Collider Phenomenology (1)

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Plan for the lectures

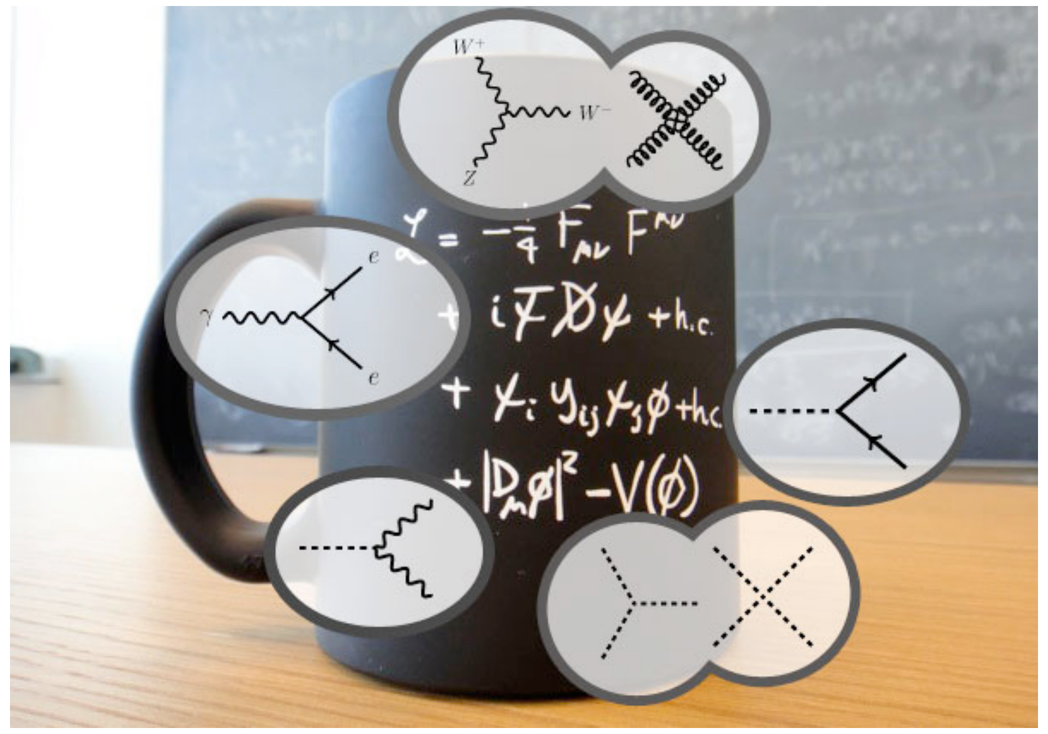
- Basics of collider physics
- Basics of QCD
    - DIS and the Parton Model
    - Higher order corrections
    - Asymptotic freedom
    - QCD improved parton model
- State-of-the-art computations for the LHC
- Monte Carlo generators
- Higgs phenomenology
- Top phenomenology
- Searching for New Physics: EFT

Basics of collider physics

Goals of collider physics:

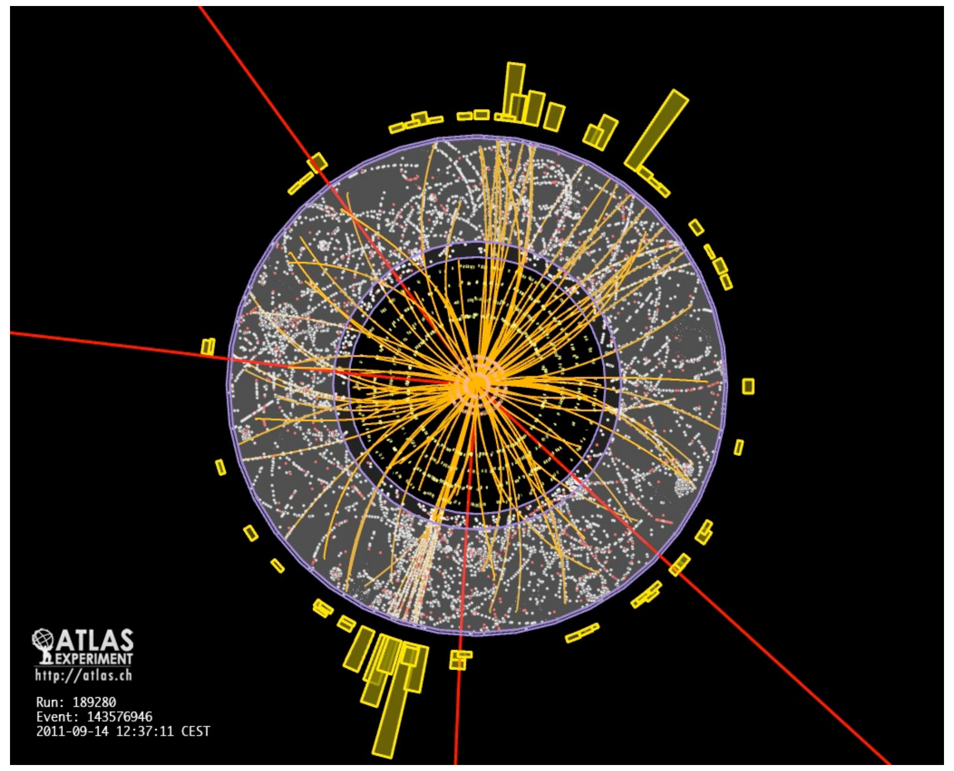
- Test theoretical predictions: Standard Model and New Physics
- Hopefully find the unexpected!

Collider physics

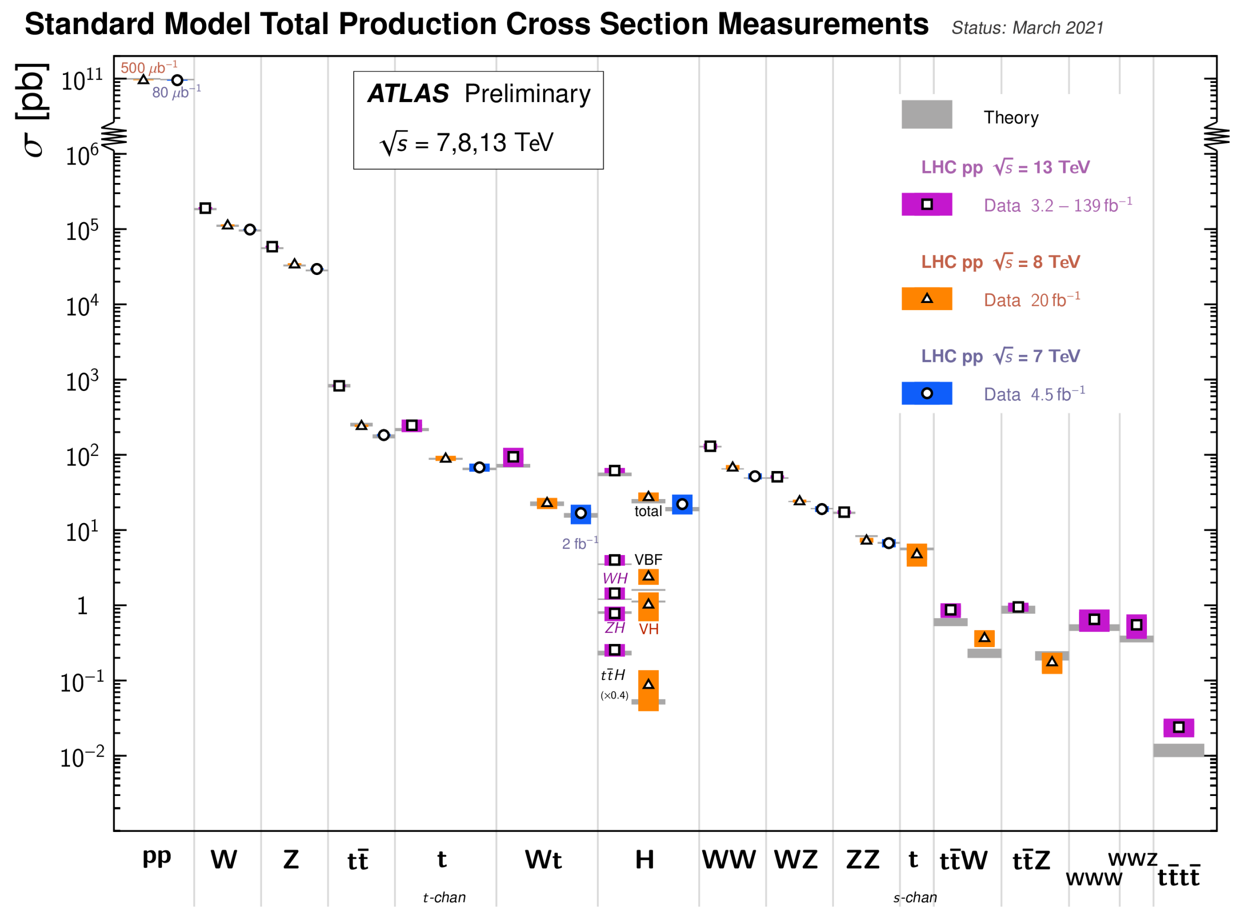


Theory

Interpretation



Experiment



Need good control of every step

Historical perspective

Why bother? Because it works!

Collider	When	What particle	Energy	Main Impact
SPS-CERN	1981-1984	pp	600 GeV	W/Z bosons
Tevatron	1983-2011	ppbar	2 TeV	Top quark
LEP-CERN	1989-2000	e+e-	210 GeV	Precision EW
HERA-DESY	1992-2007	ep	320 GeV	QCD/PDFs
BELLE	1999-2010	e+e-	10 GeV	Flavour physics
LHC	2009-Today	pp	7/8/13 TeV	Higgs…

Future of collider physics?

FCC
hh ee he

clic

International
MUON Collider
Collaboration

ilc
international linear collider

CEPC

Collider reach

How heavy a particle can be produced?

$$A + B \to X \qquad\qquad M_X^2 = (p_1 + p_2)^2$$

Fixed target experiment:

$$p_1 \simeq (E,0,0,E)$$
$$p_2 = (m,0,0,0)$$

$$M_X \simeq \sqrt{2mE}$$

Collider experiment:

$$p_1 \simeq (E,0,0,E)$$
$$p_2 \simeq (E,0,0,-E)$$

$$M_X \simeq 2E$$

Better energy scaling for collider experiment

Note: fixed target can benefit from dense target

Collider aspects

Luminosity: rate of particles in colliding bunches

$$\mathscr{L} = \frac{N_1 N_2 f}{A}$$

N_i number of particles in bunches

f bunch collision rate

A transverse bunch area

Integrated Luminosity: $L = \int \mathscr{L} dt$

Number of events for process with cross-section σ: $L\sigma$

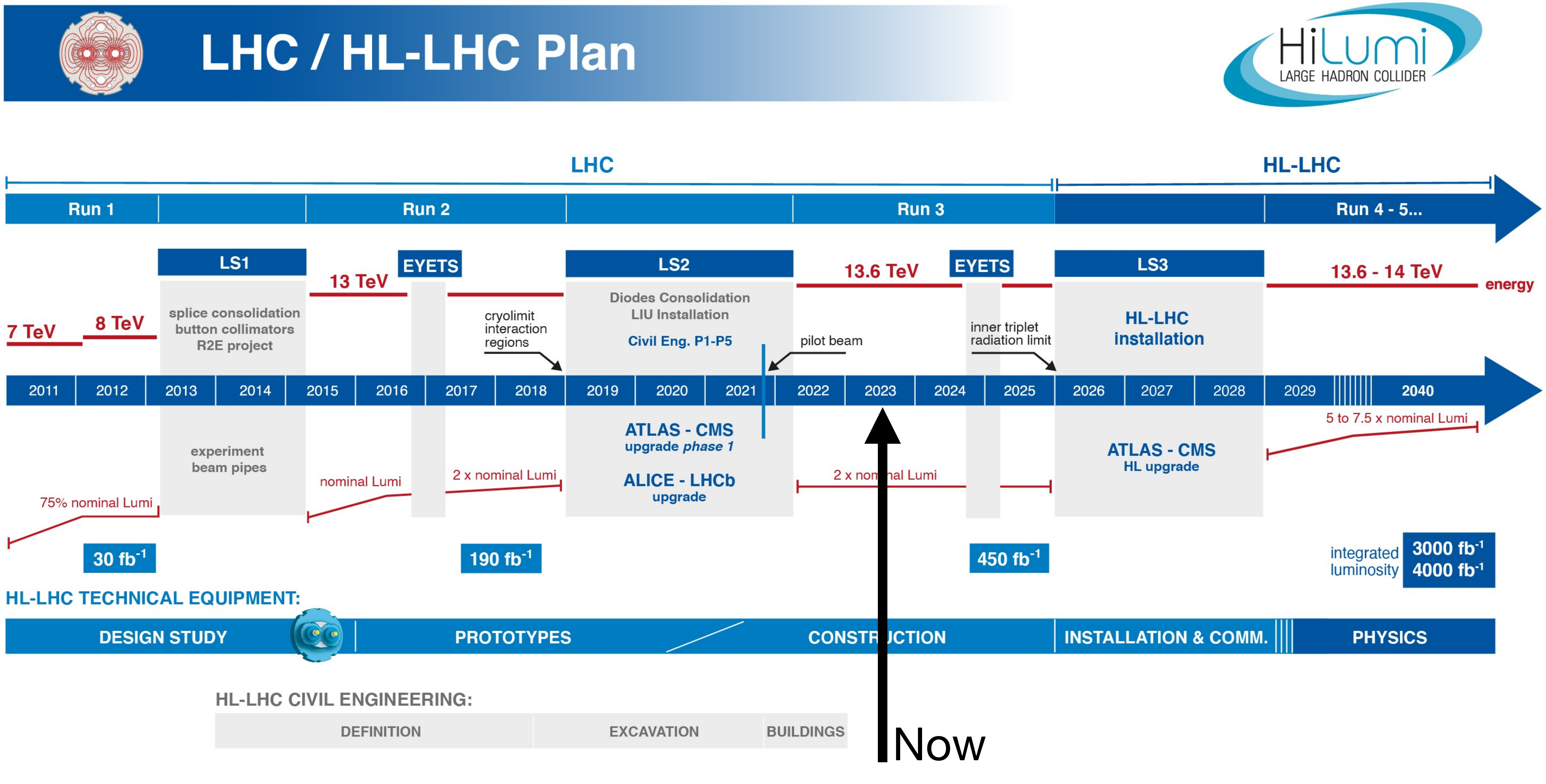
LHC luminosity Run II $L = 300\ \text{fb}^{-1}$

Circular vs linear: circular colliders are compact, but suffer from synchrotron radiation

Lepton vs Hadron: Lepton colliders, all energy available in the collision

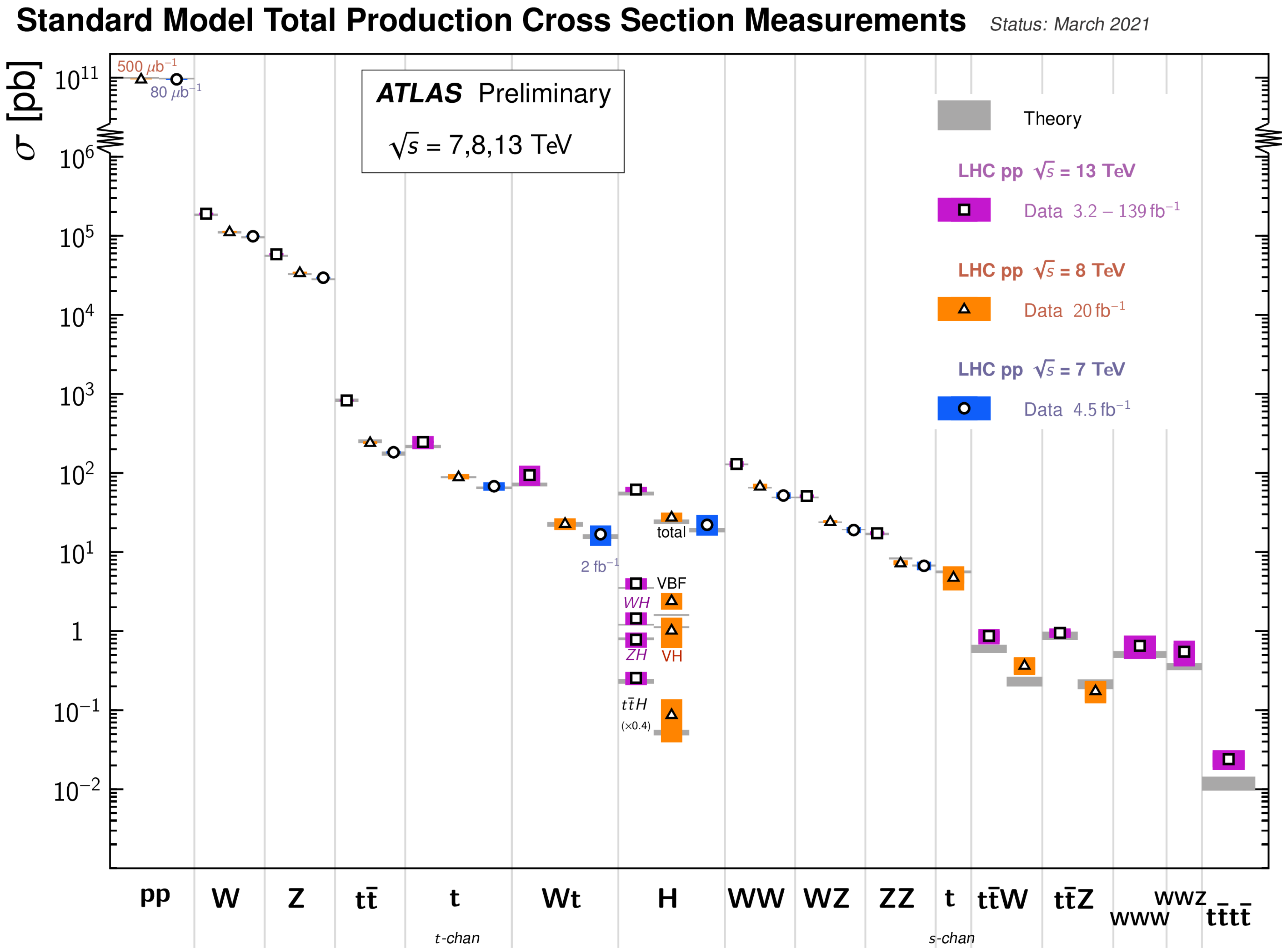
Hadron colliders, energy available determined by PDFs but can generally reach higher energies

LHC: a hadron collider

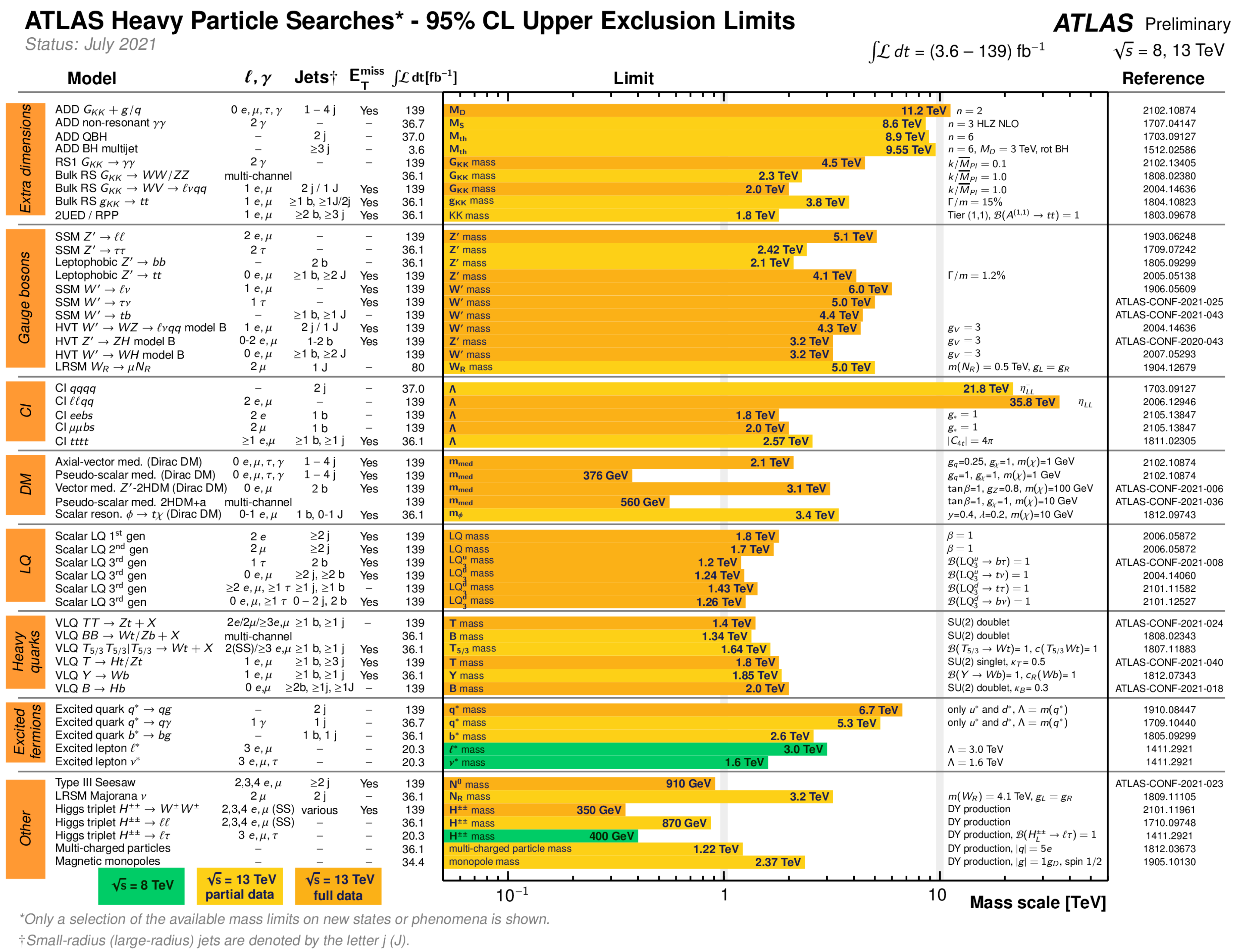


LHC status

Rediscovering the SM



Searching for the unknown



Good agreement with the SM

LHC physics

What's next?

No sign of new physics! Searches for deviations continue

New Physics can be:
Weakly coupled: Small rates means that more Luminosity can help

Exotic: Need new ways to search for it, going beyond standard searches or even beyond high-energy colliders

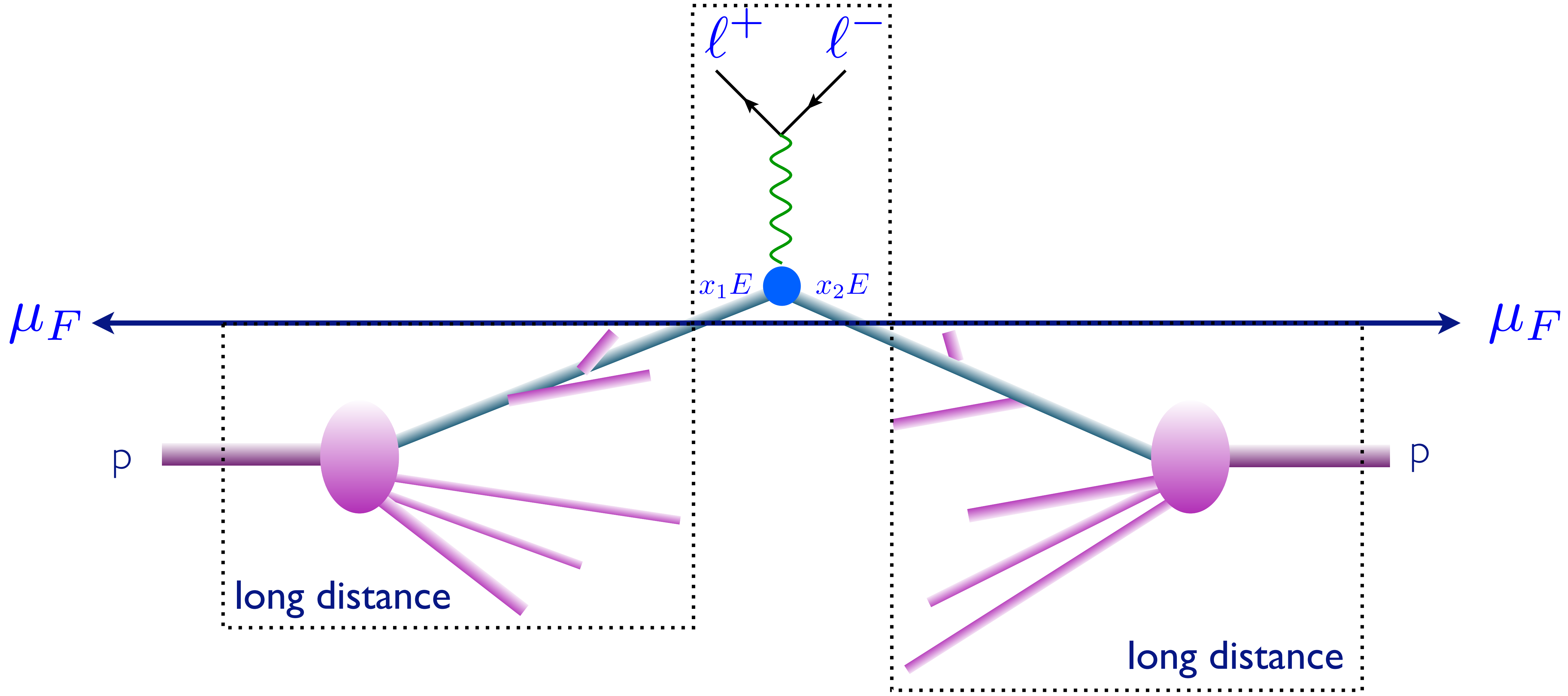
Heavy: Not enough energy to produce it
Need indirect searches: SMEFT

What is next for LHC physics

- New Physics is hiding well!
- Need to probe small deviations from the Standard Model using very precise predictions.
- Precise predictions are needed for both the SM and BSM.

In this course we will study the ingredients which enter in theoretical predictions and interpretations of LHC data!

How to compute cross-sections for the LHC?



$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Phase-space integral Parton density functions Parton-level cross section

Master formula for LHC physics

$$\sum_{a,b}\int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Phase-space integral

Important aspect of a Monte Carlo generator

Parton density functions

Universal:

~Probabilities of finding given parton with given momentum in proton

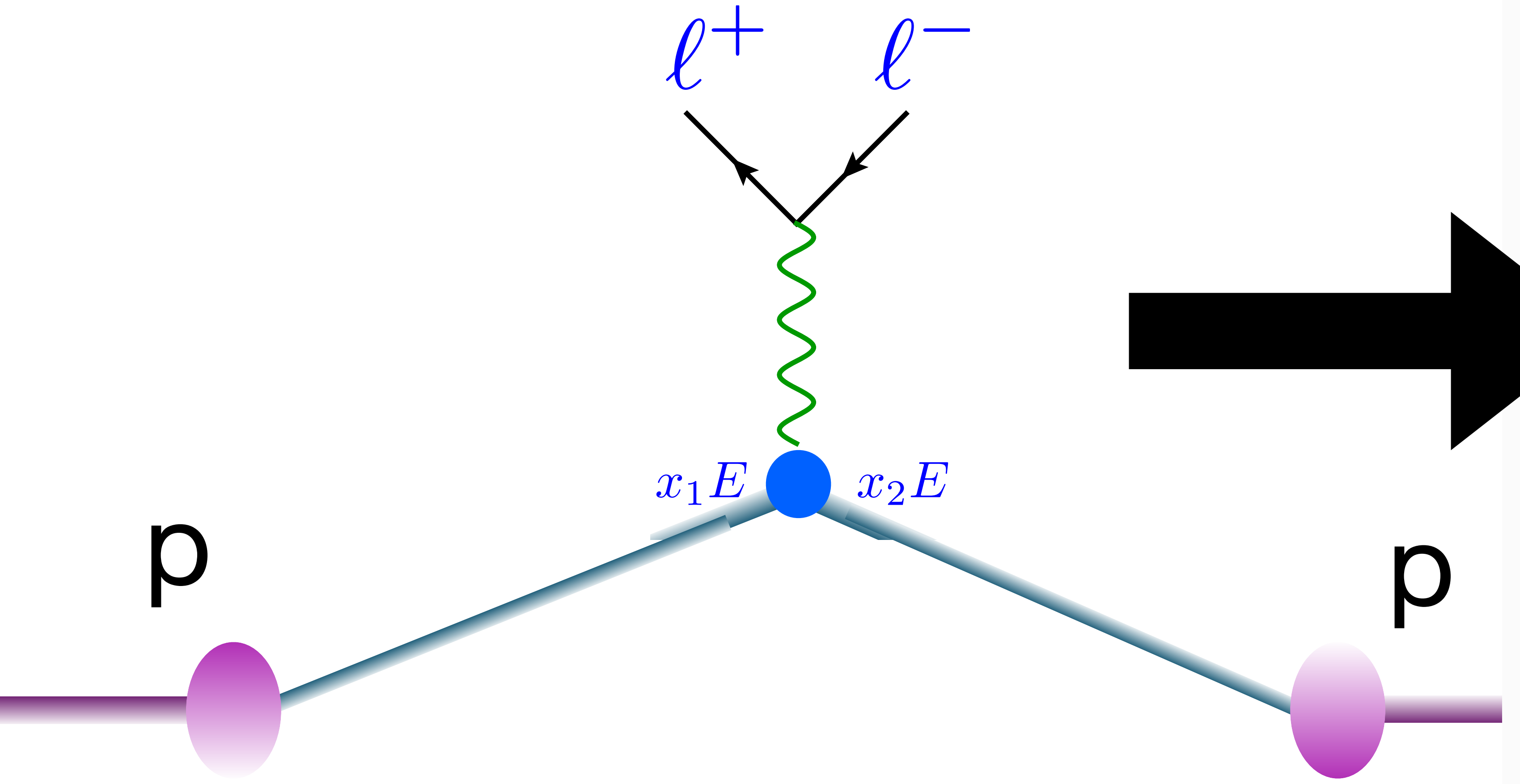
Extracted from data

Parton-level cross section

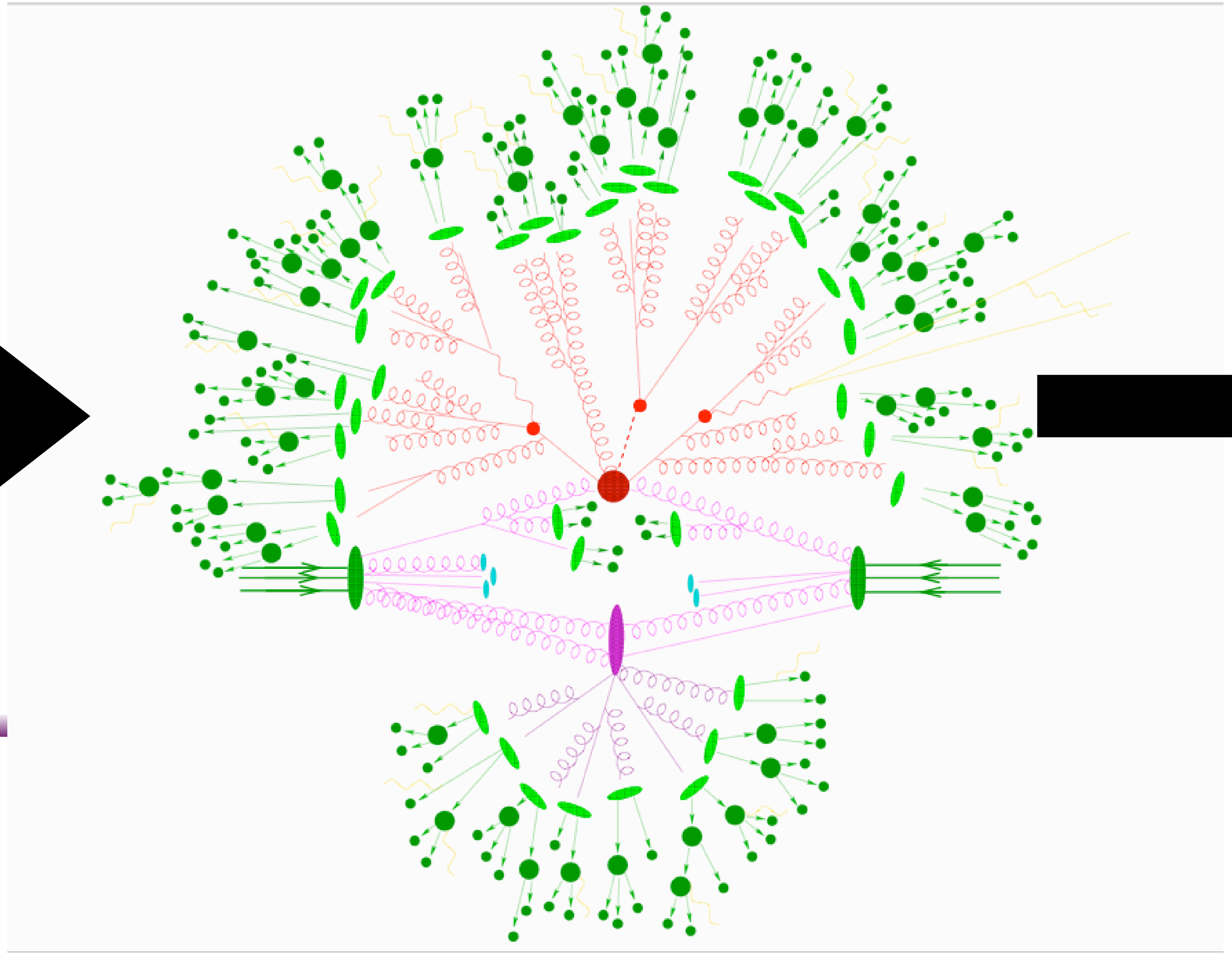
Subject of huge efforts in the LHC theory community to systematically improve this

We will study in detail this formula this week!

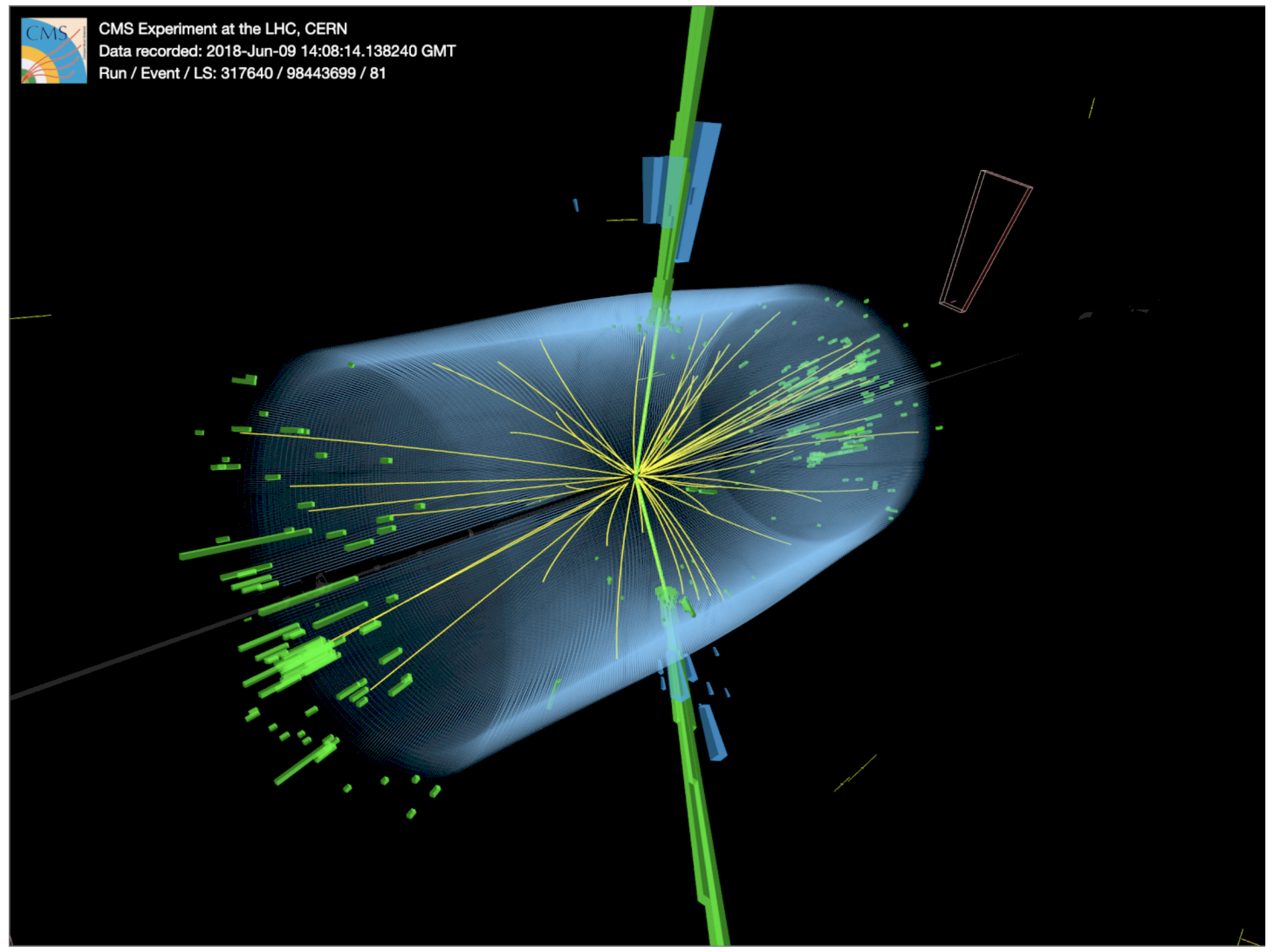
From the hard scattering to events



Ideally

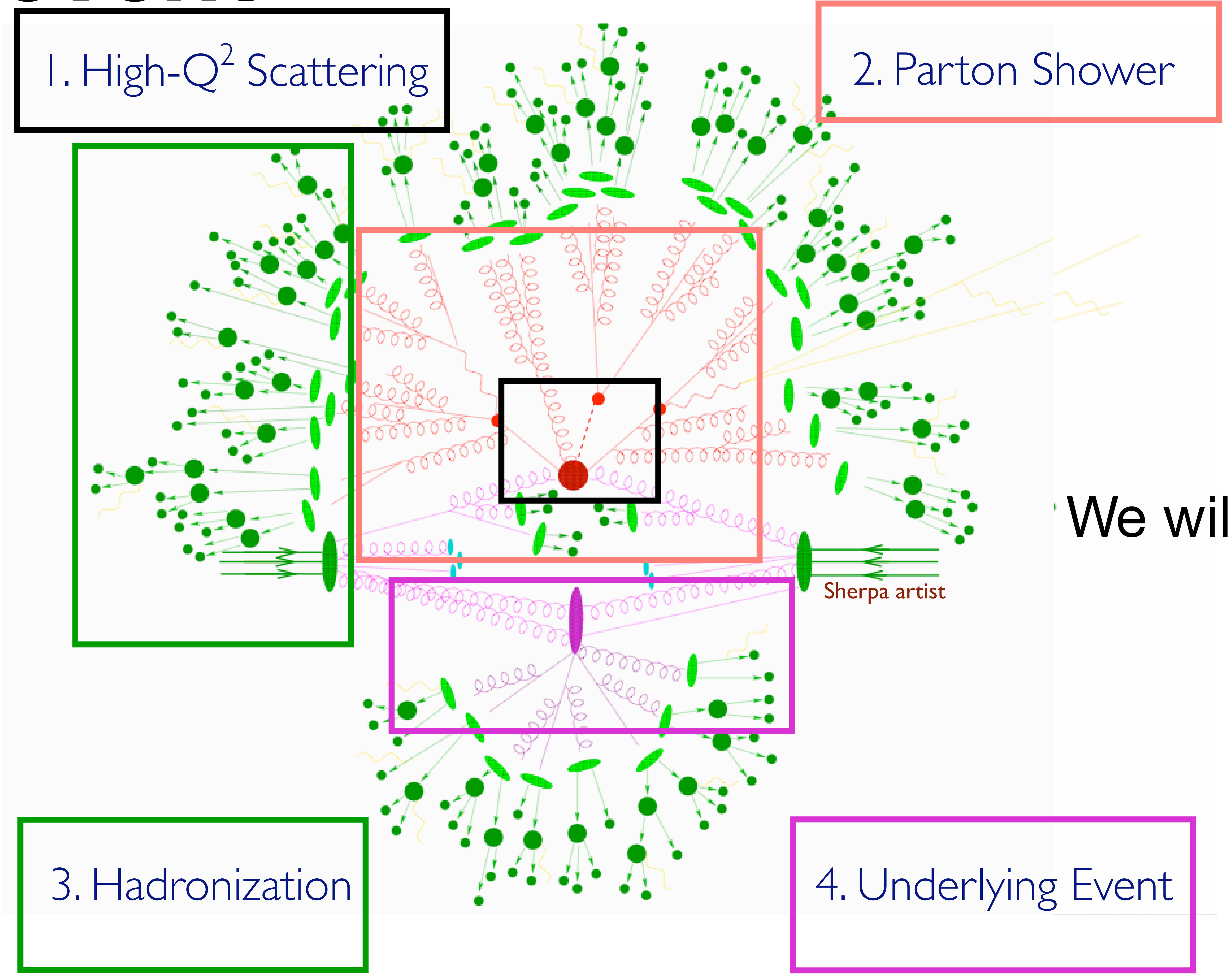


Artist's impression of reality



Reality

An LHC event



We will discuss all of these!

QCD…

LHC is a proton-proton collider:

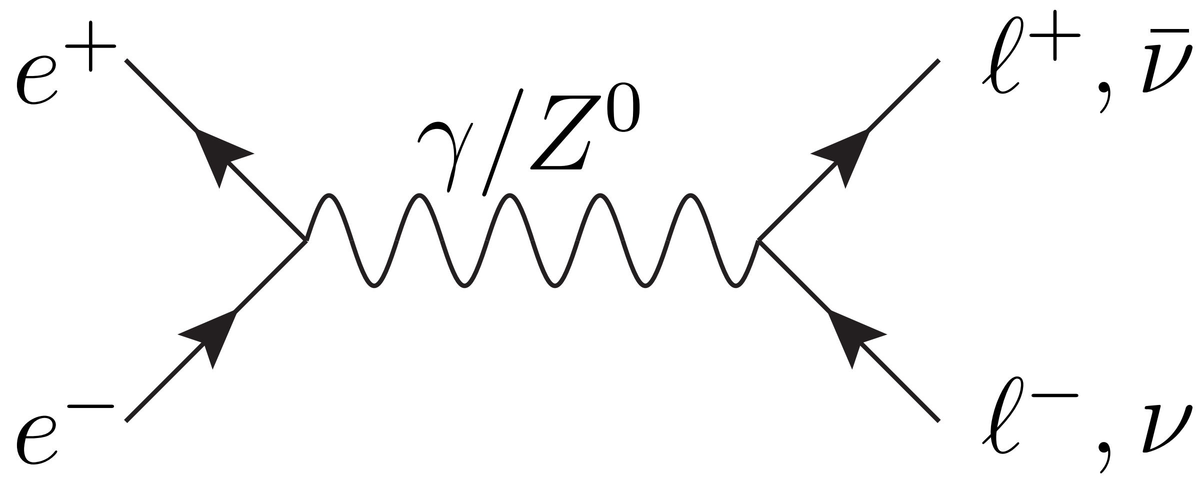
- colliding particles are proton constituents which are coloured particles

QCD plays a crucial role in what we eventually observe in the detectors

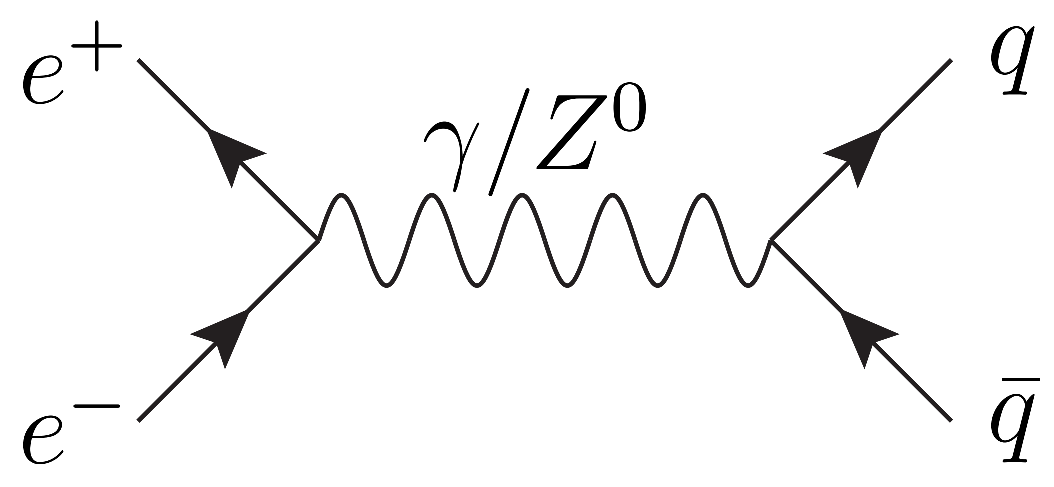
Why is QCD “special”? Let’s compare it to what we know best: QED

From QED to QCD

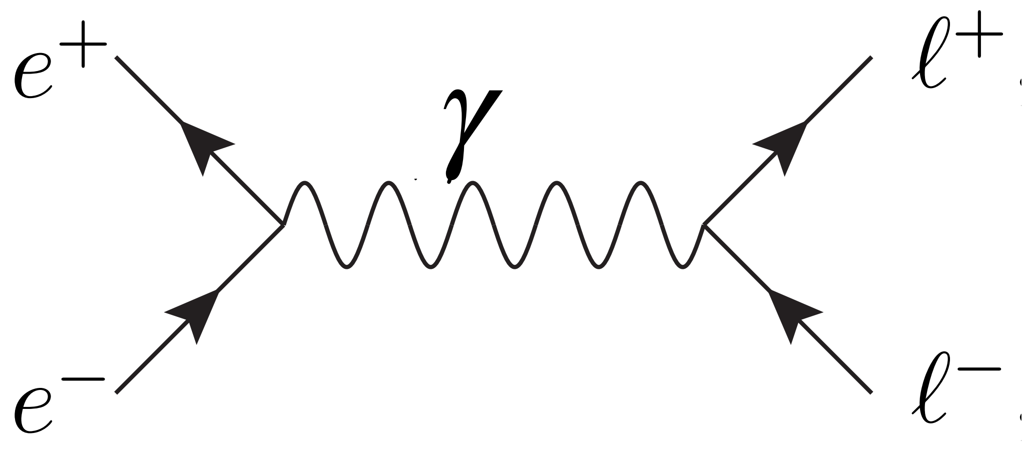
Example 1: R-ratio

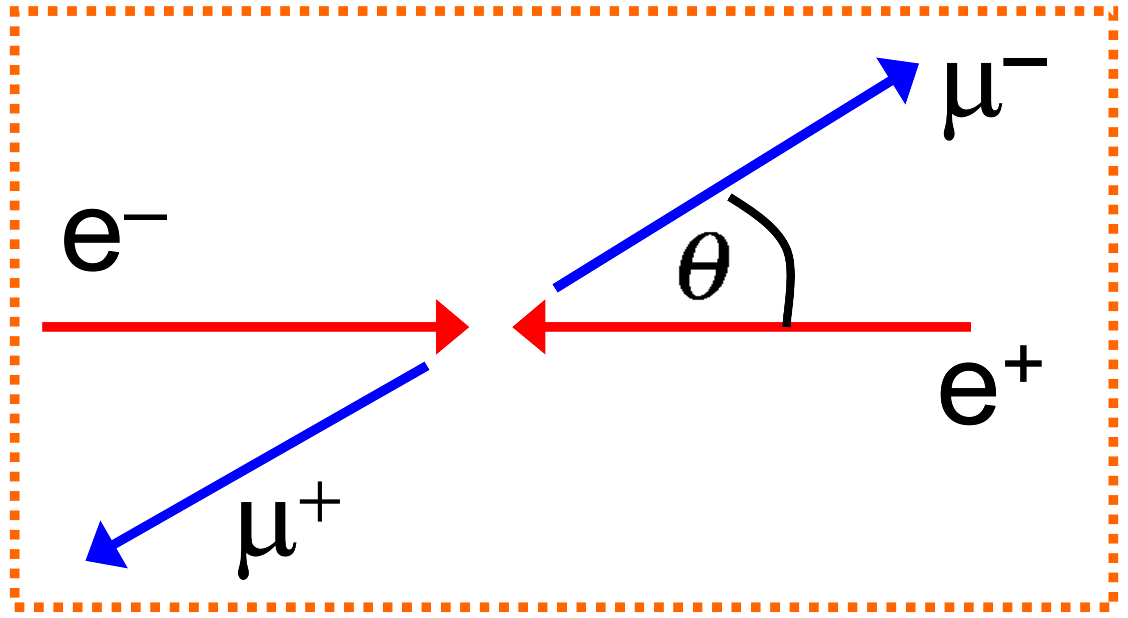


vs



Let's compute the matrix element for:





Summing and averaging:

$$\bar{\sum} |M|^2 = \frac{2e^4}{s^2}[t^2 + u^2]$$

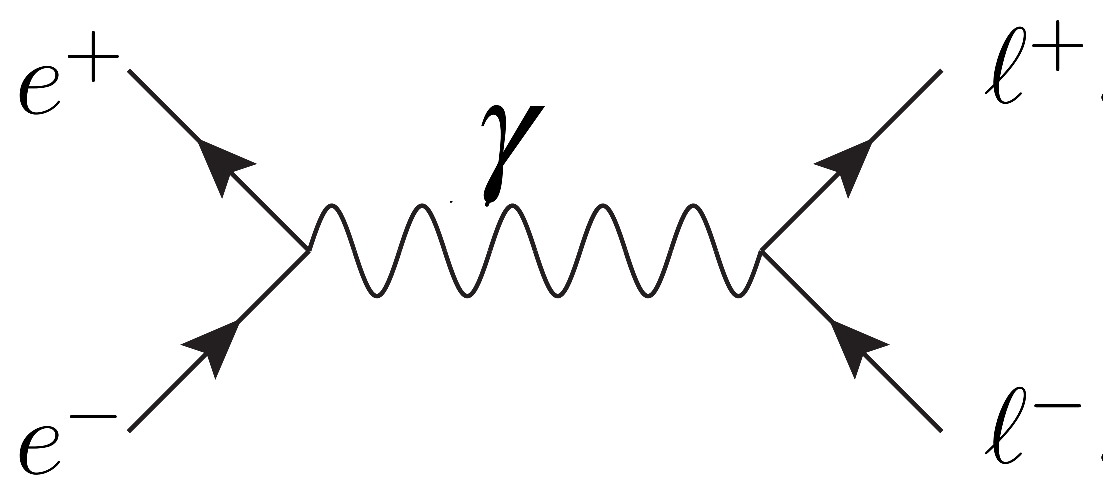
Try this out!

Mandelstam variables: $s = (p_{e+} + p_{e-})^2$ $\quad t = (p_{e+} - p_{\mu+})^2 = -\frac{s}{2}(1 - cos\theta)$

Why? $s + t + u = 0$ $\quad u = (p_{e+} - p_{\mu-})^2 = -\frac{s}{2}(1 + cos\theta)$

From QED to QCD

Example 1: R-ratio



$$\bar{\sum} |M|^2 = \frac{2e^4}{s^2}[t^2 + u^2]$$

$$\bar{\sum} |M|^2 \propto (1 + \cos^2\theta)$$

Cross-section:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \bar{\sum} |M|^2$$

2-body phase-space+Momentum conservation

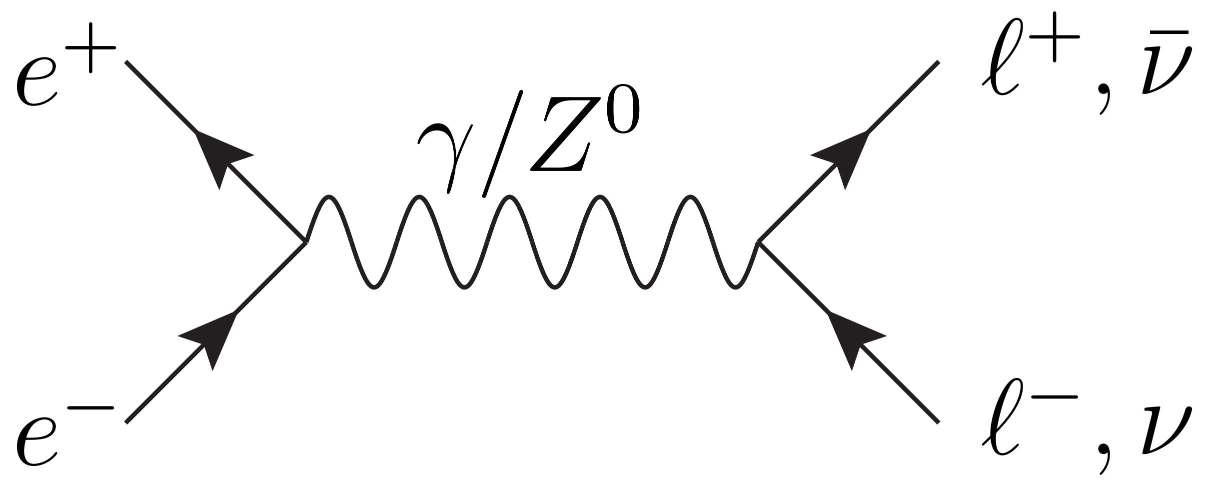
$$d\Omega = d\phi\, d\cos\theta$$

$$\sigma_{e^+e^- \to \mu^+\mu^-} = \frac{4\pi\alpha^2}{3s}$$

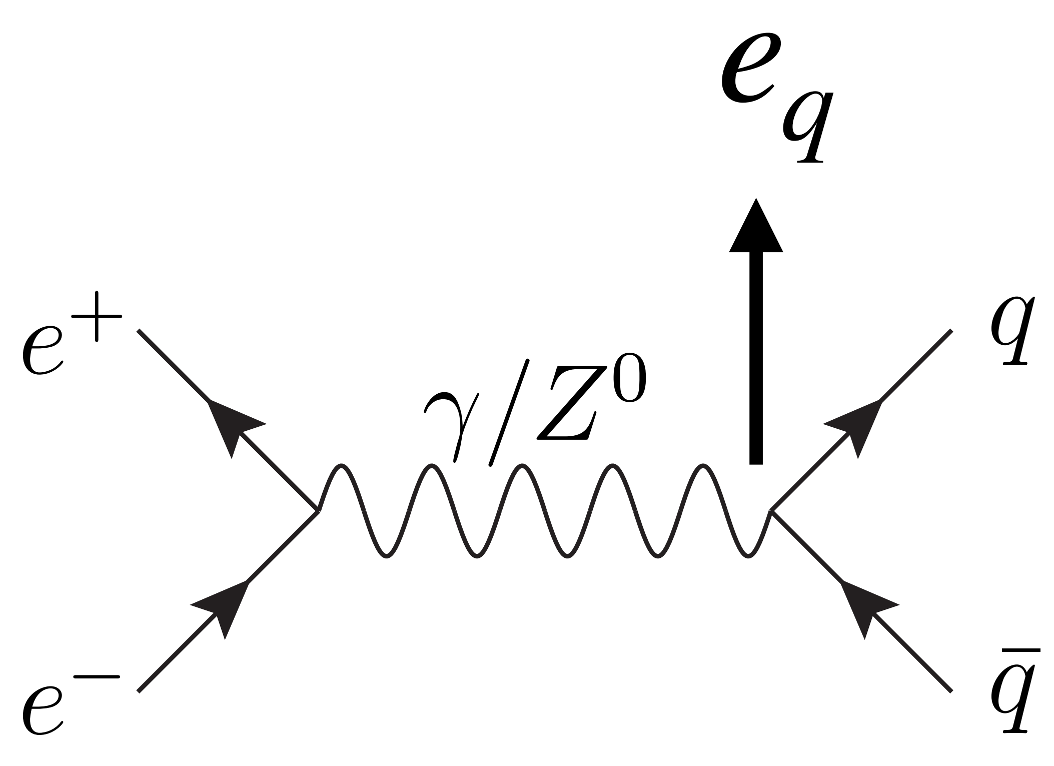
Try this out!

From QED to QCD

Example 1: R-ratio



vs



$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \sim N_c \sum_q e_q^2$$

$$= 2(N_c/3) \quad q = u, d, s$$

$$= 3.7(N_c/3) \quad q = u, d, s, c, b$$

Difference due to colour!!!

Quark—anti-pair can be one of $r\bar{r}, g\bar{g}, b\bar{b}$

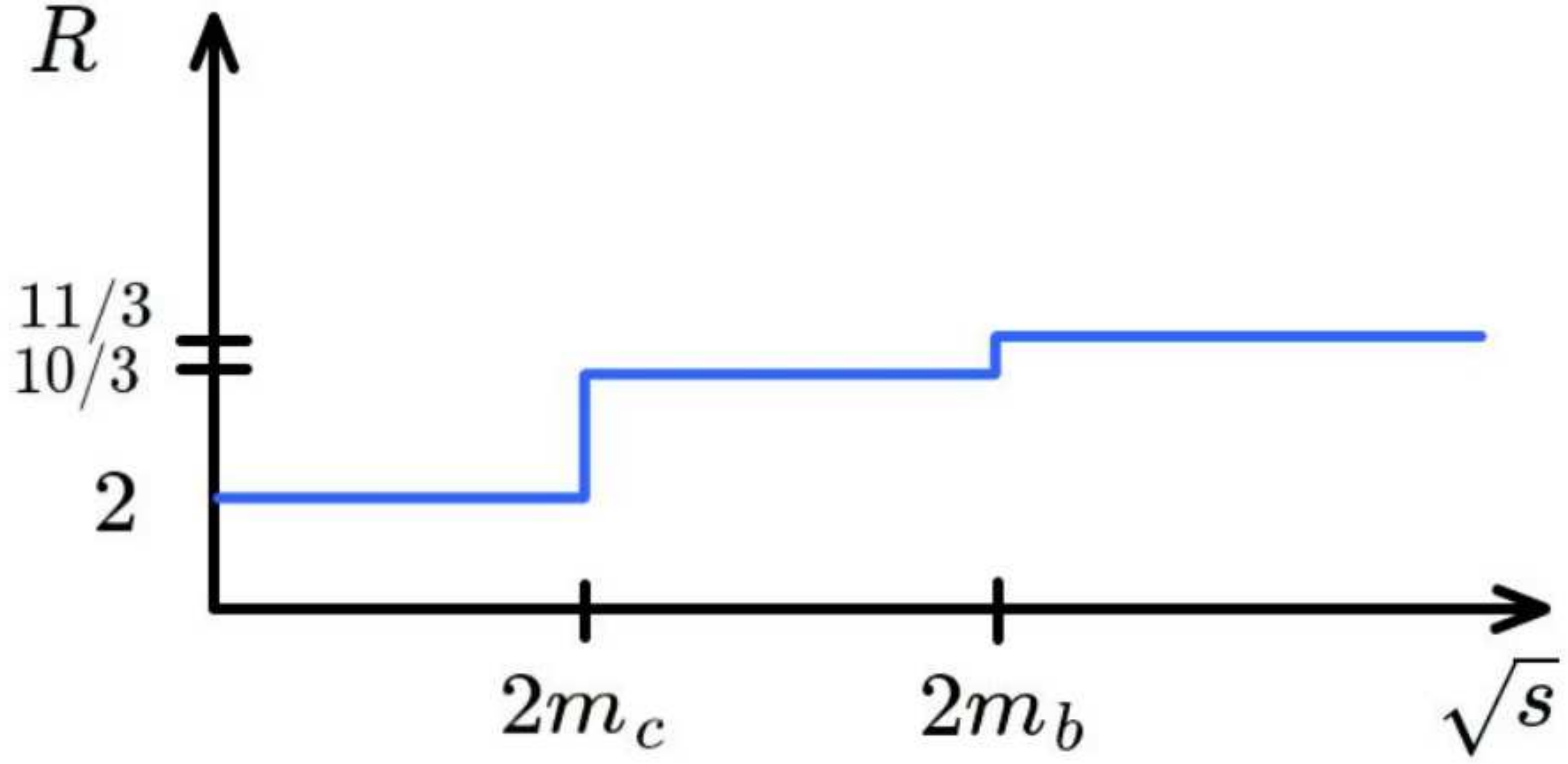
Why did we pick $\mu^+\mu^-$?

Experimental evidence for colour!

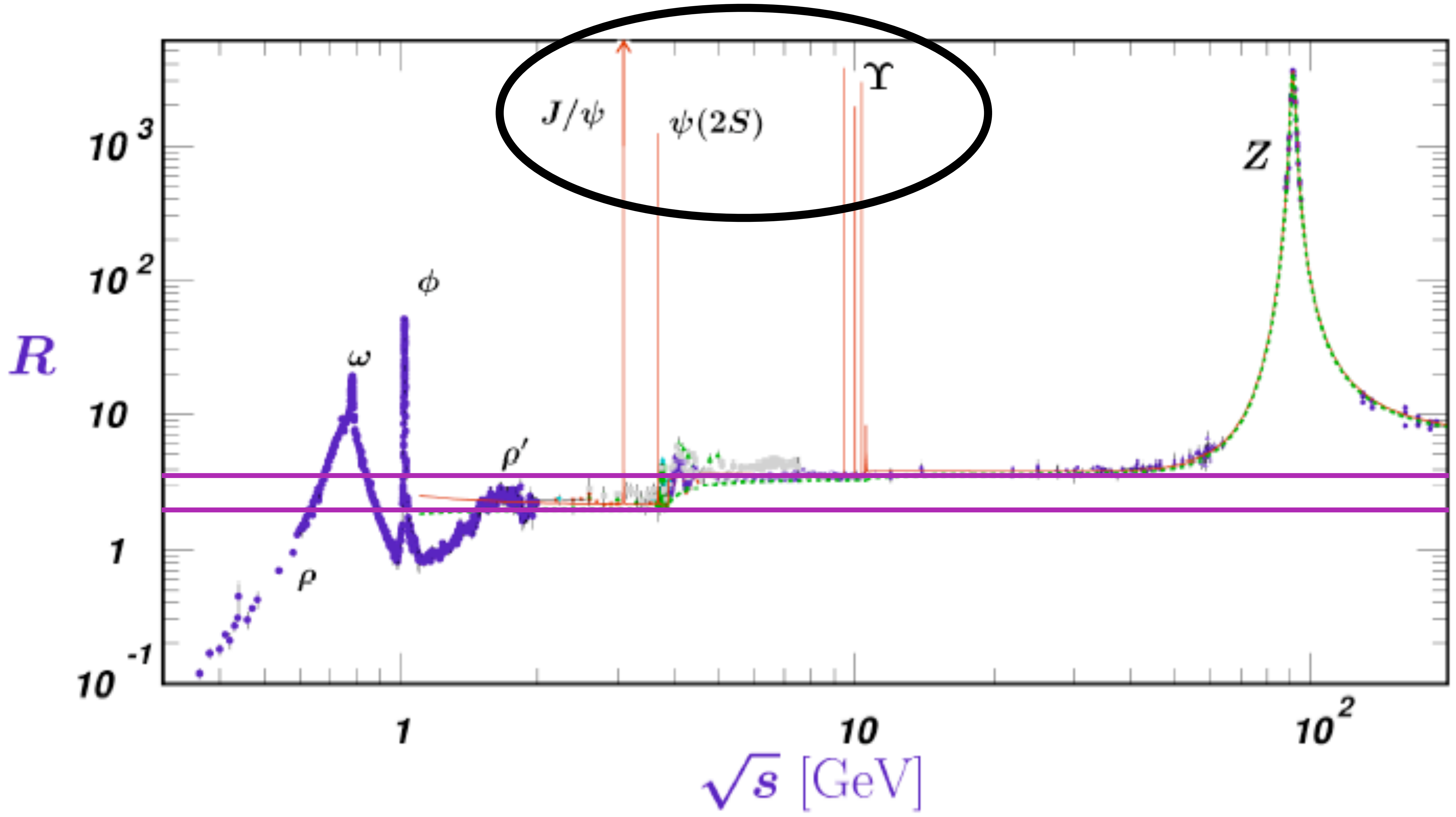
From QED to QCD

Example 1: R-ratio

R-ratio computation



Expected

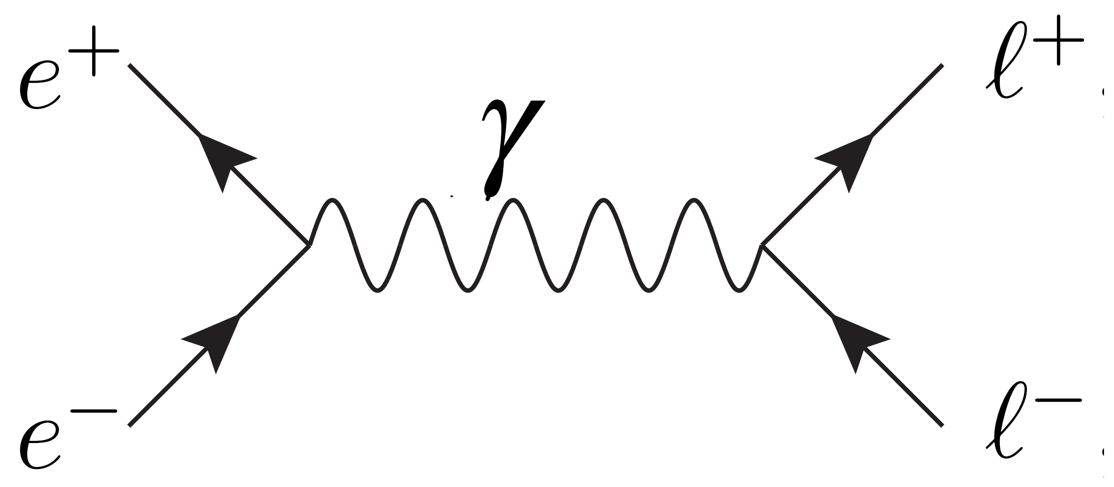


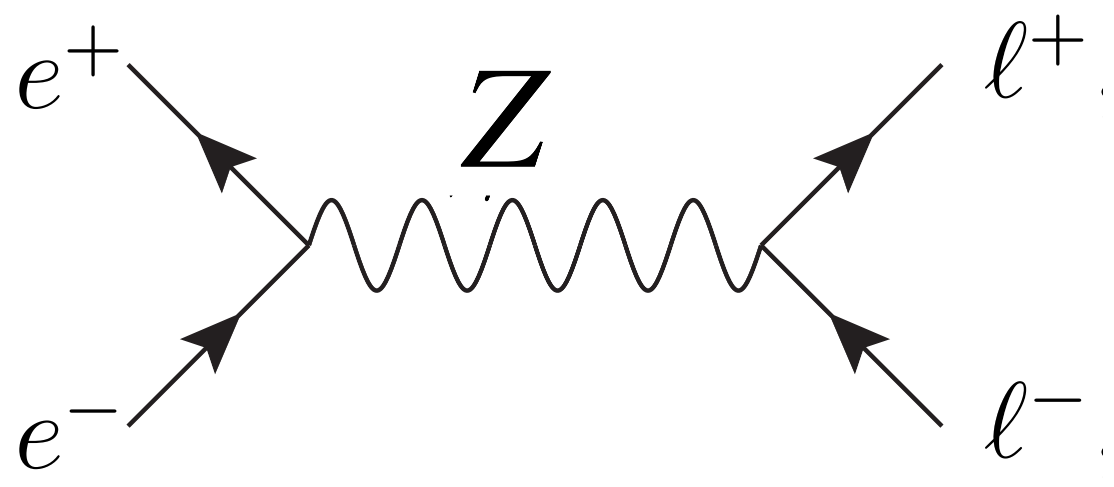
Measured

Quarkonium states: very small width, very long lived states

A few words about the Z-resonance

Breit -Wigner





Z contribution becomes relevant when $\sqrt{s} \sim M_Z$

We then need both diagrams and their interference

See exercise!

Z-resonance

Breit-Wigner and Narrow Width Approximation

Z is an unstable particle, we can't simply use $\dfrac{1}{s - M_Z^2}$

Breit-Wigner propagator: $\dfrac{1}{s - M_Z^2 + i\Gamma M}$

Narrow width approximation:

$$\frac{1}{(\hat{s} - M_Z^2)^2 + M_Z^2\Gamma_Z^2} \approx \frac{\pi}{M_Z\Gamma_Z}\delta(\hat{s} - M_Z^2) \quad \text{if } \Gamma_Z/M_Z \ll 1$$

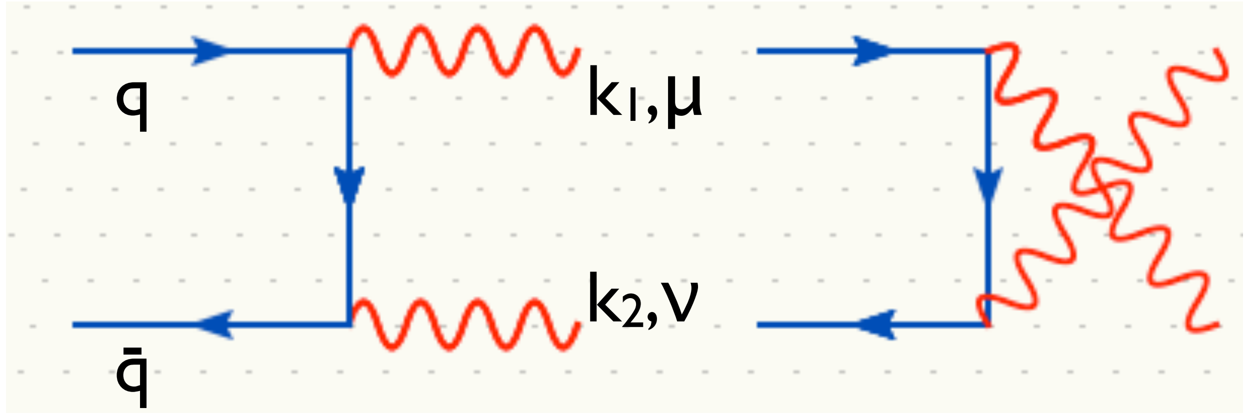
$$\sigma_{e^+e^-\to Z\to\mu^+\mu^-} \simeq \sigma_{e^+e^-\to Z} \times Br(Z \to \mu^+\mu^-) \text{ with } Br(Z \to \mu^+\mu^-) = \Gamma_{Z\to\mu^+\mu^-}/\Gamma_Z$$

Simplifies computations for particles with narrow width (e.g. Higgs)

From QED to QCD

Example 2: QCD and gauge invariance

Let's compute the amplitude for $q\bar{q} \to \gamma\gamma$



$$i\mathcal{M} = \mathcal{M}_{\mu\nu}\epsilon_1^{*\mu}\epsilon_2^{*\nu} = D_1 + D_2 = e^2\left(\bar{v}(\bar{q})\not{\epsilon}_2\frac{1}{\not{q}-\not{k}_1}\not{\epsilon}_1 u(q) + \bar{v}(\bar{q})\not{\epsilon}_1\frac{1}{\not{q}-\not{k}_2}\not{\epsilon}_2 u(q)\right)$$

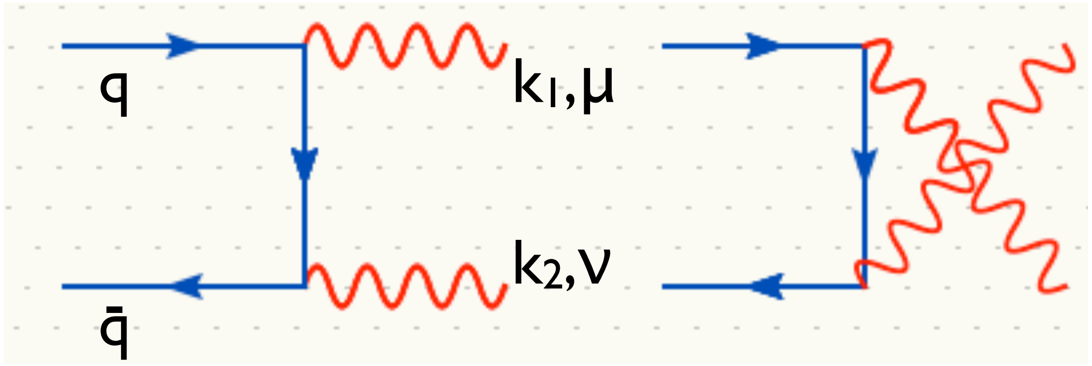
Gauge invariance requires: $\epsilon_1^{*\mu}k_2^{\nu}\mathcal{M}_{\mu\nu} = \epsilon_2^{*\nu}k_1^{\mu}\mathcal{M}_{\mu\nu} = 0$

$$\mathcal{M}_{\mu\nu}k_1^{*\mu}\epsilon_2^{*\nu} = D_1 + D_2 = e^2\left(\bar{v}(\bar{q})\not{\epsilon}_2\frac{1}{\not{q}-\not{k}_1}(\not{k}_1-\not{q})u(q) + \bar{v}(\bar{q})(\not{k}_1-\not{\bar{q}})\frac{1}{\not{k}_1-\not{q}}\not{\epsilon}_2 u(q)\right)$$
$$= -\bar{v}(\bar{q})\not{\epsilon}_2 u(q) + \bar{v}(\bar{q})\not{\epsilon}_2 u(q) = 0$$

Works fine!

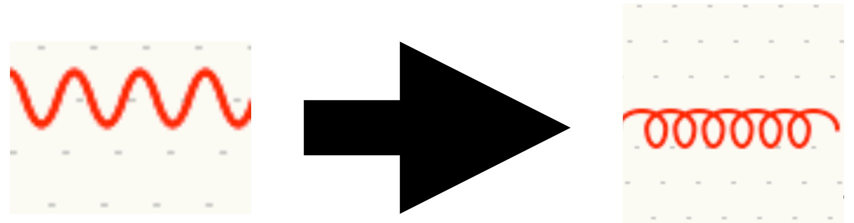
From QED to QCD

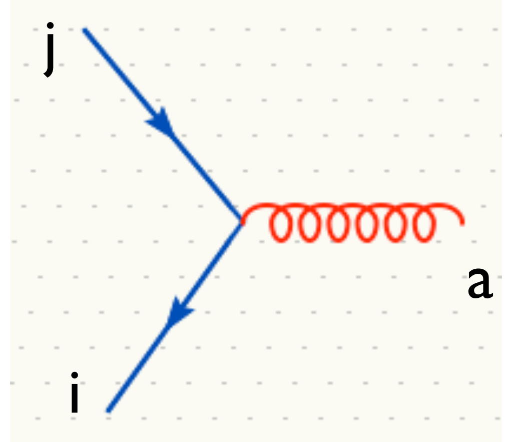
Example 2: QCD and gauge invariance



$$i\mathcal{M} = \mathcal{M}_{\mu\nu}\epsilon_1^{*\mu}\epsilon_2^{*\nu} = D_1 + D_2 = e^2\left(\bar{v}(\bar{q})\not{\epsilon}_2\frac{1}{\not{q}-\not{k}_1}\not{\epsilon}_1 u(q) + \bar{v}(\bar{q})\not{\epsilon}_1\frac{1}{\not{q}-\not{k}_2}\not{\epsilon}_2 u(q)\right)$$

Let's do the same for $q\bar{q} \to gg$





$$-ig_s t^a_{ij}\gamma^\mu$$

$$\frac{i}{g_s^2}M_g \equiv (t^b t^a)_{ij}D_1 + (t^a t^b)_{ij}D_2$$

$$M_g = (t^a t^b)_{ij}M_\gamma - g^2 f^{abc}t^c_{ij}D_1$$

$$[t^a, t^b] = if^{abc}t^c$$

Is this gauge invariant?

$$k_{1\mu}M_g^\mu = -g_s^2 f^{abc}t^c_{ij}\bar{v}_i(\bar{q})\not{\epsilon}_2 u_i(q)$$

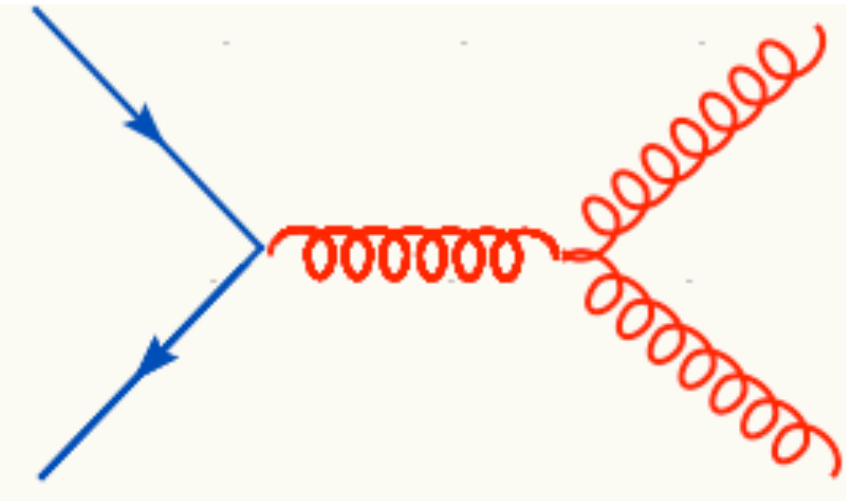
We don't get zero anymore!

$$k_{1\mu}M_g^\mu = i(-g_s f^{abc}\epsilon_2^\mu)(-ig_s t^c_{ij}\bar{v}_i(\bar{q})\gamma_\mu u_i(q))$$

From QED to QCD

Example 2: QCD and gauge invariance

What are we missing?



$$-ig_s^2 D_3 = \left(-ig_s t^a_{ij} \bar{v}_i(\bar{q})\gamma^\mu u_j(q)\right) \times \left(\frac{-i}{p^2}\right) \times \left(-g f^{abc} V_{\mu\nu\rho}(-p, k_1, k_2)\epsilon_1^\nu(k_1)\epsilon_2^\rho(k_2)\right)$$

- Lorentz invariant
- Anti-symmetry
- Dimensional analysis

$$V_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) = V_0\left[(p_1 - p_2)_{\mu_3} g_{\mu_1\mu_2} + (p_2 - p_3)_{\mu_1} g_{\mu_2\mu_3} + (p_3 - p_1)_{\mu_2} g_{\mu_3\mu_1}\right]$$

$$k_1 \cdot D_3 = g^2 f^{abc} t^c V_0 \left[\bar{v}(\bar{q})\not{\epsilon}_2 u(q) - \frac{k_2 \cdot \epsilon_2}{2k_1 \cdot k_2}\bar{v}(\bar{q})\not{k}_1 u(q)\right]$$

Gauge invariant IFF the other gluon is physical!

An empirical way to write down the triple gluon vertex!

QCD Lagrangian

$$\mathcal{L} = \underbrace{-\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}}_{\text{Gauge Fields}} + \sum_f \underbrace{\bar{\psi}_i^{(f)}(i\partial\!\!\!/ - m_f)\psi_i^{(f)}}_{\text{Matter}} - \underbrace{\bar{\psi}_i^{(f)}(g_s t^a_{ij} A\!\!\!/_a)\psi_j^{(f)}}_{\text{Interaction}}$$

Gauge Fields

Matter

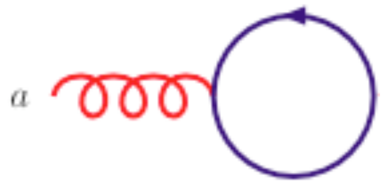
Interaction

$$\Rightarrow F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

See QCD-QED course!

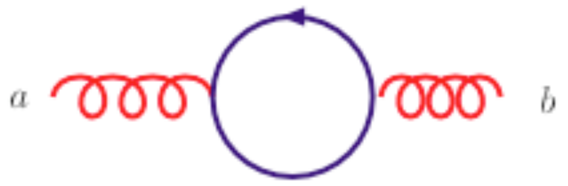
Colour algebra

$$\mathrm{Tr}(t^a) = 0$$



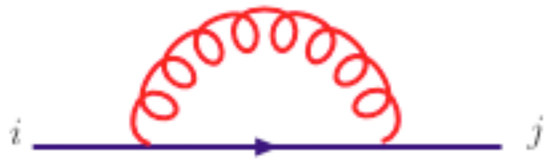
= 0

$$\mathrm{Tr}(t^a t^b) = T_R \delta^{ab}$$



= T_R *

$$(t^a t^a)_{ij} = C_F \delta_{ij}$$



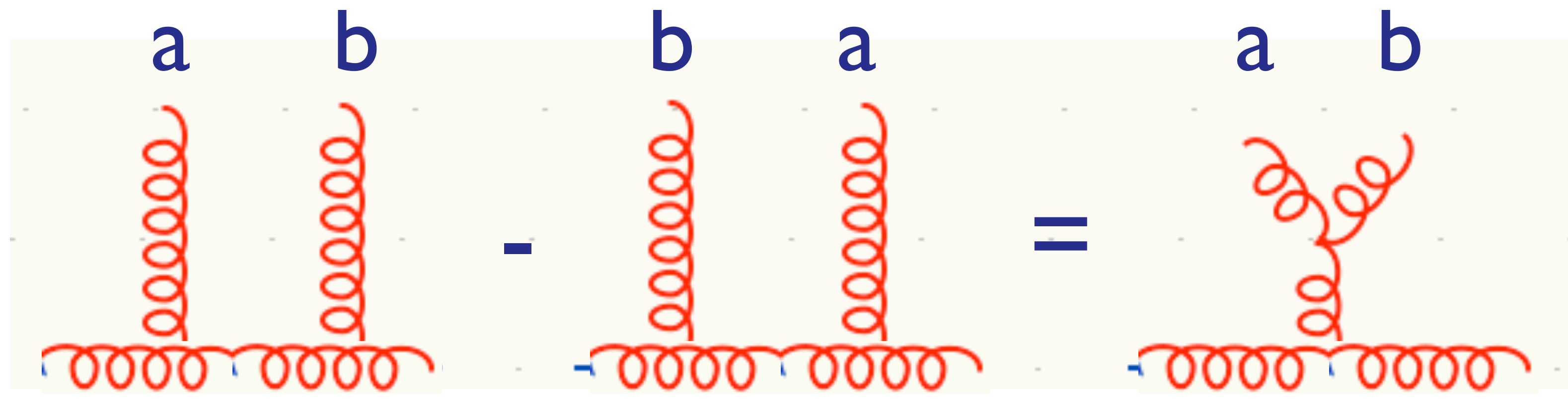
= C_F *

$$\sum_{cd} f^{acd} f^{bcd}$$
$$= (F^c F^c)_{ab} = C_A \delta_{ab}$$

= C_A*

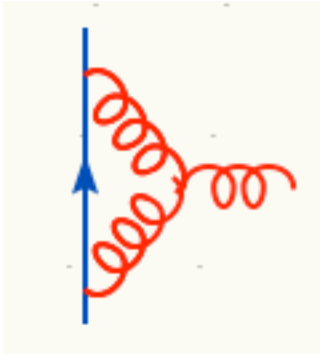
$$[t^a, t^b] = i f^{abc} t^c$$

$$[F^a, F^b] = i f^{abc} F^c$$

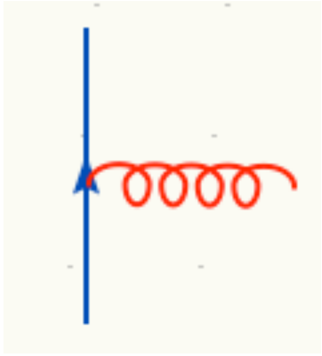


1-loop vertices

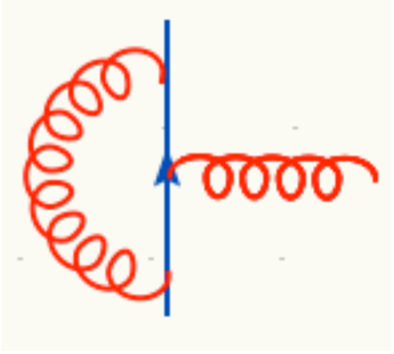
$$i f^{abc} (t^b t^c)_{ij} = \frac{C_A}{2} t^a_{ij}$$



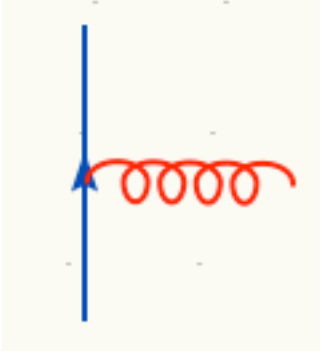
= $C_A/2$ *



$$(t^b t^a t^b)_{ij} = (C_F - \frac{C_A}{2}) t^a_{ij}$$

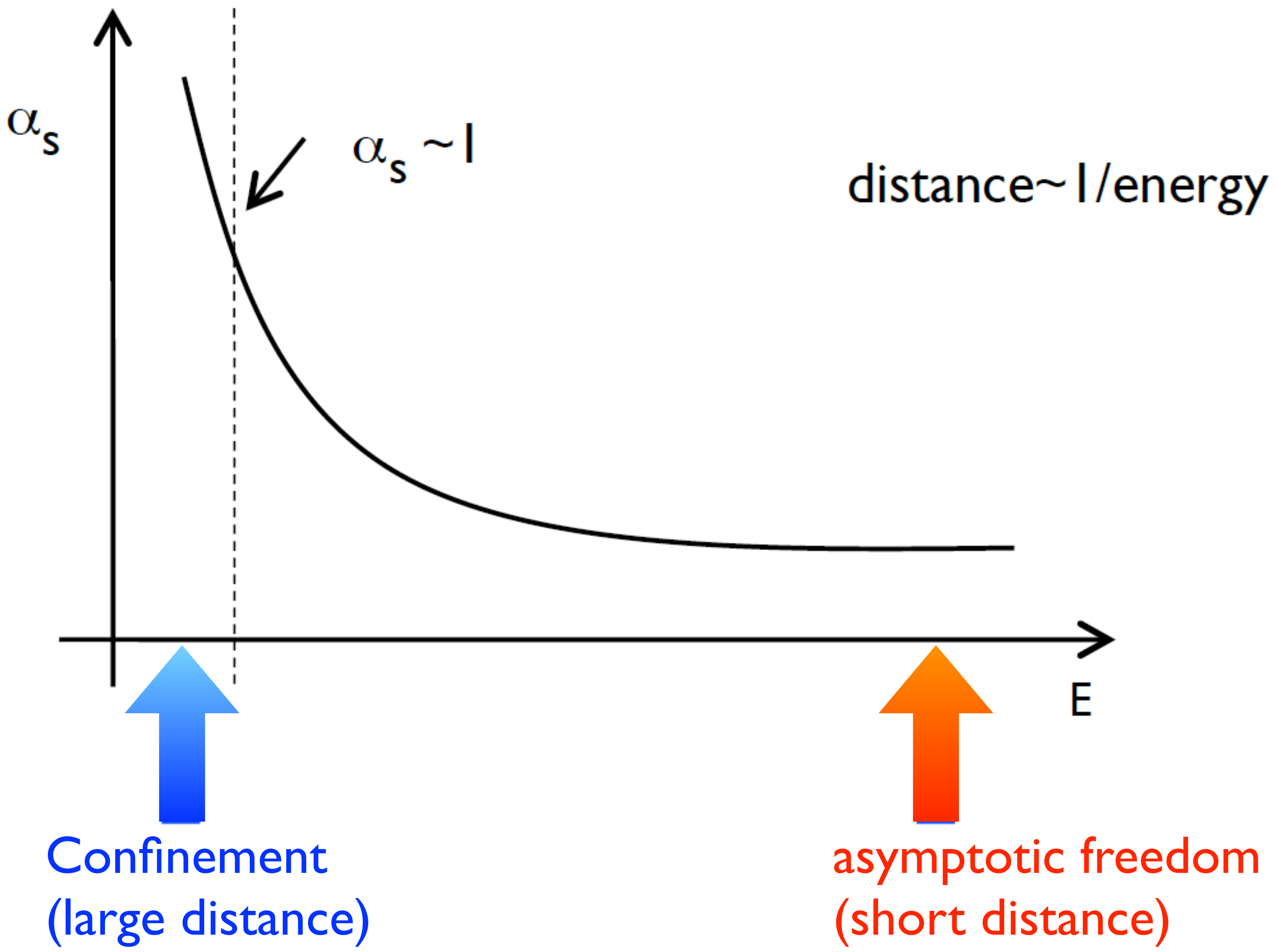


= -1/2/Nc *



Can be a bottleneck for higher order computations! People always on the lookout for simplifications! Quite a few computations are done in the large N_c limit.

Properties of QCD

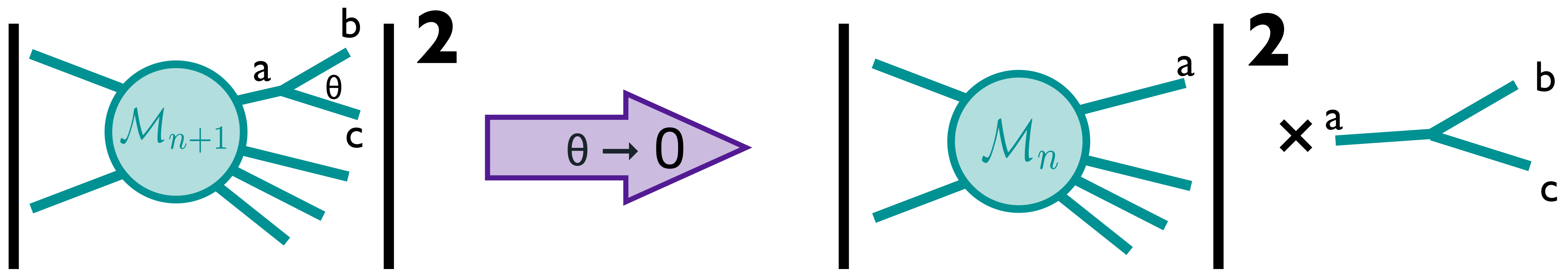


UV: Asymptotic freedom

- Perturbative computations
- Parton model

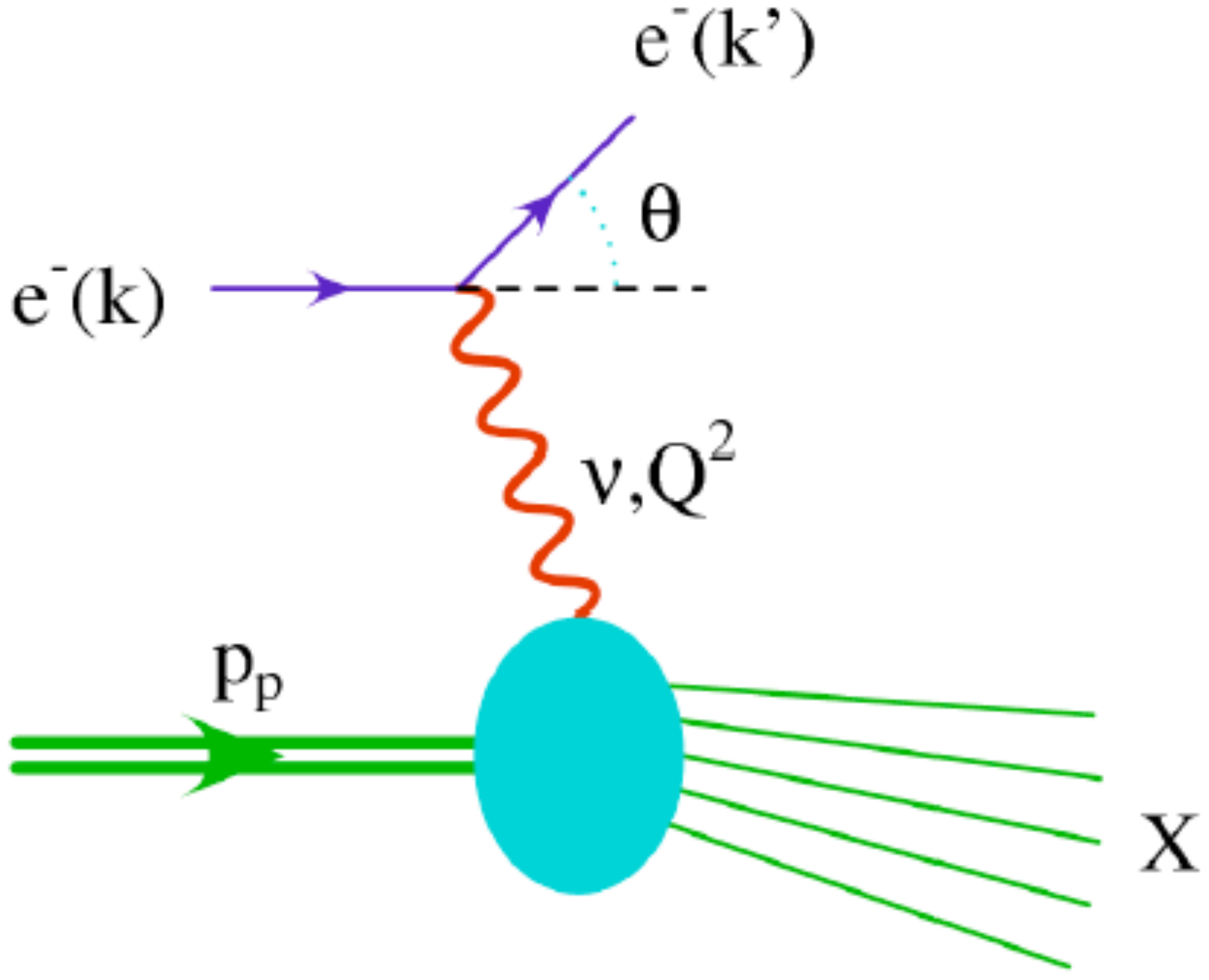
IR: Universality

- Collinear Factorisation
- Parton showers



The parton model of QCD

Deep Inelastic Scattering



$$s = (P+k)^2$$ CoM energy

$$Q^2 = -(k-k')^2$$ momentum transfer^2

$$x = Q^2/2(P\cdot q)$$ scaling variable

$$\nu = (P\cdot q)/M = E - E'$$ energy loss

$$y = (P\cdot q)/(P\cdot k) = 1 - E'/E$$ relative energy loss

$$W^2 = (P+q)^2 = M^2 + \frac{1-x}{x}Q^2$$ recoil mass

$$\frac{d\sigma_{\text{elastic}}}{dq^2} = \left(\frac{d\sigma}{dq^2}\right)_{\text{point}} \cdot F^2_{\text{elastic}}(q^2)\,\delta(1-x)\,dx$$

$$\frac{d\sigma_{\text{inelastic}}}{dq^2} = \left(\frac{d\sigma}{dq^2}\right)_{\text{point}} \cdot F^2_{\text{inelastic}}(q^2,x)\,dx$$

Can we guess what F looks like?

Deep Inelastic scattering

What can $F^2(q^2)$ look like?

1. Proton charge is smoothly distributed (probe penetrates proton like a knife through butter)

$$F^2_{elastic}(q^2) \sim F^2_{inelastic}(q^2, x) \ll 1$$

2. Proton consists of tightly bound charges (quarks hit as single particles, but cannot fly away because tightly bound)

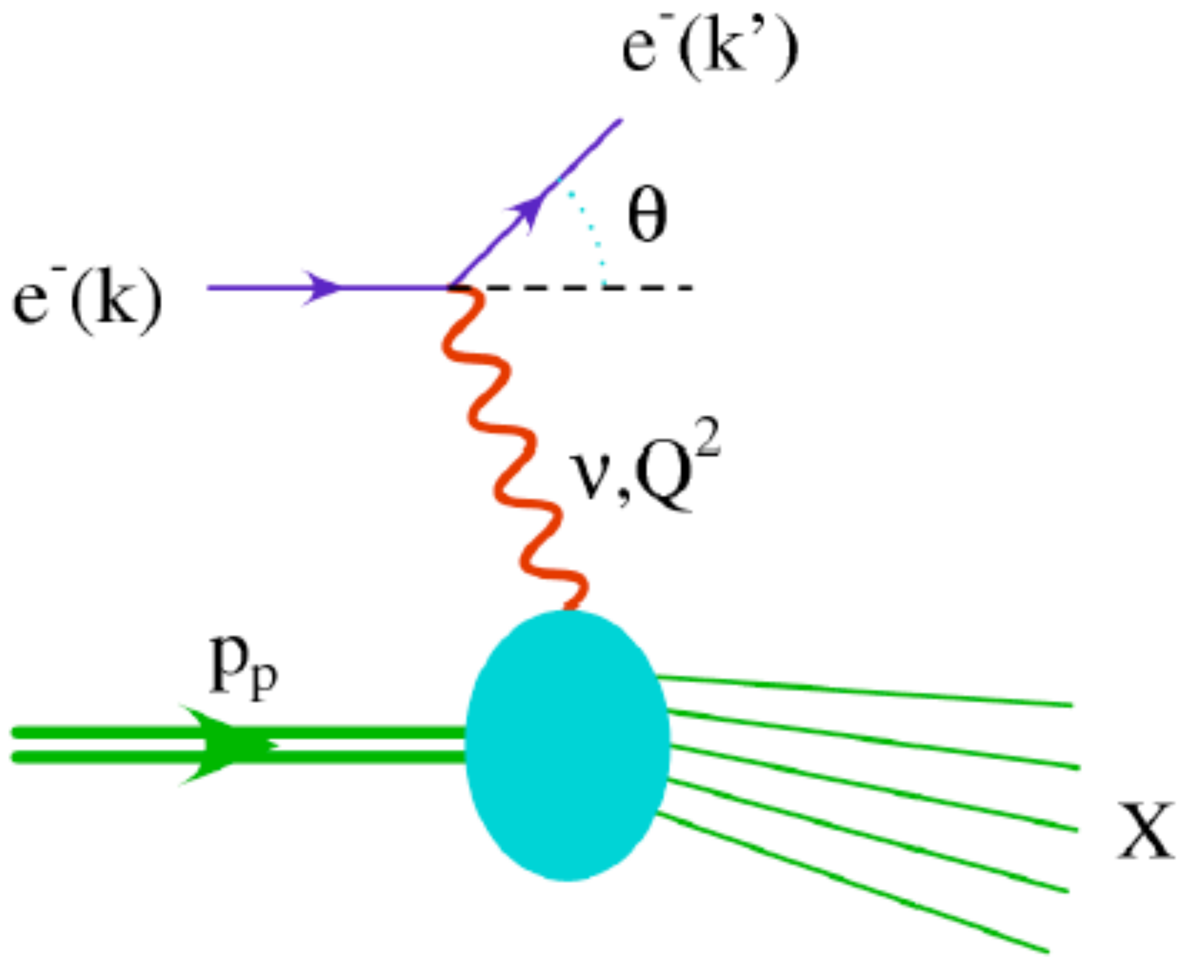
$$F^2_{elastic}(q^2) \sim 1 \qquad F^2_{inelastic}(q^2, x) \ll 1$$

!!!3. $F^2_{elastic}(q^2) \ll 1 \qquad F^2_{inelastic}(q^2, x) \sim 1$

Quarks are free particles which fly away without caring about confinement!

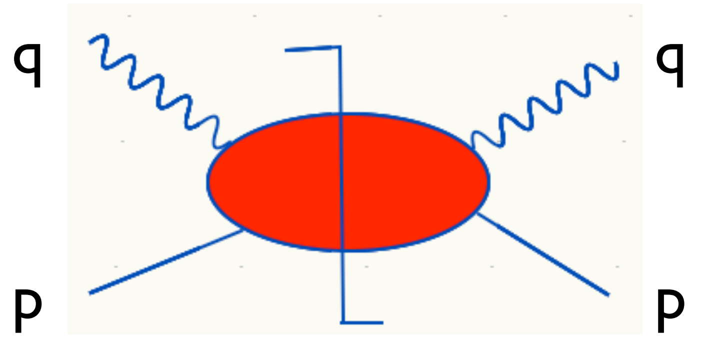
Parton Model

DIS cross-section



$$d\Phi = \frac{d^3k'}{(2\pi)^3 2E'} d\Phi_X = \frac{ME}{8\pi^2} y\, dy\, dx\, d\Phi_X$$

$$\frac{1}{4}\sum |\mathcal{M}|^2 = \frac{e^4}{Q^4} L^{\mu\nu} h_{X\,\mu\nu}$$



Why $1/Q^4$?

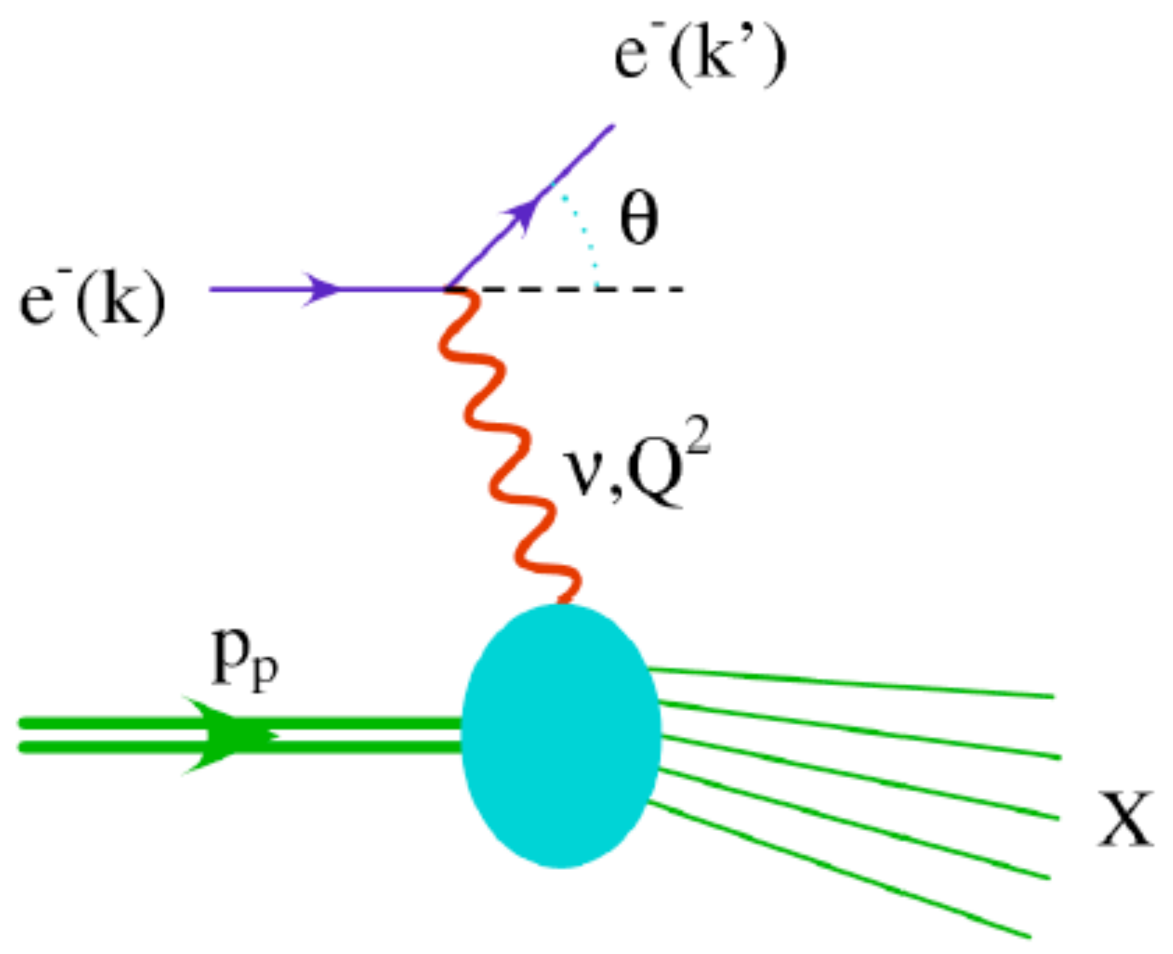
$$L^{\mu\nu} = \frac{1}{4}\mathrm{tr}[\not{k}\gamma^\mu \not{k}'\gamma^\nu] = k^\mu k'^\nu + k'^\mu k^\nu - g^{\mu\nu} k\cdot k'$$

Based on Lorentz and gauge invariance

$$W^{\mu\nu} = \sum_X \int d\Phi_X h_{X\,\mu\nu}$$

$$W_{\mu\nu}(p,q) = \left(-g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right)\frac{1}{p\cdot q} F_2(x,Q^2)$$

Parton Model



$$\sigma^{ep\to eX} = \sum_X \frac{1}{4ME}\int d\Phi\,\frac{1}{4}\sum_{\text{spin}}|\mathcal{M}|^2$$

After a bit of maths (good exercise!), we get:

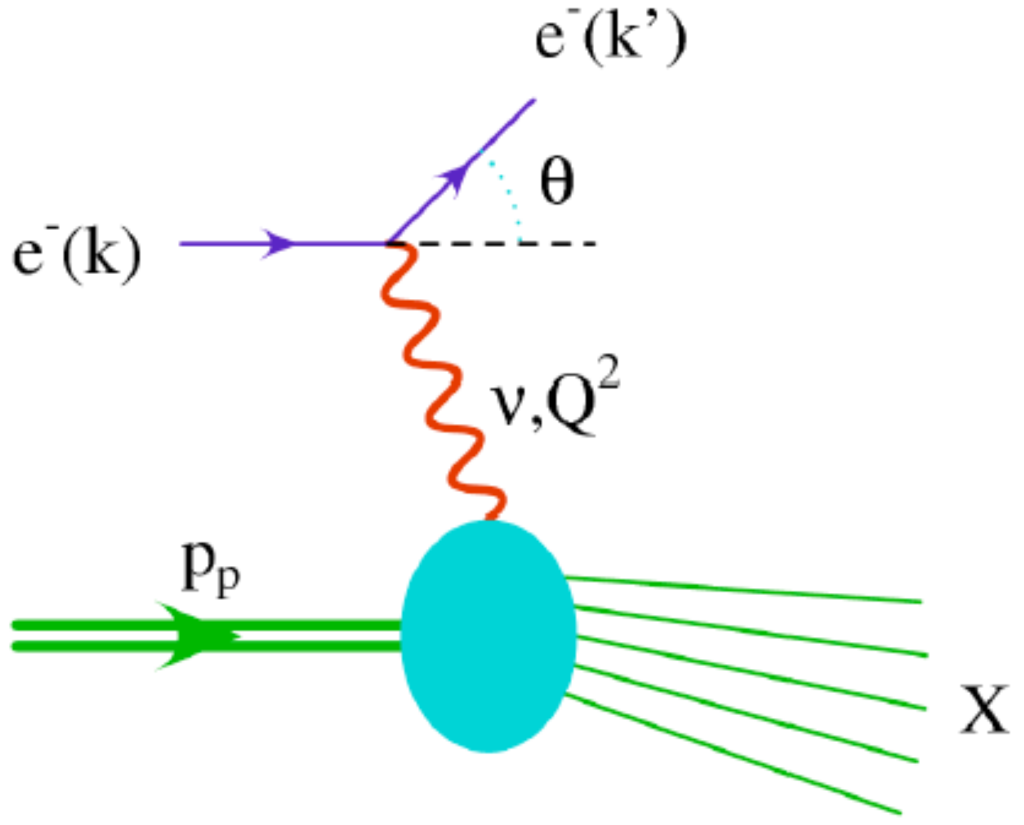
$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left\{[1+(1-y)^2]F_1(x,Q^2) + \frac{1-y}{x}\left[F_2(x,Q^2) - 2xF_1(x,Q^2)\right]\right\}$$

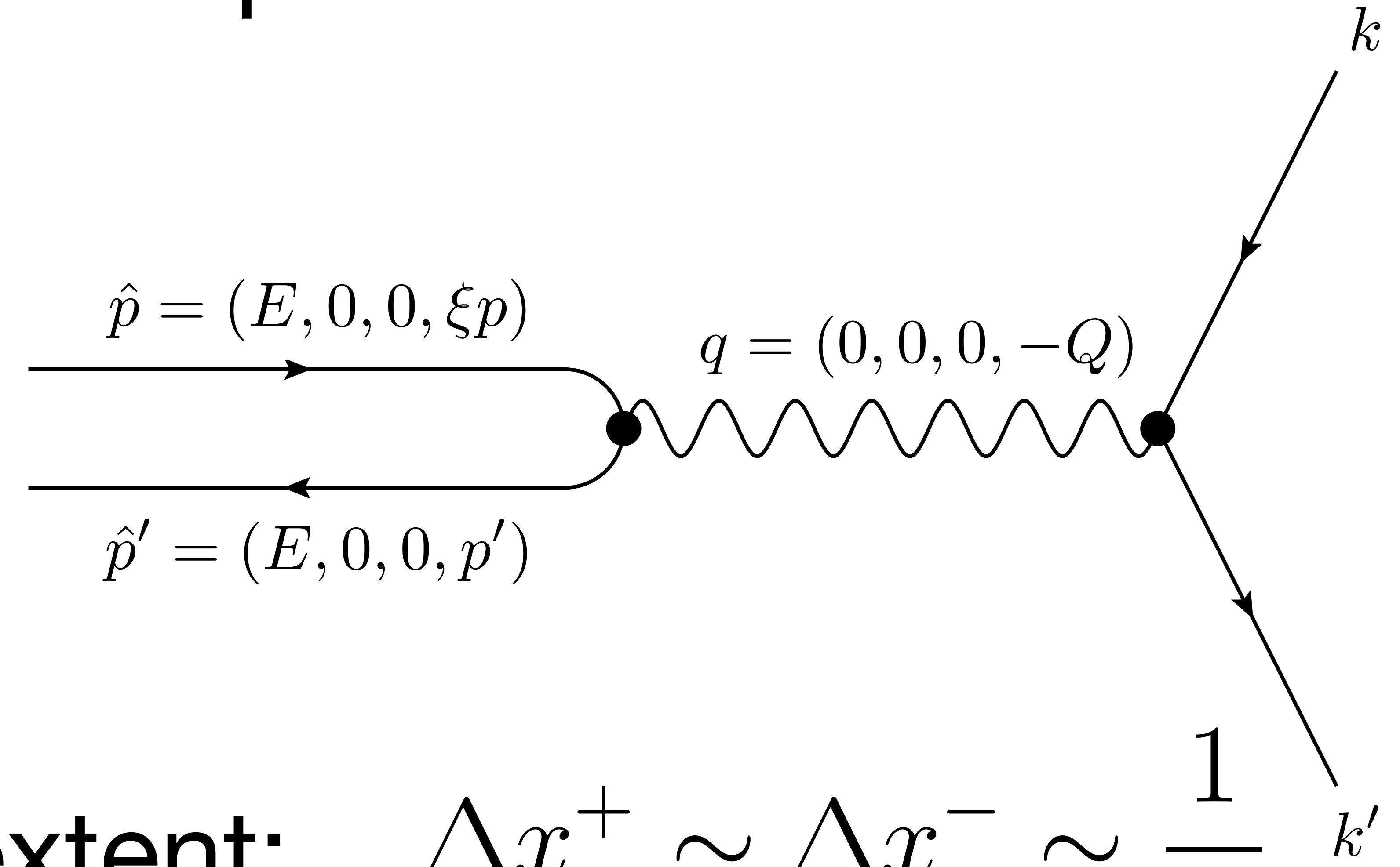
Transverse photon Longitudinal photon

Parton Model

Breit frame

The proton moves fast and the photon has zero energy





$$p \equiv \left(\sqrt{\frac{Q^2}{4x^2} + m^2}, \frac{Q}{2x}, \vec{0}_\perp\right) \approx \left(\frac{Q}{2x} + \frac{xm^2}{Q}, \frac{Q}{2x}, \vec{0}_\perp\right)$$

$$q \equiv \left(0, -Q, \vec{0}_\perp\right).$$

Rest frame: Proton extent: $\Delta x^+ \sim \Delta x^- \sim \dfrac{1}{m}$

Breit frame: Proton extent: $\Delta x^+ \sim \dfrac{Q}{m^2}, \qquad \Delta x^- \sim \dfrac{1}{Q}$

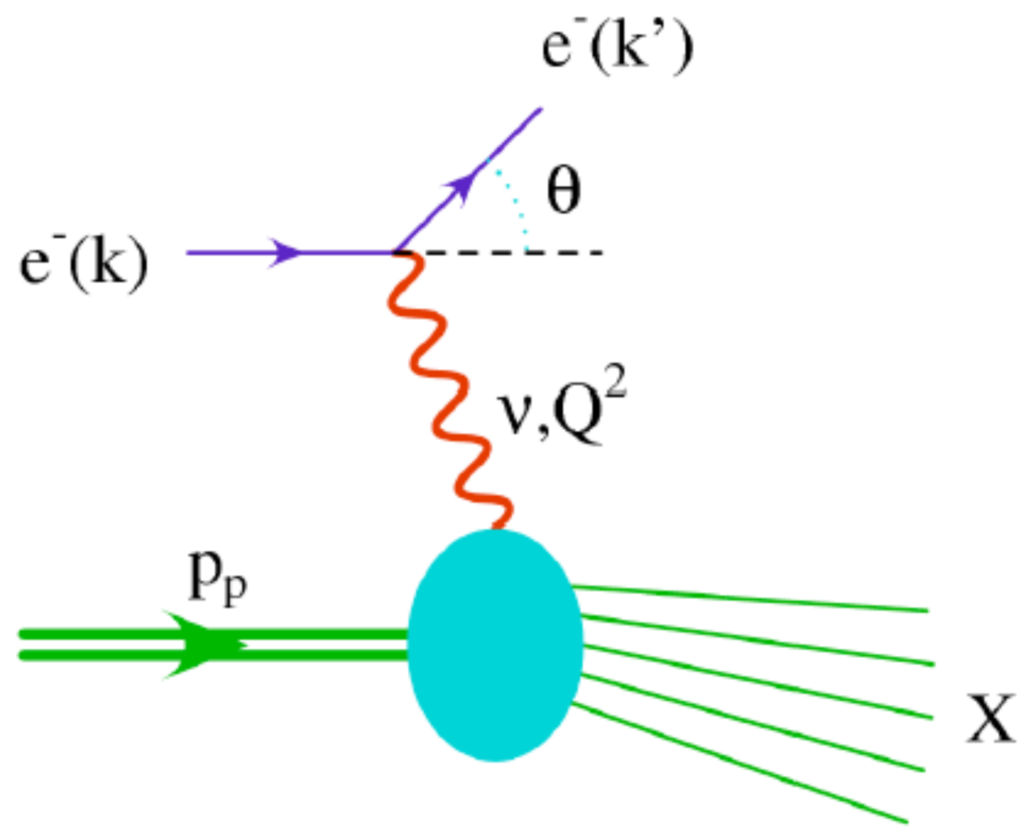
Photon extent: $\Delta x^+ \sim 1/Q,$

$\Rightarrow \quad \left(\Delta x^+\right)_{\text{photon}} \ll \left(\Delta x^+\right)_{\text{proton}}$

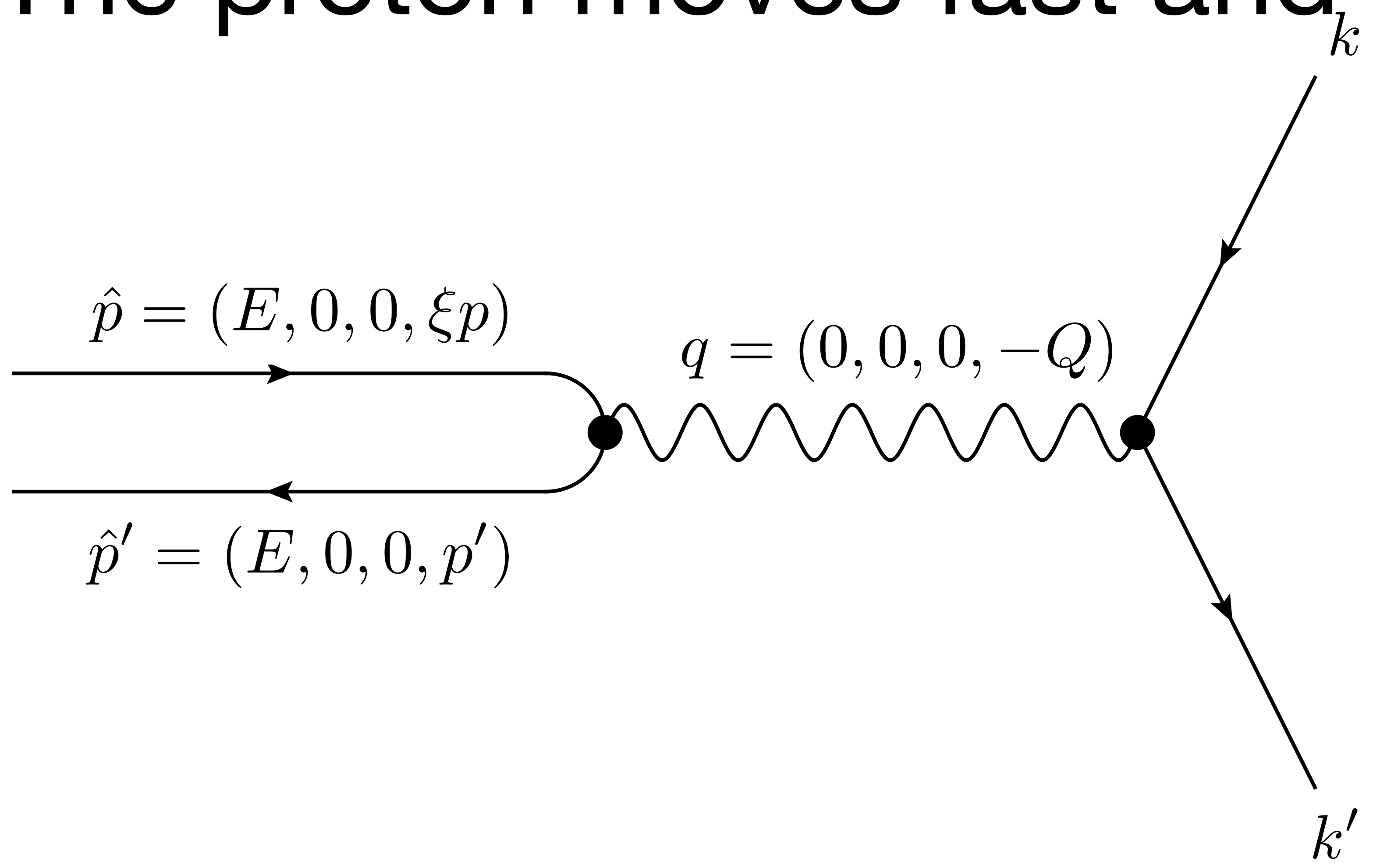
The time scale of a typical parton-parton interaction is much larger than the hard interaction time.

Parton Model

Breit frame



The proton moves fast and the photon has zero energy



$$\left(\Delta x^{+}\right)_{\mathrm{photon}} \ll \left(\Delta x^{+}\right)_{\mathrm{proton}}$$

- The time scale of a typical parton-parton interaction is much larger than the hard interaction time.
- Schematically: in the Breit frame the proton moves very fast towards the photon, and is therefore Lorentz contracted to a kind of pancake.
- The photon interaction then takes place on the very short time scale when the photon passes that pancake.
- During the short interaction time, the struck quark thus does not interact with the spectator quarks and can be regarded as a free parton.