Quark Flavour Physics - Status and Outlook

(Justus) Tobias Tsang
j.t.tsang@cern.ch

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Introduction

Disclaimer

(near) impossible Task:

"*contribute a 45-minute rapporteur-style talk on* **Quark Flavour Physics**."

- 51 parallel talks in "Quark and Lepton Flavour Physics"!
- + additional talks in "Standard Model Parameters" and posters

My interpretation:

Quark flavour physics

=

"processes with a change in quark flavour"

=

"occurrence of Cabibbo-Kobayashi-Maskawa matrix elements"

☺ Thanks to everyone who has made me aware of their latest work!

☹ Too much material to cover in 40+5 minutes ⇒ Needed to make a selection!

☺ 1st talk of Lattice24 ⇒ all quark-flavour parallel talks are yet to come!

Flavour Physics: The Cabibbo-Kobayashi-Maskawa matrix

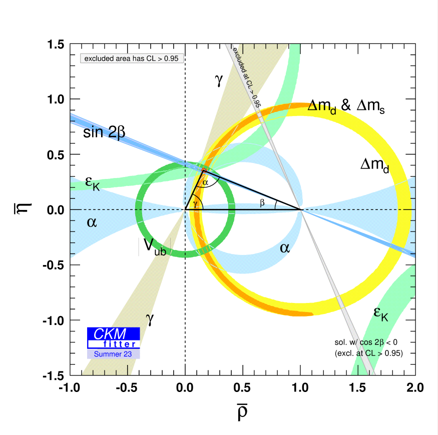
C[Cabibbo '63]KM[Kobayashi, Maskawa '73]

- parameterises transitions up-type ↔ down-type

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- **unitary matrix in the SM**
- **fundamental SM parameters**
- bands: different constraints on the same parameters (→)
- unitarity ⇔ meet in one apex

Unitarity Triangle [CKMfitter'05 + web updates]



Non-unitarity of CKM ⇔ New Physics Beyond the SM

CKM $=$ theory $\otimes$ experiment

$$\text{experiment} \approx \left|V_{qq'}\right|^n \sum_i \text{kinematic} \times \text{non-perturbative}$$

$$\Gamma(B \to \ell\nu) \approx |V_{ub}|^2 \, \mathcal{K} f_B$$

$$\frac{\mathrm{d}\Gamma(B \to \pi\ell\nu)}{\mathrm{d}q^2} \approx |V_{ub}|^2 \left(\mathcal{K}_1 f_+^{B\to\pi}(q^2) + \mathcal{K}_2 f_0^{B\to\pi}(q^2)\right)$$

$$\Delta m_d \approx \left|V_{tb}^* V_{td}\right|^2 \mathcal{K} f_{B_d}^2 \hat{\mathcal{B}}_{B_d}^{(1)}$$

- CKM is extracted as combination of experiment and theory
- Precision of CKM matrix element depends on knowledge of theory AND experiment.
- e.g. decay constants easy for lattice, hard for experiment
- Can access the same CKM matrix element from
  - different processes
  - different experimental bins
  - different lepton final states
  - ...

Structure of this talk

Many ways to order this talk! By

- flavour content (light; strange; charm; bottom)
- type of process (leptonic; semileptonic; mixing; radiative; ...)
- hadron type (mesonic; baryonic)
- "size" of the decay (tree; loop or 'rare')
- complexity of calculation ("established"; novel; exploratory; future prospects)
- reported precision
- lattice parameters (N_f, choice of discretisation, parameter ranges)
- CKM-unitarity test (row/column of the CKM matrix; triangles)
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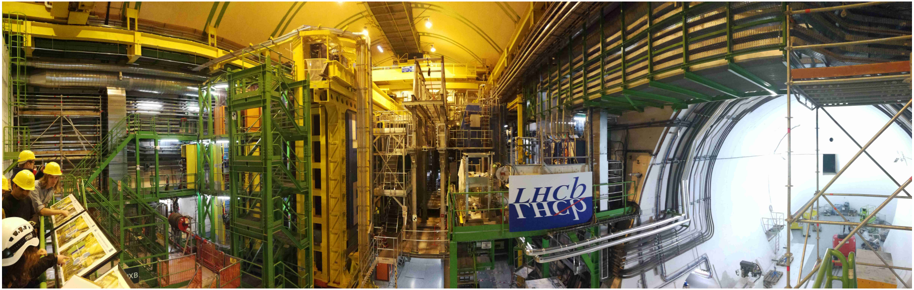
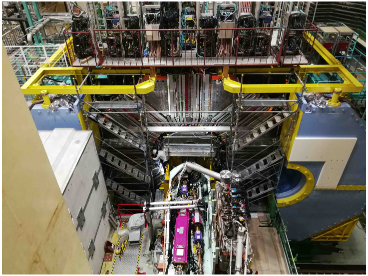
(We will see what "established" means later on)

Current heavy flavour experiments





top: LHCb at LHC, CERN (next talk!)

left: Belle II at SuperKEKb, KEK

⇒ Huge experimental efforts!
+ BES-III and other LHC experiments

⇒ <u>B-**factory**</u>; <u>**hadron machine**</u>
Very complementary

+ older data from BaBar, Belle, Cleo, ...

b-decays as "sweet spot" for experiments

Properties of b-decays [PDG'20]

1. $\overline{m}_b(\overline{m}_b) = 4.18(3)\,\mathrm{GeV} \gg \overline{m}_c(\overline{m}_c) = 1.27(2)\,\mathrm{GeV} \gg m_s, m_u, m_d$
   $\rightarrow$ many different decay products
2. b hadrons have *relatively long* lifetime of $\tau_b \sim 10^{-12}s$ ($\tau_t \sim 10^{-25}s$)
   $\rightarrow$ b hadronises and b-jets travel some distance before decaying
   $\rightarrow$ but not far enough to escape the detector
   $\rightarrow$ allows for $b-$**tagging**

$\Rightarrow$ **Plethora of accessible decay channels for hadrons with b-quarks**

Distinguish two categories:

Charged currents

- Present at *tree level* in the SM
  e.g. $B^0 \rightarrow D^+\ell^-\nu_\ell$
  $\Rightarrow$ Precision tests of the SM

Flavour changing neutral currents

- Only at *loop level* in the SM
  e.g. $B \rightarrow K\ell^+\ell^-$
  $\Rightarrow$ Sensitive to NP searches

Outline

1. Introduction
2. Heavy decays: “bread and butter” (?)
3. Heavy decays: “suggested benchmarks”
4. Heavy decays: “tackling systematics”
5. Selection of other new works and ongoing efforts

Heavy decays: "bread and butter" (?)

Wealth of observables (incomplete list)

$b \to u$ (tree)

- $B \to \ell\nu$
- $B \to \pi\ell\nu$
- $B_s \to K\ell\nu$
- $\Lambda_b \to p\ell\nu$
- $\dots$

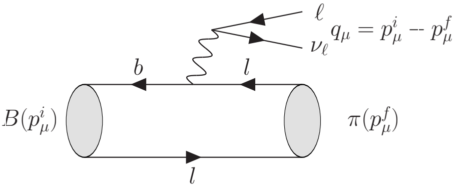
$b \to c$ (tree)

- $B \to D^{(*)}\ell\nu$
- $B_s \to D_s^{(*)}\ell\nu$
- $\Lambda_b \to \Lambda_c\ell\nu$
- $B_c \to J/\Psi\ell\nu$
- $\dots$

$b \to s$ (loop)

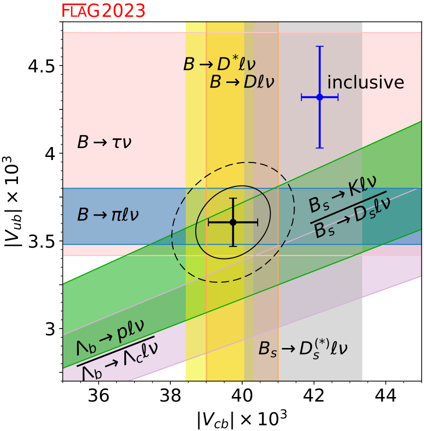
- $B \to K^{(*)}\ell\ell$
- $B \to \rho\ell\ell$
- $B_s \to \phi\ell\ell$
- $B \to \pi\ell\ell$
- $\dots$

In particular: Many different semi-leptonic decays



- In the SM:
  - PS $\to$ PS (tree): 2 ffs
  - PS $\to$ V (tree): 4 ffs
  - PS $\to$ PS (loop): 3 ffs
  - PS $\to$ V (loop): 7 ffs
- Each ff depends on momentum transfer q^2 to the lepton pair

What is the status of V_{ub} and V_{cb}? Let's look at FLAG!



Consistency between different determinations ✓(or is it?!)

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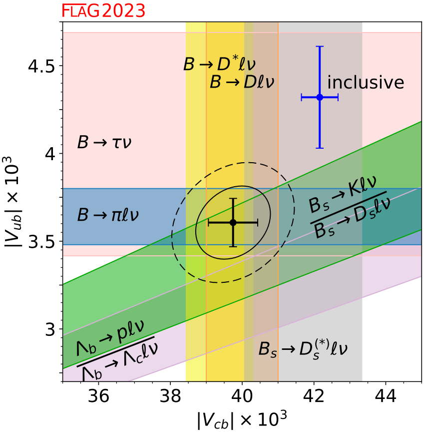

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$B \to \tau\nu$: good agreement! ✓

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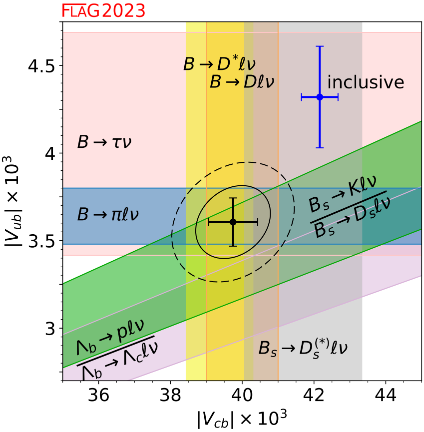

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$\Lambda_b \to p$: Only a single result ✓(?)

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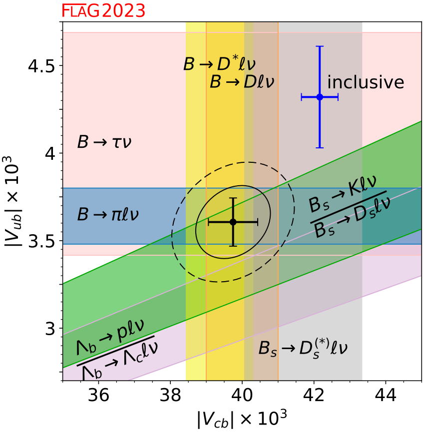

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$B \to \pi\ell\nu$: $p \sim 2 \times 10^{-5}$ ✗

Y. Aoki et al. *FLAG Review 2021* 2111.09849

$B \to \pi$ ($N_f = 2+1$)

	Central Values	Correlation Matrix				
a_0^+	0.423 (21)	1	-0.00466	-0.0749	0.402	0.0920
a_1^+	−0.507 (93)	-0.00466	1	0.498	-0.0556	0.659
a_2^+	−0.75 (34)	-0.0749	0.498	1	-0.152	0.677
a_0^0	0.561 (24)	0.402	-0.0556	-0.152	1	-0.548
a_1^0	−1.42 (11)	0.0920	0.659	0.677	-0.548	1

Table 46: Coefficients and correlation matrix for the $N^+ = N^0 = 3$ z-expansion fit of the $B \to \pi$ form factors f_+ and f_0. The coefficient a_2^0 is fixed by the $f_+(q^2 = 0) = f_0(q^2 = 0)$ constraint. The chi-square per degree of freedom is $\chi^2/\text{dof} = 43.6/12$ and the errors on the z-parameters have been rescaled by $\sqrt{\chi^2/\text{dof}} = 1.9$. The lattice calculations that enter this fit are taken from FNAL/MILC 15 [58], RBC/UKQCD 15 [59] and JLQCD 22 [60]. The parameterizations are defined in Eqs. (533) and (534).

Consistency between different determinations ✓(or is it?!)

$\exists$ sys errors $\Rightarrow$ p only indicative

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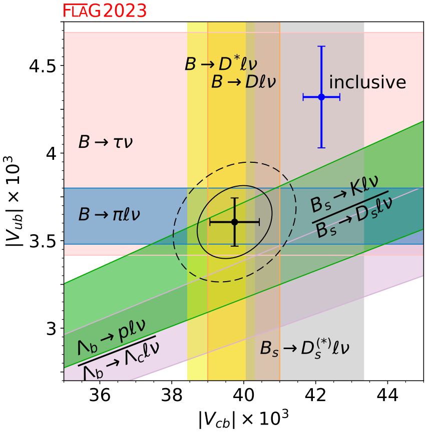

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$B \to \pi\ell\nu$: $p \sim 2 \times 10^{-5}$ ✗

$B_s \to K\ell\nu$: $p \sim 7 \times 10^{-6}$ ✗

Y. Aoki et al. *FLAG Review 2021* 2111.09849

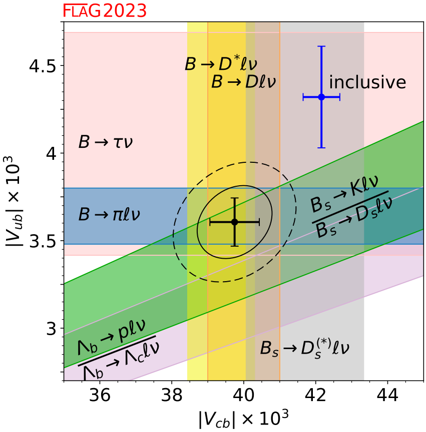
$B_s \to K$ ($N_f = 2+1$)

	Central Values	Correlation Matrix						
a_0^+	0.370(21)	1.	0.2781	−0.3169	−0.3576	0.6130	0.3421	0.2826
a_1^+	−0.68(10)	0.2781	1.	0.3672	0.1117	0.4733	0.8487	0.8141
a_2^+	0.55(48)	−0.3169	0.3672	1.	0.8195	0.3323	0.6614	0.6838
a_3^+	2.11(83)	−0.3576	0.1117	0.8195	1.	0.2350	0.4482	0.4877
a_0^0	0.234(10)	0.6130	0.4733	0.3323	0.2350	1.	0.6544	0.5189
a_1^0	0.135(86)	0.3421	0.8487	0.6614	0.4482	0.6544	1.	0.9440
a_2^0	0.20(35)	0.2826	0.8141	0.6838	0.4877	0.5189	0.9440	1.

Table 48: Coefficients and correlation matrix for the $N^+ = N^0 = 4$ z-expansion of the $B_s \to K$ form factors f_+ and f_0. The coefficient a_3^0 is fixed by the $f_+(q^2=0) = f_0(q^2=0)$ constrain. The chi-square per degree of freedom is $\chi^2/\mathrm{dof} = 3.82$ and the errors on the z-parameters have been rescaled by $\sqrt{\chi^2/\mathrm{dof}} = 1.95$.

(I counted 7 fit parameters and 19 datapoints ⇒ 12 dof's)

What is the status of V_{ub} and V_{cb}? Let's look at FLAG!



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$|V_{ub}|\,(B \to \pi)$: $p \sim 3 \times 10^{-5}$ ✗

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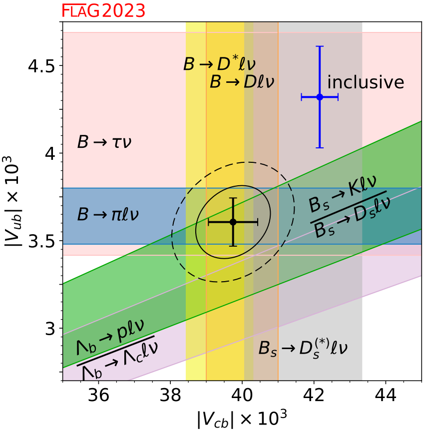
$B \to \pi\ell\nu$ ($N_f = 2+1$)

	Central Values	Correlation Matrix					
$\lvert V_{ub}\rvert \times 10^3$	3.64 (16)	1	-0.812	-0.108	0.128	-0.326	-0.151
a_0^+	0.425 (15)	-0.812	1	-0.188	-0.309	0.409	0.00926
a_1^+	−0.441 (39)	-0.108	-0.188	1	-0.498	-0.0343	0.150
a_2^+	−0.52 (13)	0.128	-0.309	-0.498	1	-0.190	0.128
a_0^0	0.560 (17)	-0.326	0.409	-0.0343	-0.190	1	-0.772
a_1^0	−1.346 (53)	-0.151	0.00926	0.150	0.128	-0.772	1

Table 57: $|V_{ub}|$, coefficients for the $N^+ = N^0 = N^T = 3$ z-expansion of the $B \to \pi$ form factors f_+ and f_0, and their correlation matrix. The chi-square per degree of freedom is $\chi^2/\mathrm{dof} = 116.7/62 = 1.88$ and the errors on the fit parameters have been rescaled by $\sqrt{\chi^2/\mathrm{dof}} = 1.37$. The lattice calculations that enter this fit are taken from FNAL/MILC [58], RBC/UKQCD [59] and JLQCD [60]. The experimental inputs are taken from BaBar [161, 162] and Belle [163, 164].

What is the status of V_{ub} and V_{cb}? Let's look at FLAG!



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$|V_{ub}|\,(B \to \pi)$: $p \sim 3 \times 10^{-5}$ ✗

$B \to K\ell\ell$: $p \sim 0.046$ (✓)

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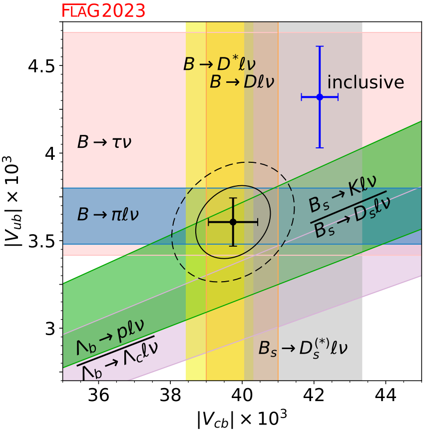
$B \to K$ ($N_f = 2+1$)

	Central Values	Correlation Matrix							
a_0^+	0.471 (14)	1	0.513	0.128	0.773	0.594	0.613	0.267	0.118
a_1^+	−0.74 (16)	0.513	1	0.668	0.795	0.966	0.212	0.396	0.263
a_2^+	0.32 (71)	0.128	0.668	1	0.632	0.768	-0.104	0.0440	0.187
a_0^0	0.301 (10)	0.773	0.795	0.632	1	0.864	0.393	0.244	0.200
a_1^0	0.40 (15)	0.594	0.966	0.768	0.864	1	0.235	0.333	0.253
a_0^T	0.455 (21)	0.613	0.212	-0.104	0.393	0.235	1	0.711	0.608
a_1^T	−1.00 (31)	0.267	0.396	0.0440	0.244	0.333	0.711	1	0.903
a_2^T	−0.9 (1.3)	0.118	0.263	0.187	0.200	0.253	0.608	0.903	1

Table 51: Coefficients and correlation matrix for the $N^+ = N^0 = N^T = 3$ z-expansion of the $B \to K$ form factors f_+, f_0 and f_T. The coefficient a_2^0 is fixed by the $f_+(q^2=0) = f_0(q^2=0)$ constraint. The chi-square per degree of freedom is $\chi^2/\text{dof} = 1.86$ and the errors on the z-parameters have been rescaled by $\sqrt{\chi^2/\text{dof}} = 1.36$.

(does not include HPQCD'23 $N_f = 2+1+1$ yet)
(I counted 8 fit parameters and 18 datapoints $\Rightarrow$ 10 dof's)

What is the status of V_{ub} and V_{cb}? Let's look at FLAG!



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We need to scrutinise this!

How to simulate the b-quark?

$m_b/M_\pi \approx 30$ and we want to be "far" away from IR and UV cut-offs.
$\Rightarrow$ Need to simultaneously satisfy: $(am_b)^{-1} \gg 1$, $M_\pi L \geq 4$
$\Rightarrow (am_b)^{-1} M_\pi L \gg 4$, so we require $L/a \gg 120$ (for multiple choices of a!)
Currently computationally impossible at physical quark masses!

Effective action for b

- Can tune to $m_b \sim m_b^{\rm phys}$
- comes with systematic errors which are hard to estimate/reduce

(HQET, NRQCD, Fermilab, RHQ,...)

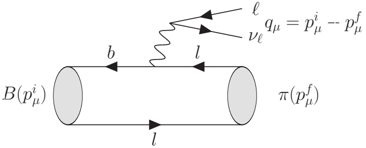
Relativistic action for b

- Theoretically cleaner and systematically improvable
- $m_b < m_b^{\rm phys}$: control extrapolation to $m_b^{\rm phys}$

(HISQ, DWF, TM, Wilson,...)

- relativistic will win in the long term
- for now, settle on a compromise.
- different systematics but should produce complementary results ($\Rightarrow$ reminiscent of (light) fermion discretisations...)

Challenges in computing $f_X(q^2)$: example $B \to \pi\ell\nu$



- $q^\mu = p_B^\mu - p_\pi^\mu$
- $M_B \approx 5.28\,\mathrm{GeV},\ M_\pi \approx 0.14\,\mathrm{GeV}$
- Semileptonic region $q^2 \in [0, q^2_{\mathrm{max}}]$
- $q^2_{\mathrm{max}} \equiv (M_B - M_\pi)^2 \sim 26.4\,\mathrm{GeV}^2$

- physical kinematics in the B rest-frame: $q^2 = 0 \Leftrightarrow |p_\pi|^2 = 6.96\,\mathrm{GeV}^2$
- Assuming $M_\pi L = 4$ and physical pion masses implies:
  $\Rightarrow$ final state momentum of $\vec{p}_\pi \approx \frac{2\pi}{L}(7,7,7)$ to reach $q^2 \sim 0$.
- typical simulations cannot achieve (i.e. control) this
  $\Rightarrow$ compromise in at least one of the following:
  - $M_\pi > M_\pi^{\mathrm{phys}}$ ($\Rightarrow$ need chiral extrapolation)
  - $m_b < m_b^{\mathrm{phys}}$ ($\Rightarrow$ need heavy quark mass extrapolation)
  - $q^2_{\mathrm{min}} \gg 0$ ($\Rightarrow$ need kinematic extrapolation)

From correlators to the physical world

Extrapolations are based on theoretical foundations...

- Extraction of ground state parameters see also → Tue 12:15 (Antoine Geradin)
- $M_\pi^{\rm phys}$ (chiral) extrapolation guided by heavy meson chiral perturbation theory (HMχPT)
- $m_b^{\rm phys}$ (heavy quark) extrapolation guided by HQET
- $q^2 = 0$ (kinematic) extrapolation guided by model independent z-expansion (BGL, BCL) [or $(w-1)$ for heavy to heavy]
  - Physical q^2 dependence can be mapped to interval $z(q^2) \in [-z_{\rm max}, z_{\rm max}]$ with $0 < z_{\rm max} \ll 1$
  - BGL expansion: $f_X(z) = \frac{1}{B_X \phi_X} \sum_i a_i z^i$, unitarity bounds $\sum_i |a|_i^2 < 1$.
- $a \to 0$ (continuum limit) extrapolation guided by Symanzik E.T.

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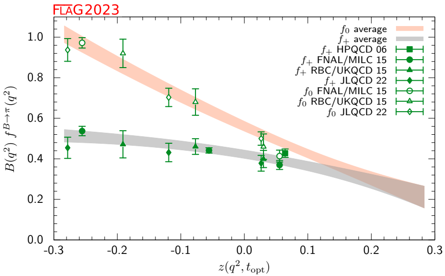
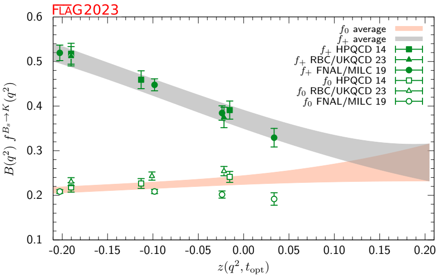
... but they are intertwined and difficult

and all of them come with systematic uncertainties - are they controlled?

FLAG's summary of $B \to \pi$ and $B_s \to K$



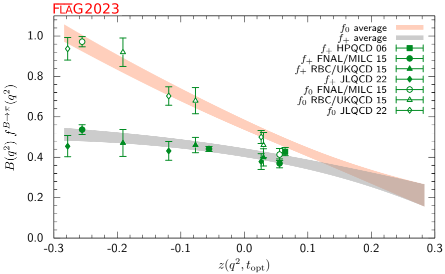


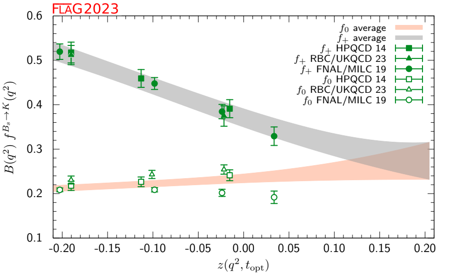
- f_+ looks fine, f_0 shows some tensions
- Most experimental data obtained for $\ell \in \{e, \mu\}$, so $m_\ell \sim 0$ and recall:

$$\frac{\mathrm{d}\Gamma(B_{(s)} \to P\ell\nu_\ell)}{\mathrm{d}q^2} = |V_{ub}|^2 \mathcal{K} \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) \left|f_+(q^2)\right|^2 + \mathcal{K}_2 m_\ell^2 \left|f_0(q^2)\right|^2 \right]$$

Does that mean V_{ub} should be fine?

FLAG's summary of $B \to \pi$ and $B_s \to K$





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Does that mean V_{ub} should be fine? ✗

- kinematic extrapolation (z-expansion) stabilised by kinematic constraint $f_0(0) = f_+(0)$, so f_0 does impact CKM determinations!

Some insights from $B_s \to K$ (1)

$$f_X^{B_s \to K} = \frac{\Lambda}{E_K + \Delta_X} \times \left[\chi(M_\pi^2) + k(E_K) + d((a\Lambda)^2)\right]$$

where Δ_X is the "relevant pole mass"

$$\Delta_+ = M_{B^*(1^-)} - M_{B_s}, \quad M_{B^*(1^-)} = 5.32471\,\mathrm{GeV} \quad \text{(exp.)}$$
$$\Delta_0 = M_{B^*(0^+)} - M_{B_s}, \quad M_{B^*(0^+)} = 5.63\,\mathrm{GeV} \quad \text{(the.)}$$

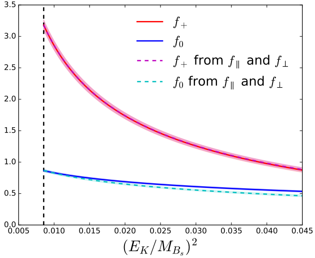
RBC/UKQCD'15 and FNAL/MILC'19 strategy:

1. Assume $f_\parallel$ dominated by f_0 and $f_\perp$ dominated by f_+.
2. HMχPT fit to $f_\parallel$, $f_\perp$ using $\Delta_\parallel \sim \Delta_0$, $\Delta_\perp \sim \Delta_+$
3. converting to f_+, f_0 in the continuum

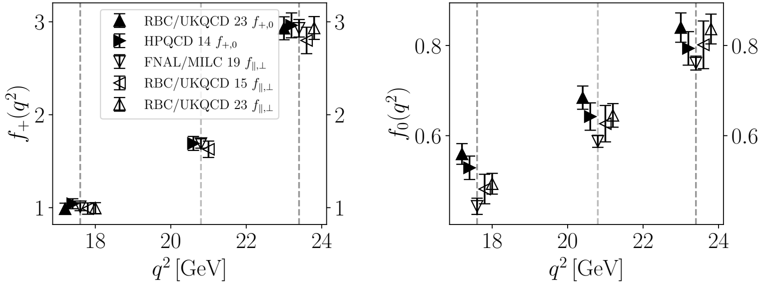
Is this justified?

And how to deal with poles when $m_h \neq m_b$?

Some insights from $B_s \to K$ (2) [JTT, RBC/UKQCD PRD 107 (2023) 114512]



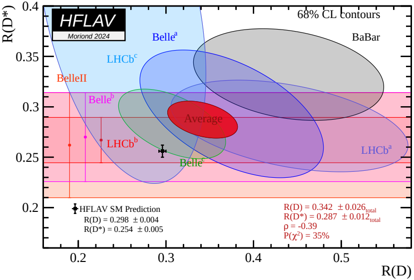
- ← All fine for f_+ (red vs magenta)✓
- ← Several (stat) sigmas difference for f_0 ✗!!
- ← Discrepancy gets worse with increasing energy ⇒ easy to miss!
- ↓ picture persists with full error budget



⇒ Not unique to $B_s \to K$, same strategy was used for $B \to \pi$

What about $b \to c$? $B \to D^* \ell \nu$ and $R(D^*)$

$$R(D^{(*)}) = \frac{\int \mathrm{d}q^2 \mathrm{d}\Gamma(B \to D^{(*)}\tau\nu)/\mathrm{d}q^2}{\int \mathrm{d}q^2 \mathrm{d}\Gamma(B \to D^{(*)}\ell\nu)/\mathrm{d}q^2}$$



- Long standing "tension"
- SM here not from lattice (yet!)
- $B \to D^*$ also important for V_{cb}

☺ 3 recent results away from q^2_{max} [FNAL/MILC'21, HPQCD'23, JLQCD'23]

☺ different ensembles, different actions, different analyses

☺ jointly fittable with $p > 5\%$ [Bordone, Jüttner '24].

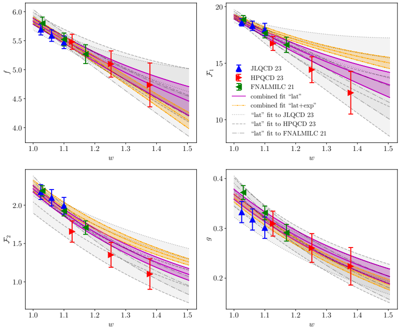
☺ Quoted $R(D^*)$ values:
0.265(13) [FNAL/MILC'21]
0.273(15) [HPQCD'23]
0.252(22) [JLQCD'23]

? using known constraints: [Martinelli, Simula, Vittorio'23] [Bordone, Jüttner '24]

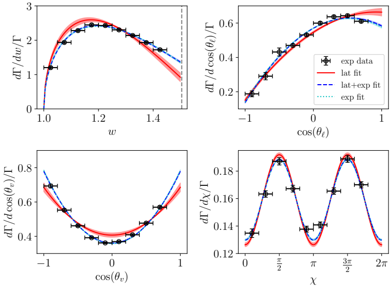
	FNAL/MILC'21	HPQCD'23	JLQCD'23
$[MSV]$	0.275(8)	0.266(12)	0.247(8)
$[BJ]$	0.2748(89)	0.270(13)	0.2482(81)

There is more than just $R(D^*)$. Comparison to experimental shapes?

Situation in $B \to D^*\ell\nu$ — shapes

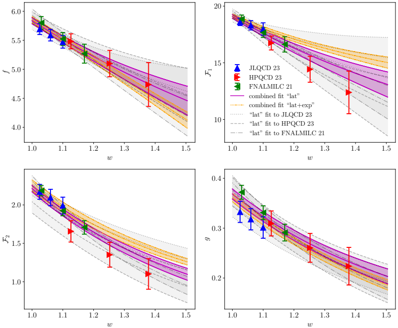


FNAL/MILC'21 + HPQCD'23

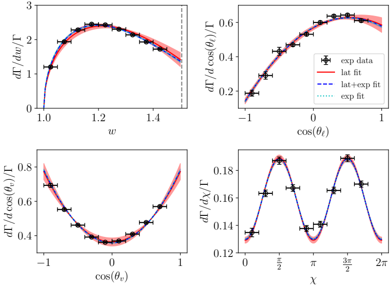


- Plots taken from [Bordone, Jüttner'24]; see also [Martinelli, Simula, Vittorio'23]
- Experimental data from HFLAV'24 (Belle + Belle II combination)
- V_{cb} from ratio of lattice and experiment...

Situation in $B \to D^*\ell\nu$ — shapes



JLQCD'23



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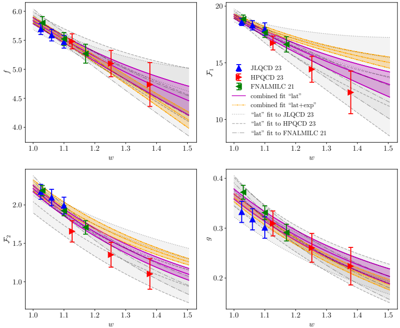
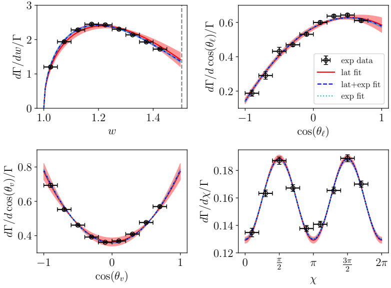
Situation in $B \to D^*\ell\nu$ — shapes



JLQCD'23



- Plots taken from [Bordone, Jüttner'24]; see also [Martinelli, Simula, Vittorio'23]
- Experimental data from HFLAV'24 (Belle + Belle II combination)
- V_{cb} from ratio of lattice and experiment...
- The jury is still out $\Rightarrow$ I am looking forward to many updates!

exclusive (semi-)leptonic decay at Lattice'24

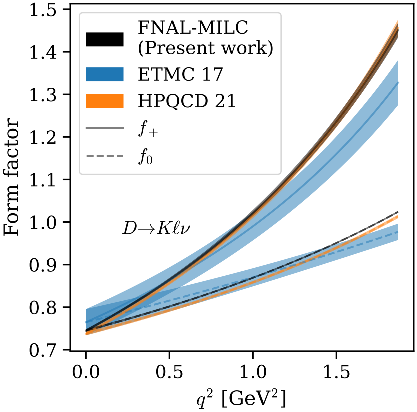
☺ Great to see a lot of ongoing work:

- Tue 13:45 (Yu Meng): $J/\Psi \to D, D_s\ell\nu$
- Tue 14:05 (Anastasia Bouchmelev): $B_{(s)} \to D^*_{(s)}\ell\nu$
- Tue 14:25 (Pietro Butti): $B_{(s)} \to D^*_{(s)}\ell\nu$
- Tue 14:45 (Logan Roberts): $B_{(s)} \to \pi, K, D_{(s)}\ell\nu$ and $D_{(s)} \to \pi, K\ell\nu$
- Tue 16:35 (Judd Harrison): $B_c \to J/\Psi\ell\nu$
- Tue 16:35 (Tinghong Shen): semileptonic D decays [SM params]
- Tue 11:35 (Callum Radley-Scott): Form factor curves consistent with unitarity for semileptonic decays
- N/A (Andrew Lytle, FNAL/MILC): $B_s \to K, D_s$ $B_{(s)} \to D^*_{(s)}$, all HISQ
- Tue 16:55 (Kerr Miller): $B^{(*)}_{(s)}$ and $D^{(*)}_{(s)}$ decay constants + hyperfine splittings
- Tue 16:15 (Wolfgang Soeldner): charmed decay constants [SM params]
- as well as posters!!

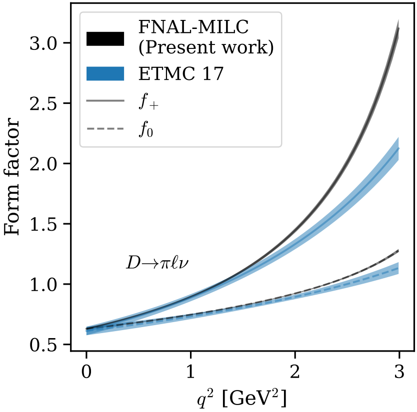
What about charm decays?

Not unique to b-decays: Similar tensions for $c \to s$ and $c \to d$

[FNAL/MILC'22]



[FNAL/MILC'22]



Puzzling: Tensions at large q^2 where data should be most precise.

$\Rightarrow$ We need to resolve these discrepancies

Heavy decays: “suggested benchmarks”

How to scrutinise results?

- these analyses are **hard** and very time consuming!
- many dependencies (sources of systematics) to consider:
  - excited states (particularly when approaching M_π^{phys} ensembles)
  - chiral (M_π)
  - heavy quark (m_b)
  - kinematic (q^2)
  - discretisation, improvement and renormalisation (a)
- limited data to control all of these
- many choices to make and/or parameters to fit

$\Rightarrow$ **easy to miss something!** (...and given the spread of results, we might be...)

Furthermore different works have

- data sets with different parameter coverage
- data sets with different statistical and systematic properties
- different approaches

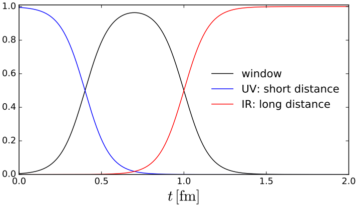
Are there "simpler" quantities to compare?

Learning from our community: Similarities with $g-2$

- the stakes are high ✓
- if final results disagree, it is very hard to pin down why ✓
- potential for an "analyst bias" ✓
- "easy" to blind ✗
  (normalisation vs shapes!)
  but we should still do it!

"Resolved" by **simpler quantities**, less susceptible to some systematics

["windows" - JTT, RBC/UKQCD, PRL 121 (2018) 2, 022003]

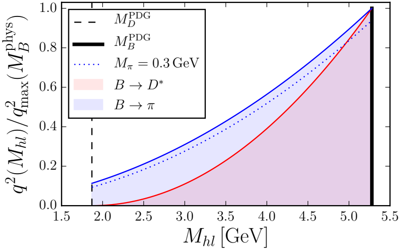


Is there something similar we can do for exclusive heavy decays? In particular it would be great to have:

⇒ **quantities which can be compared relying on fewer extrapolations**
⇒ **publish enough information to reproduce the analyses**

Interplay between extrapolations

Target: solid black line



Reality:

- Effective b: top right along the line
- Relativistic b: some data points near the solid line to the left

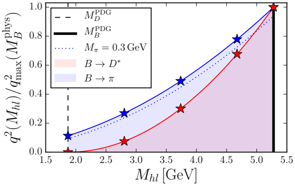
varying m_h changes

- q^2 range
- size of $(am_h)^n$ cut-off effects
- position of poles

Further difficulties:

- Signal to Noise — gets worse as $m_b \uparrow$ and $m_\pi \downarrow$ and $|\vec{p}| \uparrow$.
  $\Rightarrow$ Most precise data far away from desired kinematics.
- Continuum limit — fewer values of a as m_h is increased
  $\Rightarrow$ CL relies on data from smaller m_h
- Dependence on ansatz ($E_{\rm final}$ vs q^2 vs z vs $w-1$ expansion)?
- Parameter counting: many effects $\Rightarrow$ many parameters

Suggestion: benchmarks & checks [JTT, Della Morte: EPJ-ST 233 (2024), 253-270]

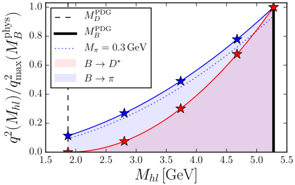


Goal: Test how well a global fit works/what data drives result?

Simplest benchmarks: **Full error budgets** for

1. separate continuum limits at q^2_{max} at fixed m_h
   - $\Rightarrow$ Directly accessible (some ffs)
   - $\Rightarrow$ everything at rest (no kinematic extraps.)
   - $\Rightarrow$ fixed $(am_h)^n$ terms

Suggestion: benchmarks & checks [JTT, Della Morte: EPJ-ST 233 (2024), 253-270]

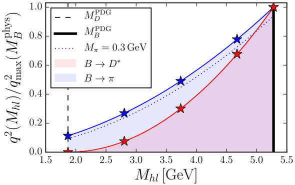


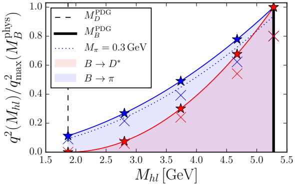
Goal: Test how well a global fit works/what data drives result?

Simplest benchmarks: **Full error budgets** for

1. separate continuum limits at $q^2_{\max}$ at fixed m_h
2. separate m_h extrap. at fixed $q^2_{\max}$ in the continuum vs simultaneous m_h-continuum extrap.
   $\Rightarrow$ everything at rest (no kinematic extraps.)
   $\Rightarrow$ assess $(am_h)^n$ terms

Suggestion: benchmarks & checks [JTT, Della Morte: EPJ-ST 233 (2024), 253-270]



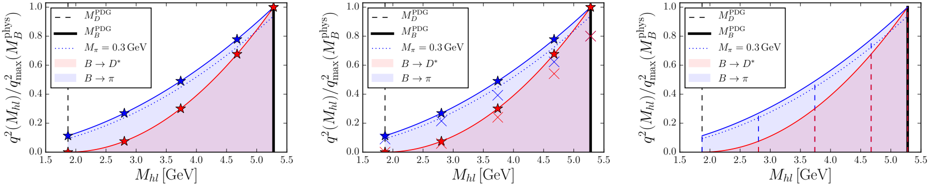


Goal: Test how well a global fit works/what data drives result?

Simplest benchmarks: **Full error budgets** for

1. separate continuum limits at $q^2_{\rm max}$ at fixed m_h
2. separate m_h extrap. at fixed $q^2_{\rm max}$ in the continuum vs simultaneous m_h-continuum extrap.
3. separate continuum limits at $q^2 < q^2_{\rm max}$ at fixed m_h
   - $\Rightarrow$ only small kinematic interpolations
   - $\Rightarrow$ probes information content of simulated data
   - $\Rightarrow$ no heavy-quark extrapolation

Suggestion: benchmarks & checks [JTT, Della Morte: EPJ-ST 233 (2024), 253-270]



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Simplest benchmarks: **Full error budgets** for

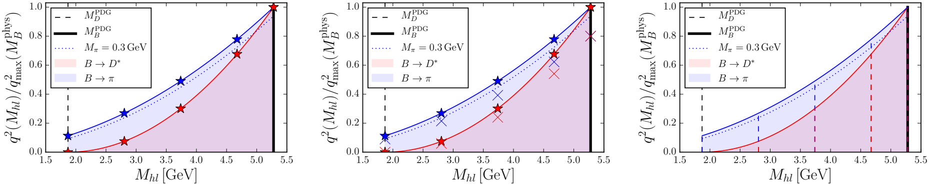
1. separate continuum limits at q^2_{max} at fixed m_h
2. separate m_h extrap. at fixed q^2_{max} in the continuum vs simultaneous m_h-continuum extrap.
3. separate continuum limits at $q^2 < q^2_{\mathrm{max}}$ at fixed m_h
4. separate continuum-kinematic extrapolation at fixed m_h

   $\Rightarrow$ no heavy-quark extrapolation

   $\Rightarrow$ fixed $(am_h)^n$ terms

   $\Rightarrow$ errors come from data in the simulated region

Suggestion: benchmarks & checks [JTT, Della Morte: EPJ-ST 233 (2024), 253-270]



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1. separate continuum limits at $q^2_{\rm max}$ at fixed m_h
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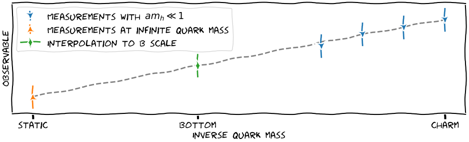
3. separate continuum limits at $q^2 < q^2_{\rm max}$ at fixed m_h
4. separate continuum-kinematic extrapolation at fixed m_h

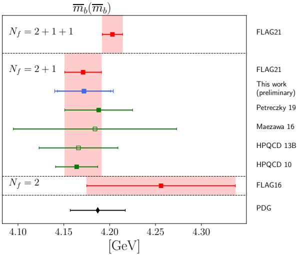
- Publish reference q^2 value data **before** z-expansion
  $\Rightarrow$ no unitarity imposed yet, no error reduction from z-expansion
- **publish fit coefficients** & **correlations** (for all fits)

Heavy decays: "tackling systematics"

Controlling the b: extrapolation $\to$ interpolations

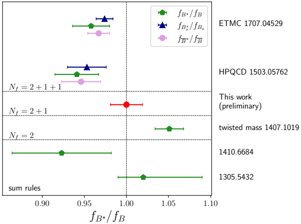
[Sommer, Conigli, ALPHA, PoS LATTICE2023 (2024) 268+237; S. Kuberski: Lattice@CERN'24]





(Rough) idea: Interpolate between results from

- small volumes $\sim 0.5\,\mathrm{fm}$, but $am_b \ll 1$
- large scale simulations at $m_h \lesssim m_b$.



after continuum extrapolation$^{(\dagger)}$. Consider observables s.t. matching and renormalisation cancel + simple $1/m_h^n$ scaling (no log corrections).
First (preliminary) applications m_b and $f_{B^*}/f_B \to$

($\dagger$) different works can be combined!

Use this to design optimal normalisation of benchmark observables.

Taming discretisation effects with massive NPR?

- RI/MOM and RI/SMOM are defined in the massless limit of QCD

$\Rightarrow$ mass independent renormalisation constants Z

$\Rightarrow$ introduces discretisation effects scaling with $(am_q)^n$.

$\Rightarrow$ on typical lattices $am_c \sim 0.2$ $am_b \lesssim 1$. Large cut-off effects!

- extension of RI/SMOM away from chiral limit: renormalisation conditions at finite renormalised mass $\overline{m}$ suggested in [Boyle et al., 2016],

<u>ADVANTAGE:</u> Different masses at which the scheme is defined.
Different approaches to the continuum limit?
Possible to choose this to reduce cut-off effects?

<u>PRACTICAL TEST:</u> First numerical implementation of mSMOM applied to computation of the charm quark mass [JTT, RBC/UKQCD, arxiv:2407.18700]

Renormalisation conditions in RI/(m)SMOM

RI/SMOM [Sturm et al, PRD 80 (2009) 014501]

$$1 = \lim_{m_R \to 0} \frac{1}{12p^2} \mathrm{Tr}\left[-iS_R(p)^{-1} \not{p}\right]$$

$$1 = \lim_{m_R \to 0} \frac{1}{12m_R} \left\{ \mathrm{Tr}\left[S_R(p)^{-1}\right] + \frac{1}{2} \mathrm{Tr}\left[\left(iq \cdot \Lambda_{\mathrm{A},R}\right) \gamma_5\right] \right\}$$

$$1 = \lim_{m_R \to 0} \frac{1}{12q^2} \mathrm{Tr}\left[\left(q \cdot \Lambda_{\mathrm{V},R}\right) \not{q}\right]$$

$$1 = \lim_{m_R \to 0} \frac{1}{12q^2} \mathrm{Tr}\left[q \cdot \Lambda_{\mathrm{A},R} \gamma_5 \not{q}\right]$$

$$1 = \lim_{m_R \to 0} \frac{1}{12i} \mathrm{Tr}\left[\Lambda_{\mathrm{P},R} \gamma_5\right]$$

$$1 = \lim_{m_R \to 0} \frac{1}{12} \mathrm{Tr}\left[\Lambda_{\mathrm{S},R}\right].$$

RI/mSMOM [Boyle et al, PRD 95 (2017) 054505]

$$1 = \lim_{m_R \to \overline{m}} \frac{1}{12p^2} \mathrm{Tr}\left[-iS_R(p)^{-1} \not{p}\right]$$

$$1 = \lim_{m_R \to \overline{m}} \frac{1}{12m_R} \left\{ \mathrm{Tr}\left[S_R(p)^{-1}\right] + \frac{1}{2} \mathrm{Tr}\left[\left(iq \cdot \Lambda_{\mathrm{A},R}\right) \gamma_5\right] \right\}$$

$$1 = \lim_{m_R \to \overline{m}} \frac{1}{12q^2} \mathrm{Tr}\left[\left(q \cdot \Lambda_{\mathrm{V},R}\right) \not{q}\right]$$

$$1 = \lim_{m_R \to \overline{m}} \frac{1}{12q^2} \mathrm{Tr}\left[\left(q \cdot \Lambda_{\mathrm{A},R} + 2m_R \Lambda_{P,R}\right) \gamma_5 \not{q}\right]$$

$$1 = \lim_{m_R \to \overline{m}} \frac{1}{12i} \mathrm{Tr}\left[\Lambda_{\mathrm{P},R} \gamma_5\right]$$

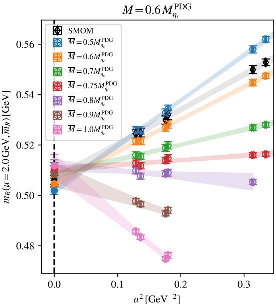
$$1 = \lim_{m_R \to \overline{m}} \left\{ \frac{1}{12} \mathrm{Tr}\left[\Lambda_{\mathrm{S},R}\right] + \frac{1}{6q^2} \mathrm{Tr}\left[2m_R \Lambda_{P,R} \gamma_5 \not{q}\right] \right\}.$$

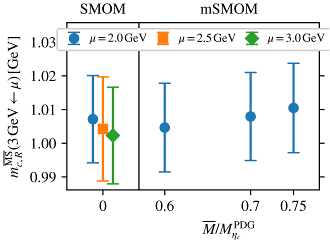
both ensure continuum WIs hold, yielding

$Z_V = Z_A = 1 \qquad Z_P = Z_S \qquad Z_m Z_P = 1$

- evaluated at arbitrary mass scale $m_R = \overline{m}$, which defines the scheme.
- $m_R \to 0$ limit reproduces RI/SMOM.
- linear system of equations for Z_q, Z_m, Z_A, Z_V, Z_S, Z_P.

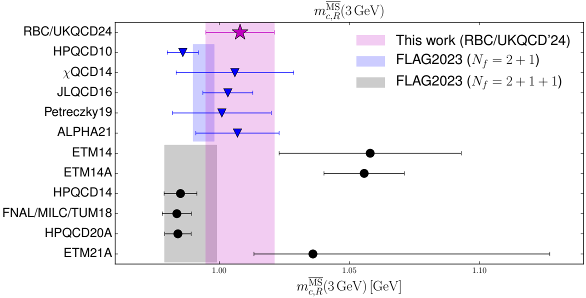
modified approach to the continuum





← continuum limit results still in different schemes! Values cannot be directly compared from plot.

- Very different CL approaches,
- arXiv:2407.18700 (today) +
  Fri 15:55 (Rajnandini Mukherjee):

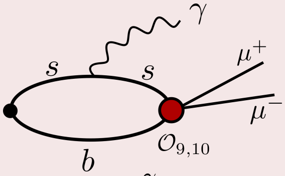


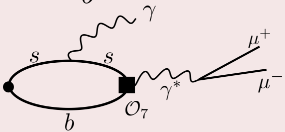
Selection of other new works and ongoing efforts

$B_s \to \mu\mu\gamma$ [Rome123 + Soton, PRD 109 (2024) 11, 114506] (1)

Helicity suppression lifted $\Rightarrow$ comparable to $B_s \to \mu\mu$.

semi-leptonic

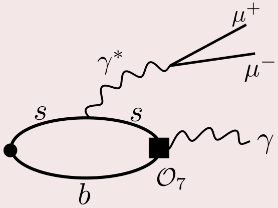




4

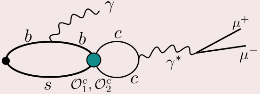
ffs: F_V, F_A, F_{TV}, F_{TA}
+ diagrams with $b \leftrightarrow s$
ETMC, $N_f = 2+1+1$
(4) $a \in [0.06 - 0.09]\,\mathrm{fm}$
(5) $m_h \in [M_{D_s}, 2M_{D_s}]$
(4) $x_\gamma \equiv 2E_\gamma / M_{Hs} \in [0.1, 0.4]$

photon-penguins



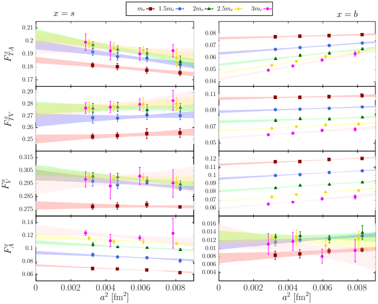
1 ff: $\bar{F}_T$ but ✗analytical continuation
$\Rightarrow$ spectral density reconstruction [ETMC, PRD 108 (2023) 074510] ["HLT" PRD 99 (2019) 094508]
subset of $\leftarrow$, but numerically smaller

charm-loops



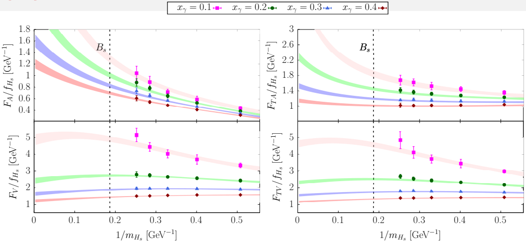
Included as phenomenologically parameterised shift in Wilson coefficient C_9 [Kozachuk et al, PRD 97, 053007 (2018); Guadagnoli et al JHEP 07 (2023) 112; Guadagnoli et al JHEP 11 (2017) 184]
Studies impact of unknown parameters to estimate uncertainty

$B_s \to \mu\mu\gamma$ [PRD 109 (2024) 11, 114506] (2)

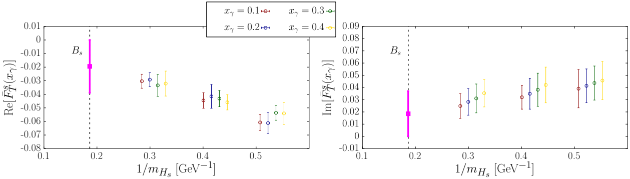


160 CLs:

$\#_{E_\gamma} \otimes \#_{m_h} \otimes \#_{ff} \otimes \#_{\text{b vs. s}}$

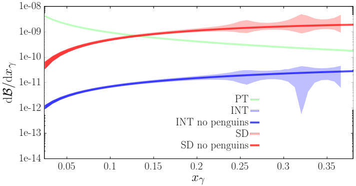


- Scaling laws [Beneke et al, EPJC 2011, JHEP 2020] up to $O(E_\gamma^{-1}, M_{H_s}^{-1})$, but assume large E_γ and large M_{H_s}
- supplemented by VMD model for small E_γ (80 data point, max 14 parameters)



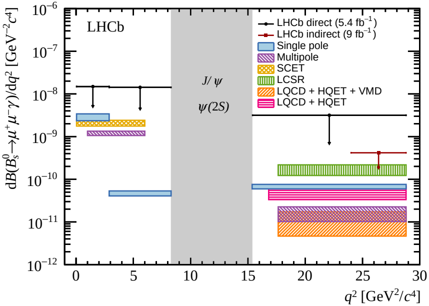
$\bar{F}_T$ found to be small (via spectral reconstruction)

$B_s \to \mu\mu\gamma$ [PRD 109 (2024) 11, 114506] (3)



charming penguin error ↑ as x_γ ↑, dominated by structure dep.

[LHCb, JHEP 07 (2024) 101]

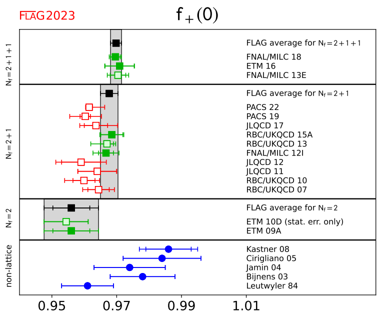


- impressive calculation, combining many novel methods and ideas.
- form factors F_V, F_A, F_{TA} and F_{TV} discrepant with previous non-lattice predictions [Janowski et al. JEHP12 (2021) 008; Kozachuk et al, PRD 97, (2018) 053007; Guadagnoli, JHEP 07 (2023) 112]
- error dominated by resonance + long-distance effects inc. charming penguins

Tue 16:15 (Francesco Sanfilippo): $B_s \to \mu\mu\gamma$ and $B_s \to \phi\gamma$

Kaons (1) — $K_{\ell 3}$

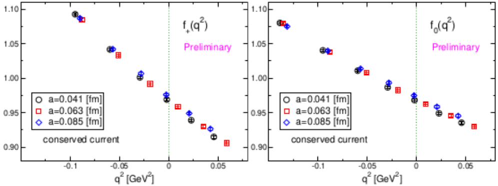
$K \to \pi \ell \nu$



- $K_{\ell 3}$ typically at $q^2 = 0$
- $|V_{us}|\, f_+^{K\to\pi}(0) = 0.2165(4)$
  [PoS CKM2016 (2017) 033]
- Interesting information also in form factor shape

Ongoing work by PACS

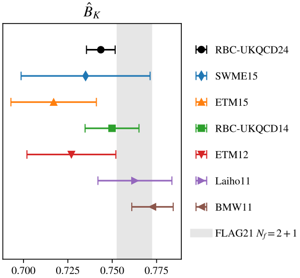
- 3 lattice spacings
- $O(a)$-improved Wilson
- $L \sim 10\,\mathrm{fm}$
- $\approx$ physical quark masses

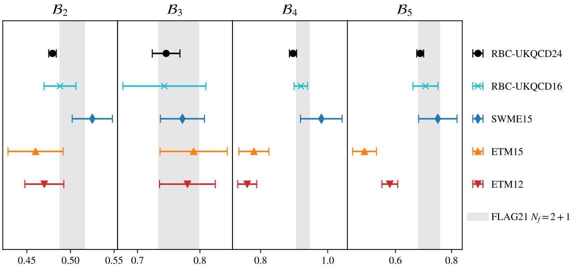


Fri 11:15 (Takeshi Yamazaki): Update of kaon semileptonic form factor using $N_f = 2+1$ PACS10 configurations

Kaons (2) — BSM kaon mixing

- BSM bag parameters $\mathcal{B}_4$, $\mathcal{B}_5$ are in tension between results using RI/MOM and RI/SMOM.
- tension recently confirmed! [JTT, RBC/UKQCD, 2404.02297 to appear in PRD]
  Full error budget based on 3 lattice spacings, 2 $m_\pi^{\rm phys}$ ensembles, domain-wall fermions + RI/SMOM, comprehensive estimates of all sources of uncertainties, several cross checks for continuum limit,...





$\Rightarrow$ Foundation for $B_{(s)} - \overline{B}_{(s)}$ mixing programme [JTT, RBC/UKQCD + JLQCD PoS LATTICE2021 (2022) 224]

Aside: Flow time expansion for flavour Tue 11:55 (Matthew Black):

- mixing: $\Delta F = 2$ 4-quark ops which mix amongst each other
- life times: $\Delta F = 0$ 4-quark ops also mix with dim 3 ops

IDEA: use gradient flow [Lüscher, CMP. 293 (2010) 899 + JHEP 08 (2010) 071] + small flow time expansion [Lüscher, Weisz, JHEP 02 (2011) 051; Suzuki PTEP (2013) 083B03] [Black et al., PoS LATTICE2023 263]

$\Rightarrow$ the flow removes divergences

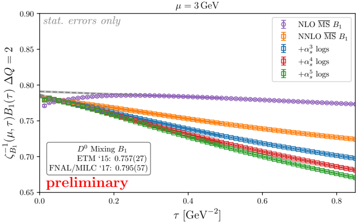
$$\mathcal{H}_{\rm eff} = \sum_m C_m \mathcal{O}_m = \sum_m \underbrace{\tilde{C}_m(\tau)\tilde{\mathcal{O}}_m(\tau)}_{\text{flowed}}$$

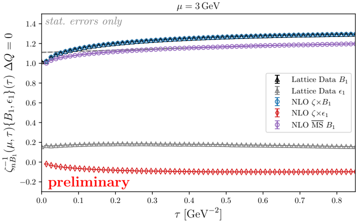
C and $\tilde{C}$ calculable in PT. Write

$$\tilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

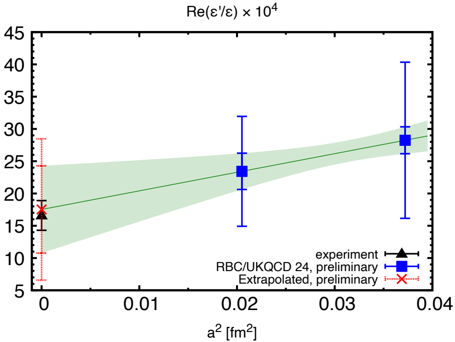
and diagonalise and invert:

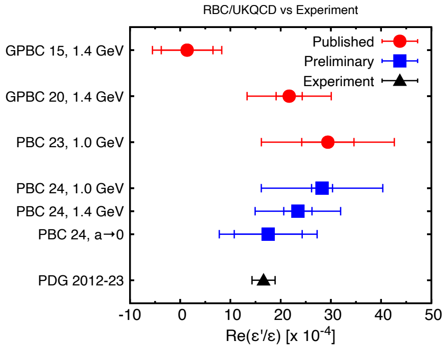
$$\left\langle \mathcal{O}_m^{\overline{\rm MS}} \right\rangle(\mu) = \sum_n \zeta_{nm}^{-1}(\mu,\tau)\left\langle \mathcal{O}_m^{\rm GF} \right\rangle(\tau) + O(\tau)$$





Kaons (3) — $K \to \pi\pi$ update by RBC/UKQCD





Fri 12:35 (Masaaki Tomii): $\Delta I = 1/2$ process of $K \to \pi\pi$ decay on multiple ensembles with periodic boundary conditions

See also: Poster (Seungyeob Jwa): 2024 Update on ϵ_K with lattice QCD inputs

☹ Further talks and topics I could not cover (1)

- Tue 15:05 (Alessandro De Santis): Inclusive semileptonic $D_s \mapsto X\ell\nu$ decays from lattice QCD
- Tue 15:25 (Christiane Groß): Semileptonic Inclusive Decay of the D_s meson
- Thu 11:30 (Zhi Hu): Study on the P-wave form factors of the B_s to D_s semi-leptonic decaus from inclusive lattice simulations
- Thu 11:50 (Joshua Lin): Spectator effects in inclusive lifetimes of heavy hadrons
- Thu 12:10 (Ryan Kellermann): Systematic effects in the lattice calculation of inclusive semileptonic decays
- Tue 16:55 (Giuseppe Gagliardi): The Cabibbo Angle from Inclusive τ decays
- Treating vector final states beyond the narrow width approximation (covered in Felix Erben's plenary tomorrow)
- Tue 12:15 (Antoine Geradin): $B^*\pi$ excited-state contamination in B-physics observables
- Tue 11:15 (Roberto Di Palma): Virtual radiative Leptonic decays of charged Kaons

☹ Further talks and topics I could not cover (2)

- Fri 11:35 (Raoul Hodgson): Split-even approach to the rare kaon decay $K \to \pi\ell^+\ell^-$
- Fri 11:55 (En-Hung Chao): Two photon contribution to the $K \to \mu\mu$ decay amplitude on a $1/a \approx 1$GeV lattice
- Fri 12:15 (Ceran Hu): Contribution of the eta to a lattice calculation of $K \to \mu\mu$ decay
- Fri 12:55 (Yikai Huo): Enhanced Lattice QCD Studies on ϵ_K and ΔM_K
- Fri 14:15 (Peng-Xiang Ma): Lattice QCD Calculation of Electroweak Box Contributions to Superallowed Nuclear and Neutron Beta Decays
- Fri 14:35 (Marios Costa): 4-quark operators with $\Delta F = 2$ in the GIRS scheme
- Fri 14:55 (Ryan Hill): Bringing near-physical QCD+QED calculations beyond the electro-quenched approximation
- Fri 15:15 (Xinyu Tuo): Finite-volume formalism for physical processes with an electroweak loop integral
- Fri 15:35 (Matteo Di Carlo): On-shell derivation of QED finite-volume effects

Summary

- ☹ Several tensions in 'standard' b and c decays — requires scrutiny!
- ☺ Ongoing work on all of these decays!
- ☹ Non-trivial analyses, intertwined extrapolations:
  - simplify analyses to check individual "directions"
  - suggestion of cross checks ("windows")
  - ☺ many parameters are physics, i.e. should be discretisation independent $\Rightarrow$ comparable!
- ☺ ongoing work on all relevant extrapolations (heavy-quark, excited states, continuum limit)
- ☹ many topics I could not cover due to time constraints.
- ☺ some might yet be covered, e.g. vector final states beyond the narrow width approximation by Felix Erben (tomorrow).

☺ new exciting results and ongoing work in all aspects of flavour physics!

A lot to look forward to this week — enjoy the conference!