On Holographic Vacuum Misalignment

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Based on [arXiv:2405.08714] Daniel Elander, AF, Maurizio Piai
Dataset available on [10.5281/zenodo.11774202]

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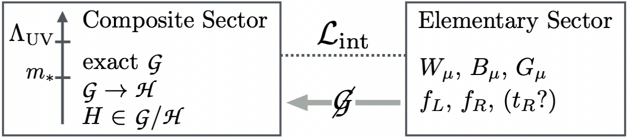
Overview

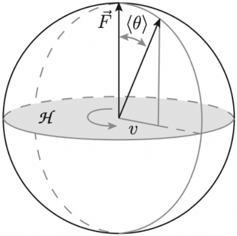
Composite Higgs Models

$$\frac{m_H^2}{\Lambda_{\rm SM}^2} \sim 10^{-28} \lll 1 \quad \rightarrow \text{Naturalness problem}$$

- Investigations into the fundamental nature and origin of the Higgs boson are among the topical subjects in theoretical physics.
- The Higgs being a composite object with a compositeness scale of TeV order, is one of the few options for "Naturally" generating its mass.
- The Higgs boson itself, as a pseudo-Nambu-Goldstone boson associated with symmetry breaking pattern, is a reasonable candidate.

Vacuum misalignment





$\mathcal{G} \to \mathcal{H}$ spontaneous breaking $\to$ massless NGBs in the coset $\mathcal{G}/\mathcal{H}$

$$G_{\text{EW}} = \text{SU}(2)_L \times \text{U}(1)_Y \subseteq \mathcal{H}$$

G is large enough to contain at least one Higgs doublet in the coset

[Panico, Wulzer '15]

Correspondance Dictionary

CHM	**Gravity**
D=4 ↓ Confining	D=5 ↓ D=6 compactified on a circle *[Witten '98]*
Global SO(5) ↓ SO(4)	Local SO(5) isometry ↓ Local SO(4)
Weakly gauged SO(4) (misaligned)	**Boundary terms**

Instead of $SU(2) \times U(1)$, our study is gauging an SO(4) group.

6D garvity action

$$\mathcal{S}_6 = \mathcal{S}_6^{(bulk)} + \sum_{i=1,2} \mathcal{S}_{5,i} \,,$$

$$\mathcal{S}_6^{(bulk)} = \frac{1}{2\pi} \int \mathrm{d}^6 x \sqrt{-\hat{g}_6} \left\{ \frac{\mathcal{R}_6}{4} - \frac{1}{2} \hat{g}^{\hat{M}\hat{N}} \left(D_{\hat{M}} \mathcal{X} \right)^T D_{\hat{N}} \mathcal{X} - \mathcal{V}_6(\mathcal{X}) - \frac{1}{2} \mathrm{Tr} \left[\hat{g}^{\hat{M}\hat{P}} \hat{g}^{\hat{N}\hat{Q}} \mathcal{F}_{\hat{M}\hat{N}} \mathcal{F}_{\hat{P}\hat{Q}} \right] \right\} ,$$

$\rho_1 < \rho < \rho_2$ and the he space-time index is $\hat{M} = 0, 1, 2, 3, 5, 6$.

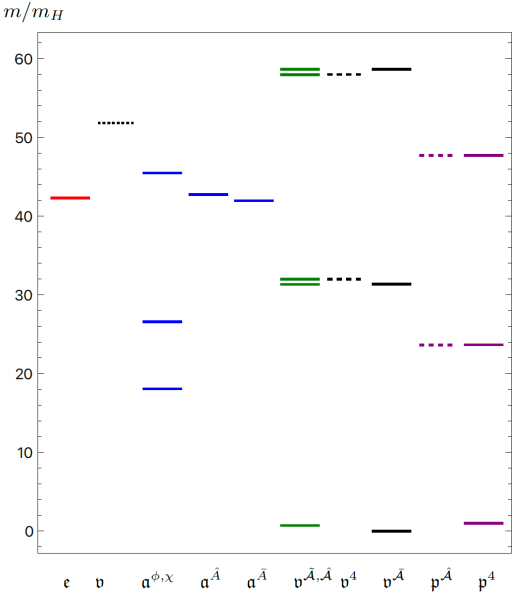
$$\mathcal{V}_6(\phi) = -5 - \frac{\Delta(5-\Delta)}{2} \phi^2 - \frac{5\Delta^2}{16} \phi^4 \,,$$

$$\mathcal{X} = \exp \left[2i \textstyle\sum_{\hat{A}} \pi^{\hat{A}} t^{\hat{A}} \right] \mathcal{X}_0 \, \phi \,, \qquad \text{where} \qquad \mathcal{X}_0 = (0,0,0,0,1)^T \,,$$

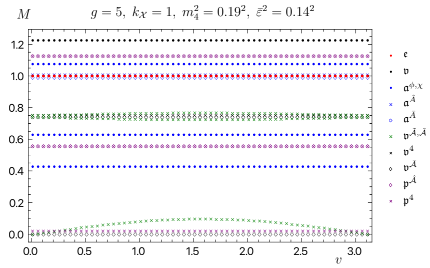
with $\hat{A} = 1, \cdots, 4$, labelling the generators of the $SO(5)/SO(4)$ coset.
For the gauge field and the scalar we have:

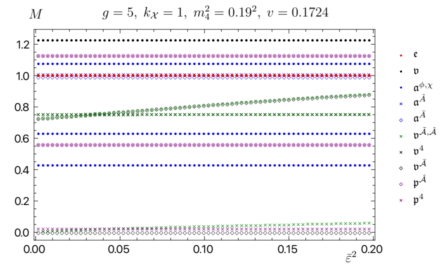
$$\boxed{10 \to 6 \oplus 4 \to 3 \oplus 3 \oplus 3 \oplus 1, \quad 5 \to 4 \oplus 1 \to 3 \oplus 1 \oplus 1.}$$

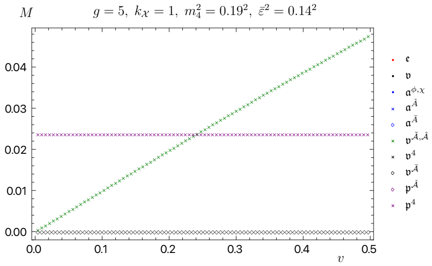
Results

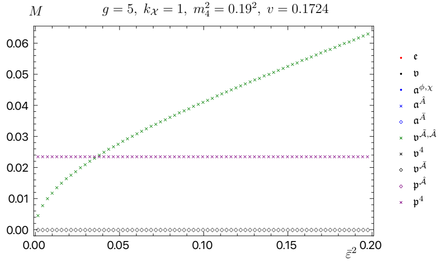


m/m_H
60
50
40
30
20
10
0
$\mathfrak{e}$
$\mathfrak{v}$
$\mathfrak{a}^{\phi,\chi}$
$\mathfrak{a}^{\hat{A}}$
$\mathfrak{a}^{\bar{A}}$
$\mathfrak{v}^{\tilde{A},\hat{A}}$
$\mathfrak{v}^{4}$
$\mathfrak{v}^{\bar{A}}$
$\mathfrak{p}^{\hat{A}}$
$\mathfrak{p}^{4}$


M
$g = 5,\ k_\chi = 1,\ m_4^2 = 0.19^2,\ \bar{\varepsilon}^2 = 0.14^2$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0
v
$\mathfrak{e}$
$\mathfrak{v}$
$\mathfrak{a}^{\phi,\chi}$
$\mathfrak{a}^{\hat{A}}$
$\mathfrak{a}^{\bar{A}}$
$\mathfrak{v}^{\bar{A},\hat{A}}$
$\mathfrak{v}^4$
$\mathfrak{v}^{\bar{A}}$
$\mathfrak{p}^{\hat{A}}$
$\mathfrak{p}^4$


M
$g = 5,\ k_\chi = 1,\ m_4^2 = 0.19^2,\ v = 0.1724$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.00 0.05 0.10 0.15 0.20
$\bar{\varepsilon}^2$
$\mathfrak{e}$
$\mathfrak{v}$
$\mathfrak{a}^{\phi,\chi}$
$\mathfrak{a}^{\hat{A}}$
$\mathfrak{a}^{\bar{A}}$
$\mathfrak{v}^{\bar{A},\hat{A}}$
$\mathfrak{v}^4$
$\mathfrak{v}^{\bar{A}}$
$\mathfrak{p}^{\hat{A}}$
$\mathfrak{p}^4$


M
$g = 5,\ k_\chi = 1,\ m_4^2 = 0.19^2,\ \bar{\varepsilon}^2 = 0.14^2$
0.04
0.03
0.02
0.01
0.00
0.0 0.1 0.2 0.3 0.4 0.5
v
$\mathfrak{e}$
$\mathfrak{v}$
$\mathfrak{a}^{\phi,\chi}$
$\mathfrak{a}^{\hat{A}}$
$\mathfrak{a}^{\bar{A}}$
$\mathfrak{v}^{\bar{A},\hat{A}}$
$\mathfrak{v}^4$
$\mathfrak{v}^{\bar{A}}$
$\mathfrak{p}^{\hat{A}}$
$\mathfrak{p}^4$


M
$g = 5,\ k_\chi = 1,\ m_4^2 = 0.19^2,\ v = 0.1724$
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.00 0.05 0.10 0.15 0.20
$\bar{\varepsilon}^2$
$\mathfrak{e}$
$\mathfrak{v}$
$\mathfrak{a}^{\phi,\chi}$
$\mathfrak{a}^{\hat{A}}$
$\mathfrak{a}^{\bar{A}}$
$\mathfrak{v}^{\bar{A},\hat{A}}$
$\mathfrak{v}^4$
$\mathfrak{v}^{\bar{A}}$
$\mathfrak{p}^{\hat{A}}$
$\mathfrak{p}^4$

Summary and Outlook

- We have studied a bottom-up model, with a completely smooth gravity background, that implements a simple realisation of the holographic description of confinement in the dual gauge theory.
- We presented the mass spectrum of bosonic states in the field theory.
- All the new particles are parametrically heavy with respect to the bosons that play the role of the Z, W, and Higgs boson
- We can further extend this six-dimensional model to more realistic composite Higgs models with gauged $SU(2) \times U(1)$ and fermions.
- Extension to a top-down model is expected.

Thank you.

Asymptotics and dimensional reduction to five dimensions

$$\mathrm{d}s_6^2 = e^{-2\chi}\mathrm{d}x_5^2 + e^{6\chi}\left(\mathrm{d}\eta + \chi_M \mathrm{d}x^M\right)^2 \,,$$

where the space-time index is $M = 0, 1, 2, 3, 5$.
We consider background solutions in which $\chi_M = 0$, while the metric g_{MN}, ϕ, and χ depend on the radial coordinate only. The metric in five dimensions takes the domain-wall (DW) form

$$\mathrm{d}s_5^2 = \mathrm{d}r^2 + e^{2A(r)}\mathrm{d}x_{1,3}^2 = e^{2\chi(\rho)}\mathrm{d}\rho^2 + e^{2A(\rho)}\mathrm{d}x_{1,3}^2 \,,$$

with $\mathrm{d}\rho = e^{-\chi}\mathrm{d}r$.

UV expansions

Asymptotically in the UV, the dual field theory flows towards a CFT in 5 dimensions, deformed by the insertion of operators $\mathcal{O}$ The two parameters appearing in the solution of the corresponding second-order classical equations correspond in field-theory terms to the coupling and condensate associated with $\mathcal{O}$.

$$\rho \to +\infty, \quad \phi = 0 \quad \text{critical point of } \mathcal{V}_6, \quad \chi \simeq \frac{1}{3}\rho, \quad A \simeq \frac{4}{3}\rho$$

We classify all the solutions in terms of a power expansion in the small parameter $z \equiv e^{-\rho}$. The expansion depends on five free parameters.

$$\begin{aligned}
\phi(z) &= \phi_J z^{\Delta_J} + \cdots + \phi_V z^{\Delta_V} + \cdots , \\
\chi(z) &= \chi_U - \frac{1}{3}\log(z) + \cdots + (\chi_5 + \cdots) z^5 + \cdots , \\
A(z) &= A_U - \frac{4}{3}\log(z) + \cdots .
\end{aligned}$$

Background solutions

Confining solutions The confining solutions are such that the circle parametrised by η shrinks to zero size at some point ρ_o of the radial direction ρ and there is no conical singularity. For small $(\rho - \rho_o)$, we find that such solutions have the following form

$$\phi(\rho) = \phi_I - \frac{1}{16}\Delta\phi_I \left(20 + \Delta\left(5\phi_I^2 - 4\right)\right)(\rho - \rho_o)^2 + \mathcal{O}\left((\rho - \rho_o)^2\right) ,$$
$$\chi(\rho) = \chi_I + \frac{1}{3}\log(\rho - \rho_o) + O\left((\rho - \rho_o)^4\right) ,$$
$$A(\rho) = A_I + \frac{1}{3}\log(\rho - \rho_o) + \mathcal{O}\left((\rho - \rho_o)^2\right) ,$$

Singular domain-wall solutions They obey the DW ansatz $A = 4\chi = \frac{4}{3}\mathcal{A}$. In six dimensions, they take the form of Poincaré domain walls.

$$\phi(\rho) = \phi_I - \sqrt{\frac{2}{5}}\log(\rho - \rho_o) + \mathcal{O}((\rho - \rho_o)^2)\,,$$
$$\mathcal{A}(\rho) = \frac{1}{5}\log(\rho - \rho_o) + \mathcal{O}((\rho - \rho_o)^2)\,.$$

The system of equations is symmetric under the change $\phi \to -\phi$, hence a second branch of solutions can be obtained by just changing the sign of ϕ. These solutions are singular at the end of space.

Boundary-localised interactions

We add to the bulk action, $\mathcal{S}_5^{(bulk)}$, several boundary terms—denoted as in order to obtain the desired five-dimensional action,

$$\mathcal{S}_5 = \mathcal{S}_5^{(bulk)} + \sum_{i=1,2} \left(\mathcal{S}_{\mathrm{GHY},i} + \mathcal{S}_{\lambda,i} \right) + \mathcal{S}_{P_5,2} + \mathcal{S}_{\mathcal{V}_4,2} + \mathcal{S}_{\mathcal{A},2} + \mathcal{S}_{\chi,2} + \mathcal{S}_{\mathcal{X},2} \,,$$

$$\mathcal{S}_{P_5,2} = \int \mathrm{d}^4x \sqrt{-\tilde{g}} \left\{ -\frac{1}{2} K_5 \, \tilde{g}^{\mu\nu} \left(D_\mu P_5 \right) D_\nu P_5 - \lambda_5 \left(P_5^T P_5 - v_5^2 \right)^2 \right\} \Big|_{\rho=\rho_2} \,,$$

$$\mathcal{S}_{\mathcal{V}_4,2} = - \int \mathrm{d}^4x \sqrt{-\tilde{g}} \, \mathcal{V}_4(\mathcal{X}, \chi, P_5) \Big|_{\rho=\rho_2} \,,$$

$$\mathcal{S}_{\mathcal{A},2}\Big|_{P_5=\overline{P_5}} = \int \mathrm{d}^4x \sqrt{-\tilde{g}} \left\{ -\frac{1}{4} \hat{D}_2 \, \tilde{g}^{\mu\rho} \tilde{g}^{\nu\sigma} \mathcal{F}_{\mu\nu}^{\hat{A}} \mathcal{F}_{\rho\sigma}^{\hat{A}} - \frac{1}{4} \bar{D}_2 \, \tilde{g}^{\mu\rho} \tilde{g}^{\nu\sigma} \mathcal{F}_{\mu\nu}^{\bar{A}} \mathcal{F}_{\rho\sigma}^{\bar{A}} \right\} \Big|_{\rho=\rho_2} \,.$$

$$\mathcal{S}_{\mathcal{X},2} = \int \mathrm{d}^4x \sqrt{-\tilde{g}} \left\{ -\frac{1}{2} K_{\mathcal{X},2} \, \tilde{g}^{\mu\nu} (D_\mu \mathcal{X})^T D_\nu \mathcal{X} \right\} \Big|_{\rho=\rho_2} \,.$$

Results