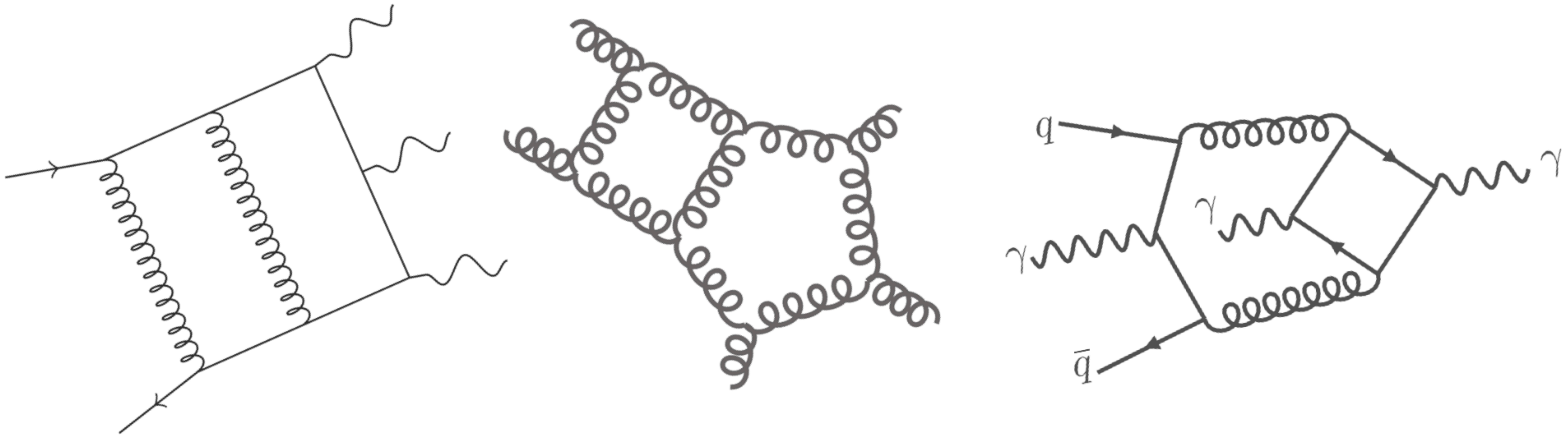
TWO-LOOP FIVE-POINT MASSLESS QCD AMPLITUDES IN FULL COLOR

In collaboration with: B.Agarwal, F.Buccioni, G.Gambuti, A.Von Manteuffel, L.Tancredi



S. Badger, D. Chicherin, T. Gehrmann, G. Heinrich, J.M. Henn, T. Peraro, P. Wasser, Y. Zhang, S. Zoia: **1905.03733**

Herschel A. Chawdhry, Michal Czakon, Alexander Mitov, Rene Poncelet: **2012.13553**

Bakul Agarwal, Federico Buccioni, Andreas von Manteuffel, Lorenzo Tancredi: **2102.01820**

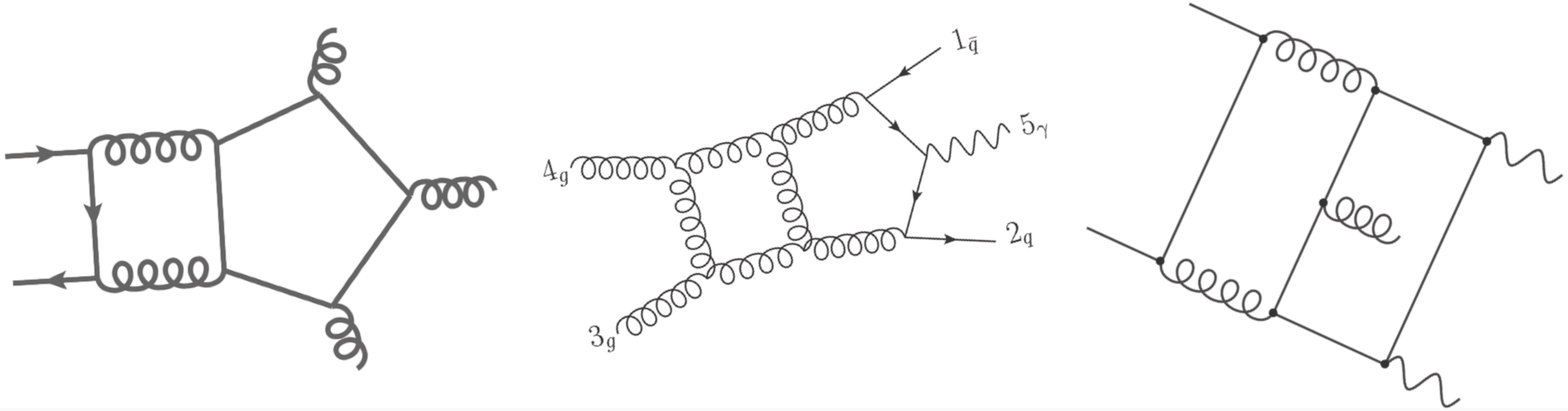
S. Abreu, F. Febres Cordero, H. Ita, B. Page, V. Sotnikov: **2102.13609**

Bakul Agarwal, Federico Buccioni, Andreas von Manteuffel, Lorenzo Tancredi: **2105.04585**

Michal Czakon, Alexander Mitov, Rene Poncelet: **2106.05331**

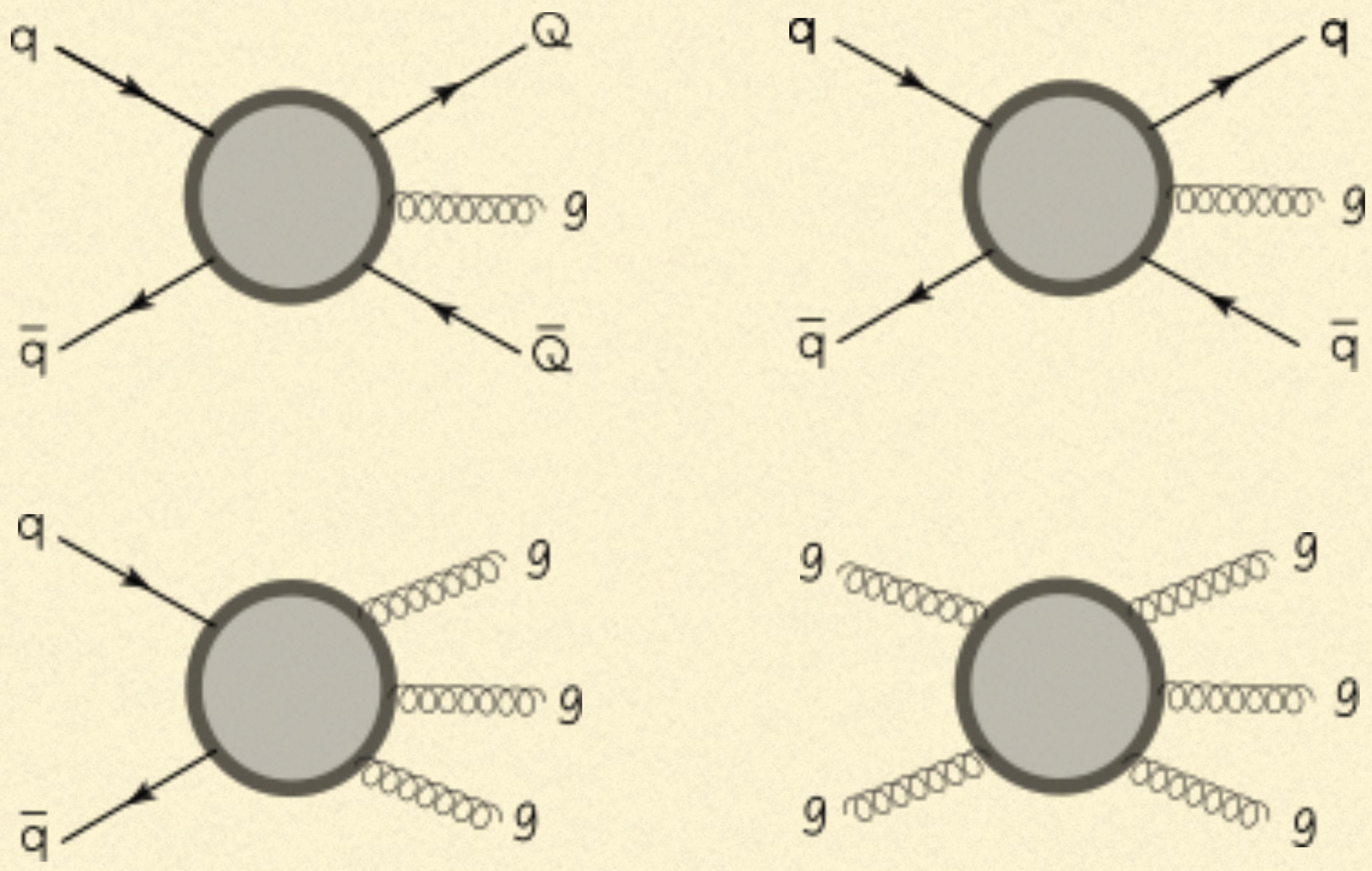
Simon Badger, Michał Czakon, Heribertus Bayu Hartanto, Ryan Moodie, Tiziano Peraro: **2304.06682**

Samuel Abreu, Giuseppe De Laurentis, Harald Ita, Maximillian Klinkert, Ben Page, Vasily Sotnikov: **2305.17056**



Slide by G.Gambuti

In this talk:



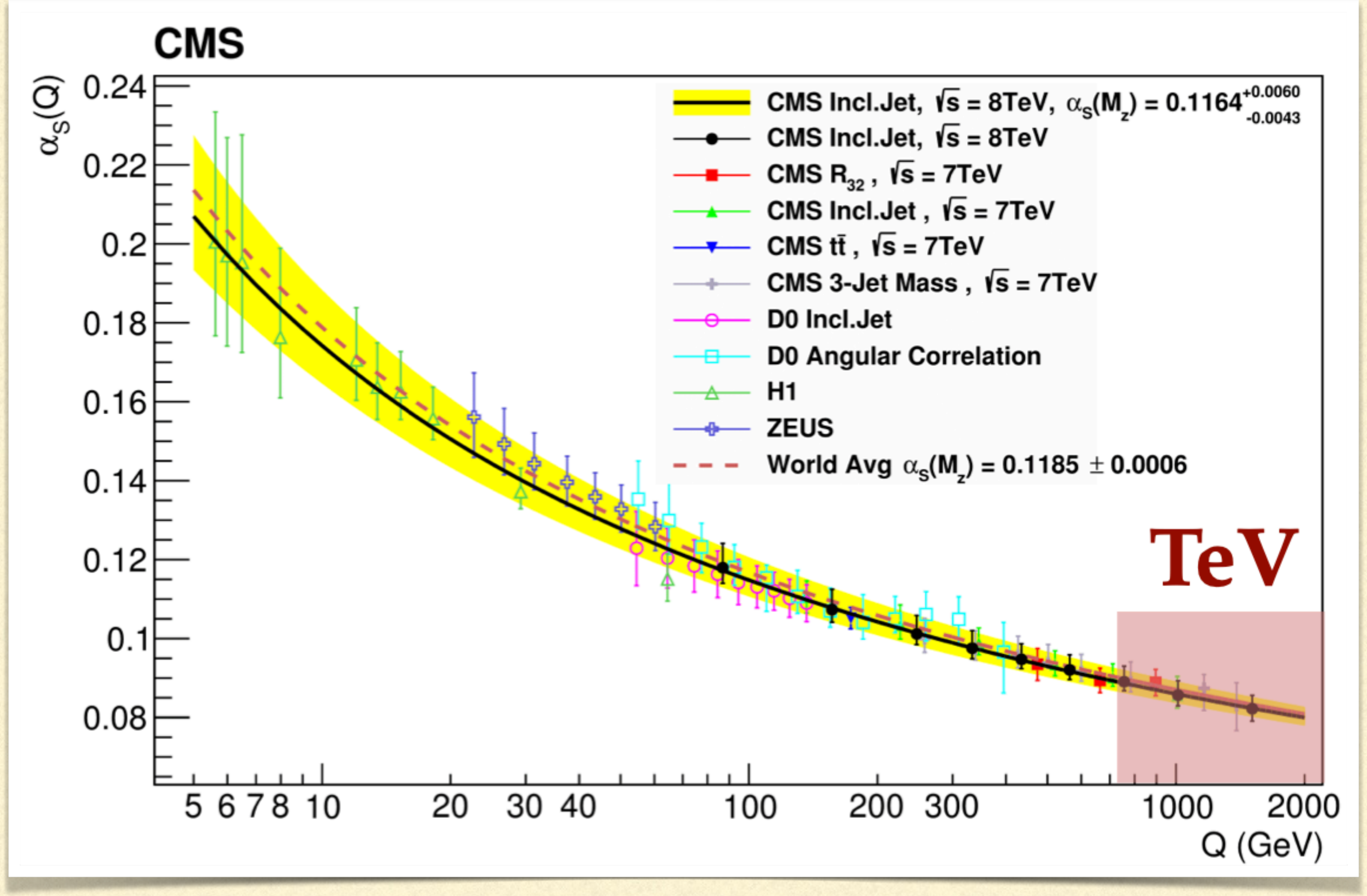
- Relevance: phenomenology + "formal" aspects
- Outline of the calculation
- Summary and outlook

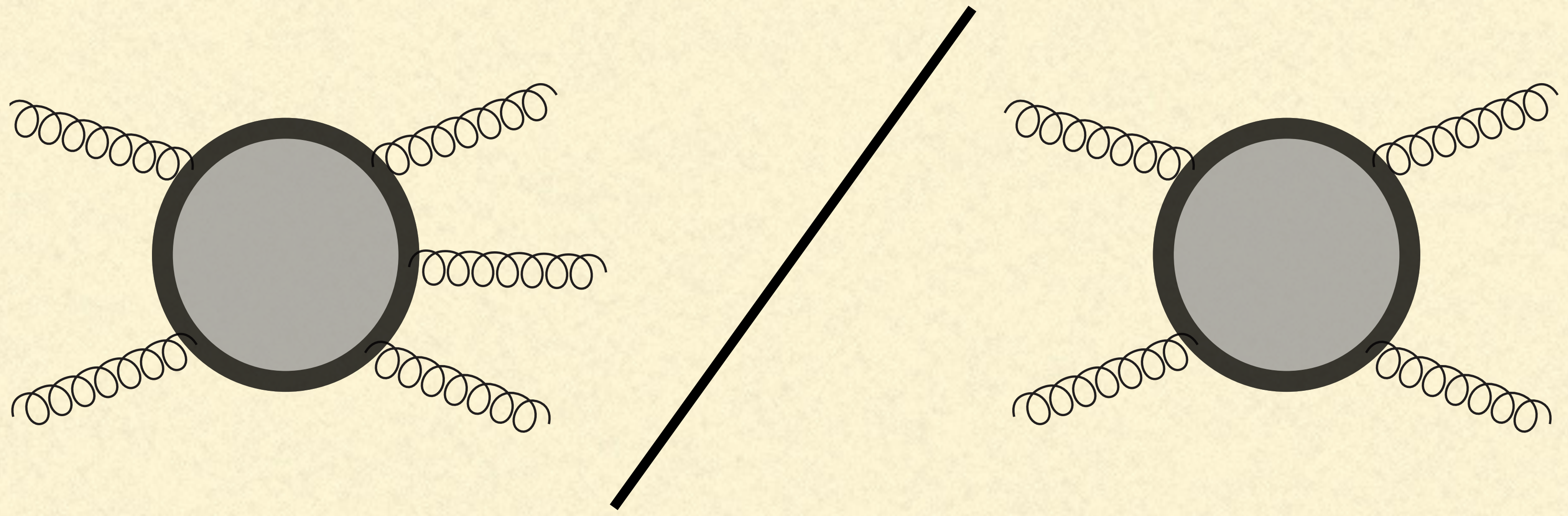
Computed in leading color approximation by Abreu et al. in [2102.13609]

This talk:
Full color

Three-jets @ LHC

- Three-to-two jet rates $R_{3/2}$ → **Extraction of α_s at LHC**



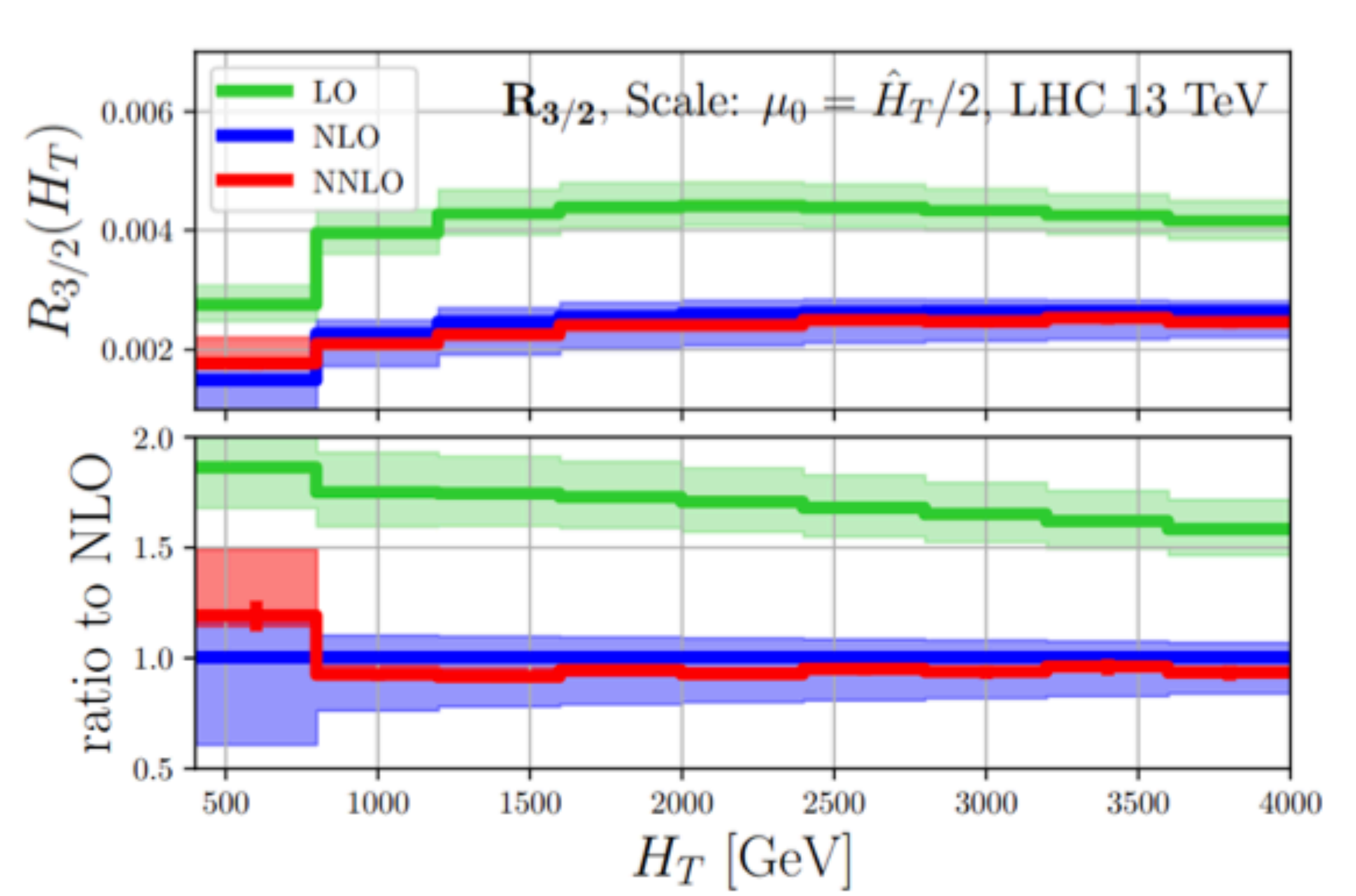


$\sim \alpha_s(Q)$

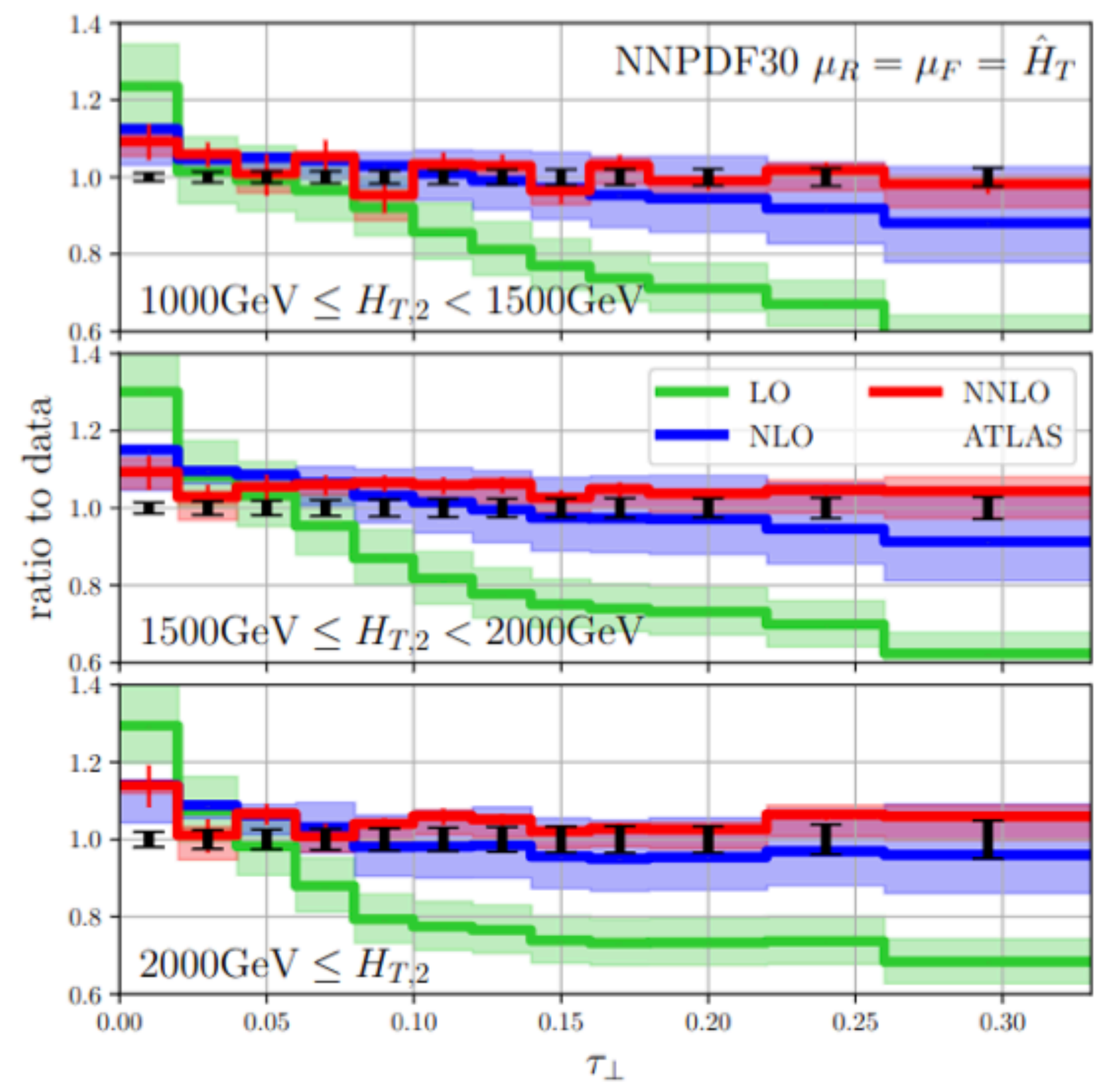
Running of strong coupling at TeV scale

Three-jets@NNLO QCD

First (impressive!) calculation of NNLO QCD corrections for 3 jets production at the LHC by Czakon, Mitov, Poncelet [2106.05331] & [2301.01086]



[Czakon, Mitov, Poncelet 2106.05331]



[Czakon, Mitov, Poncelet 2301.01086]

Caveat: double virtual contributions in leading color approximation

Study of QCD dynamics

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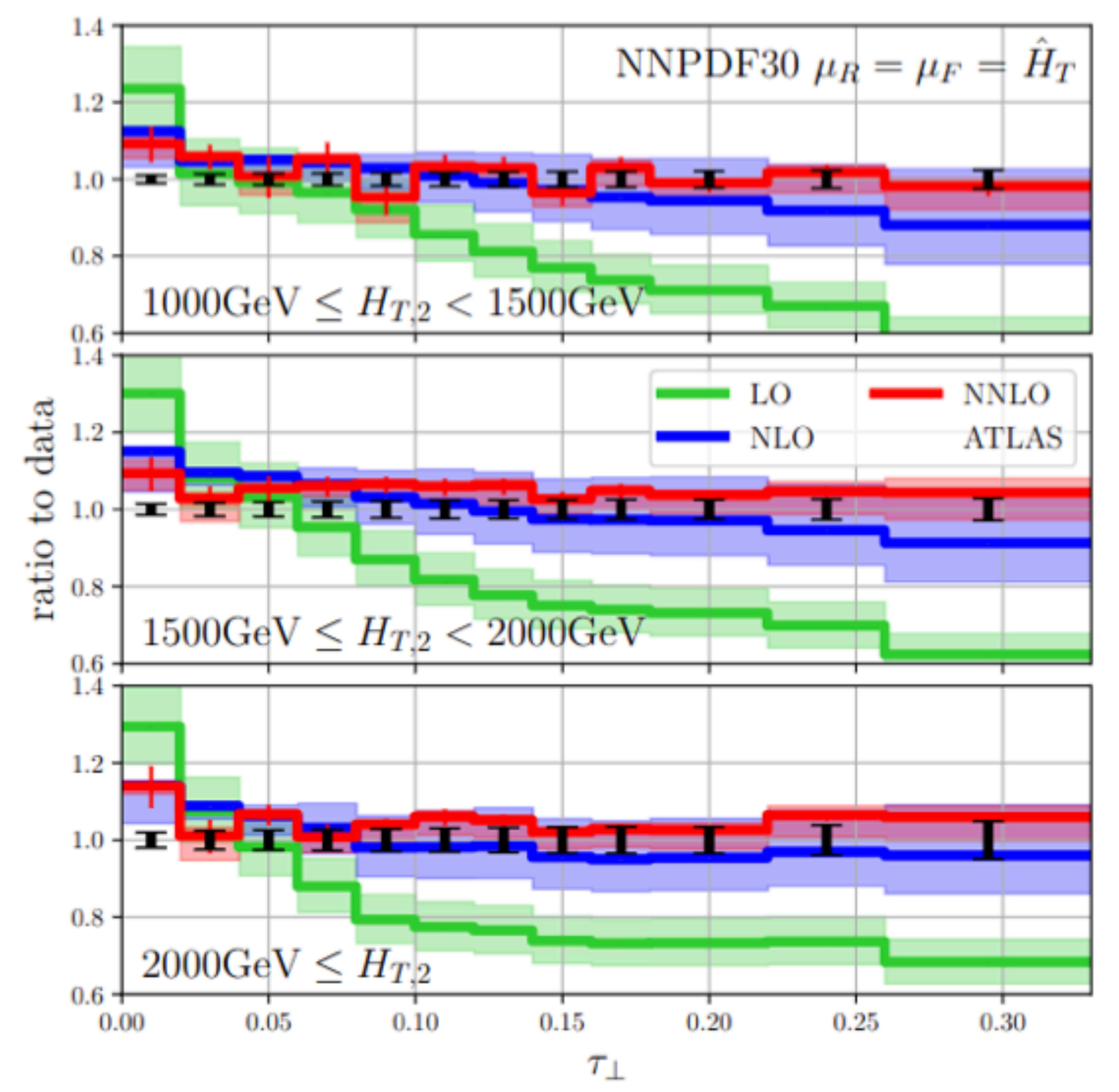

[Czakon, Mitov, Poncelet 2301.01086]

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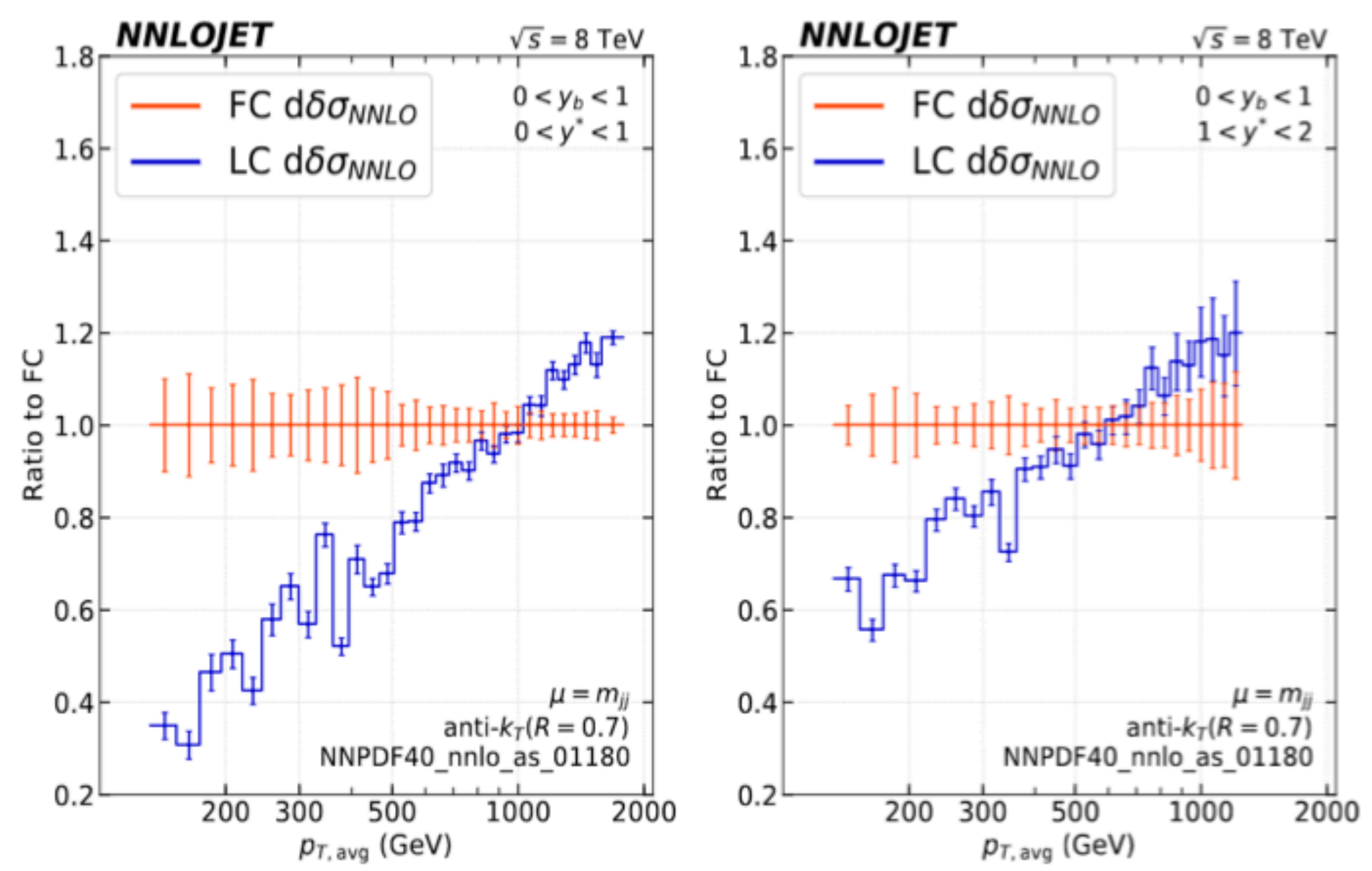

[Chen, Gehrmann, Glover, Huss, Mo 2204.10173]

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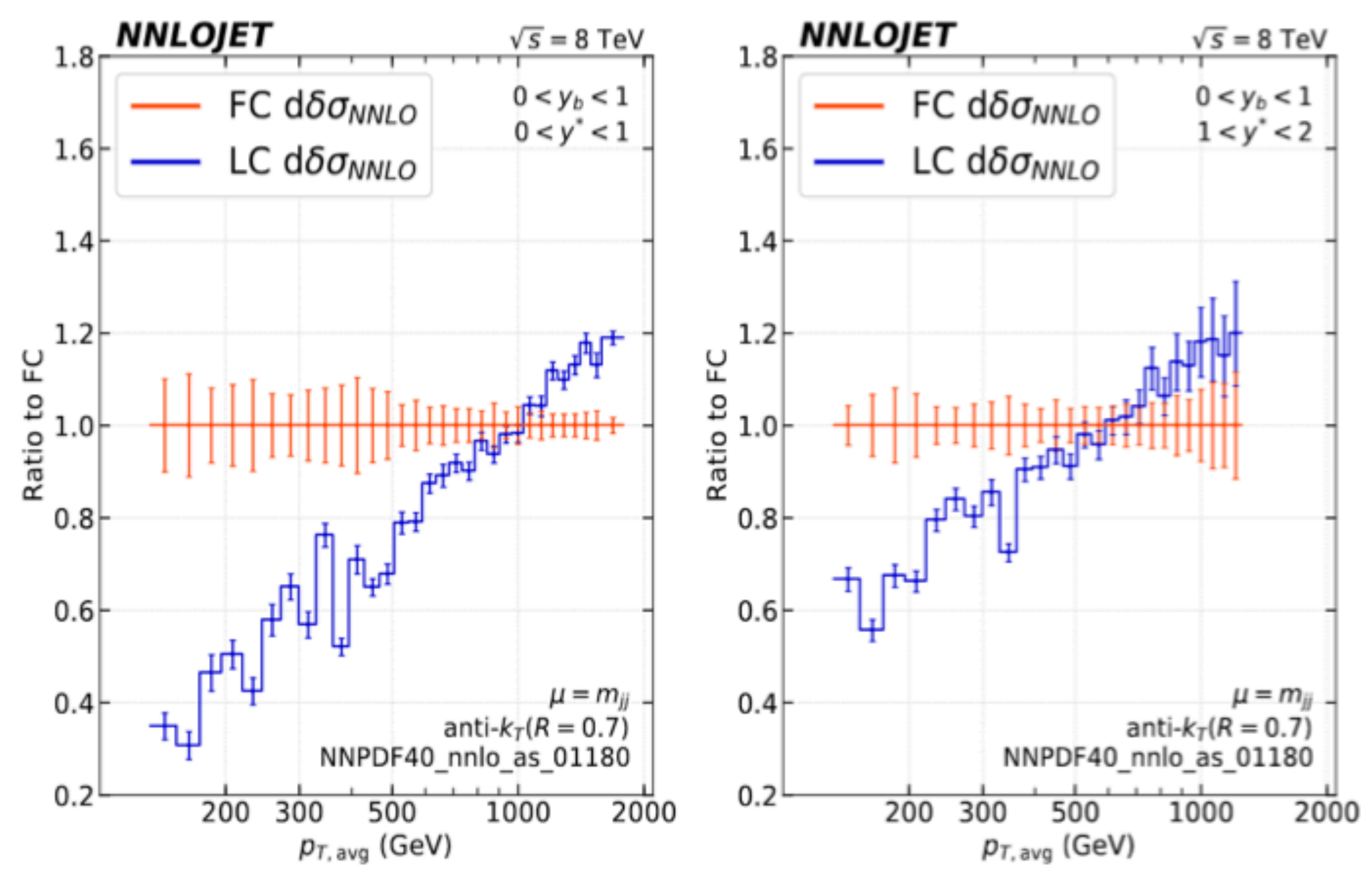


[Chen, Gehrmann, Glover, Huss, Mo 2204.10173]

Caveat: double virtual contributions in leading color approximation

Study of QCD dynamics

Triply differential dijet cross-section

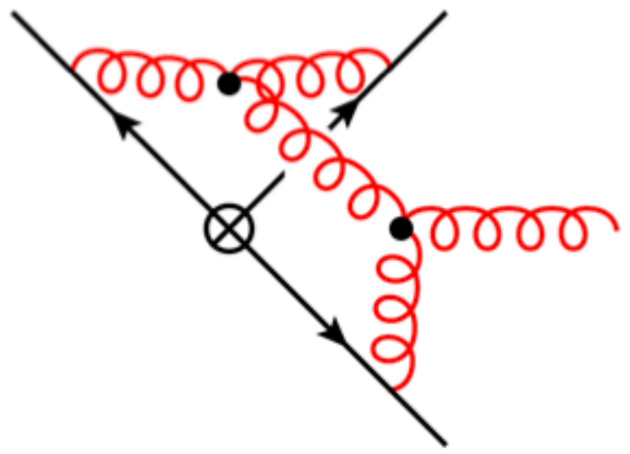
Structure of QCD amplitudes

Explore QCD non-planar sector

Most of these structures have been studied in more symmetric theories such as N=4 sYM

IR limits: soft/collinear
Example: "tripole" soft gluon current

$$\boldsymbol{S}^{+,(2)}_{a,ikj} = (V^q_{ij})^2 f^{aa_kb} f^{ba_ia_j} \boldsymbol{T}_i^{a_i} \boldsymbol{T}_j^{a_j} \boldsymbol{T}_k^{a_k} \left[\frac{\langle ik\rangle}{\langle iq\rangle\langle qk\rangle} F(z_k^{ij},\epsilon) - \frac{\langle jk\rangle}{\langle jq\rangle\langle qk\rangle} F(z_k^{ji},\epsilon)\right]$$



Purely non-planar!

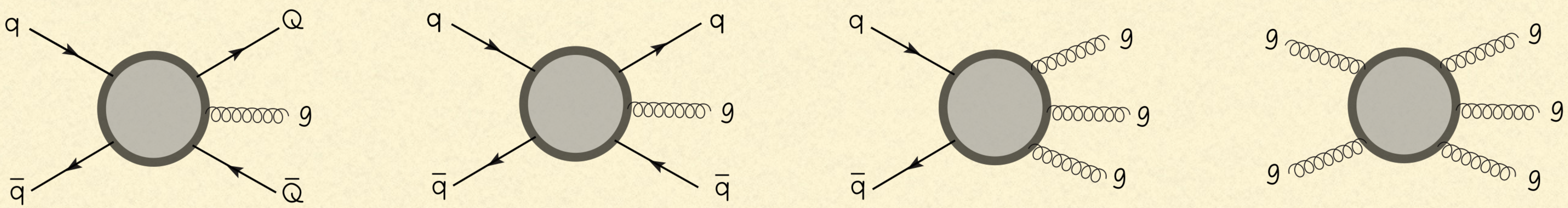
[Dixon, Herrmann, Yan, Zhu: 1912.09370]

Image taken from G. Gambuti

High energy limits:
multi-Regge
kinematics

DETAILS OF THE CALCULATION

$$\mathscr{A} = \sum \text{Feynman diagrams}$$



Feynman Diagrams	2522	4258	9136	28020
Helicities	8	8	16	32
Dimension colour space	4	4	11	22
Tot # colour structures	24	24	54	75

$$\mathscr{A} = \sum \text{Feynman diagrams}$$

Color decomposition

$$\mathscr{A}^{c_1 \dots c_5} = \sum_i^n A_i | c_1 \dots c_5 \rangle_i = \sum_i^n A_i | \mathscr{C}_i \rangle$$

Helicity decomposition

$$A_i \equiv A_i^{h_1 \dots h_5} = A_i^{\mu_1 \dots \mu_5} \epsilon_{\mu_1}^{h_1} \dots \epsilon_{\mu_5}^{h_5}$$

Projection to scalar form factors

$$A_i^{\mu_1 \dots \mu_5} = \sum_j F^j T_j^{\mu_1 \dots \mu_5}$$

IBP identities

$$F^j = \sum_k R^k I_k$$

Rational coefficients

Master integrals (MIs)

Algebraic manipulations + computation of MIs

Final result

$$\mathscr{A} = \sum \text{Feynman diagrams}$$

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Algebraic

This is where the magic happens

Final result

Full color vs Leading color

Decompose amplitude in color space

$$\mathscr{A}^{c_1\ldots c_5} = \sum_i^n A_i \,|\, c_1 \ldots c_5\rangle_i = \sum_i^n A_i \,|\, \mathscr{C}_i\rangle$$

A_i → Polynomials in Nc, nf

$|\mathscr{C}_i\rangle$ → Color basis

Example @ 2-loops:

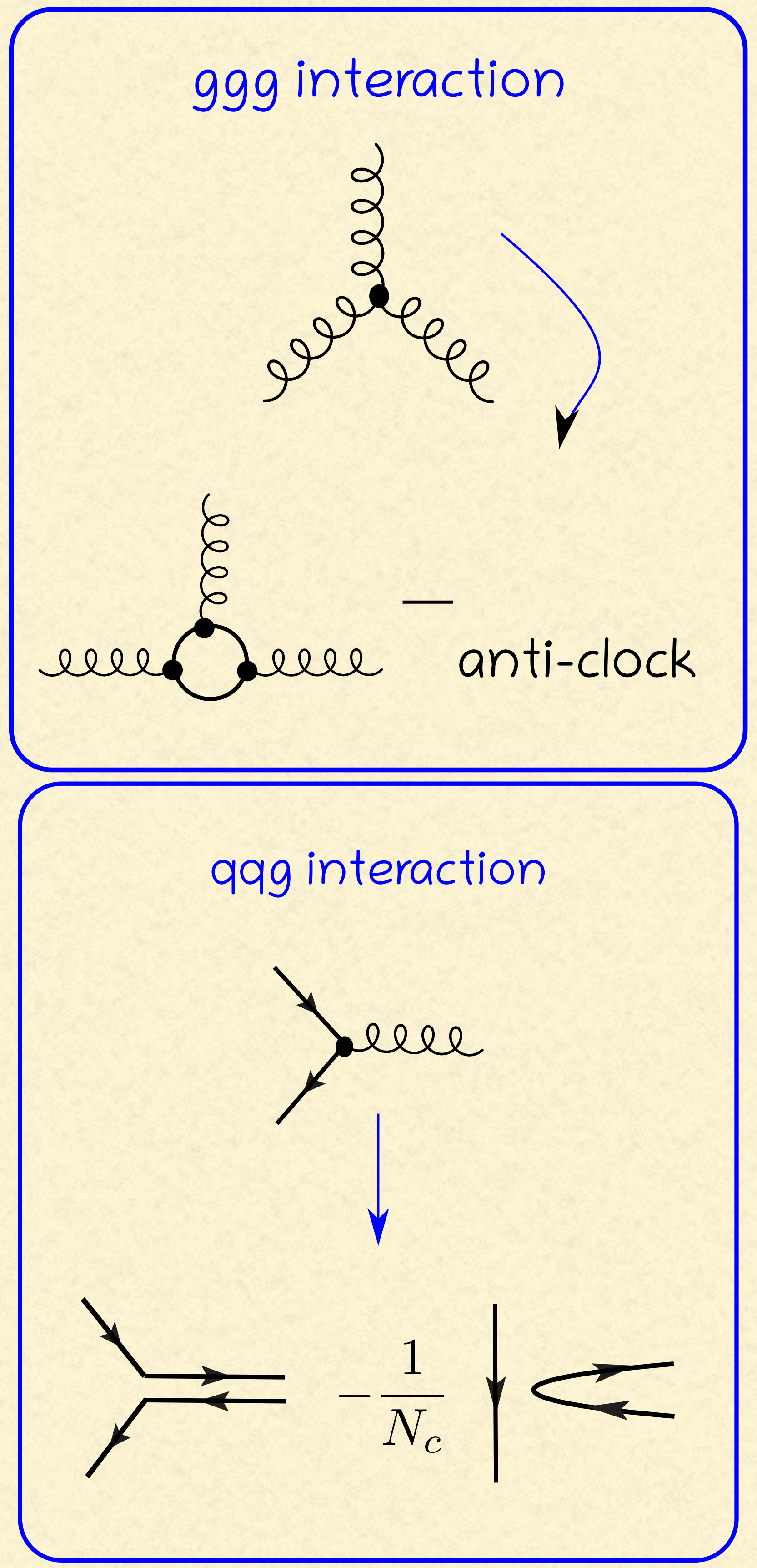
Subleading color

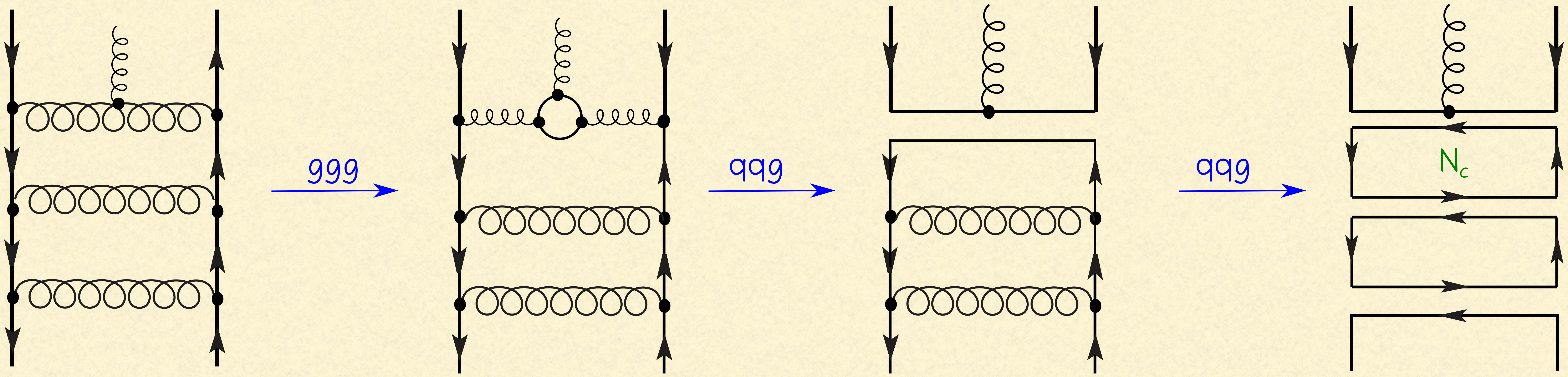
$$A_i = b_i^{(2,0)} N_c^2 + b_i^{(1,0)} N_c + b_i^{(0,0)} 1 + b_i^{(-1,0)} N_c^{-1} + b_i^{(-2,0)} N_c^{-2} + b_i^{(1,1)} N_c n_f + \ldots$$

Leading color

Full color vs Leading color - II

In QCD: **Leading color** ⟺ **Planar topologies** ['t Hooft]



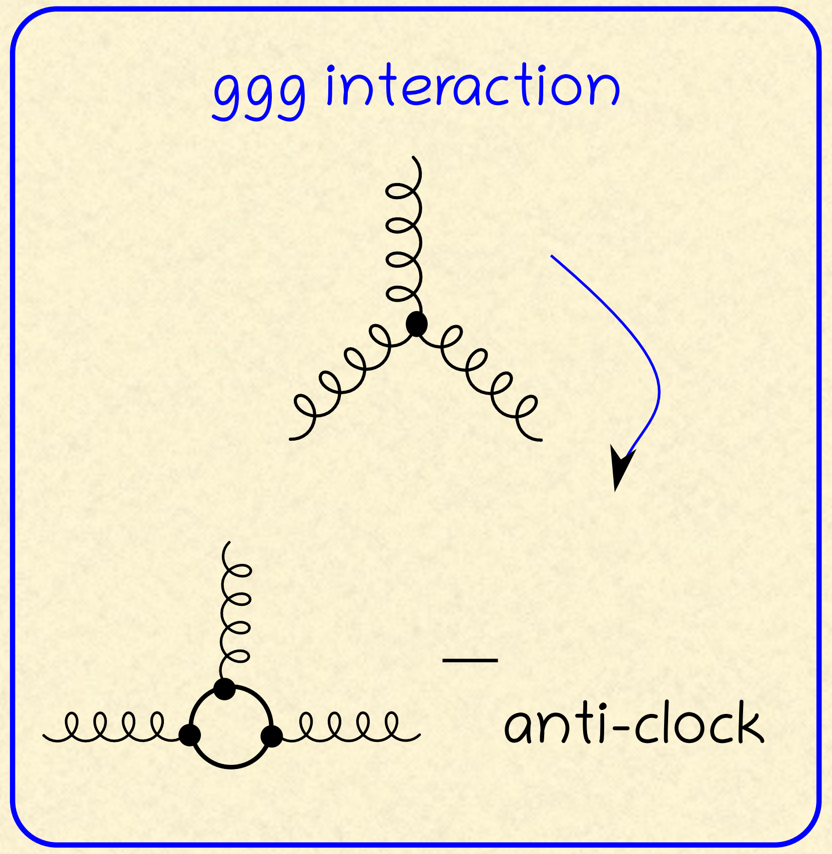


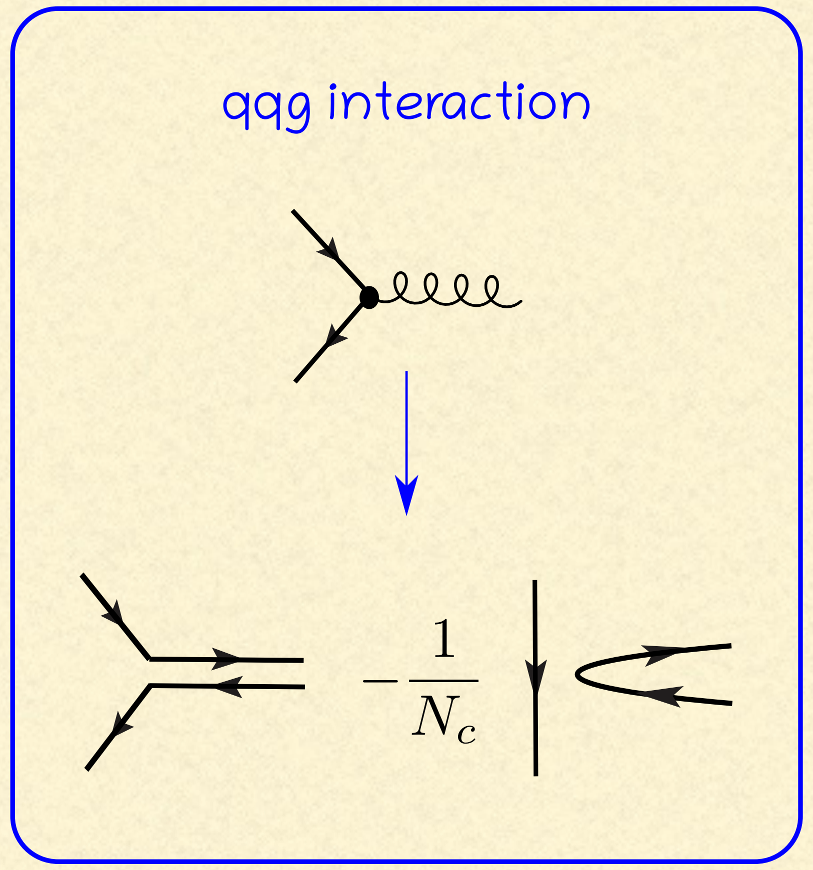
$\sim N_c^2$

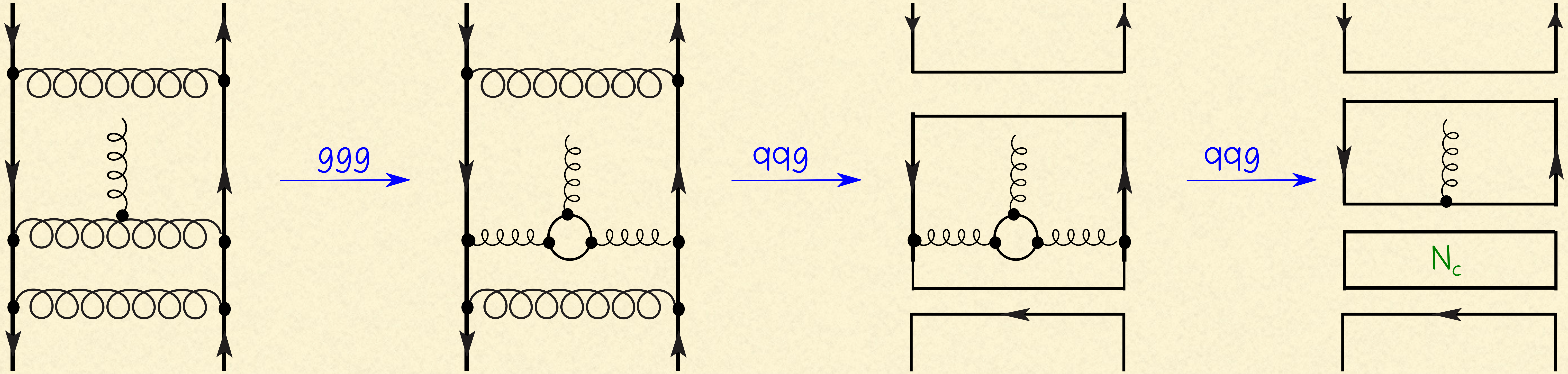
Full color vs Leading color - II

In QCD: **Leading color** → **Planar topologies**

['t Hooft]







$\sim N_c^{-2}$

Color decomposition

$$\mathscr{A}^{c_1\ldots c_5} = \sum_i^n A_i \,|\, c_1 \ldots c_5\rangle_i = \sum_i^n A_i \,|\, \mathscr{C}_i\rangle$$

Full color: span the whole color space

A_i → "Partial amplitudes"

$\vert\mathscr{C}_i\rangle$	$ggggg$	$q\bar{q}ggg$	$q\bar{q}Q\bar{Q}g$
Tree level	$\mathrm{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4}T^{a_5}) - \mathrm{Tr}(T^{a_5}T^{a_4}T^{a_3}T^{a_2}T^{a_1})$ + permutations	$(T^{a_1}T^{a_2}T^{a_3})_{ji}$ + permutations	$T^a_{ij}\delta_{kl}$ $T^a_{ik}\delta_{jl}$
Beyond tree	$\mathrm{Tr}(T^{a_1}T^{a_2}) \times (\mathrm{Tr}(T^{a_3}T^{a_4}T^{a_5}) - \mathrm{Tr}(T^{a_5}T^{a_4}T^{a_3}))$ + permutations	$\mathrm{Tr}(T^{a_1}T^{a_2})T^{a_3}_{ij}$ $(\mathrm{Tr}(T^{a_3}T^{a_4}T^{a_5}) - \mathrm{Tr}(T^{a_5}T^{a_4}T^{a_3}))\delta_{ij}$ $(\mathrm{Tr}(T^{a_3}T^{a_4}T^{a_5}) + \mathrm{Tr}(T^{a_5}T^{a_4}T^{a_3}))\delta_{ij}$	Same as tree

Helicity decomposition

$$A_i \equiv A_i^{h_1 \ldots h_5} = A_i^{\mu_1 \ldots \mu_5} \epsilon_{\mu_1}^{h_1} \ldots \epsilon_{\mu_5}^{h_5}$$

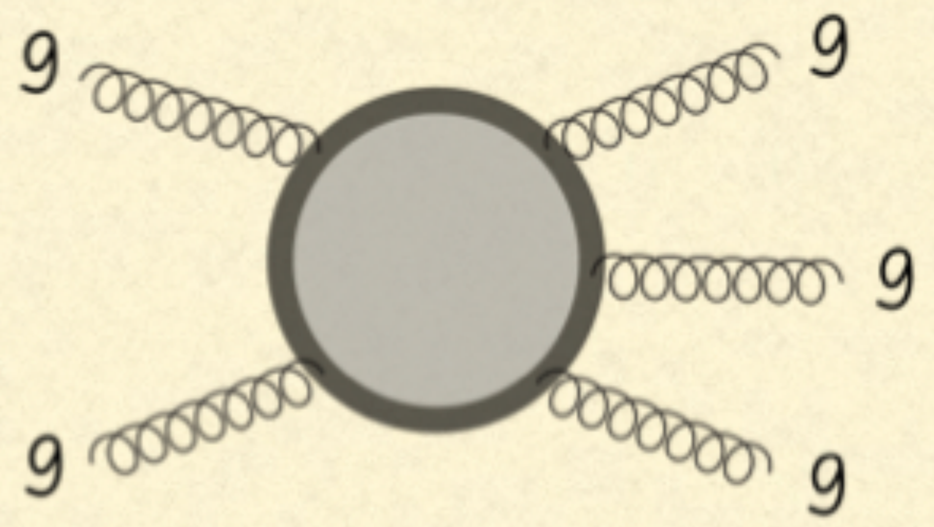
For n>4 can be constructed in terms of external momenta!

Independent in d=4

[Peraro, Tancredi 1906.03298, 2012.00820]

Transversality + choice reference vector

# of independent structures = # helicity configurations !



tHV scheme: external particles in d=4

$$A_i^{\mu_1 \ldots \mu_5} = \sum_{j=1}^{32} F^j \, T_j^{\mu_1 \ldots \mu_5}$$

Form Factors

Tensors

$$F^j = \sum_{pol} \mathscr{P}^j(\mathbf{h}) A(\mathbf{h})$$

Projectors

Reduction to master integrals

$$A_i^{\mu_1\ldots\mu_5} = \sum_{j=1}^{32} F^j T_j^{\mu_1\ldots\mu_5} \quad \longrightarrow \quad F^j = \sum_k R^k I_k$$

Scalar objects

IBP reduction

Master Integrals

Wise choice of MIs?

FinRed

[A.v. Manteuffel]

How FinRed overcomes the complexity:

- Finite-fields arithmetics [von Manteuffel, Schabinger 1406.4513; Peraro 1905.08019]
- Syzygy techniques [Gluza, Kadja, Kosower 1009.0472, Ita 1510.05626; Larsen, Zhang, 1511.01071, Agarwal, Jones, von Manteuffel 2011.15113]
- Denominator guessing [Abreu, Dormans, Febres Cordero, Ita, Page 1812.04586; Heller, von Manteuffel 2101.08283]

Reduction to master integrals

$$A_i^{\mu_1 \dots \mu_5} = \sum_{j=1}^{32} F^j \, T_j^{\mu_1 \dots \mu_5}$$

Scalar objects

Major bottleneck

eduction

FinRed

[A.v. Manteuffel]

$$F^j = \sum_k R^k I_k$$

Master Integrals

Wise choice of MIs?

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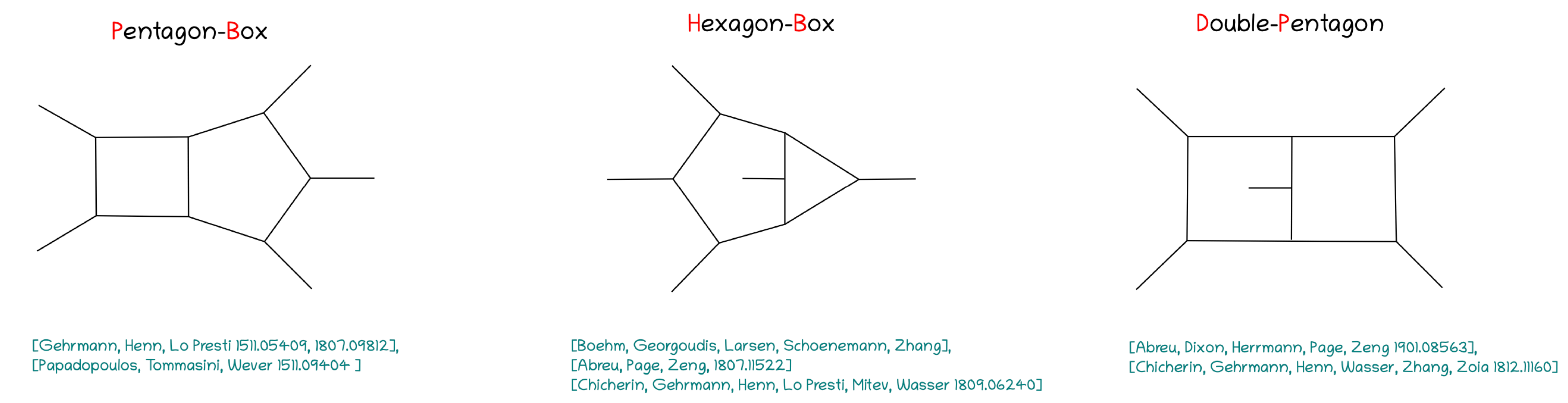
MIs: Pentagon Functions



Expressed as Chen iterated integrals, full set available

[Chicherin, Sotnikov 2009.07803]

- Canonical basis
- UT weight integrals

$$f^{(\omega)}(\mathbf{x}) = \int_\gamma \mathrm{d}\log W_{i_1} \ldots \mathrm{d}\log W_{i_n}$$

Evaluation time: $\sim 1s$

Reduction to master integrals - II

$$I(s_{ij}, d) = \sum_{k=1}^{M} a_k(s_{ij}, d) \mathcal{I}(s_{ij}, d)$$

Common denominator

$$a_k(s_{ij}, d) = \frac{N(s_{ij}, d)}{Q(d) D(s_{ij})}$$

Partial-fractioned form

$$a_k(s_{ij}, d) = \sum_{l} g_l(d) R_l(s_{ij})$$

Crucial step: drastic reduction in size!

We employ MultivariateApart for multivariate partial fractioning: [Heller, von Manteuffel 2101.08283]

- Avoids spurious denominators
- Produces unique results when applied to terms of a sum separately

Simplifying the rational coefficients

$$a_k(s_{ij}, d) = \frac{N(s_{ij}, d)}{Q(d)D(s_{ij})} \longrightarrow a_k(s_{ij}, d) = \sum_l g_l(d)\, R_l(s_{ij})$$

MultivariateApart + Singular

[Heller, von Manteuffel 2101.08283][Decker, Greuel, Pfister, Schoenemann]

Drastic simplifications occur:

	GCD		PF
PB: INT[TA,8,255,8,5,{1,1,1,1,1,1,1,1,-5,0,0}]	162 MB	→	3.9 MB
HB: INT[TB,8,255,8,5,{1,1,1,1,1,1,1,1,-4,0,-1}]	513 MB	→	9.9 MB
DP: INT[TB,8,510,8,5,{0,1,1,1,1,1,1,1,1,-5,0}]	2.9 GB	→	24 MB

$\sim \mathcal{O}(100)$!!

Final result: infrared structure

$$A_i(\mathbf{h}) = \sum_l r_l(s_{ij}, \epsilon_5) f_l(s_{ij}, \epsilon_5)$$

UV and IR subtraction

$$\mathbf{Z}^{-1} = 1 - \frac{\alpha_s}{2\pi}\mathbf{I}_1 - \left(\frac{\alpha_s}{2\pi}\right)^2 \mathbf{I}_2$$

$$A_{ren}(\epsilon, p_i) = \mathbf{Z}(\epsilon, p_i, \mu) A_{fin}(p_i, \mu)$$

I_1 and I_2 diagonal in LC approximation, but not in full color

$$\mathbf{I}_1(\epsilon) = \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \sum_i \left(\frac{1}{\epsilon^2} - \frac{\gamma_0^i}{2\epsilon}\frac{1}{\mathbf{T}_i^2}\right) \sum_{j\neq i} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \left(\frac{\mu^2}{-s_{ij}}\right)^\epsilon$$

$$\mathbf{I}_2(\epsilon) = \frac{e^{-\epsilon\gamma_E}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \sum_i \left(\frac{\gamma_1^{cusp}}{8} + \frac{\beta_0}{2\epsilon}\right) \mathbf{I}_1(2\epsilon) - \frac{1}{2}\mathbf{I}_1(\epsilon)\left(\mathbf{I}_1(\epsilon) + \frac{\beta_0}{\epsilon}\right) + \mathbf{H}_2(\epsilon)$$

Contains non-trivial triple color correlated contributions beyond LC

Final result: checks

1-loop:

- Match prediction for IR poles @ NLO
- For 5 gluon amplitudes: check color trace identities [Bern, Kosower Nucl.Phys.B 362 (1991)]
- Reproduce available results and checked vs OpenLoops

2-loop:

- ggggg $\to 0, q\bar{q}ggg \to 0, q\bar{q}Q\bar{Q}g \to 0$ completed
- Match prediction for IR poles @ NNLO
- For ggggg: agreement with LC result [Abreu, Febres Cordero, Ita, Page, Sotnikov: 2102.13609]
- For ggggg: agreement with all-plus full color Yang Mills result [Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia 1905.03733]

TO DO:

- Check against LC result for other partonic channels
- Check color trace identities for ggggg

Outlook and conclusions

- Calculation of 5-points 2-loops QCD massless amplitudes in full color
- In principle everything is there to study numerical impact and analyse structure
- In practice, final result still needs to be massaged and simplified (understand/remove spurious singularities, stability tests in soft/collinear configurations, representation of rational functions)

For the future…

- Make amplitude available and use it for pheno applications (study FC vs LC)
- Multi-Regge kinematics of QCD amplitudes + IR limits (soft-gluon current etc.)

THANK YOU!