Adjoint chromoelectric correlators for heavy quarkonium diffusion

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Motivation

- Diffusion of particles in Quark Gluon Plasma (QGP) described by a set of equations
- The equations depend on transport coefficients such as κ, γ, and related quantities
- Transport coefficients can be related to experimental observables (Nuclear modification factor R_{AA}, Elliptic flow ν_2)
  $\rightarrow$ theoretical predictions for high-precision measurements required
- However: Finite T requires non-perturbative calculations since PT results do not converge
  $\rightarrow$ use Lattice QCD
- How to connect: relate lattice quantities with a proper EFT (NRQCD, pNRQCD)

Lattice Setup

- Lattice configurations at $T = 1.5\,T_c$ and $T = 10^4\,T_c$
- Quenched lattices produced with heatbath and overrelaxation
- Gradient flow: new scale $\sqrt{8\tau_F}$, reference scale t_0
- Gradient flow improves chromo field strength components
  $\rightarrow$ requires $\sqrt{8\tau_F} > a$
- Gradient flow improves signal-to-noise ratio
  $\rightarrow$ LPT indicates to stay below $\sqrt{8\tau_F} < \frac{\tau - a}{3}$
- Gradient flow scale regulates divergences
- t_0 can be used for scale setting
  (Francis et.al. Phys. Rev. D 91, 096002 (2015))

We use gradient flow to improve field insertions and signal-to-noise ratio within $a < \sqrt{8\tau_F} < \frac{\tau - a}{3}$

$T = 1.5\,T_c$

N_s	N_τ	β	N_{conf}
48^3	16	6.872	1000
48^3	20	7.044	1705
48^3	24	7.192	2060
56^3	28	7.321	1882
68^3	34	7.483	900

$T = 10^4\,T_c$

N_s	N_τ	β	N_{conf}
48^3	16	14.443	1000
48^3	20	14.635	450
48^3	24	14.792	398

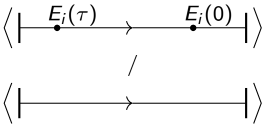
Recall: Fundamental κ^{fund}

- Related EFT: NRQCD (in HQET description)
- Describes heavy quark diffusion
- Related Euclidean correlator and relation to κ^{fund}:

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\langle \mathrm{ReTr}[U(\beta,\tau)gE_i(\tau)U(\tau,0)gE_i(0)]\rangle}{\langle \mathrm{ReTr}(L_3)\rangle}$$

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi}\rho(\omega)K(\omega), \quad K(\omega) = \frac{\cosh(\beta/2-\tau)\omega}{\sin\beta\omega/2}$$

$$\kappa^{\mathrm{fund}} = \lim_{\omega\to 0}\frac{2T}{\omega}\rho(\omega)$$



(Moore and Teaney PRC71 (2005)), (Caron-Huot and Moore JHEP02 (2008)), (A. Bouttefeux and M. Laine JHEP 12 (2020))

- This operator was computed over many years by different groups and works better than J-J-correlators

Recall: Fundamental κ^{fund}

- Divergence in the static Wilson lines:

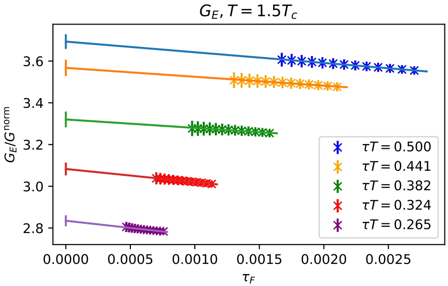
$$U(\beta,\tau).U(\tau,0). \propto e^{-\delta m/T}$$
$$L_3 \propto e^{-\delta m/T}$$

$\rightarrow$ divergence cancels in the ratio of both quantities

- Gives a linear flow time dependence
  $\rightarrow$ zero flow-time limit is save
- Extract κ^{fund}:

$$T = 1.5\,T_c : 1.7 \leq \frac{\kappa^{\text{fund}}}{T^3} \leq 3.12$$

$$T = 10^4\,T_c : 0.02 \leq \frac{\kappa^{\text{fund}}}{T^3} \leq 0.16$$



(Brambilla et.al. PRD107,054508(2023))

other Refs: (Banerjee et.al. PRD85,014510(2012)), (Francis et.al. PRD92,116003(2015)), (Brambilla et.al. PRD102,074503(2020)), (Banerjee et.al. JHEP08,128(2022)), unquenched: Altenkort et.al. PRD130,232902(2023)

Next: Adjoint κ (κ^{adj})

- Related EFT: pNRQCD
- Describes Quarkonium dynamics: two heavy quark systems
  $\rightarrow$ Quarkonium diffusion
- This study calculates first time correlator describing Quarkonium diffusion and needed to study the non-equilibrium evolution of Quarkonium in medium

  (Brambilla et.al. PRD96, 034021(2017)), (Brambilla et.al. JHEP05,282021 29136(2021)),(Brambilla et.al. PRD108, L011502(2023))

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- Two possible interactions:
  - Bound state: singlet state
  - Scatter state: octet state

Three possible processes (so far):

- singlet $\rightarrow$ octet: dissociation
- octet $\rightarrow$ singlet: recombination
- octet $\rightarrow$ octet

- Construction of the open quantum system and the Lindblad equation is pending
- But still related to chromo-electric correlators

$\Rightarrow$ calculate adjoint chromo-electric correlators to extract κ^{adj}

Adjoint correlator 1: Introducing Symmetric $G_E^{\mathrm{A\ symm}}$

- Motivated by the fundamental symmetric correlator, propose a symmetric adjoint correlator:

$$G_E^{\mathrm{A\ symm}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\langle W_{ab}(\beta,\tau) f_{bcd} E_i^d(\tau) W_{ce}(\tau,0) f_{eaf} E_i^f(0)\rangle}{\langle L_8\rangle}$$

$$E_i^a = \mathrm{Tr}(\lambda^a E_i), \quad W_{ab} = \frac{1}{2}\mathrm{Tr}(U^\dagger(\tau,0)\lambda^b U(\tau,0)), \quad f_{abc} = -\frac{i}{4}\mathrm{Tr}(\lambda^a[\lambda^b,\lambda^c])$$

$$L_8 = \frac{1}{8}\mathrm{Tr}\prod_{i=0}^{N_\tau-1} W = \frac{1}{16}\mathrm{Tr}(\mathrm{L}_3^\dagger \lambda^{\mathrm{a}} \mathrm{L}_3 \lambda^{\mathrm{a}})$$

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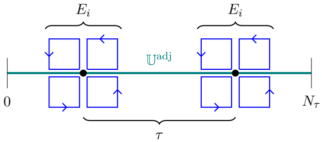
- Nominator & denominator have the same divergence $\propto e^{-\delta m_8/T}$
- Describes the process of octet-octet diffusion

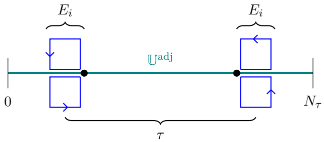
⇒ Continuum limit and zero flow time limit of $G_E^{\mathrm{A\ symm}}$ are safe

Adjoint correlator 1: Results of the symmetric $G_E^{\mathrm{A\ symm}}$

- Use clover and half-of-clover (2-plaquette) discretizations of the E-fields:



Clover



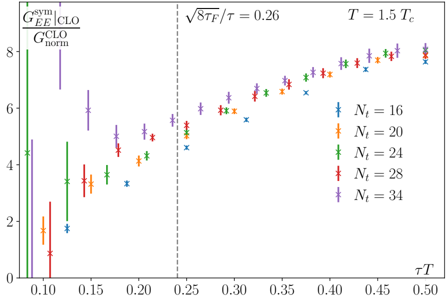
2-plaquette

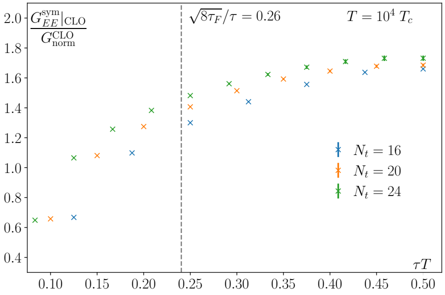
- Consider dimensionless and tree-level improved quantity $G_E^{\mathrm{Latt}}/G_{\mathrm{norm}}^{\mathrm{Latt}}$ with

$$\frac{G_{\mathrm{norm}}^{\mathrm{Latt}}(\tau T)}{T^4} = \frac{N_\tau^4}{3}\int_{-\pi}^{\pi}\frac{d^3k}{(2\pi)^3}\,\frac{\cosh zN_\tau(\frac{1}{2}-\tau T)}{\sinh zN_\tau/2}\,\frac{1}{\sinh z}\times\begin{cases}\left(1+\frac{\hat{k}^2}{4}\right)\left(\hat{k}^2-\frac{(\hat{k}^2)^2}{8}+\frac{\hat{k}^4}{8}\right) & \text{(CLO)}\\ \left(\hat{k}^2+\frac{\hat{k}^4-(\hat{k}^2)^2}{8}\right) & \text{(2PL)}\end{cases}$$

$$\hat{k}^n = \sum_i\left(2\sin\frac{k_i}{2}\right)^n, \quad \sinh\frac{z}{2} = \sqrt{\frac{\hat{k}^2}{4}}$$

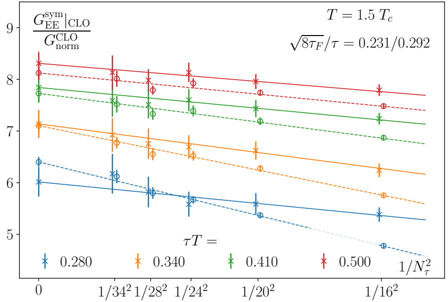
Adjoint correlator 1: Results of the symmetric $G_E^{\mathrm{A\ symm}}$

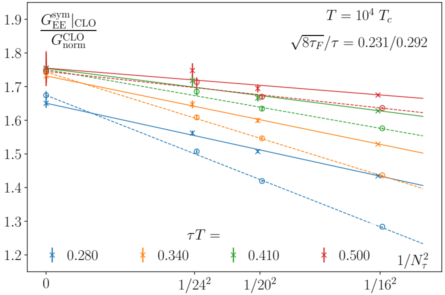




- our scale of interest is the flow time ratio $\sqrt{8\tau_F}/\tau$
- in preparation for the continuum limit: we use cubic spline interpolation with symmetric boundaries at $\tau T = 0.5$

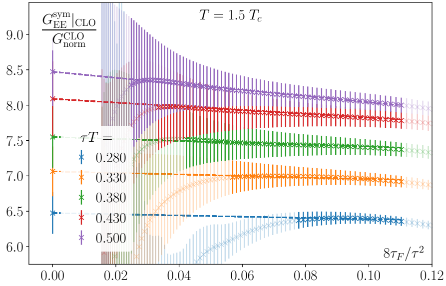
Adjoint correlator 1: Results of the symmetric $G_E^{\mathrm{A\ symm}}$

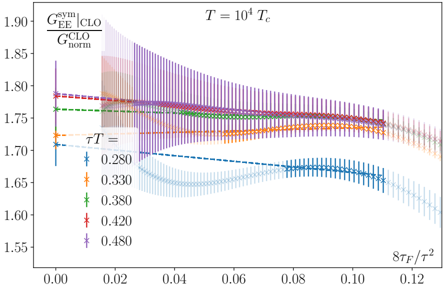




- linear in $1/N_\tau^2 \Leftrightarrow a^2$ continuum extrapolation
- $\chi^2/\mathrm{dof} = \mathcal{O}(1)$

Adjoint correlator 1: Results of the symmetric $G_E^{\mathrm{A\ symm}}$





- linear flow time dependence within a proper flow time window
  $\rightarrow$ linear zero flow time limit

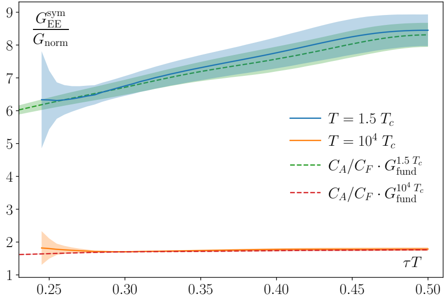
Adjoint correlator 1: Results of the symmetric $G_E^{\mathrm{A\ symm}}$

- Continuum & zero flow time extrapolated correlators show Casimir scaling within errors compared with the fundamental correlator
  (Brambilla et.al. PRD107,054508(2023))

- To extract κ, we already have the parameters for ρ from the fundamental

$$G_E(\tau) = (C_A/C_F)G^{\mathrm{fund}}(\tau)$$
$$= \int_0^\infty \frac{d\omega}{\pi}(C_A/C_F)\rho(\omega)K(\omega)$$
$$\Rightarrow \kappa_{adj}^{sym} = (C_A/C_F)\kappa^{\mathrm{fund}}$$

- Related to octet-octet diffusion



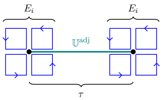
Adjoint correlator 2: Introducing non-symmetric G_E^{A}

- This correlator emerged from EFT calculations:

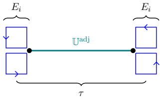
$$G_E^{\mathrm{A}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\langle E_i^a(\tau) W_{ab}(\tau,0) E_i^b(0)\rangle$$

$$E_i^a = \mathrm{Tr}(\lambda^a E_i), \quad W_{ab} = \frac{1}{2}\mathrm{Tr}(U^\dagger(\tau,0)\lambda^b U(\tau,0))$$

- $G_E^{\mathrm{A}}(\tau) = e^{-\delta m_8 \tau} G_E^{r\mathrm{A}} \to e^{\delta m_8 \tau} G_E^{\mathrm{A}}(\tau) = G_E^{r\mathrm{A}}$



Clover



2-plaquette

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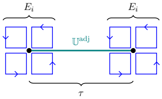
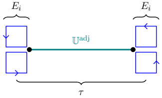
- $G_E^{\mathrm{A}}(\tau) = e^{-\delta m_8 \tau} G_E^{r\mathrm{A}} \rightarrow e^{\delta m_8 \tau} G_E^{\mathrm{A}}(\tau) = G_E^{r\mathrm{A}}$
- $L_8 = e^{-\delta m_8 / T} L_8^r$

  (Gupta et.al.PRD77,034503(2008))
- $\delta m_8(\tau_F) = T \log \frac{L_8^r}{L_8(\tau_F)}$
- Describes dissociation and recombination processes



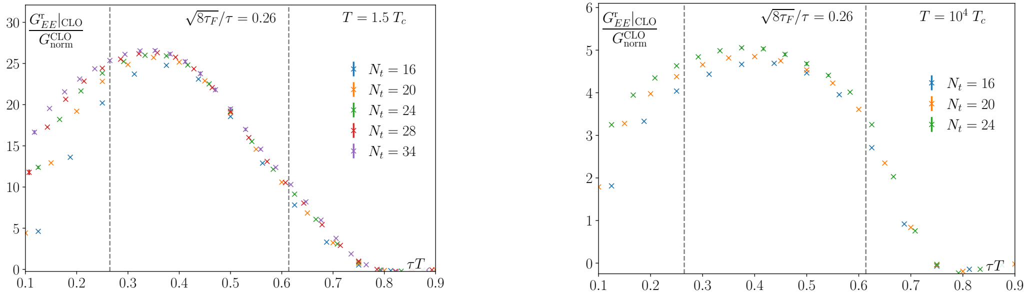
Clover



2-plaquette

→ $G_E^{r\mathrm{A}} = \left(\frac{L_8^r}{L_8(\tau_F)}\right)^{\tau T} G_E^{\mathrm{A}}$ is a divergent free quantity

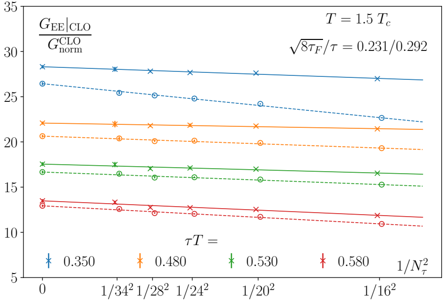
Adjoint correlator 2: Results of the non-symmetric G_E^{A}

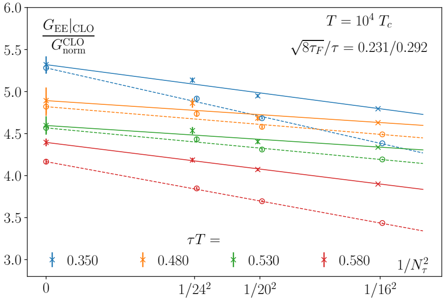


- considering dimensionless quantity $G_E^{r\mathrm{A}}/G_{\mathrm{norm}}^{\mathrm{Latt}}$
- periodicity of the lattice:

$$\sqrt{8\tau_F} < \frac{(N_t-2)a-\tau}{2} \qquad \Rightarrow \qquad \tau T < \frac{1-2/N_\tau}{2\sqrt{8\tau_F}/\tau+1}$$

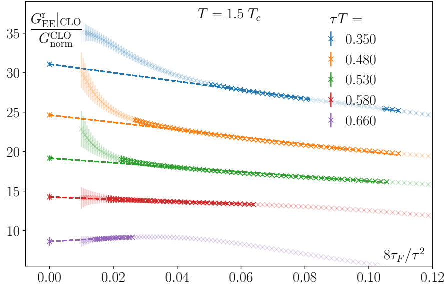
Adjoint correlator 2: Results of the non-symmetric G_E^{A}

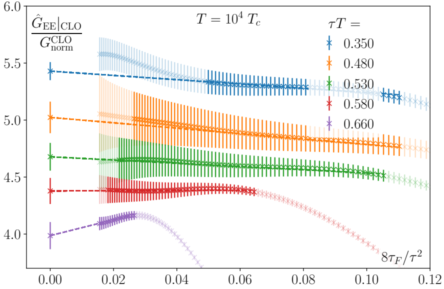




- now cubic splines with default boundary conditions
- linear in $1/N_\tau^2 \Leftrightarrow a^2$ continuum extrapolation
- $\chi^2/\mathrm{dof} = \mathcal{O}(1)$

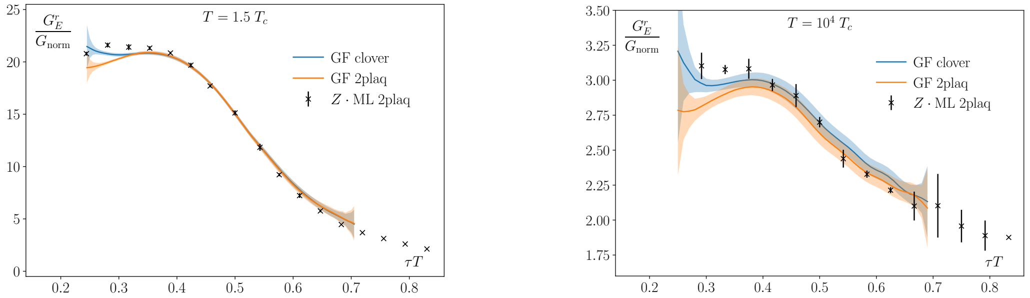
Adjoint correlator 2: Results of the non-symmetric G_E^{A}





- linear behavior in flow time
  $\rightarrow$ perform linear zero flow time limit

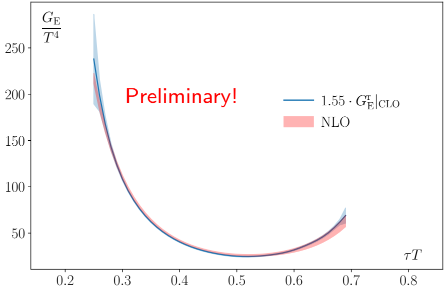
Adjoint correlator 2: Results of the non-symmetric G_E^{A}



- comparison to results with ML: tadpole improvement, same Wilson line renormalization
- GF and ML agree up to a T-dependent factor
- Clover and 2-plaquette match

Adjoint correlator 2: Results of the non-symmetric G_E^{A}

- Compare to NLO calculations at $T = 10^4\, T_c$
  (N. Brambilla, P. Panayiotou, S. Säppi, A. Vairo in preparation)
- Correlator is not symmetric: requires further studies for $\kappa_{adj}^{non-symm}$-extraction
- NLO calculation of the correlator can help to understand the non-symmetry nature in the spectral function
- G_E^{A} is related to dissociation and recombination processes



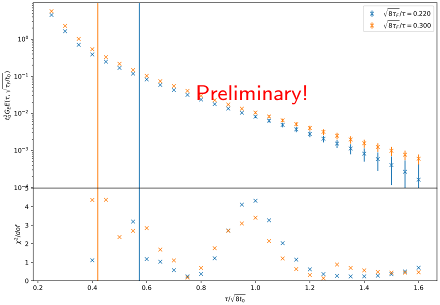
Adjoint correlator 2: Towards γ extraction

- γ encoded in G_E^{A} after $T = 0$ contribution was substracted:

$$\gamma \propto \int_0^\infty d\tau \, G_E(\tau)$$

$$G_E(\tau) = G_E^{\mathrm{A}}(\tau) - G_E^{\mathrm{A},T=0}(\tau)$$

- Only for the non-symmetric adjoint correlator since Polyakov loop normalization at $T = 0$ is inadequate
- Divergence renormalization with Polyakov loop not possible
- Gradient flow scale $\sqrt{8\tau_F}$ serves as regulator as long as $\sqrt{8\tau_F} > 0$
- $G_E^{\mathrm{A},T=0}(\tau)$ contains gluelump spectra



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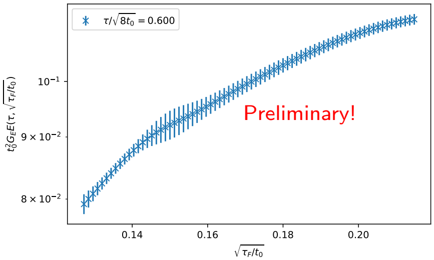

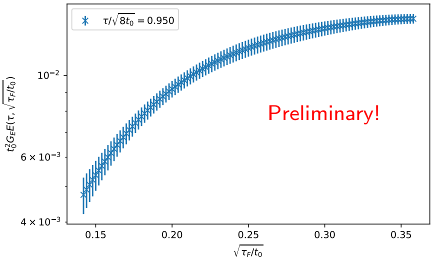

- strong flow time dependence indicates the presence of the divergence
- Polyakov loop renormalization not possible
  $\rightarrow$ requires to develop an EFT-motivated divergence removal
- requires finer lattices to match with the τ-axis at finite T

Summary and Outlook

- Summary:
  - In this study, we calculated the first time Quarkonium EE-correlators which gives first non-perturbative inputs for the study of non-equilibrium evolution of Quarkonium in QGP
  - This is in particular interesting for the phenomenological application, for the characterization of the QGP, and in general to understand QFT out of equilibrium
  - At least (!) two possible adjoint EE-correlator exist representing different processes
  - We have observed Casimir scaling for the symmetric correlator $\kappa_{adj}^{symm} = (C_A/C_F)\kappa^{\mathrm{fund}}$
  - The non-symmetric correlator is not symmetric which requires a different methodology to extract κ
  - The spectral representation with the usual kernel does not apply
  - Without Wilson line renormalization, we obtain a strong flow time dependence for $T = 0$
- Outlook:
  - To understand the non-symmetry of the non-symmetric correlator, further perturbative studies are necessary
  - In general, the inversion for the spectral function of non-symmetric correlators has to be developed
  - To subtract the $T = 0$-contribution, data from finer lattices are required
  - To extract γ, an EFT-motivated renormalization scheme has to be implemented
  - The $T = 0$ data also provides gluelump spectrum and $\mathcal{E}_3$ (related to Quarkonium decay)

Sample frame title

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➡ Knock Knock!! Who's there!?