Grassmann tensor approach for two-dimensional QCD in the strong-coupling expansion

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Outline

- Aim: Compute the partition function of LQCD (and resulting observables) in order to study the phase diagram of QCD.
- Current Restrictions:
  - Two dimensions (one time and one spatial dimension)
  - One quark flavor on the lattice using staggered fermions.
  - Expand gauge action in orders of the coupling-parameter β.
- Usual Monte-Carlo-Method does not work. (sign problem for $\mu \neq 0$)
- Method: Tensor-Network: Grassmann higher order tensor renormalization group approach (Grassmann HOTRG)
- Previous work: Infinite coupling (No gauge action) up to four dimensions[1]

[1] Bloch and Lohmayer, Grassmann higher-order tensor renormalization group approach for two-dimensional strong-coupling QCD (2022)

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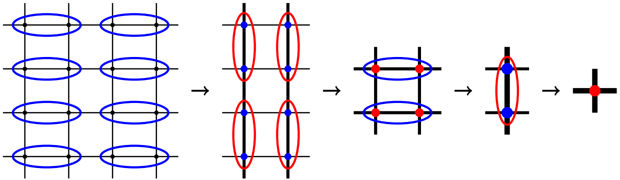
Tensor-Network approach in two dimensions

$$Z = \sum_{\{j_{(x,\nu)}\}} \prod_{x}^{V} \mathcal{T}_{j_{x,-1}, j_{x,1}, j_{x,-2}, j_{x,2}}$$

- Every site has the same tensor on it.
- Every link (x, ν) on the lattice has an index $j_{x,\nu}$. (Range D_0 depends on theory.)
- Adjacent tensors are connected via contraction, i.e. summation over link index $j_{x,\nu}$.



- Contracting two adjacent tensors leads to a new coarse grid tensor with increased size.
- HOTRG: Iterative truncation scheme which reduces the range of a "fat index" from D_0^2 to D based on SVD of unfoldings.[2]

[2] De Lathauwer et al., A multilinear singular value decomposition (2000)

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LQCD partition function

$$Z_{QCD} = \int \left[\prod_x d\psi_x d\bar{\psi}_x\right] \left[\prod_{x,\nu} dU_{x,\nu}\right] \left[\prod_{x,\nu} \mathrm{e}^{S^{\mathrm{f}}_{x,\nu}} \mathrm{e}^{S^{\mathrm{b}}_{x,\nu}}\right] \left[\prod_{x,\mu,\nu}^{\mu\neq\nu} \mathrm{e}^{S^G_{x,\mu,\nu}}\right] \mathrm{e}^{S_M}$$

Staggered fermions

$$S^{\mathrm{f}}_{x,\nu} = \eta_{x,\nu}\bar{\psi}_x \mathrm{e}^{\mu\delta_{\nu,1}} U_{x,\nu}\psi_{x+\hat{\nu}} \qquad S^{\mathrm{b}}_{x,\nu} = -\eta_{x,\nu}\bar{\psi}_{x+\hat{\nu}} \mathrm{e}^{-\mu\delta_{\nu,1}} U^\dagger_{x,\nu}\psi_x,$$

with chemical potential μ and usual staggered phases $\eta_{x,\nu}$

Wilson action

$$S^G_{x,\mu,\nu} = \frac{\beta}{2N_c}\mathrm{tr}\left[U_{x,\mu}U_{x+\hat{\mu},\nu}U^\dagger_{x+\hat{\nu},\mu}U^\dagger_{x,\nu}\right], \qquad S_M = 2m\sum_x \bar{\psi}_x\psi_x,$$

with coupling parameter β and quark mass m

- Make Taylor-expansions for all exponentials. $\rightarrow$ Summation indices will be TN-indices.

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Result of gauge integral[3,4]

- Only non-zero result when $\frac{a-p}{N_c} \equiv q \in \mathbb{Z}$, without loss of generality $a > p$

$$\int_{SU(N_c)} DU U_{i_1 j_1} \cdots U_{i_a j_a} U^{\dagger}_{k_1 l_1} \cdots U^{\dagger}_{k_p l_p} \propto \sum_{(\alpha,\beta)} \sum_{\pi,\sigma \in S_p} \varepsilon^{\otimes q}_{i_{\{\alpha\}}} \delta^{l_\pi}_{i_{\{\beta\}}} \tilde{\mathrm{Wg}}^{q,p}_{N_c}(\pi \circ \sigma^{-1}) \varepsilon^{\otimes q, j_{\{\alpha\}}} \delta^{j_{\{\beta\}}}_{k_\sigma}$$

$\rightarrow$ $\tilde{\mathrm{Wg}}$ are so-called generalized Weingarten functions.

Grassmann integration

- Grassmanns can not be integrated directly without producing non-local signs.
  $\rightarrow$ Introduce Grassmann-Network[5]

Color indices can be contracted locally and therefore are no d.o.f. of the TN!

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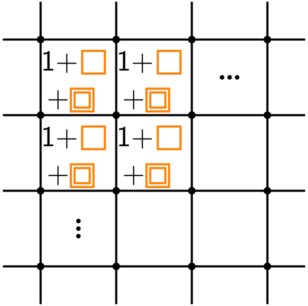
[3] Gagliardi and Unger, A new dual representation for staggered lattice QCD (2020)

[4] Borisenko, Voloshyn and Chelnokov, SU(N) polynomial integrals and some applications (2020)

[5] Shimizu and Kuramashi, Grassmann tensor renormalization group approach to one-flavor lattice Schwinger model (2014)

Separation of different orders in β



- So far: Expansion up to order $n^{\max}$ for every plaquette.
- Approach includes many terms of higher order than $n^{\max}$ that spoil applicable range in β.
- Fit of coefficients produces large errors.

$\rightarrow$ We developed a modified GHOTRG procedure, where terms of higher order than $n^{\max}$ are deleted in each step.

Translation invariance

- Many configurations differ only in the choice of the origin.
- Consider only one of those configurations and introduce combinatorial factor.

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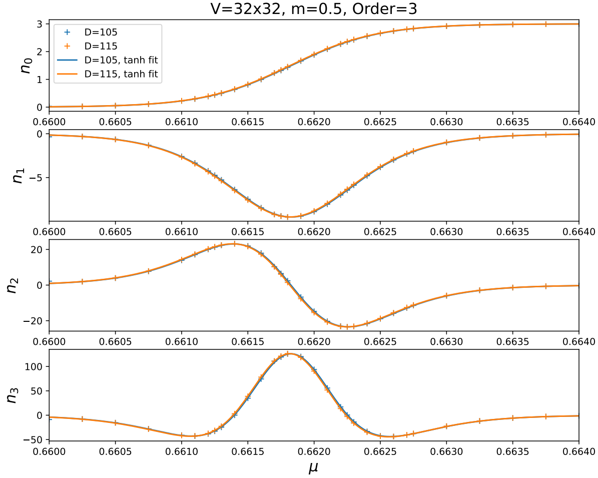
Results: Expansion of particle density



$$n(\beta,\mu) \equiv \frac{1}{V}\frac{\partial \log Z(\beta,\mu)}{\partial\mu} \approx \sum_{i=0}^{n^{\max}} n_i(\mu)\beta^i$$

- Define fit-ansatz, motivated by Fermi-Dirac statistics:

$$n(\beta,\mu) = \frac{3}{2}\left(1+\tanh[a_n(\beta)(\mu-\mu_n^c(\beta))]\right)$$

- Expand this function in β, using

$$a_n(\beta) \approx \sum_{i=0}^{n^{\max}} a_{n,i}\beta^i$$

$$\mu_n^c(\beta) \approx \sum_{i=0}^{n^{\max}} \mu_{n,i}^c\beta^i$$

- Fit the data with resulting functions.

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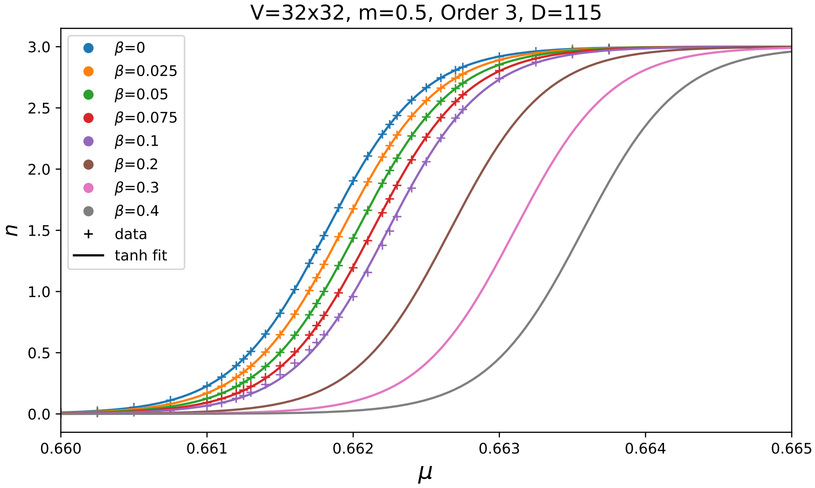
Results: Particle density



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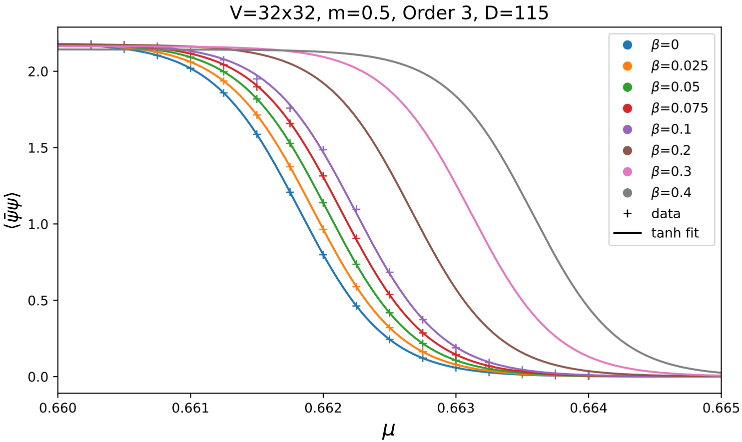
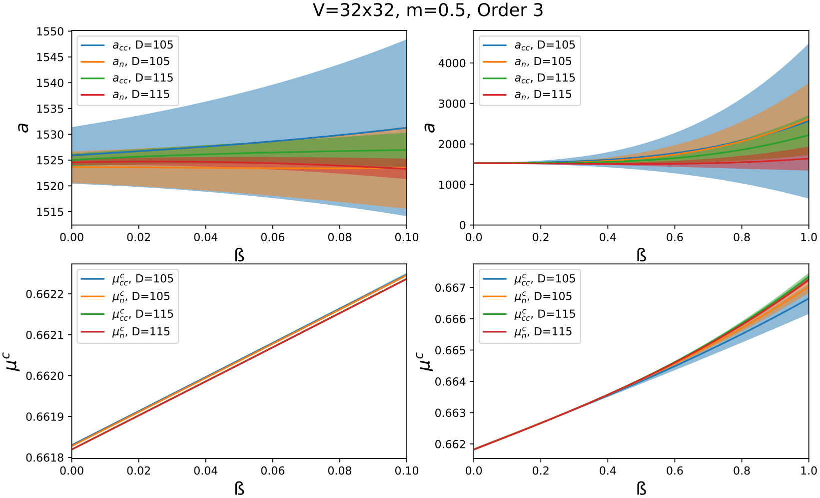
Results: Chiral condensate

Equivalent approach with fit-ansatz $\langle\bar{\psi}\psi\rangle(\mu) = b_{cc}(\beta)\left(1 - \tanh[a_{cc}(\beta)(\mu - \mu^{c}_{cc}(\beta))]\right)$



Results: critical chemical potential



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Outlook

- Apply this method in 4 dimensions.
- Add a second fermion flavor.