Electroweak phase transition and gravitational waves

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UK HEP Forum 2023: Completing the Higgs-saw puzzle
22 November 2023

Overview

1. Cosmological phase transitions
2. Gravitational waves
3. Reliable predictions and effective field theory
4. What's next?

Hot Big Bang

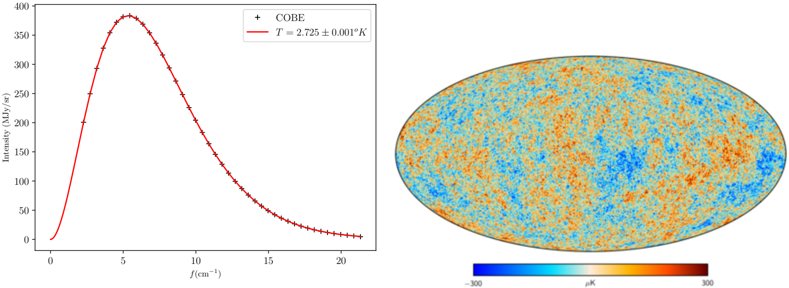
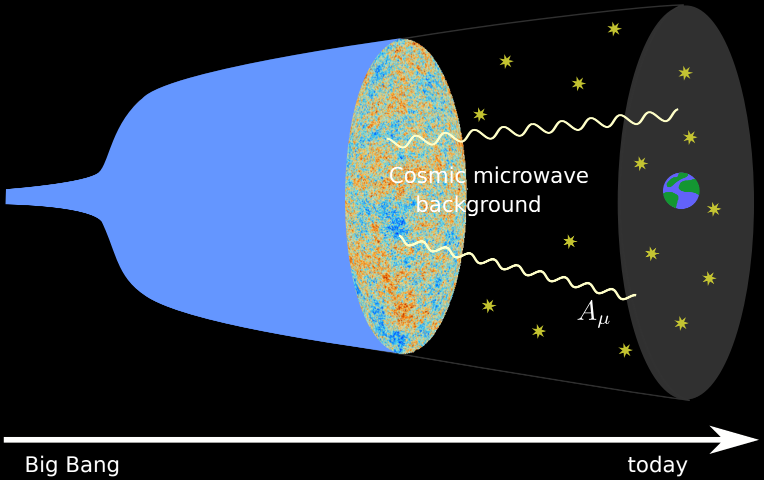
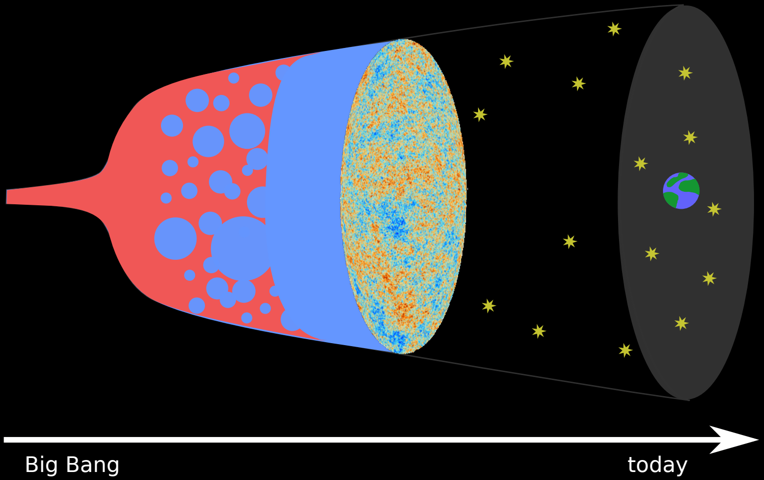


Figure: Blackbody spectrum of cosmic microwave background (COBE), and temperature anisotropies (Planck).

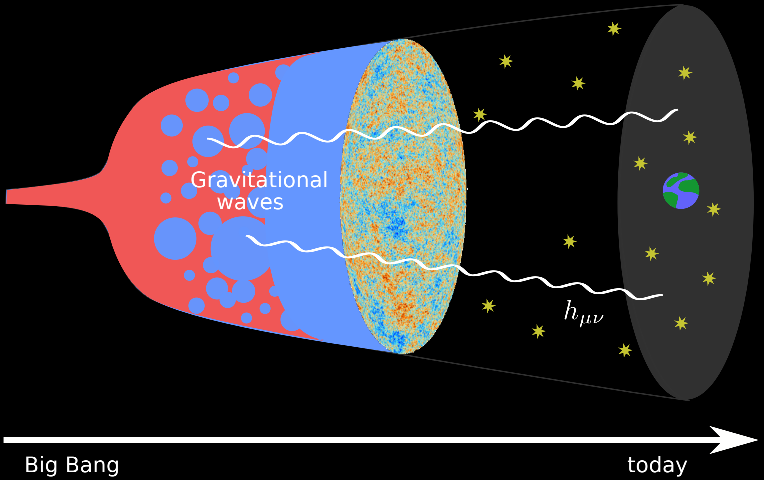
- Matter was thermal in the early universe.
- Lots of interesting thermal physics.



Cosmic microwave
background
A_μ
Big Bang
today

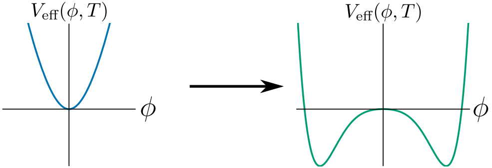


Big Bang
today


Gravitational waves
$h_{\mu\nu}$
Big Bang
today

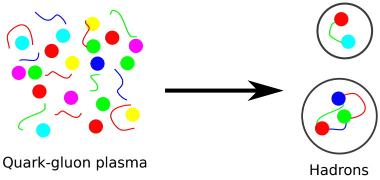
Standard Model phase transitions

- Electroweak symmetry breaking occurs at $T \sim 160$ GeV



D'Onofrio & Rummukainen 1508.07161

- Quark confinement occurs at $T \sim 155$ MeV

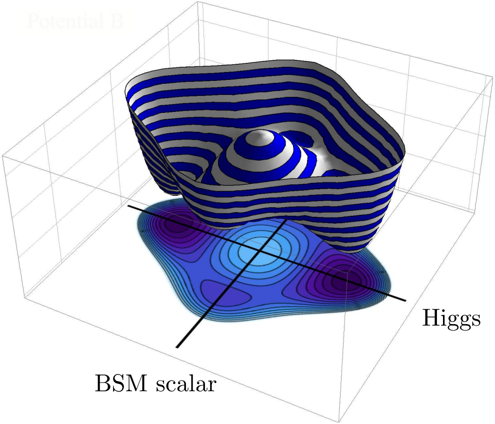


Aoki et al. hep-lat/0611014

In the minimal Standard Model (SM) both are crossovers.

Beyond SM phase transitions

- A 1st-order transition is required for successful electroweak baryogenesis.
- With new scalars, all kinds of transition patterns are possible.



Patel & Ramsey-Musolf 1212.5652

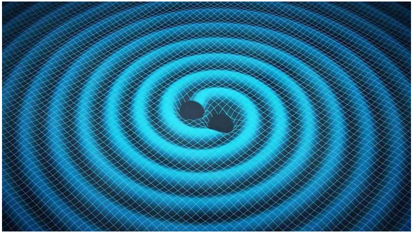
Gravitational waves

Gravitational waves

Wave-like fluctuations of the metric,

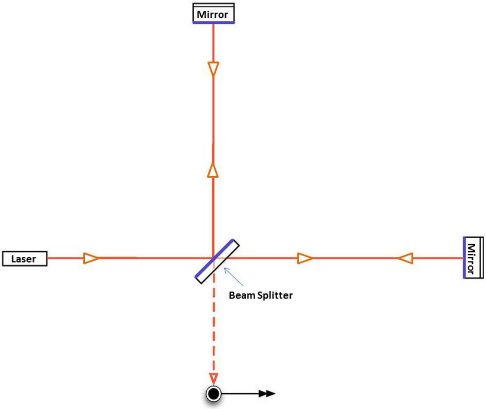
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$
$$\Box h^{\mathrm{TT}}_{\mu\nu} = 4\pi G_{\mathrm{N}}\, T^{\mathrm{TT}}_{\mu\nu},$$

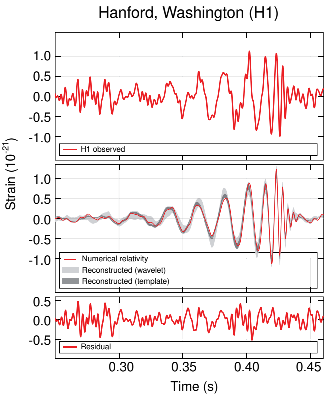
sourced by the (trasverse-traceless) energy-momentum tensor.



GW interferometers

Interferometers precisely measure arm lengths to look for wave-like deviations.

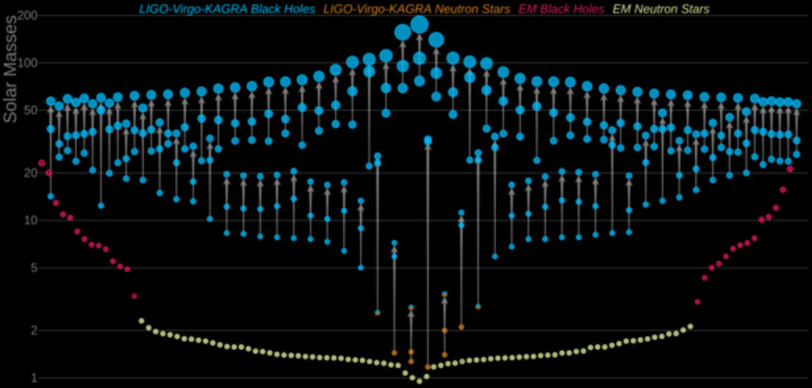




Discovery in 2015

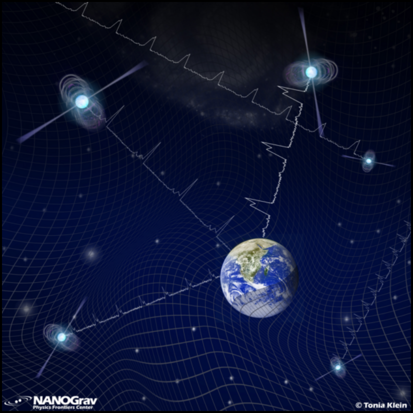
LIGO/Virgo 1602.03837

Masses in the Stellar Graveyard

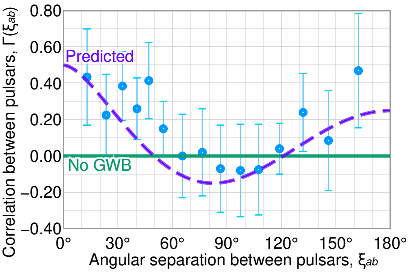


LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

Pulsar timing arrays



Discovery announced this June



Celestial correlations explained by GWs with lightyear wavelengths

European PTA, Indian PTA, NANOGrav, Parkes PTA '23

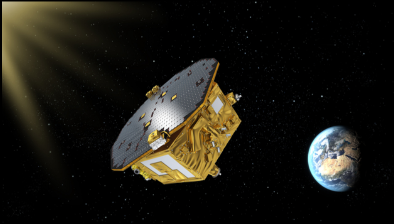
LISA, Taiji and TianQin

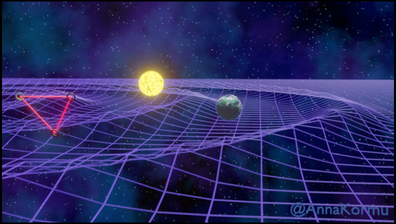
LISA timeline

- 2016: LISA Pathfinder
- Now: Detailed Design Phase
- ~2035: Launch
- 4 years data taking planned

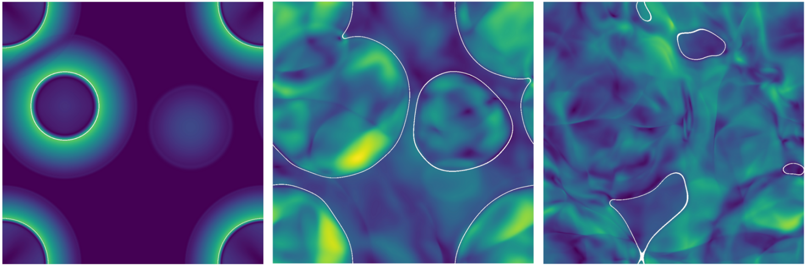
Taiji and TianQin timeline

- 2019: Taiji-1 and TianQin-1
- 2030s: Launch





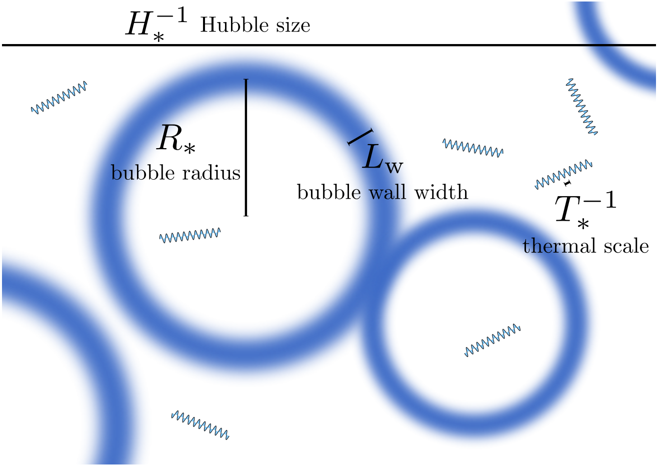
Cosmological 1st-order phase transitions



Cutting et al. 1906.00480

- Universe supercools
- Bubbles nucleate, expand and collide
- This creates long-lived fluid flows
- And creates gravitational waves: $\Box h_{ij}^{(\mathrm{TT})} \sim T_{ij}^{(\mathrm{TT})}$

Scales

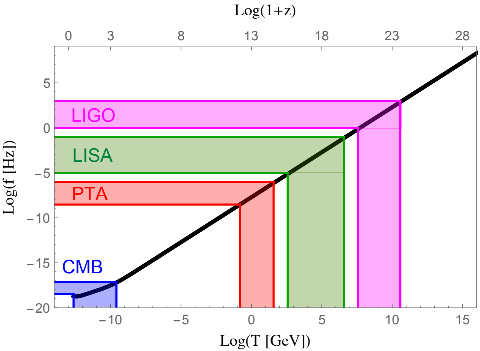


$$\underbrace{H_*^{-1} \gg R_*}_{\text{fluid}} \gg \underbrace{L_\mathrm{w} \gg T^{-1}}_{\text{microphysical}}$$

Gravitational wave frequencies

- Signal produced on frequencies $f_* \approx \frac{1}{R_*} \geq H_*$
- Red-shifting to today:

$$f_0 = \left(\frac{a_*}{a_0}\right) f_* \gtrsim \frac{10^{-3}}{R_* H_*} \left(\frac{g(T_*)}{100}\right)^{1/6} \left(\frac{T_*}{100\ \mathrm{GeV}}\right) \mathrm{Hz}$$



Caprini & Figueroa 1801.04268

The prediction pipeline

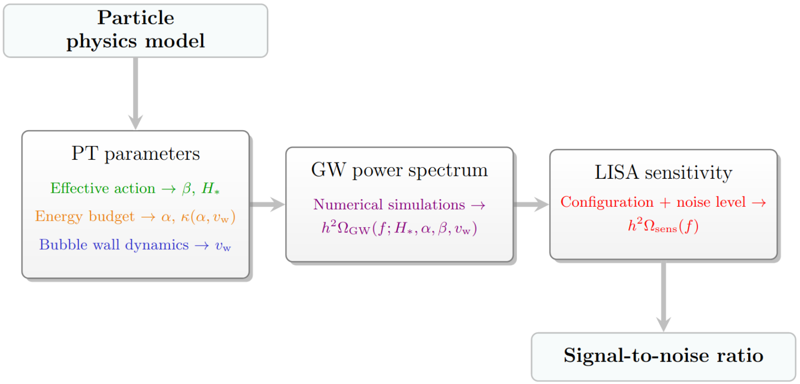


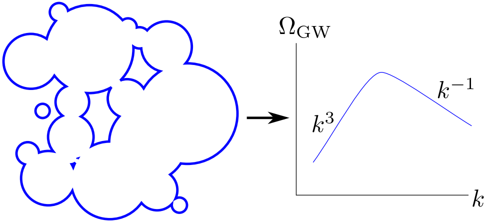
Figure: The Light Interferometer Space Antenna (LISA) pipeline $\mathscr{L} \to \mathrm{SNR}(f)$, Caprini et al. 1910.13125.

Gravitational wave spectrum

Contributions:

- Gradient energy of scalar field
  - Envelope approximation: only uncollided walls contribute

Kosowsky & Turner '93, Weir '16



Van de Vis '23

Gravitational wave spectrum

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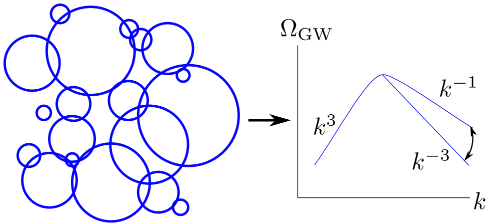
  - Contributions of collided walls can change UV power law

    Cutting, Hindmarsh & Weir '18, Konstandin '18



Van de Vis '23

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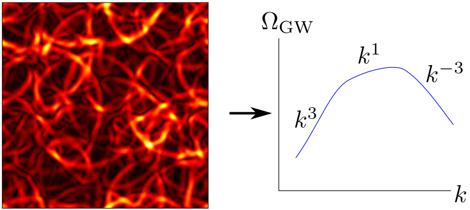
- Sound waves in fluid plasma

  Hindmarsh, Rummukainen & Weir '13, Hindmarsh '16



Jinno et al. '22

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- Sound waves in fluid plasma

  Hindmarsh, Rummukainen & Weir '13, Hindmarsh '16
- Fluid shocks, turbulence

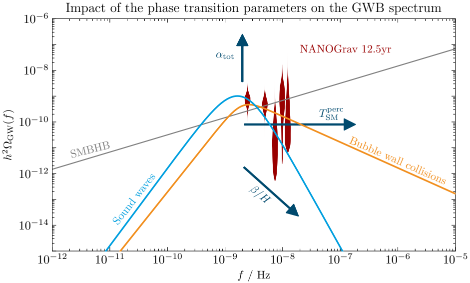
  Caprini et al. '09, Roper Pol et al. '19, Dahl et al. '22
- Feebly interacting particles — Jinno et al. '22
- Topological defects, oscillons . . .

Spectral dependence on thermodynamics

Spectrum depends most strongly on 4 quantities,

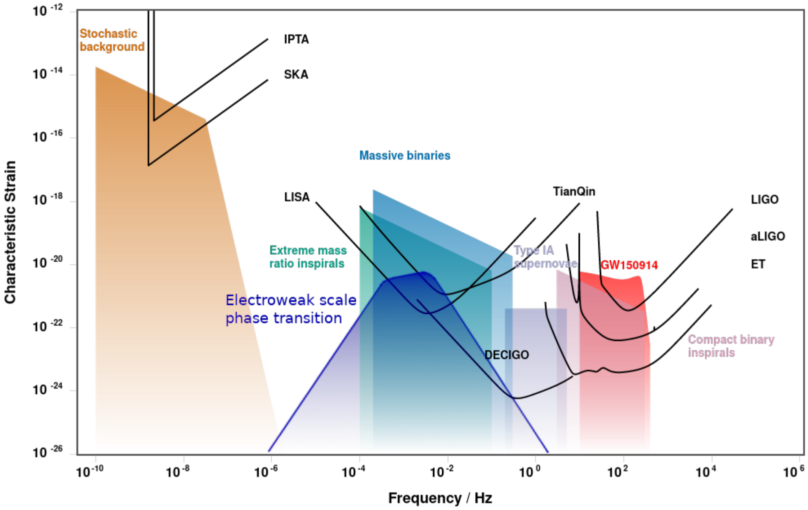
$\Omega_{\mathrm{GW}} = F(T_*, R_*, \alpha_*, v_{\mathrm{w}})$,

- T_* : transition temperature,
- R_* : bubble radius,
- α_* : transition strength,
- v_{w} : bubble wall speed.



Bringmann et al. 2306.09411

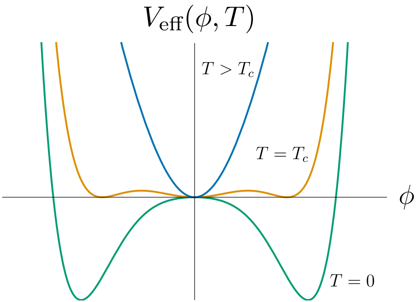
Spectrum of gravitational wave experiments



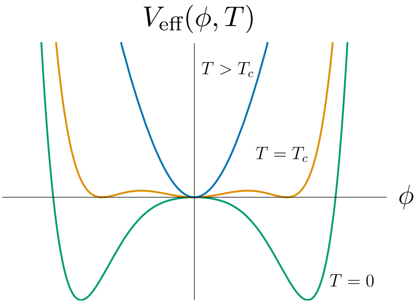
gwplotter.com

Reliable predictions and effective field theory

Effective potential

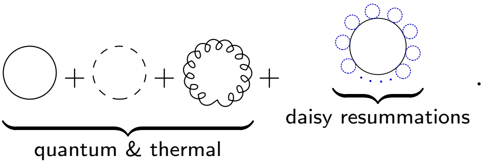


Effective potential



Typically, computed as

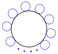
$$V_{\text{eff}}(\phi, T) \approx V_{\text{tree}}(\phi) + \ldots$$



Kapusta '79, Parwani '92, Arnold & Espinosa '92

Order of the EW phase transition? A potted history

- Leading order (LO): $V_T = V_0 +$ 

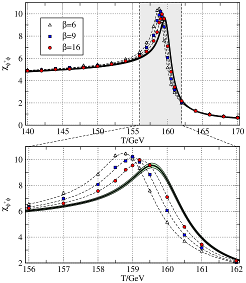
  $\Rightarrow$ 2nd order — Dolan & Jackiw '73
- NLO: $V_T = V_0 +$ 

  $\Rightarrow$ 1st order — Arnold & Espinosa '92
- Infrared problems at higher orders

  $\Rightarrow$? order — Linde '80
- EFT + lattice approach resolves all issues

  $\Rightarrow$ crossover — Kajantie et al '96
- Accurate thermodynamics for SM

  D'Onofrio & Rummukainen '15, Laine & Meyer '15



Quantum field theory at $T > 0$

- Thermodynamics $Z = \mathrm{Tr} e^{-\hat{H}/T}$ formulated in $\mathbb{R}^3 \times S^1$,

- Fields are expanded into Fourier modes:

$$\Phi(\tau, x) = \sum_n \phi_n(x) e^{i(n\pi T)\tau}$$

where n is even (odd) for bosons (fermions).

Dimensional reduction

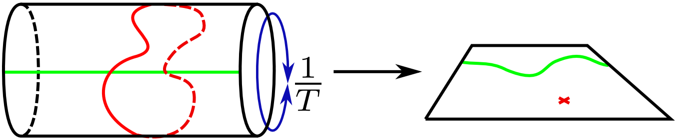
Substituting in the Fourier expansion (here for a scalar),

$$\int_\tau \int_x \left[\frac{1}{2}\Phi(\tau, x)(-\nabla^2 - \partial_\tau^2 + m^2)\Phi(\tau, x)\right] = \frac{1}{T}\sum_n \int_x \left[\frac{1}{2}\phi_n(x)(-\nabla^2 + (n\pi T)^2 + m^2)\phi_n(x)\right].$$

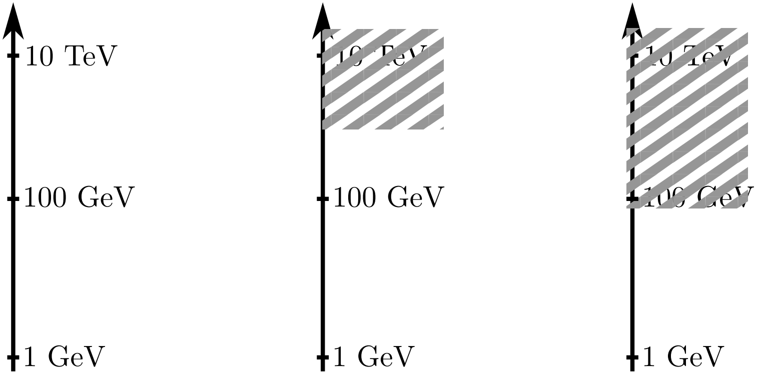
The masses of the Fourier modes are

$$m_n^2 = (n\pi T)^2 + m^2.$$

Matsubara '55

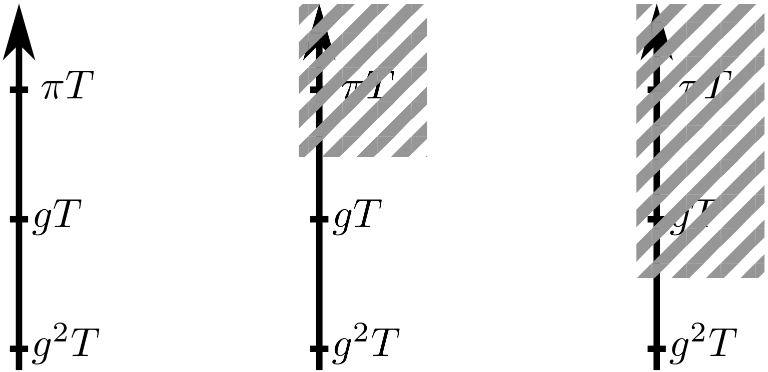


EFTs at zero temperature



$\mathscr{L} = \mathscr{L}_{\mathsf{SM}} + \mathscr{L}_{\mathsf{BSM}}$ $\mathscr{L}_{\text{eff}} \approx \mathscr{L}_{\mathsf{SMEFT}}$ $\mathscr{L}_{\text{eff}} \approx \mathscr{L}_{\mathsf{Fermi}}$

EFTs at high temperature



$$\mathscr{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi} \not{D} \psi$$

$$\mathscr{L}_{\text{eff}} \approx \frac{1}{4} F_{ij} F_{ij} + (D_i A_0)^2 + m_D^2 A_0^2 + \lambda A_0^4$$

$$\mathscr{L}_{\text{eff}} \approx \frac{1}{4} F_{ij} F_{ij}$$

Farakos et al. '94, Braaten & Nieto '95, Kajantie et al. '95

Minimal EFT for the phase transition

Start from BSM model in $T = 0$ and integrate out σ at high T,

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \mathscr{L}_\sigma + \tfrac{1}{2}a_1\sigma\phi^\dagger\phi + \tfrac{1}{2}a_2\sigma^2\phi^\dagger\phi$$

$$\downarrow$$

$$\mathscr{L}_{\rm eff} = \tfrac{1}{4}F^a_{ij}F^a_{ij} + D_i\phi^\dagger D_i\phi + m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2$$

The electroweak phase transition is first order if

$$x \equiv \frac{\lambda_3}{g_3^2} < x_* = 0.0983(15).$$

Kajantie et al '96

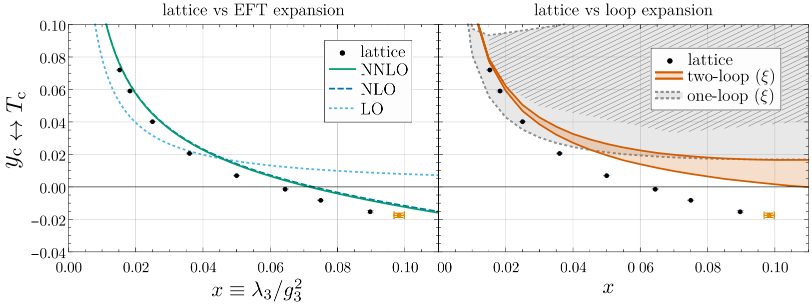
The new scalar modifies the SM value (≈ 0.3) at tree-level,

$$x \approx \frac{\lambda}{g^2} = \frac{m_H^2}{8m_W^2}\left(1 + \frac{m_\sigma^2 - m_H^2}{m_H^2}\sin^2\theta\right),$$

and at loop-level,

$$\Delta x \sim -\frac{a_2^2 T}{\pi g^2 m_{\sigma,3}}.$$

Electroweak phase diagram



OG, Güyer, Rummukainen 2205.07238,

Ekstedt, OG & Löfgren 2205.07241

- Thermodynamics of minimal EFT accurately known
- EFT solves pathologies of loop expansion
- Only the lattice can pin down the endpoint

Infrared strong coupling

Infrared bosons are highly occupied; the effective expansion parameter $\alpha_{\rm eff}$ grows

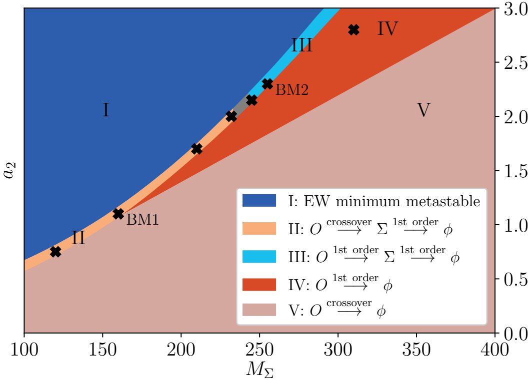
$$\alpha_{\rm eff} \sim g^2 \frac{1}{e^{E/T} - 1} \approx g^2 \frac{T}{E}$$

Softer modes are classically occupied and more strongly coupled:

$$\begin{aligned}
\text{hard}: &\qquad E \sim \pi T \Rightarrow \alpha_{\rm eff} \sim g^2, \\
\text{soft}: &\qquad E \sim gT \Rightarrow \alpha_{\rm eff} \sim g, \\
\text{supersoft}: &\qquad E \sim g^{3/2} T \Rightarrow \alpha_{\rm eff} \sim g^{1/2}, \\
\text{ultrasoft}: &\qquad E \sim g^2 T \Rightarrow \alpha_{\rm eff} \sim 1.
\end{aligned}$$

Linde '80

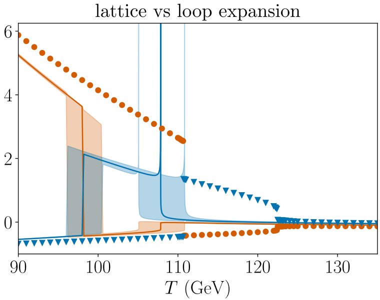
Triplet scalar extension: phase diagram

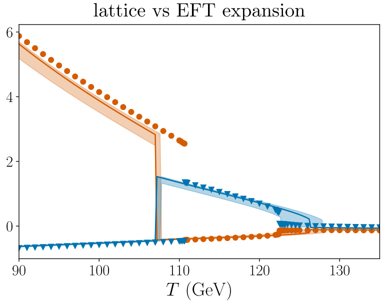


Niemi et al. 2005.11332

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \frac{a_2}{2}\Phi^\dagger\Phi\Sigma^a\Sigma^a + \frac{1}{2}D_\mu\Sigma^a D_\mu\Sigma^a + \frac{m_\Sigma^2}{2}\Sigma^a\Sigma^a + \frac{b_4}{4}(\Sigma^a\Sigma^a)^2$$

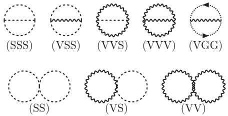
Triplet scalar extension: thermal evolution





- More loops helps, but EFT is crucial.

Niemi et al. 2005.11332, OG & Tenkanen 2309.01672



Real scalar extension: gravitational waves

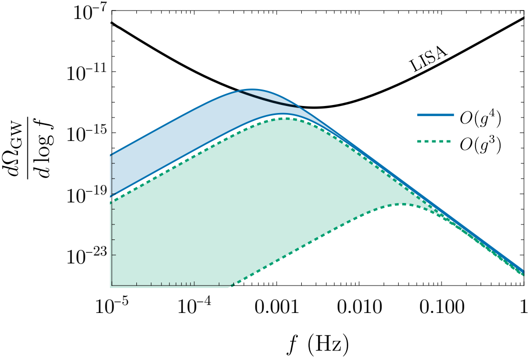
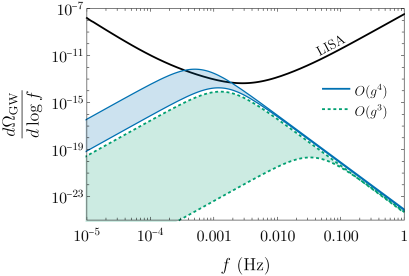


Figure: Renormalisation scale dependence of GW spectrum at one physical parameter point within perturbation theory.

$$\mathscr{L} = \mathscr{L}_{\mathsf{SM}} + \frac{a_2}{2}(\Phi^\dagger\Phi)\sigma^2 + \frac{1}{2}(\partial\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{b_4}{4}\sigma^4$$

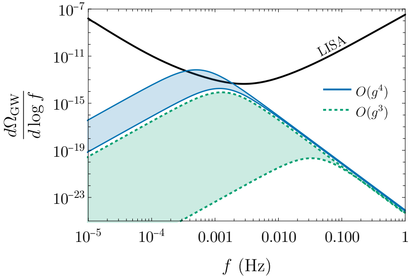
OG & Tenkanen 2104.04399

What's next?



- What about collider/gravitational wave complementarity?
- Why are the uncertainties so large?
- Why does perturbation theory work at all?
- What about nonequilibrium quantities, such as v_w?
- Where is electroweak baryogenesis in this story?

What's next?



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Thanks for listening!

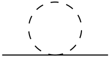
Backup slides

Introducing thermal scales

hard: $E \sim \pi T$

$$m_n^2 = (n\pi T)^2 + m^2 \text{ with } n \neq 0$$

soft: $E \sim gT$

$m_{\text{eff}}^2 \sim$  $\sim g^2 T^2$

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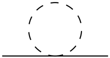
$m_{\text{eff}}^2 \sim$  $\sim g^2 T^2$

supersoft: $E \sim g^{3/2} T$

$$m_{\text{eff}}^2 = -\mu^2 + g^2 T^2$$

ultrasoft: $E \sim g^2 T$

$$A_i^a$$

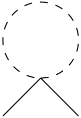
A hierarchy problem

Let's assume there is some very massive particle χ, $M_\chi \gg m_H$, coupled to the Standard Model Higgs Φ like

$$\mathscr{L} = \mathscr{L}_{\rm SM} + g^2 \Phi^\dagger \Phi \chi^\dagger \chi + \mathscr{L}_\chi .$$

If we integrate out χ, we find that the Higgs mass parameter gets a correction of the form

$$(\Delta m_H^2) \Phi^\dagger \Phi \; = $$

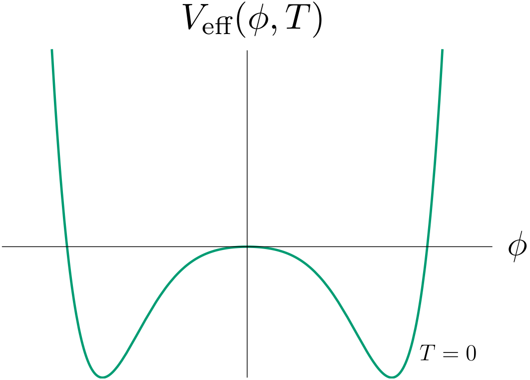


$$\sim g^2 M_\chi^2 \Phi^\dagger \Phi \; .$$

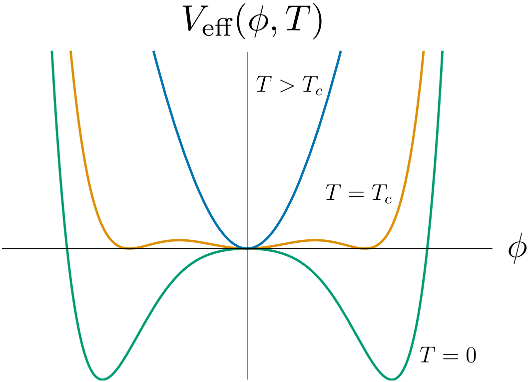
Relevant operators in the IR get large contributions from the UV,

$$\frac{\Delta m_H^2}{m_H^2} \sim g^2 \left(\frac{M_\chi}{m_H} \right)^2 .$$

Phase transitions



Phase transitions



For there to be a phase transition, thermal/quantum fluctuations should modify the potential at leading order,

$$V_{\text{eff}} = V_{\text{tree}} + \Delta V_{\text{fluct}}.$$

Hierarchies in phase transitions

So, for there to be a phase transition, we need

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim g^2 N \left(\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}}\right)^{\sigma} \overset{!}{\sim} 1,$$

where $\sigma > 0$ for relevant operators.

$\Rightarrow$ either:

(i) $\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}} \sim \frac{1}{(g^2 N)^{1/\sigma}} \gg 1$, i.e. scale hierarchy

(ii) $g^2 N \gtrsim 1$, i.e. strong coupling

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Perturbative phase transitions require scale hierarchies

UV and IR problems

There are two main difficulties

- large UV effects break loop expansion
- IR becomes more strongly coupled

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim \alpha_{\text{eff}} \left(\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}}\right)^{\sigma}$$