

Electroweak and QED effects in PDF determination

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Factorization and universality

Collinear factorization enables the prediction of cross-sections

$$\sigma_{\text{LHC}}(M, s) \propto \sum_{ij} \int_{M^2}^s d\hat{s} \mathcal{L}_{ij}(\hat{s}, s) \hat{\sigma}_{ij}(\hat{s}, \alpha_s(M)) \quad i, j = \{g, u, d, \dots\}$$

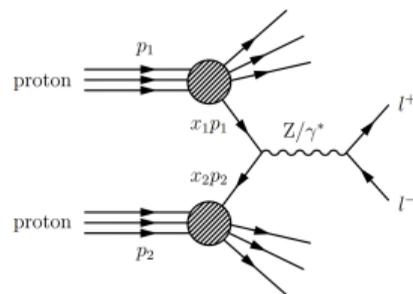
Partonic luminosity

$$\mathcal{L}_{ij}(Q, s) = \frac{1}{s} \int_{Q^2/s}^1 \frac{dx}{x} f_i\left(\frac{Q^2}{sx}, Q\right) f_j(x, Q)$$

Parton Distribution Function:

$$f_j(x, Q)$$

- Non-perturbative QCD
- Extracted from experimental data
- Universal

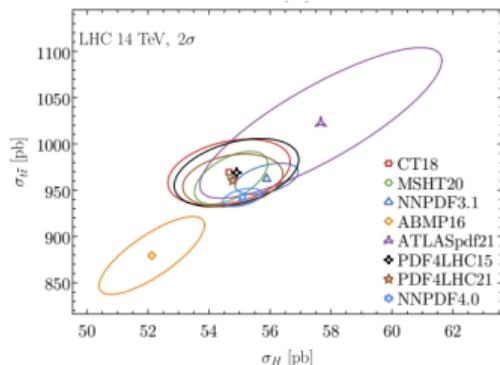


$\hat{\sigma}_{ij}(\hat{s}, \alpha_s(M))$ denotes the hard-scattering coefficient function

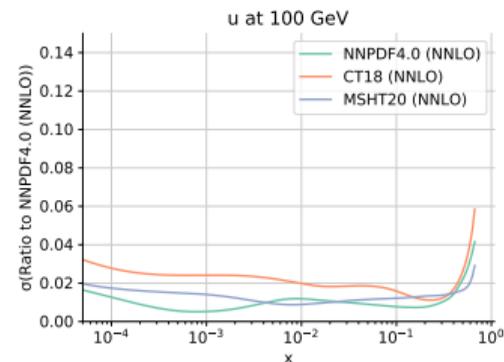
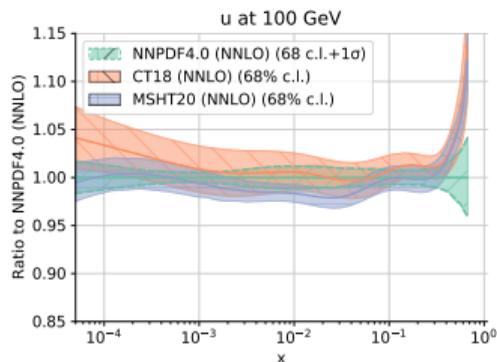
- Calculable in perturbative QCD
- Independent of the initial hadron

Status of modern PDF sets

PDF sets are provided by various collaborations, each using slightly different theory choices, datasets, and methodologies



[Snowmass 2021: 2203.13923]



- Current standard is NNLO in QCD
- Results are generally consistent, but uncertainties differ
- Approaching percent-level accuracy

- CT: Hessian + tolerance
- MSHT: Hessian + dynamic tolerance
- NNPDF: MC + closure tests

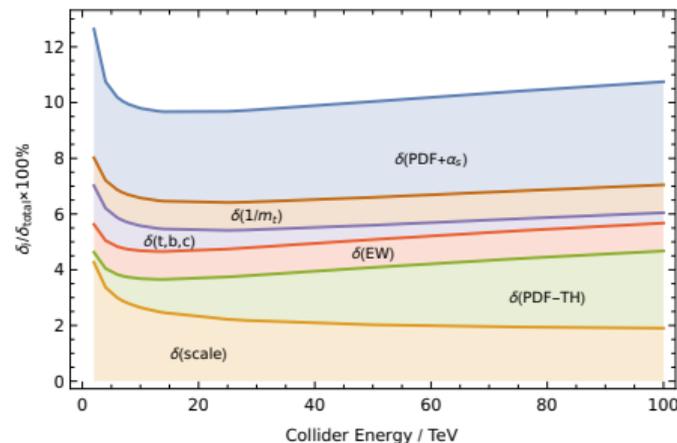
Challenges

PDFs, along with α_s , are often a dominant source of uncertainty for predictions of LHC cross-sections

Requirements for the next generation of PDFs are threefold:

- To exploit the impressive progress in N3LO calculations we require PDFs of the same order
- Missing higher order uncertainties (MHOUs) for some observables are larger than the experimental uncertainty and can thus no longer be neglected
- The level of precision aimed for at the LHC no longer allows neglecting EW corrections

Focus on QED, but first briefly mention N3LO and MHOUs



Uncertainties for inclusive Higgs production
[HL-LHC: 1902.00134]

Approximate N3LO PDFs

Towards N3LO PDFs

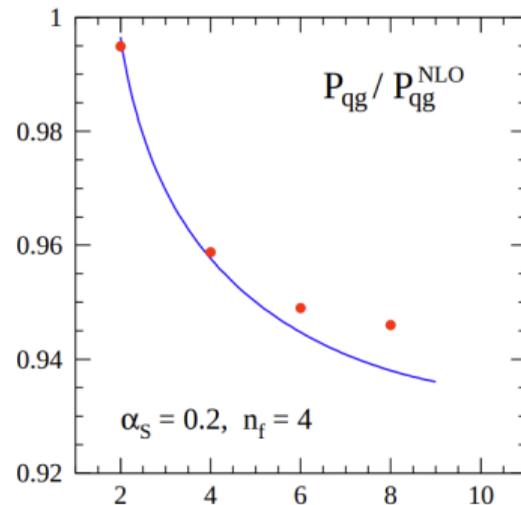
To produce PDFs at N3LO several inputs are needed at the corresponding order:

- QCD splitting functions for DGLAP evolution
- Matching conditions for heavy-quark mass schemes
- DIS partonic coefficients
- K-factors for hadronic observables

These are not completely available at N3LO

What do we know about 4-loop splitting functions?

- Singlet and non-singlet Mellin moments [Davies, Falcioni, Herzog, Kom, Moch, Ruijl, Ueda, Vermaseren, Vogt: 1707.08315, 1610.07477, 2202.10362, 2111.15561, 2302.07593, 2307.04158]
- Large- n_f limit [Davies, Vogt, Ruijl, Ueda, Vermaseren: 1610.0744]
- Small- x limits (BFKL resummation) [Bonvini and Marzani: 1805.06460] [Davies, Kom, Moch, Vogt: 2202.10362]
- Large- x limits (threshold resummation) [Duhr, Mistlberger, Vita 2205.04493], [Henn, Korchemsky, Mistlberger: 1911.10174], [Soar, Moch, Vermaseren, Vogt: 0912.0369]

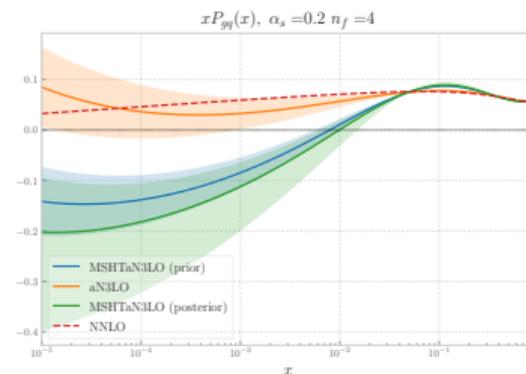
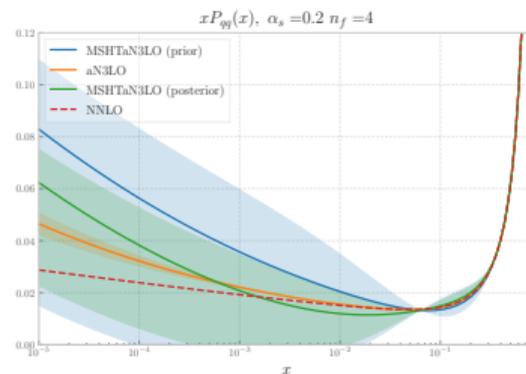


[Moch, Ruijl, Ueda, Vermaseren, Vogt: 2111.15561]

Splitting functions at aN3LO

Splitting function approximation:

- Construction of approximate splitting function susceptible to model uncertainties
- Vary interpolation basis choices to estimate model uncertainties
- MSHTaN3LO posterior uncertainties based on fitting nuisance parameter [McGowan, Cridge, Harland-Lang, Thorne: 2207.04739]
- Generally good agreement except for P_{qq} where MSHT20aN3LO appears to saturate and deviates from NNLO
- More Mellin moments available since the release of MSHT20aN3LO, explaining the large difference in uncertainty for P_{qq}



[NNPDF, Magni: 2023]

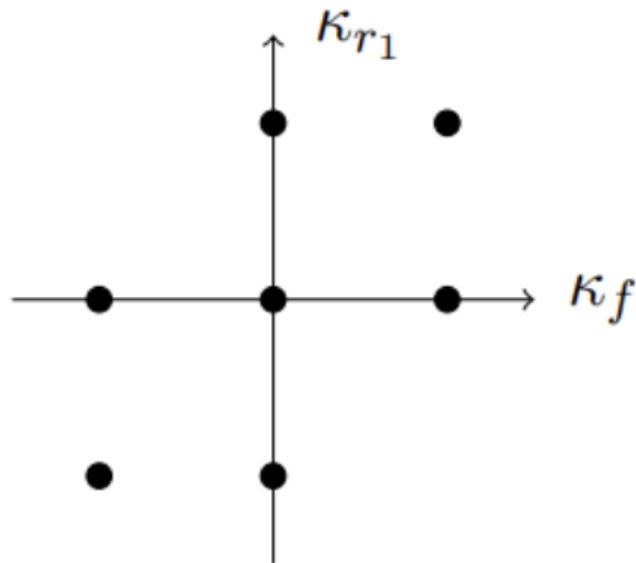
Missing higher order uncertainties

Estimating missing higher order uncertainties

Unknown residual terms have a corresponding MHOUs

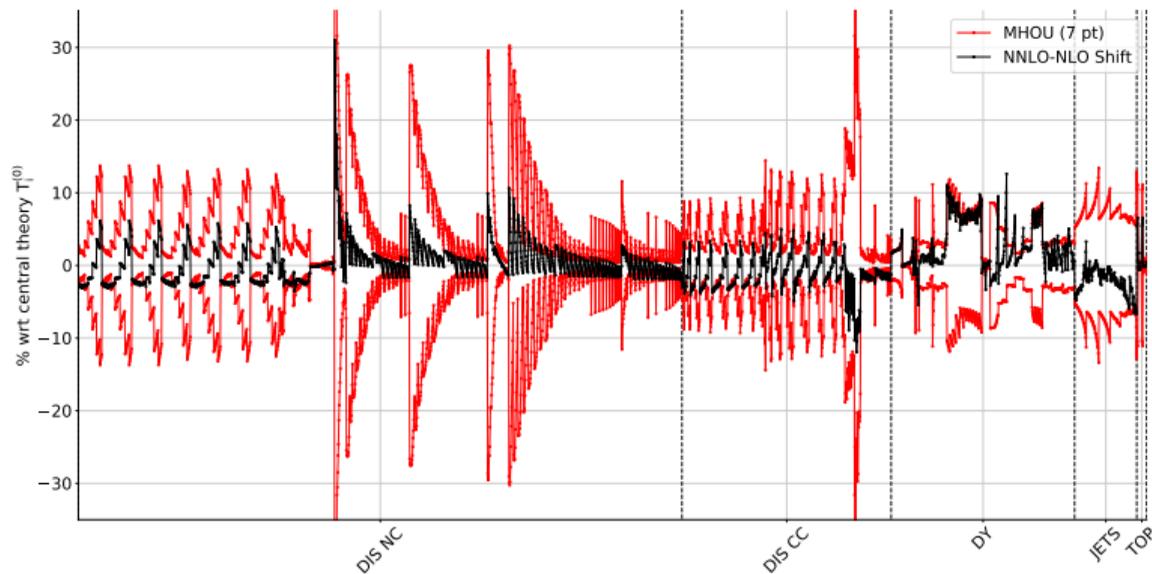
$$\hat{\sigma}(\alpha_s) = \hat{\sigma}^{(0)} \left(\sum_{k=0}^m \alpha_s^k \right) + \mathcal{O}(\alpha_s^{m+1})$$

- Commonly scale variations at N^n LO are used to estimate N^{n+1} LO MHOUs
 - factorization scale variation estimates MHOUs in DGLAP evolution
 - renormalization scale variation estimates MHOUs in hard processes
- MHOUs PDFs have been determined using the theory covariance matrix formalism (Gaussianity assumed) [NNPDF: 1906.10698]
 - $\text{COV}_{\text{exp}} \rightarrow \text{COV}_{\text{exp}} + \text{COV}_{\text{theory}}$



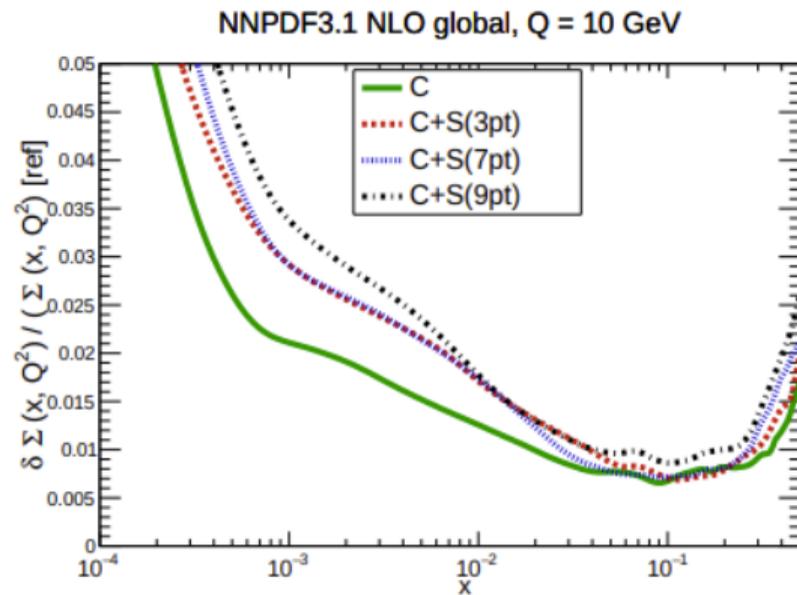
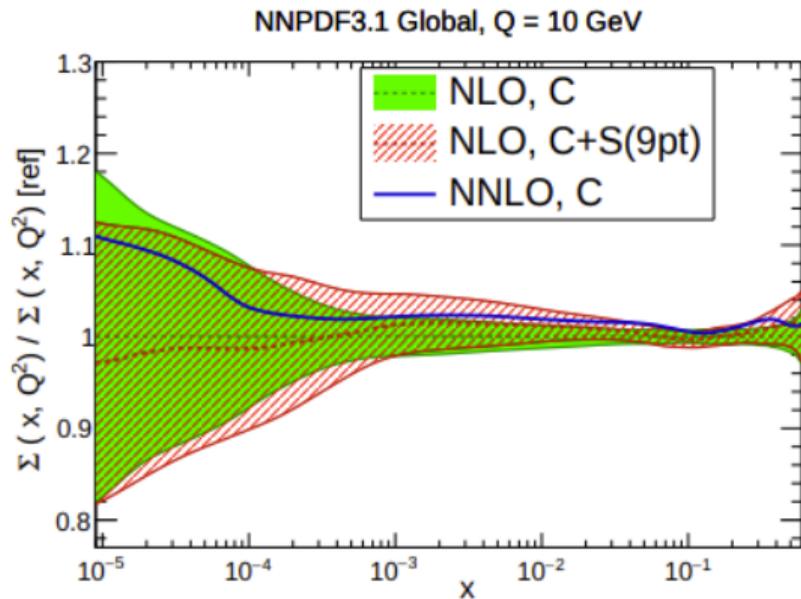
Validating missing higher order uncertainties

Compare estimated MHOU at NLO to NNLO-NLO shift:



[NNPDF: 1906.10698]

Impact of MHOU in a PDF fit



Small increase in PDF uncertainty
Soon: NNPDF4.0 with MHOU

QED effects in PDFs

Including QED corrections in a PDF set

The current standard for PDFs determination is at NNLO in QCD, however $\alpha(M_z) \sim \alpha_s^2(M_Z)$

Including QED corrections in PDFs consists of

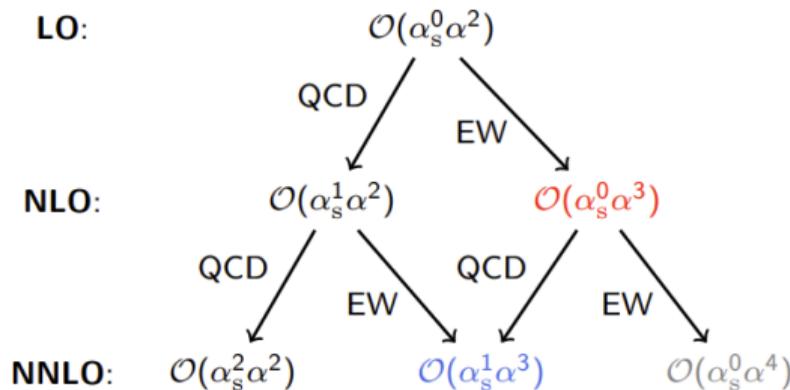
- QED corrections to DGLAP (at $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha^2)$):

$$P_{QED} = \alpha P_{ij}^{(0,1)} + \alpha\alpha_s P_{ij}^{(1,1)} + \alpha^2 P_{ij}^{(0,2)} + \dots$$

- Adding a photon PDF and including photon initiated contributions to cross-sections

The momentum sumrule is modified accordingly:

$$\int_0^1 dx (x\Sigma(x, Q^2) + xg(x, Q^2) + x\gamma(x, Q^2)) = 1$$



Example: EW corrections in DY
[C. Schwan DIS 2021]

Determination of the photon PDF

Initially the photon PDF has been determined in different ways:

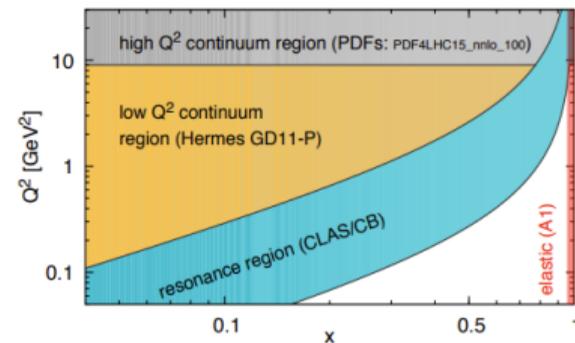
- physical model: sensitive to underlying model
- fitting: data does not provide strong constraints

However with the LUXqed approach it can be computed perturbatively based on the observation that the heavy-lepton production cross-section can be written in two ways:

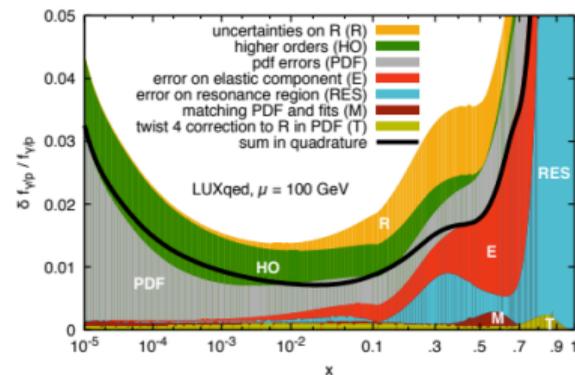
- in terms of structure functions F_2 , F_L
- in terms of PDFs (including the photon)

luxQED result [Manohar, Nason, Salam, Zanderighi: 1607.04266, 1708.01256]:

$$x\gamma(x, \mu^2) = \frac{2}{\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \left\{ \int \frac{\frac{\mu^2}{1-z}}{\frac{m_p^2 x^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \left[-z^2 F_L(x/z, Q^2) + \left(zP_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q^2) \right] - \alpha^2(\mu^2) z^2 F_2(x/z, \mu^2) \right\}$$



Input to construct F_2 and F_L



Sources of uncertainty

LUXqed PDF determinations

LUXqed has been used in all of the most recent QED PDFs:

- LUXqed_plus_PDF4LHC15 [Manohar, et al.: 1607.04266]
- LUXqed17_plus_PDF4LHC15 [Manohar, et al.: 1708.01256]
- MMHT2015qed [Harland-Land et al.: 1907.02750]
- NNPDF3.1luxQED [Bertone et al.: 1712.07053]
- CT18lux and CT18qed [Xie et al.: 2106.10299]
- MSHT20QED [Cridge et al.: 2111.05357]
- Soon: NNPDF4.0QED

The photon PDF by different groups

Differences between the studies by the different groups include:

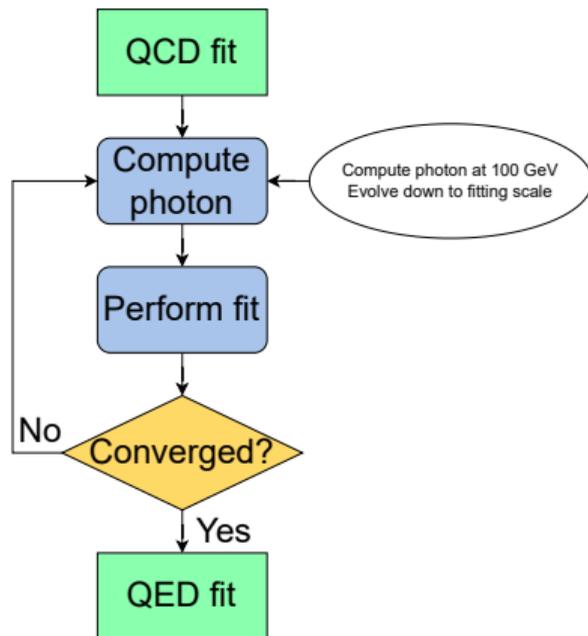
CT18qed initializes the photon at an input scale and evolves using DGLAP, while CT18lux calculates the photon at all scales

MSHT20QED has a 1 GeV input scale and includes renormalon corrections

NNPDF follow the original luxQED by calculating the photon at 100 GeV to reduce higher twist effects

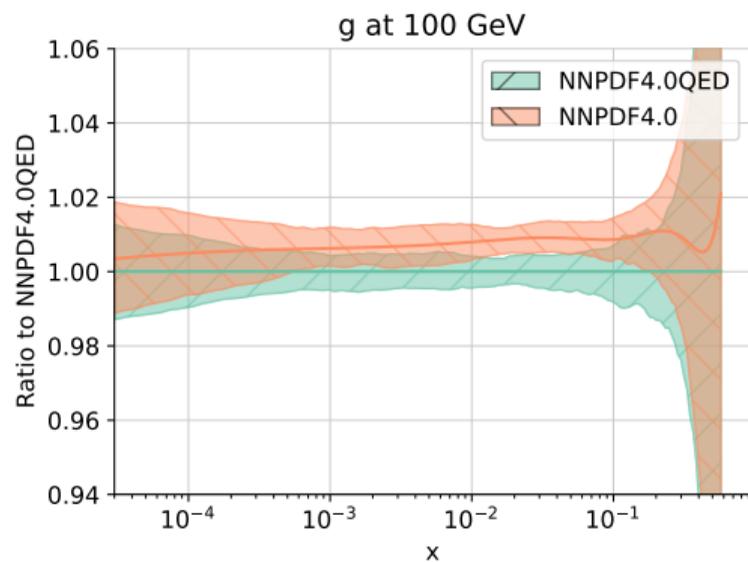
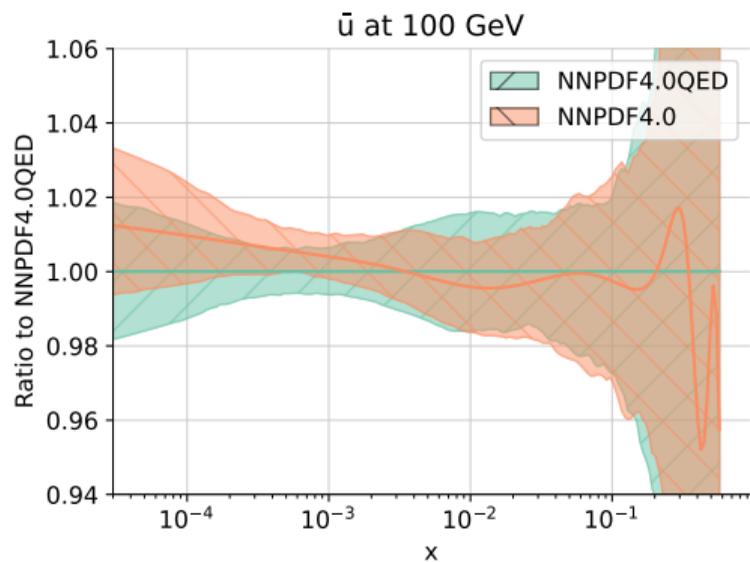
NNPDF3.1QED and NNPDF4.0QED use an iterative procedure to address the interplay between the photon and other PDFs due to the momentum sumrule

$$\int_0^1 dx (x\Sigma(x, Q^2) + xg(x, Q^2) + x\gamma(x, Q^2)) = 1$$



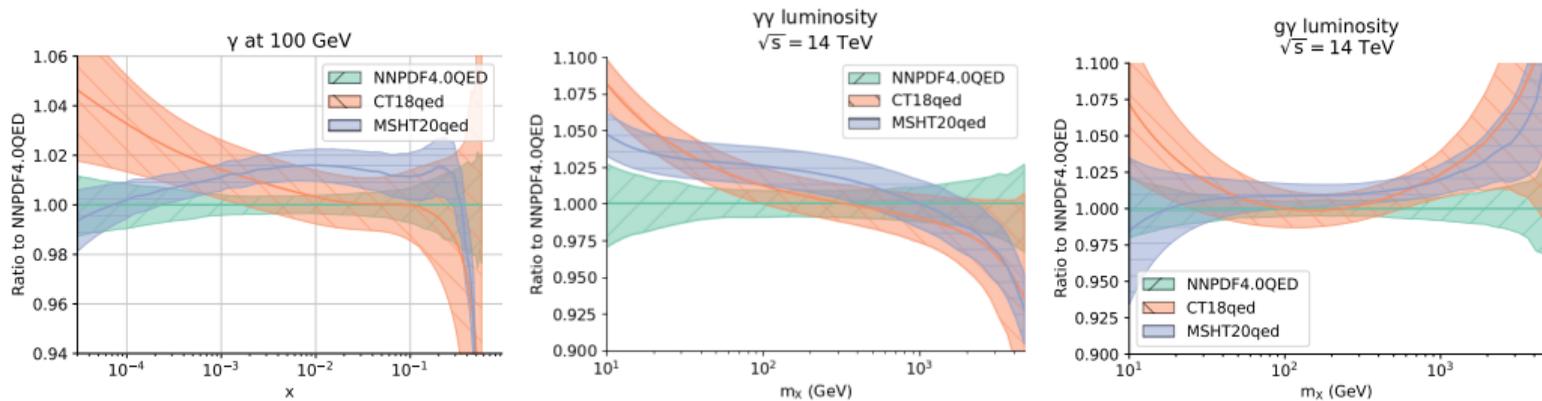
[NNPDF3.1QED: 1712.07053]

Results: Impact of the photon on other PDFs



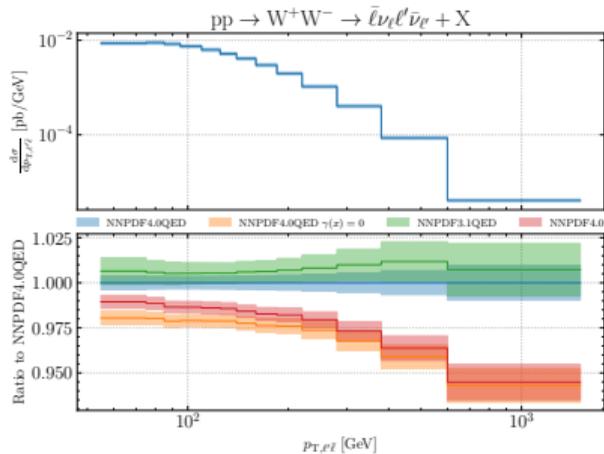
- Non-negligible impact, but PDFs are in agreement within uncertainty
- Gluon reduced due to momentum sum rule with photon carrying additional momentum

Results: photon PDF and luminosity

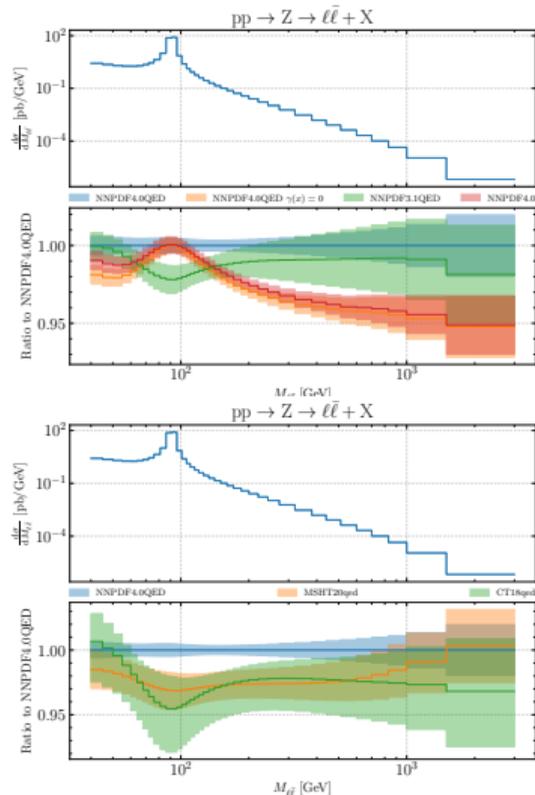


- Because all groups use the luxQED formalism, the photon PDFs agree at percent level
- Luminosity generally in agreement, but differ at very small and very large invariant mass

Results: phenomenological impact



- Here photon initiated contributions are included
- Non-negligible QED corrections in the large invariant mass and large- p_T regions relevant for new physics searches
- Negligible impact around the Z -peak
- Difference between groups driven by gluon and quark PDFs

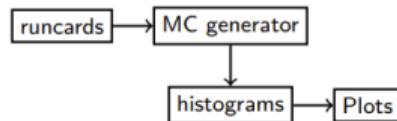


PineAPPL: combined QCD and QED interpolation tables

PineAPPL is a library for storing PDF-independent theory predictions in interpolation grids

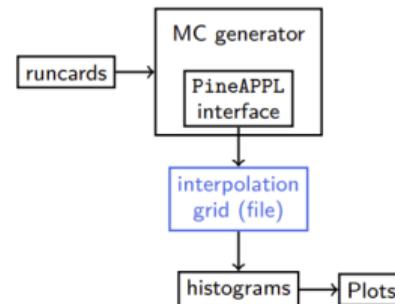
[Carazza, Nocera, Schwan, Zaro: 2008.12789]

Typical setup:



Requires rerunning MC generator for each PDF

PineAPPL:



After computing the interpolation grid once it can be convolved with an arbitrary PDF set

[C. Schwan DIS 2021]

PineAPPL: combined QCD and QED interpolation tables

Applications:

- PDF fits
- Efficient study of PDF impact on observable (e.g. for SM parameter extraction)

Other providers of fast interpolation grids:

- APPLgrid [Carli et al.: 0911.2985]
- fastNLO [Kluge, Rabbertz, Wobisch: hep-ph/0609285]

However, the need for EW corrections inspired the creation of PineAPPL

Finally, PineAPPL...

- supports up to any power in α and α_s
- is interfaced to MG5_aMC@NLO
- is interfaced to xFitter
- is open source

Summary and outlook

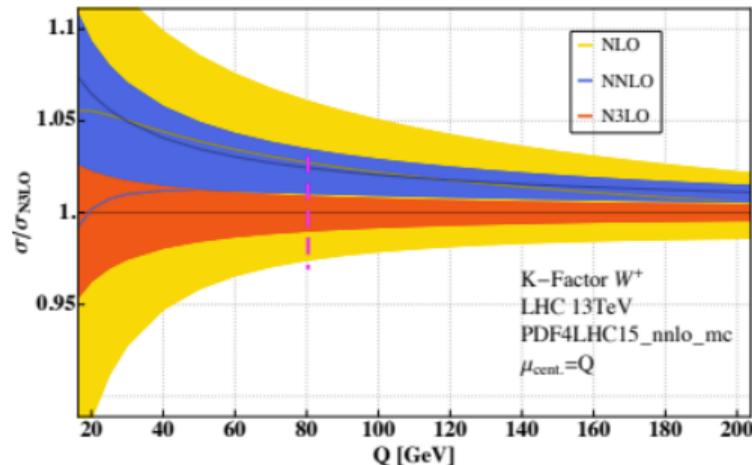
Summary and outlook

Interpretation of collider measurements depends on accurate and precise theory predictions

The last years have seen impressive progress in N3LO calculations for LHC processes, to exploit this we need N3LO PDFs

Electroweak effects can no longer be neglected thus include QED corrections in DGLAP and include a photon PDF

The target of faithful 1% uncertainty in wide kinematic range will require simultaneously including EW effects, MHOU, and N3LO!



[Duhr, Dulat, Mistleberger 2007.13313]

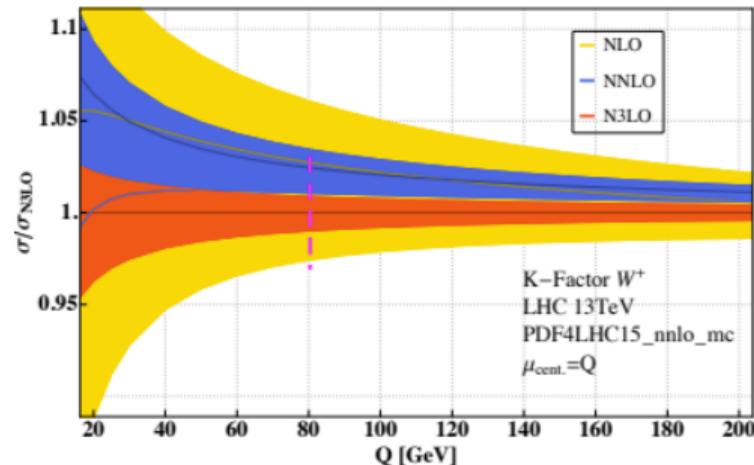
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Thank you for listening!