In- and out-of-equilibrium aspects of the Chiral Magnetic Effect from lattice QCD

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in collaboration with B. Brandt, G. Endrődi, G. Markó, D. Valois

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CRC-TR 211
Strong-interaction matter under extreme conditions

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HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

Introduction

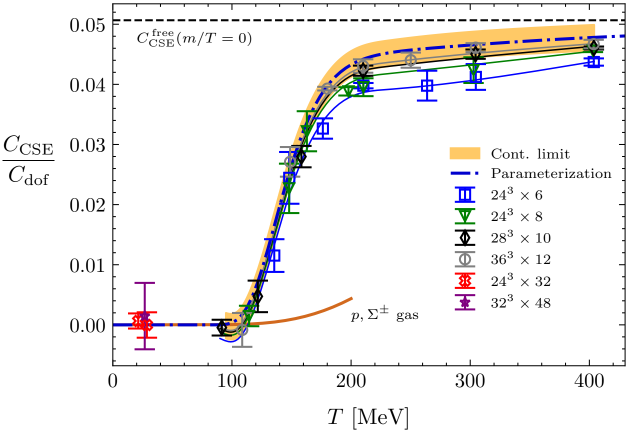
- Quantum anomalies + EM fields / Vorticity → **Anomalous transport phenomena**
- Examples:
  - Chiral Magnetic Effect (CME)
  - Chiral Separation Effect (CSE)
  - Chiral Vortical Effect (CVE)
  - ...
- Study the effect of strong interactions in the conductivities using **Lattice QCD**

Previous results: CSE

- Magnetic field + finite density → Axial current

$$J^A_{\mathsf{CSE}} = C_{\mathsf{CSE}}\, eB\mu + \mathcal{O}(\mu^3)$$

- First full QCD result Brandt, Endrődi, EGV, Markó '23



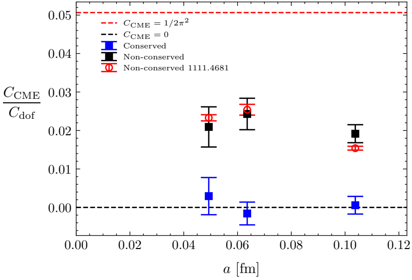
Previous results: CME

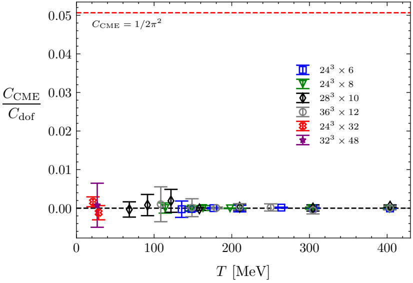
- Magnetic field + chiral density → Vector current

$$J^V_{\mathsf{CME}} = C_{\mathsf{CME}}\, eB\mu_5 + \mathcal{O}(\mu_5^3)$$

- Using conserved vector current is crucial 🔗 Brandt, Endrődi, EGV, Markó '24

  🔗 A. Yamamoto '11



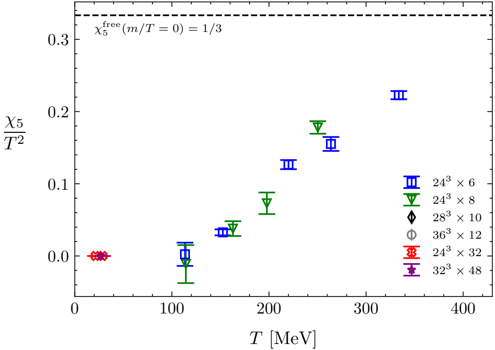


- **CME vanishes** in equilibrium, also in QCD

Previous results: CME

- Chiral density is **finite** at $\mu_5 \neq 0$

$$n_5(\mu_5) = \frac{T}{V} \left. \frac{\mathrm{d}^2 \log \mathcal{Z}}{\mathrm{d}\mu_5^2} \right|_{\mu_5=0} \mu_5 + \mathcal{O}(\mu_5^3) = \chi_5 \mu_5 + \mathcal{O}(\mu_5^3)$$



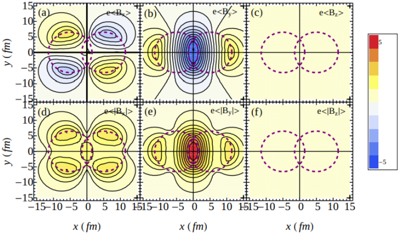
- Non-trivial absence of CME!

CME: beyond homogeneous static setups

In-equilibrium

- Magnetic fields in heavy-ion collisions are far from being uniform
- How is CME affected by an **inhomogenous** magnetic field?

Deng, Huang '12



Out-of-equilibrium

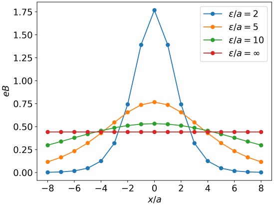
- Linear response theory: perturbation $\delta\mu_5(t)$ with *homogeneous* B
- How does the system respond? $\rightarrow C_{\mathrm{CME}}^{\mathrm{neq}}$

In-equilibrium + inhomogeneous B

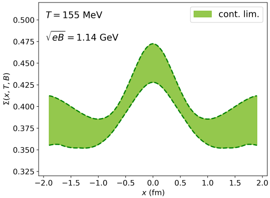
Inhomogeneous magnetic field

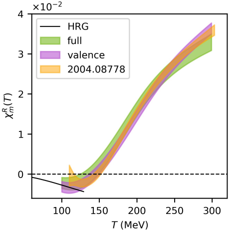
- Magnetic field profile:

$$\vec{B}(x) = \frac{B}{\cosh^2\left(\frac{x_1}{\varepsilon}\right)}\,\hat{x}_3$$



- Used to check impact in phase diagram and calculate magnetic susceptibility





Brandt et al '23 Brandt, Endrődi, Markó, Valois '24

Lattice setup

- 2+1 flavors of improved staggered fermions physical point
- Observables related to

$$\frac{\partial \langle J_3 \rangle}{\partial \mu_5}\bigg|_{\mu_5=0} = \frac{T}{V}\Bigg[\frac{1}{16}\sum_{f,f'}\frac{q_f}{e}\left\langle \mathrm{Tr}\Big(\Gamma_{45}^{f'} M_{f'}^{-1}\Big)\mathrm{Tr}\Big(\Gamma_3^f M_f^{-1}\Big)\right\rangle$$
$$-\frac{1}{4}\sum_f \frac{q_f}{e}\left\langle \mathrm{Tr}\Big(\Gamma_{45}^f M_f^{-1}\Gamma_3^f M_f^{-1}\Big)\right\rangle$$
$$+\frac{1}{4}\sum_f \frac{q_f}{e}\left\langle \mathrm{Tr}\left(\frac{\partial \Gamma_3^f}{\partial \mu_5} M_f^{-1}\right)\right\rangle\Bigg]$$

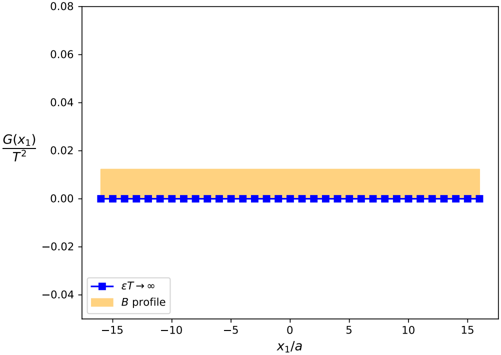
- Wall-sources to get spatial dependence

Free fermions

- Local linear response of current profile $J(x_1)$ to *homogeneous* μ_5

$$G(x_1) \equiv \left. \frac{\mathrm{d}\langle J_3^V(x_1)\rangle}{\mathrm{d}\mu_5} \right|_{\mu_5=0}$$

- Free staggered fermions



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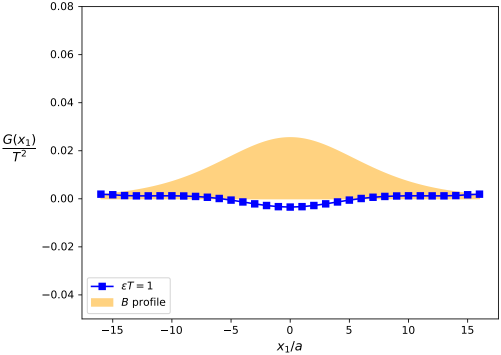

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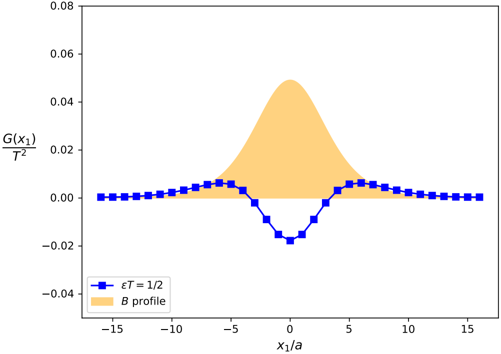

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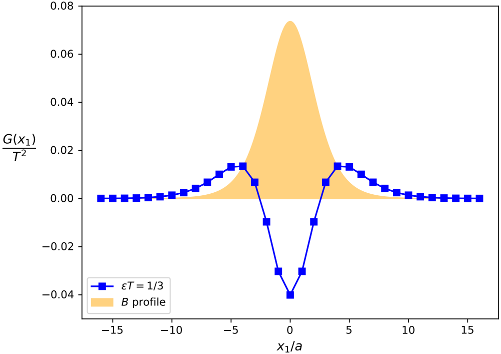
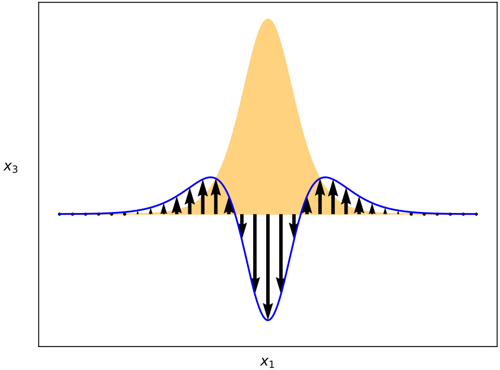


Free fermions

- Local linear response of current profile $J(x_1)$ to *homogeneous* μ_5

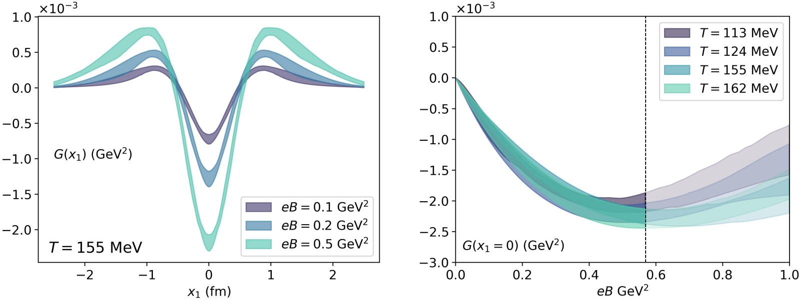
$$G(x_1) \equiv \left. \frac{\mathrm{d}\langle J_3^V(x_1)\rangle}{\mathrm{d}\mu_5} \right|_{\mu_5=0}$$

- x_1-dependent current flowing in x_3 $\rightarrow$ integrates to zero



- **Non-trivial localized CME in QCD!** (full QCD staggered)

  Brandt, Endrődi, EGV, Markó, Valois (in prep.)

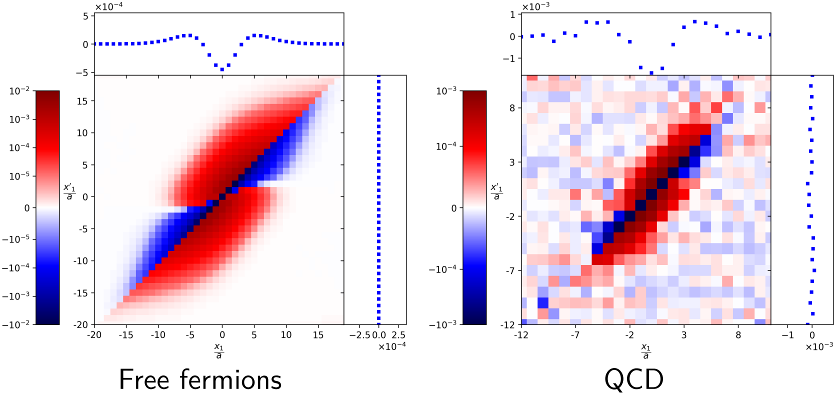


- Possible implications for CME search?

QCD

- Local linear response of current profile $J(x_1)$ to *inhomogeneous* $\mu_5(x_1')$

$$H(x_1, x_1') \equiv \left. \frac{\delta \langle J_3^V(x_1) \rangle}{\delta \mu_5(x_1')} \right|_{\mu_5=0}$$



Out-of-equilibrium (Homogeneous B)

Linear response theory

- Linear response theory → Kubo formulas → transport coefficients ξ

$$\xi = \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

- Consider system at finite B → perturbation $\delta\mu_5(t)$
- Kubo formula for $G_R(t) = i\theta(-t) \left\langle [j_{45}(t), j_3(0)] \right\rangle$

$$C_{\mathrm{CME}}^{\mathrm{neq}} \sim \frac{1}{eB} \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

- Access out-of-equilibrium C_{CME} directly! 🔗 Buividovich '24

Spectral reconstruction

- Spectral representation of Euclidean correlators

$$G_E(\tau) = \int \mathrm{d}\omega \, \frac{\rho(\omega)}{\omega} \, K(\omega, \tau)$$

Lattice — Want — Known

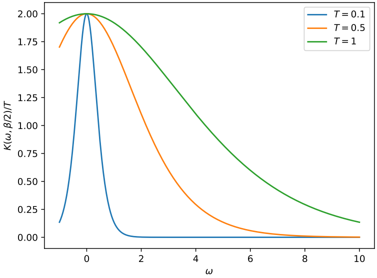
- On the lattice: $N_t \sim \mathcal{O}(10)$ ill-posed inverse problem
- Many methods on the market $\rightarrow$ applied to get other transport coefficients
- We use a simple method to get a first estimate

Midpoint method

- Kernel evaluated at the midpoint

$$K(\omega,\tau) = \omega\frac{\cosh[\omega(\tau-\beta/2)]}{\sinh[\omega\beta/2]} \to K(\omega,\tau=\beta/2) = \frac{\omega}{\sinh[\omega\beta/2]}$$

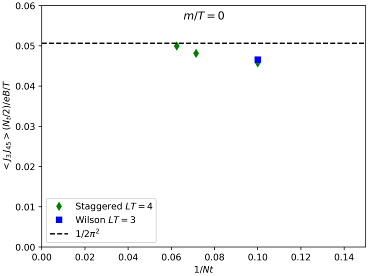
- Behaves as a smeared $\delta(\omega)$ when $T \to 0$

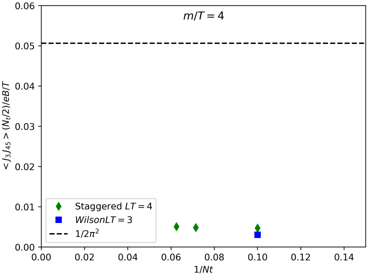


- $G_E(\tau=\beta/2)$ carries first estimate!

Preliminary results

- Free fermions





- Dependence on m/T

Summary & Outlook

Take-home results

- Localized CME signal in QCD
- Opportunity to study out-of-equilibrium CME on the lattice

Next steps

- Use more precise spectral reconstructions method to extract $C_{\mathrm{CME}}^{\mathrm{neq}}$
- Study $C_{\mathrm{CME}}^{\mathrm{neq}}$ in QCD

Backup slides

Anomalous transport

- Transport effects:

$$\begin{pmatrix} \vec{J} \\ \vec{J}_5 \end{pmatrix} = \begin{pmatrix} \sigma_{\mathsf{Ohm}} & \sigma_{\mathsf{CME}} \\ \sigma_{\mathsf{CESE}} & \sigma_{\mathsf{CSE}} \end{pmatrix} \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix}$$

- Chiral Vortical Effect: vector/axial current generated by rotation + $\mu + \mu_5$:

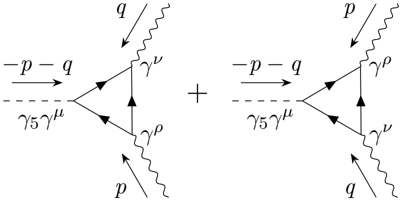
$$\vec{J} = \frac{1}{\pi^2} \mu_5 \mu \vec{\omega}$$

$$\vec{J}_5 = \left[\frac{1}{6} T^2 + \frac{1}{2\pi^2} (\mu_5^2 + \mu^2) \right] \vec{\omega}$$

Regulator sensitivity

- Well known example: triangle anomaly (with massive fermions)

$$\Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) =$$



- No/Wrong regularization

$$(p+q)_\mu \Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) = mP_5(p,q)$$

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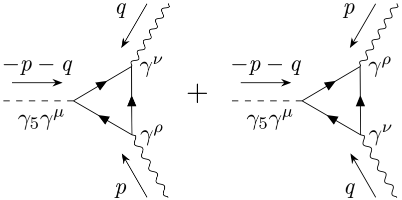


- No/Wrong regularization

$$(p+q)_\mu \Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) = mP_5(p,q)$$

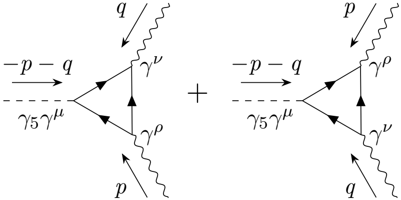
- Pauli-Villars regularization: new particles, with coeffs c_s ($s = 0, 1, 2, 3$, $s = 0$ physical fermion $m_0 \equiv m$) and masses $m_{s>0} \to \infty$

$$\begin{aligned}(p+q)_\mu \Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) =& mP_5(p,q) + \sum_{s=1} c_s m_s P^{\nu\rho}_{5,s}(p,q) \\ \to& mP_5(p,q) + \frac{\epsilon^{\alpha\beta\nu\rho} q_\alpha p_\beta}{4\pi^2}\end{aligned}$$

Regulator sensitivity

- $C_{\mathsf{CME}}/C_{\mathsf{CSE}}$ can also be written with the triangle diagram

$$\Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) =$$



with $J_3 \sim A_3, \quad J_3^5 \sim A_3^5, \quad \boldsymbol{B}_3 \sim q_1 A_2, \quad \mu = A_0, \quad \mu_5 = A_0^5$

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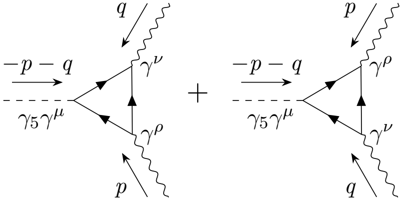

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- Kubo formulas

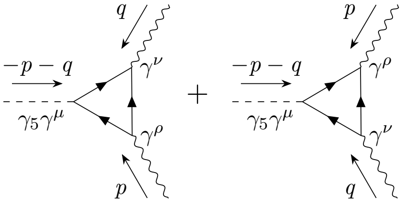
$$C_{\mathsf{CME}} = \lim_{p,q,p+q\to 0} \frac{1}{q_1}\Gamma^{023}_{AVV}(p+q,p,q) = \frac{1}{2\pi^2} + \sum_{s=1} \frac{c_s}{2\pi^2} = 0$$

$$C_{\mathsf{CSE}} = \lim_{p,q,p+q\to 0} \frac{1}{q_1}\Gamma^{320}_{AVV}(p+q,p,q) = -\frac{1}{\pi^2}\int \mathrm{d}k\, \frac{\partial n_F(E_k)}{\partial E_k}$$

Regulator sensitivity

- $C_{\mathsf{CME}}/C_{\mathsf{CSE}}$ can also be written with the triangle diagram

$$\Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) =$$



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- C_{CME} is zero due to anomalous contribution!
- C_{CSE} agrees with known results Metlitski, Zhitnitsky '05

Currents in staggered

- Staggered "gammas" (free fermions and quark chemical potential):

$$\Gamma_\nu(n,m) = \frac{1}{2}\eta_\nu(n)[e^{a\mu\delta_{\nu,4}}\delta_{n+\hat{\nu},m} + e^{-a\mu\delta_{\nu,4}}\delta_{n-\hat{\nu},m}]$$

$$\Gamma_5(n,m) = \frac{1}{4!}\sum_{i,j,k,l}\epsilon_{ijkl}\Gamma_i\Gamma_j\Gamma_k\Gamma_l$$

$$\Gamma_{\nu 5}(n,m) = \frac{1}{3!}\sum_{i,j,k}\epsilon_{ijk}\Gamma_i\Gamma_j\Gamma_k \quad i,j,k \neq \nu$$

- Conserved vector current and anomalous axial current:

$$j_\nu^V = \bar{\chi}\Gamma_\nu\chi$$

$$j_\nu^A = \bar{\chi}\Gamma_{\nu 5}\chi$$

- Staggered observable has a tadpole term, for example CSE

$$\left.\frac{\mathrm{d}\left\langle J_3^A\right\rangle}{\mathrm{d}\mu}\right|_{\mu=0} \sim \left\langle J_4^V J_3^A\right\rangle_{\mu=0} + \left\langle \frac{\partial J_3^A}{\partial\mu}\right\rangle_{\mu=0}$$

Currents in Wilson

- Local currents (don't fulfill a WI/AWI)

$$j_\mu^{VL} = \bar{\psi}\gamma_\mu\psi$$
$$j_\mu^{AL} = \bar{\psi}\gamma_\mu\gamma_5\psi$$

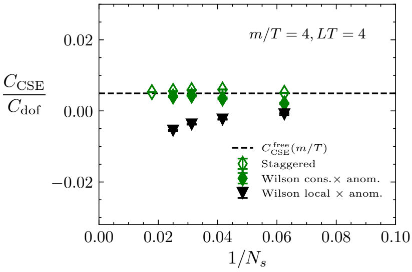
- Conserved vector current and anomalous axial current:

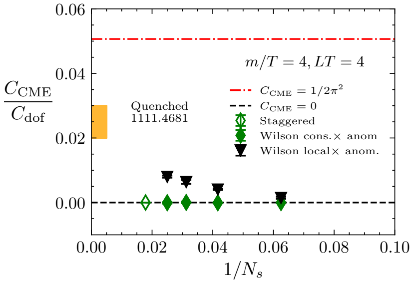
$$j_\mu^{VC}(n) = \frac{1}{2}\left[\bar{\psi}(n)(\gamma_\mu - r)\psi(n+\hat{\mu})) + \bar{\psi}(n)(\gamma_\mu + r)\psi(n-\hat{\mu}))\right]$$
$$j_\mu^{AA}(n) = \frac{1}{2}\left[\bar{\psi}(n)\gamma_\mu\gamma_5\psi(n+\hat{\mu})) + \bar{\psi}(n)\gamma_\mu\gamma_5\psi(n-\hat{\mu})\right]$$

- For correlators like $\left\langle J_4^V J_3^A \right\rangle$ we can use different combinations, for example $\left\langle J_4^{VC} J_3^{AA} \right\rangle$, $\left\langle J_4^{VL} J_3^{AA} \right\rangle$, ...

Results for free fermions

- Consistency check in the free case
- For $m/T = 4$ (similar behavior for other m/T's)





- Using the correct currents is **crucial**

Chirality free fermions

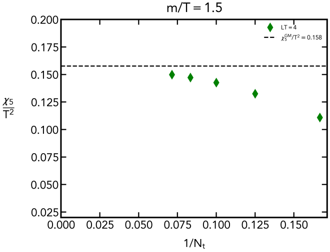
- Naturally

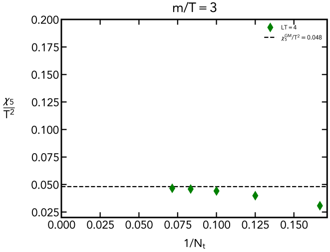
$$n5\Big|_{\mu_5=0} = \frac{T}{V}\frac{\mathrm{d}\log\mathcal{Z}}{\mathrm{d}\mu_5} = 0$$

- But

$$n_5(\mu_5) = \frac{T}{V}\left.\frac{\mathrm{d}^2\log\mathcal{Z}}{\mathrm{d}\mu_5^2}\right|_{\mu_5=0}\mu_5 + \mathcal{O}(\mu_5^2) = \chi_5\mu_5 + \mathcal{O}(\mu_5^2)$$

- χ_5 can be calculated with PV and compared to lattice





- μ_5 does induce chirality in our system

Inhomogenous B free fermions