlying event: low Q^2, involves multiple interactions



Parton Shower

What does the parton shower do/should do?

- Dress partons with radiation with an arbitrary number of branchings

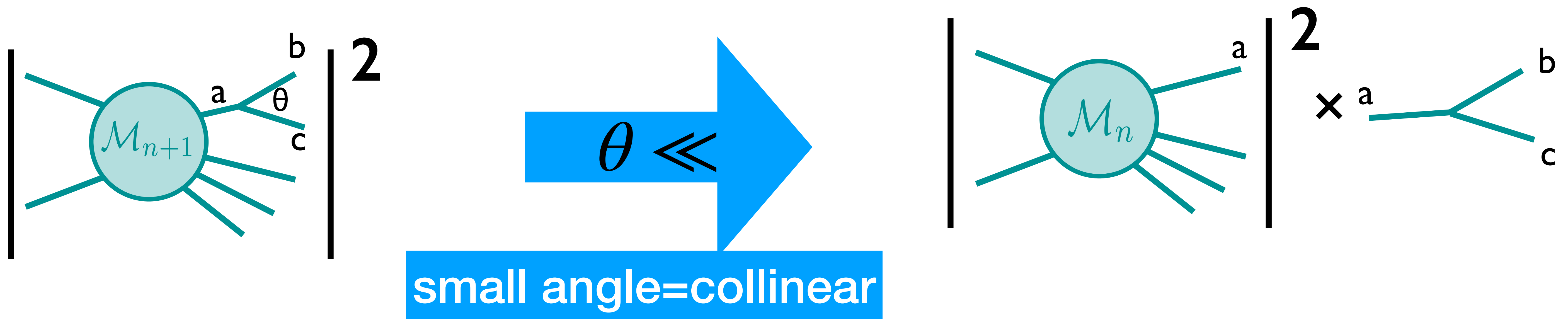
- Preserve the inclusive cross-section: unitary

- Needs to evolve in scale from Q~1TeV (hard scattering) down to ~GeV

Basics of parton shower

Collinear factorisation

Starting with one splitting



- Time scale associated with splitting much longer than the one of the hard scattering
- This kind of splitting should be described by a branching probability
- The parton shower will quantify the probability of emission

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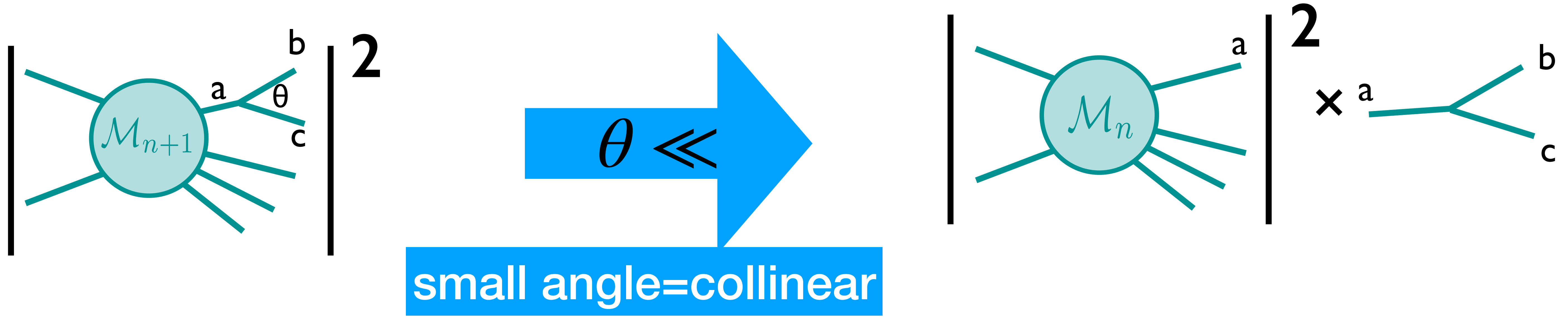

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Collinear factorisation:

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

Collinear factorisation and splitting functions

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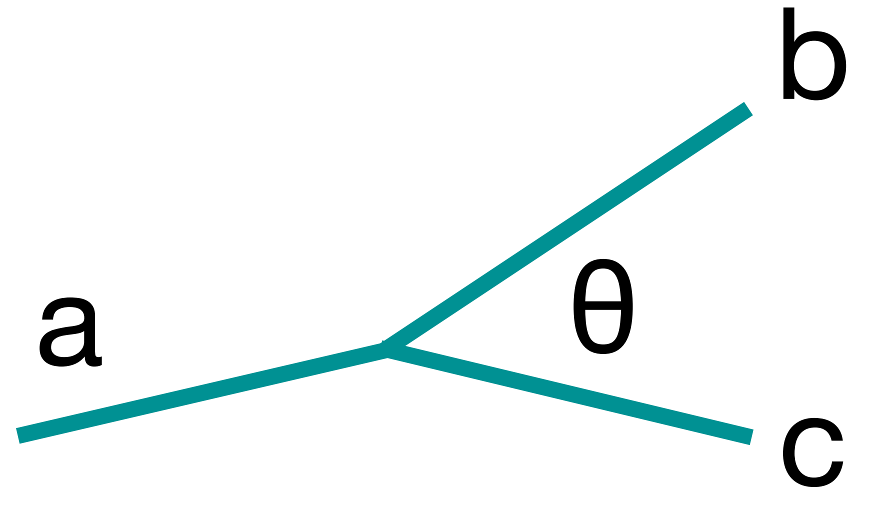

- t is the evolution variable
  - t tends to zero in the collinear limit (this factor is singular)
- z energy fraction transferred from parton a to parton b in splitting ($z \to 1$ in the soft limit)
- ϕ azimuthal angle

The branching probability has the same form for all quantities $\propto \theta^2$

- transverse momentum $k_\perp \sim z^2(1-z)^2\theta^2E^2$
- invariant mass $Q^2 \sim z(1-z)\theta^2E^2$

Collinear factorisation and splitting functions

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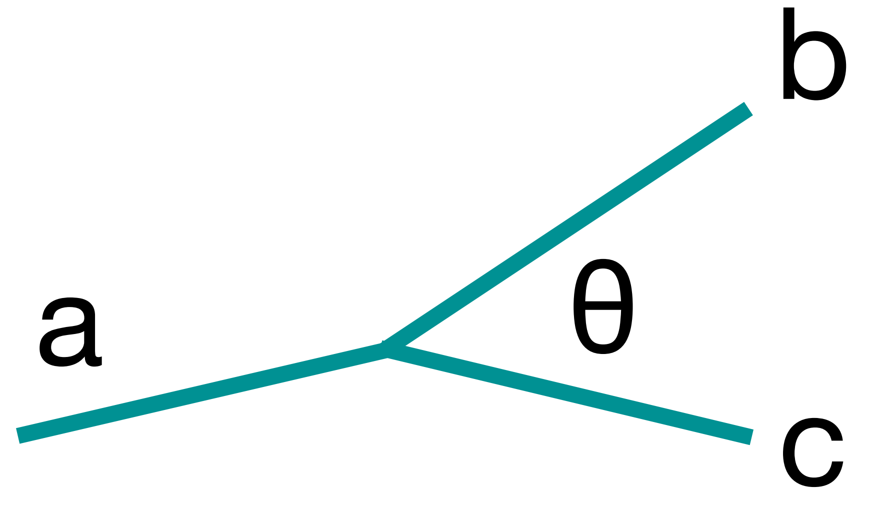

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$$\frac{d\theta^2}{\theta^2} = \frac{dk_\perp^2}{k_\perp^2} = \frac{dQ^2}{Q^2}$$

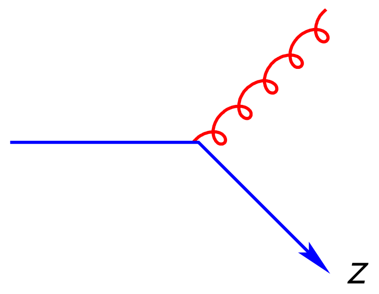
$$t \in \{\theta^2, k_\perp^2, Q^2\}$$

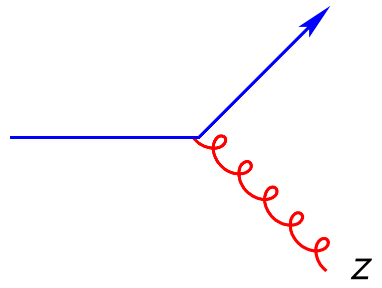
Altarelli-Parisi Splitting functions

Branching has a universal form given by the Altarelli-Parisi splitting functions (as we saw in DIS)

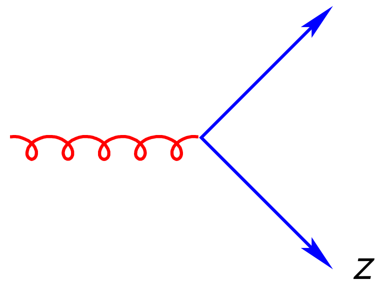
$$\frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a \to bc}(z)$$

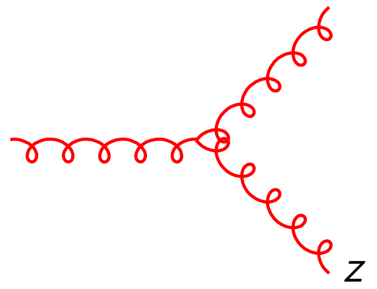
$$P_{q \to qg}(z) = C_F \left[\frac{1+z^2}{1-z} \right] , \qquad P_{q \to gq}(z) = C_F \left[\frac{1+(1-z)^2}{z} \right] .$$





$$P_{g \to qq}(z) = T_R \left[z^2 + (1-z)^2 \right] , \qquad P_{g \to gg}(z) = C_A \left[z(1-z) + \frac{z}{1-z} + \frac{1-z}{z} \right]$$





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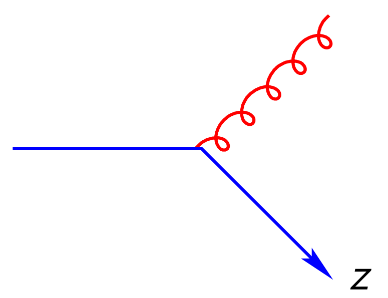

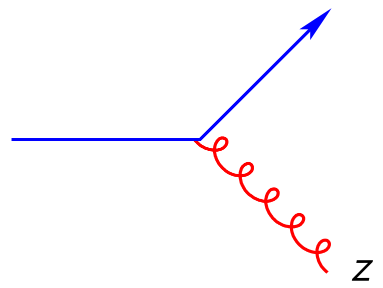

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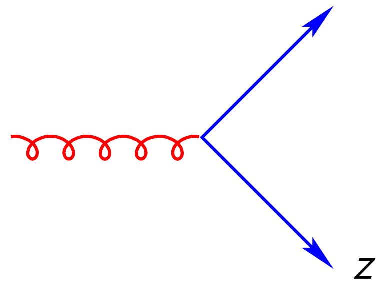
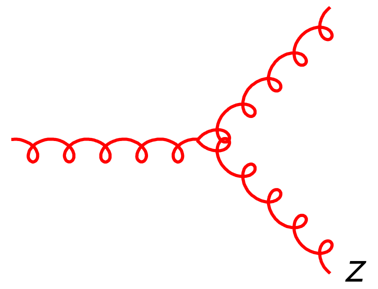
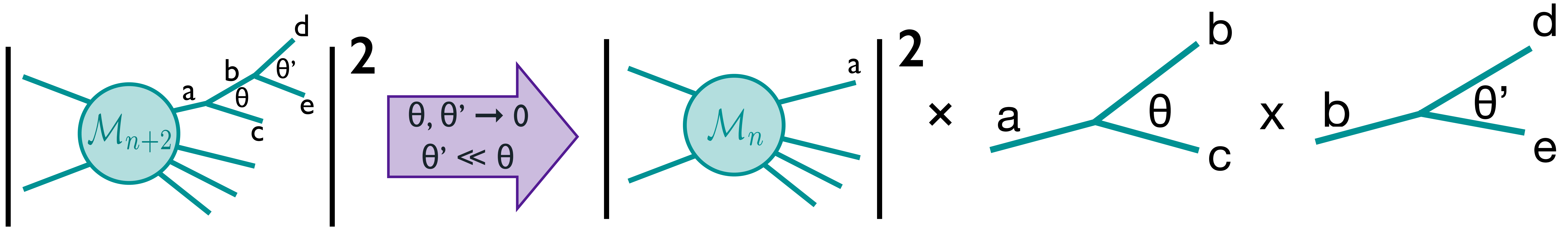




These functions are universal for each type of splitting

Multiple emissions

How does this change with multiple emissions?



$$|\mathcal{M}_{n+2}|^2 d\Phi_{n+2} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\to bc}(z) \times \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_s}{2\pi} P_{b\to de}(z')$$

We can generalise this for an arbitrary number of emissions

Iterative sequence of emissions which does not depend on the history (Markov Chain)

Multiple emissions

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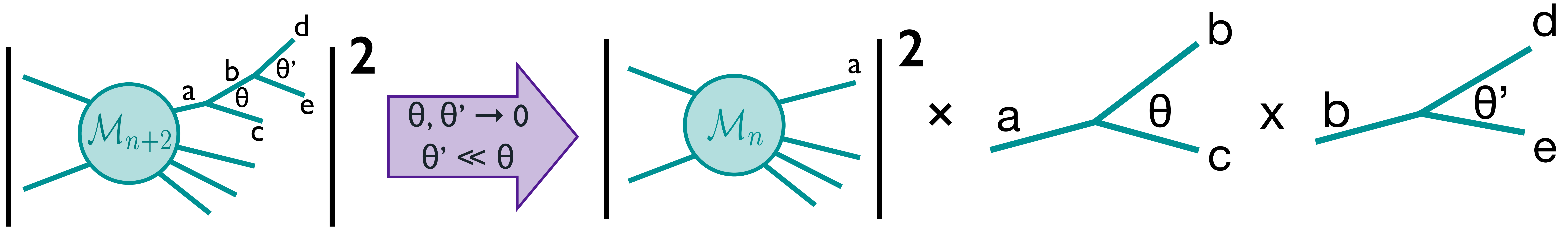

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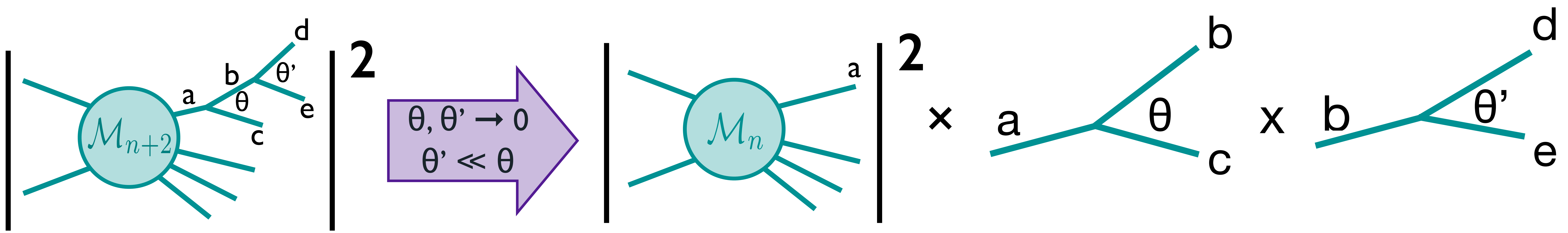

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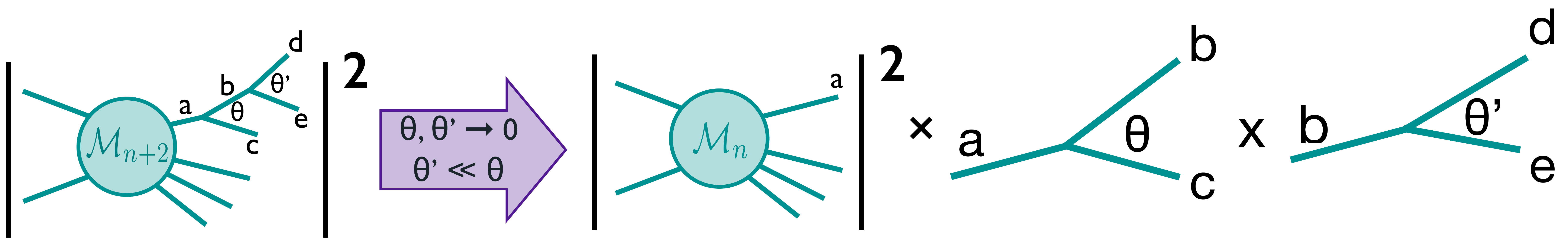
Iterative sequence of emissions which does not depend on the history (Markov Chain)

➡ **No interference: Classical**

Multiple emissions

How does this change with multiple emissions?



Dominant contribution comes from subsequent emissions which satisfy strong ordering $\theta \gg \theta' \gg \theta''$

For k emissions the rate takes the form:

$$\sigma_{n+k} \propto \alpha_S^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^{t} \frac{dt'}{t'} \ldots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_S}{2\pi}\right)^k \log^k(Q^2/Q_0^2)$$

- Q is the hard scale and Q_0 is an infrared cut off (separating non-perturbative regime)
- Each power of α_s comes with a logarithm (breakdown of perturbation theory when large)

Basics of PS

What we saw so far

- Collinear factorisation allows subsequent branchings from the hard process scale down to the non-perturbative regime
- Different legs and subsequent emissions are uncorrelated
- No interference effects
- Captures leading contributions
  - Resummed calculation
  - Bridge between fixed order and hadronisation

Sudakov form factor

We need to take the survival probability into account, i.e. a parton can split at scale t if it has not branched at $t' > t$

The probability of branching between scale t and $t + dt$ (with no emission before) is:

$$dp(t) = \sum_{bc} \frac{dt}{t} \int dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

The no-splitting probability between scale t and $t + dt$ is $1 - dp(t)$

The probability of no emission between Q^2 and t is:

$$\Delta(Q^2, t) = \prod_k \left[1 - \sum_{bc} \frac{dt_k}{t_k} \int dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)\right] =$$

$$\exp\left[-\sum_{bc} \int_t^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)\right] = \exp\left[-\int_t^{Q^2} dp(t')\right]$$

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Sudakov form factor

Sudakov form factor = No emission probability

Sudakovs

The Sudakov is used to create the branching tree of a parton

The probability of k ordered splittings form a leg at given scale is

$$\begin{aligned}
dP_1(t_1) &= \Delta(Q^2, t_1)\ dp(t_1)\Delta(t_1, Q_0^2), \\
dP_2(t_1, t_2) &= \Delta(Q^2, t_1)\ dp(t_1)\ \Delta(t_1, t_2)\ dp(t_2)\ \Delta(t_2, Q_0^2)\Theta(t_1 - t_2) \\
\ldots &= \ldots \\
dP_k(t_1, \ldots, t_k) &= \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)
\end{aligned}$$

The shower selects the t_i scales for the splitting randomly but weighted with no emission probability (before or after)

Unitarity

The parton shower is unitary. Sum of all possibilities should be 1.

Probability of k ordered splittings:

$$dP_k(t_1, ..., t_k) \quad = \quad \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)$$

Integrating this gives us:

$$P_k \equiv \int dP_k(t_1, ..., t_k) = \Delta(Q^2, Q_0^2) \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k , \quad \forall k = 0, 1, ...$$

Summing over all possible numbers of emissions (0 to ∞):

$$\sum_{k=0}^{\infty} P_k = \Delta(Q^2, Q_0^2) \sum_{k=0}^{\infty} \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k = \Delta(Q^2, Q_0^2) \exp \left[\int_{Q_0^2}^{Q^2} dp(t) \right] = 1$$

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Probability is conserved ✓

Evolution parameter in parton shower

A parton shower is constructed:

- Within the simplest collinear approximation, the splitting functions are universal, and fully factorized from the "hard" cross section
- Within the simplest approximation, decays are independent (apart from being ordered in a decreasing sequence of scales)

Other variables can be used as evolution parameter:

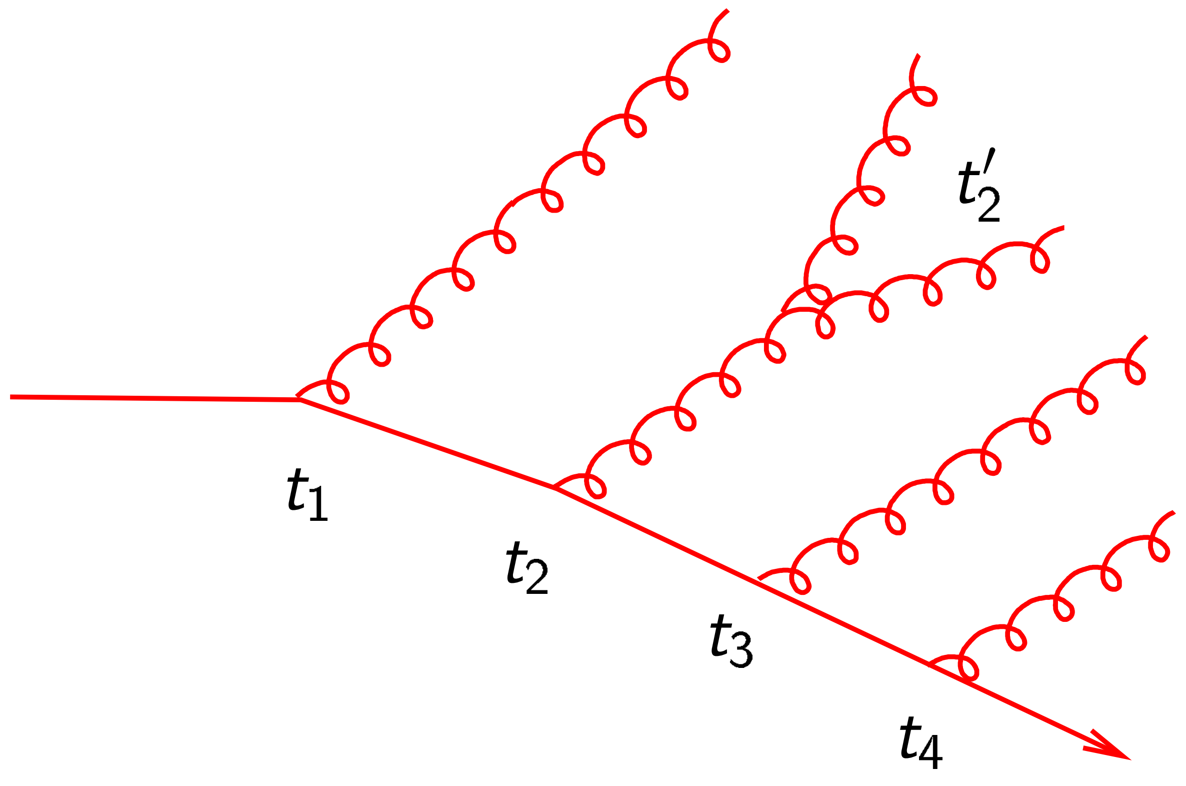
$$\frac{d\theta^2}{\theta^2} \sim \frac{dQ^2}{Q^2} \sim \frac{dp_\perp^2}{p_\perp^2} \sim \frac{d\tilde{q}^2}{\tilde{q}^2} \sim \frac{dt}{t}$$

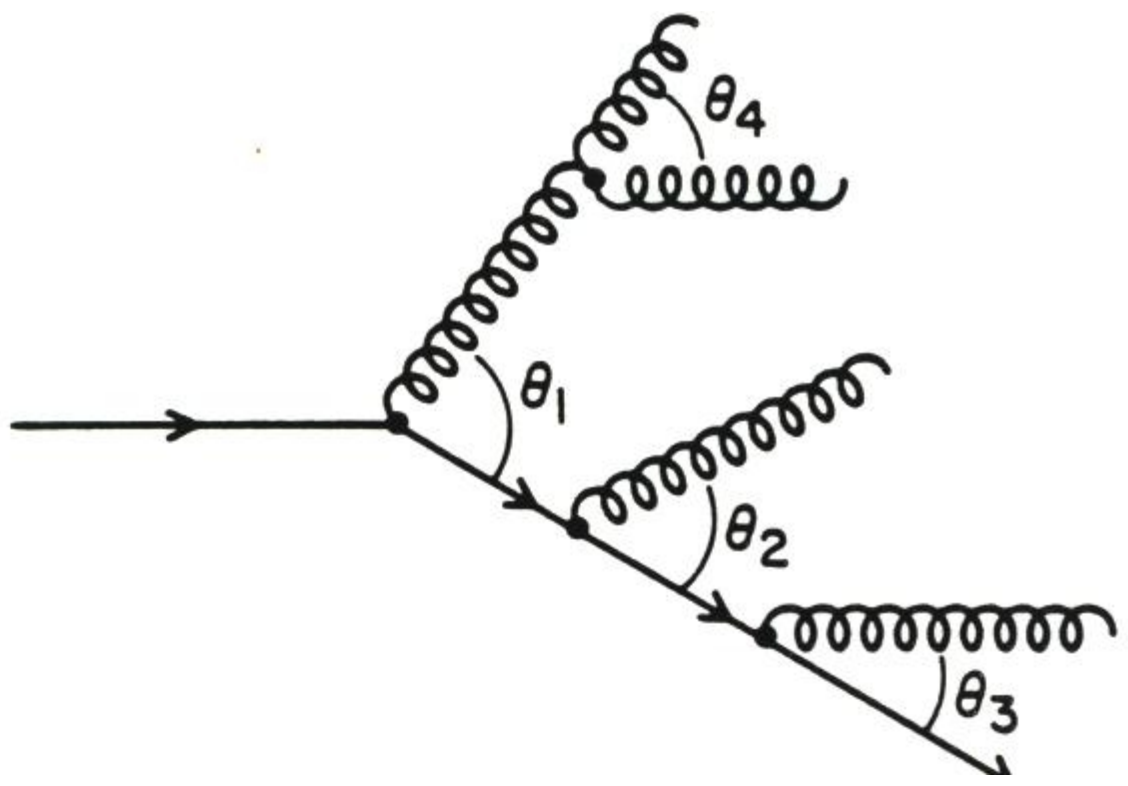
- θ: HERWIG
- Q^2: PYTHIA ≤ 6.3, SHERPA.
- $p_\perp$: PYTHIA ≥ 6.4, ARIADNE, Catani–Seymour showers.
- $\tilde{q}$: Herwig++.

Same collinear behaviour, differences in the soft limit

Ordered branchings

Angular ordering





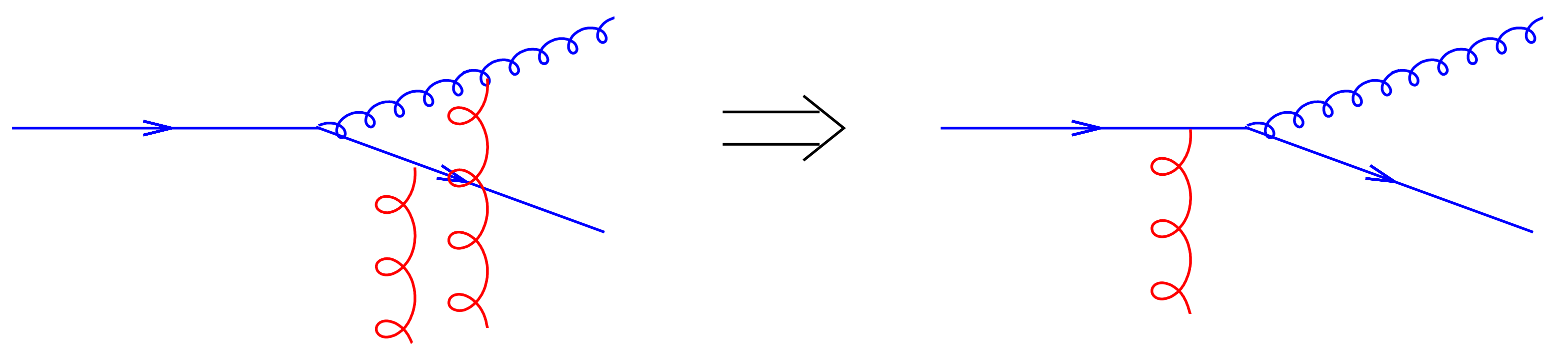
Shower is based on ordered splittings

$t_1 \gg t_2 \gg t_3 \gg t_4$ and $t_2 \gg t'_2$

Emission with smaller and smaller angles

$\theta_1 > \theta_2 > \theta_3 \qquad \theta > \theta_4$

Note:



Inside the cones partons emit as independent charges, outside radiation is **coherent** as if coming directly from the initial colour charge

Hadronisation

- Colourless hadrons observed in detectors, not partons.
- Hadronisation describes creation of hadrons in QCD at low scales where $\alpha_s \sim \mathcal{O}(1)$
- Requires non perturbative input
- Two models: cluster and string

Color-singlet parton pairs end up “close” in phase space. This is called preconfinement.

Involves collecting $q\bar{q}$ pairs into color-singlet clusters.

Cluster hadronisation

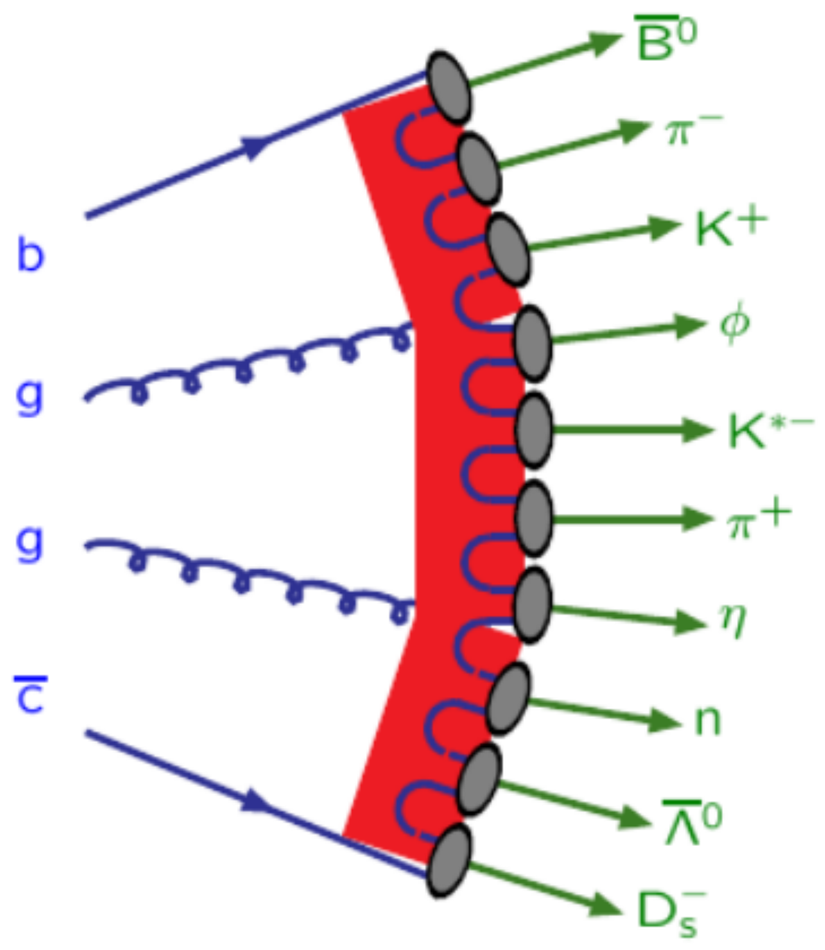
Create strings from color string, with gluons “stretching the string” locally. It uses non-perturbative insights

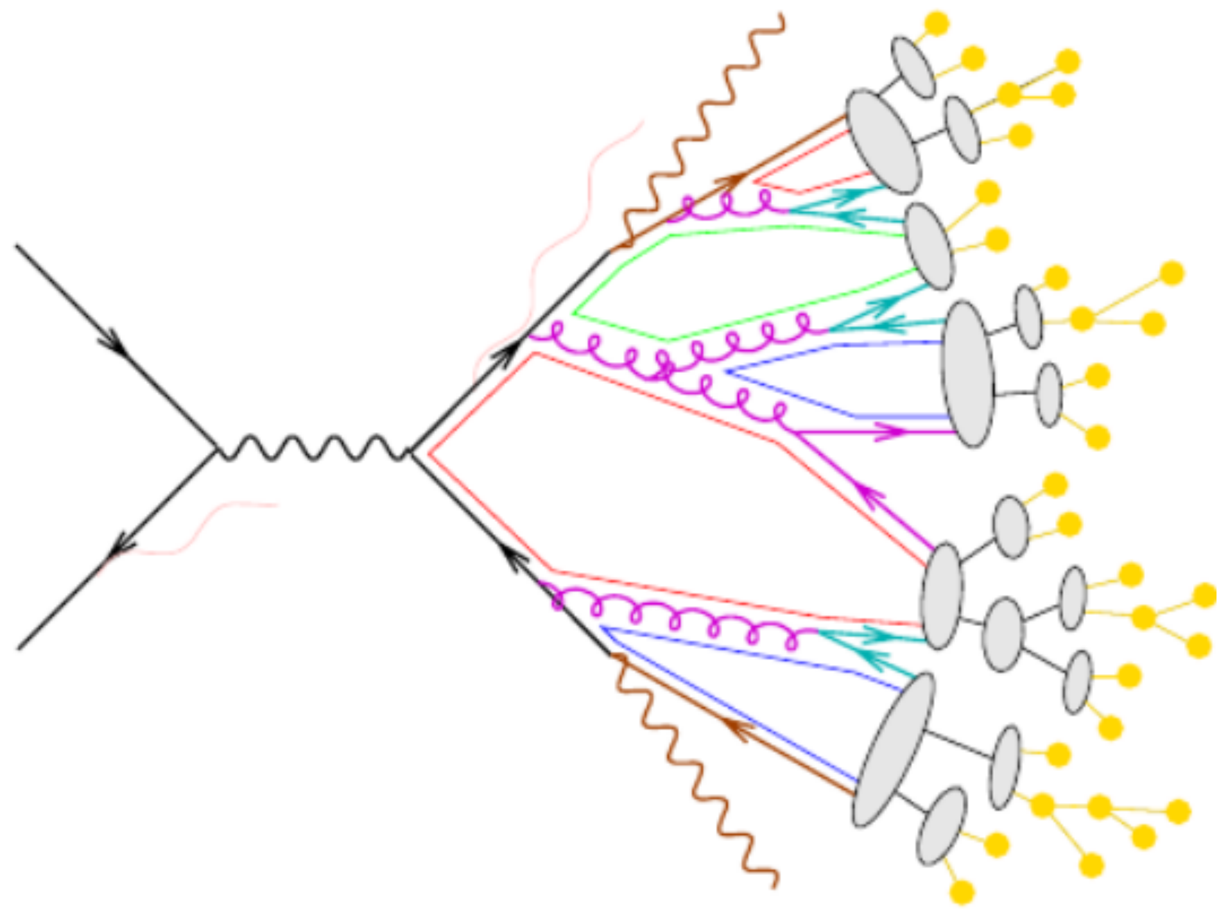
String hadronisation

Hadronisation

String vs Cluster

Sjöstrand, Durham '09





program	PYTHIA	HERWIG
model	string	cluster
energy–momentum picture	powerful	simple
	predictive	unpredictive
parameters	few	many
flavour composition	messy	simple
	unpredictive	in-between
parameters	many	few

Summary: Parton shower

- A parton shower dresses partons with radiation such that the sum of probabilities is one.
  - Predictions become exclusive.
  - General-purpose, process-independent tools
  - Based on collinear factorisation and build around the Sudakov form factors provide a resummed prediction
  - Similar ideas can be used for the initial state shower (need to account for PDFs-deconstruction of the DGLAP evolution, **backwards evolution**)
- Full description starting from hard scattering, shower and hadronisation (also underlying event)
- Move to hadronisation at a cut off at which perturbative QCD can't be trusted
  - Hadronisation is also universal and independent of the collider energy

Parton shower programs

Current release series	Hard matrix elements	Shower algorithms	MPI	Hadronization
Herwig 7	Internal, libraries, event files	QTilde, Dipoles	Eikonal	Clusters, (Strings)
Pythia 8	Internal, event files	Pt ordered, DIRE, VINCIA	Interleaved	Strings
Sherpa 2	Internal, libraries	CSShower, DIRE	Eikonal	Clusters, Strings

Herwig and Pythia use LHE files e.g. produced in MG5_aMC