

The deconfinement interface tension in $SU(N)$ gauge theories at large N

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Motivation

- ▶ Surface tension not well measured at large N
- ▶ Improving on existing data
- ▶ Amplitude of surface fluctuations in determining the interface tension
- ▶ Eases the tunneling situation due to absence of barriers

Existing Result

- ▶ Determination through tunneling probability:

$$P \propto \exp[-2\sigma A/T_c]$$

- ▶ Requires small enough volume for the tunneling to be non-negligible
- ▶ $\sigma/T_c^3 = -0.104(3) + 0.0138(3)N^2$ (Lucini, Teper, Wenger - 2005)
- ▶ $\sigma/T_c^3 = -0.333(9) + 0.118(3)N$ in same publication
- ▶ Unable to verify N^2 behaviour in study

Setup

- ▶ Plaquette action: $\beta \sum_p \{1 - \frac{1}{N} \text{ReTr}U_p\}$ where $\beta = \frac{2N}{g^2}$
- ▶ Heat bath + overrelaxation updates
- ▶ Simulations with HILA on GPUs
- ▶ Elongated system to ensure that the interface is a minimal surface

Method

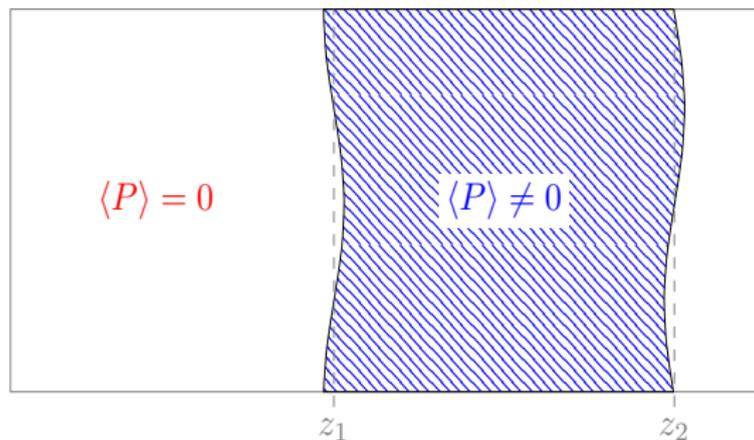
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- ▶ Generate configurations with the phases in coexistence

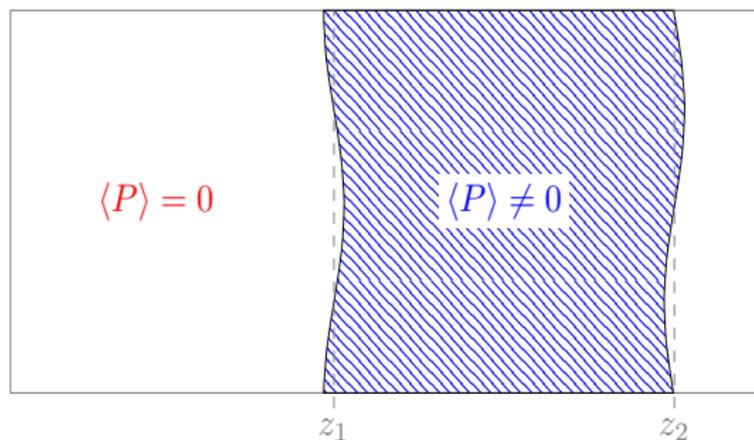
Method

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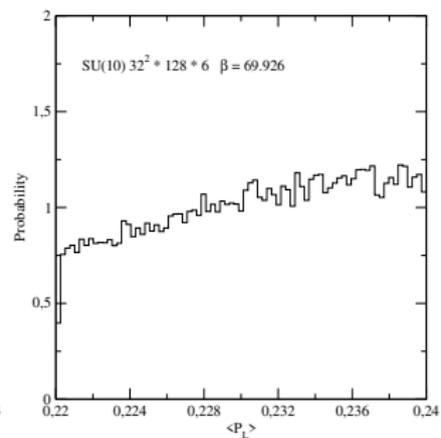
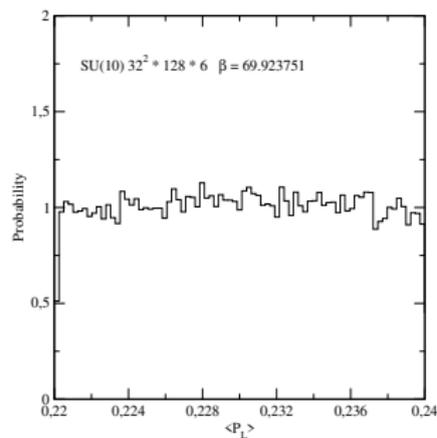
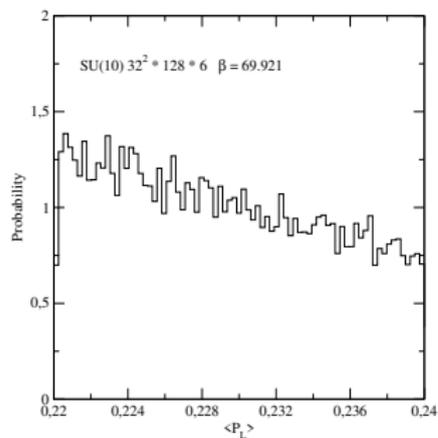
Method

- ▶ Restrict the system to the intermediate state



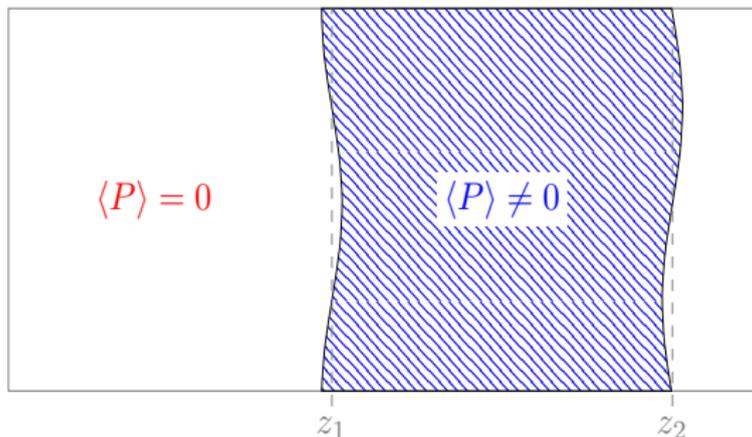
Method

- ▶ Restrict the system to the intermediate state



Method

- ▶ Location of the surface

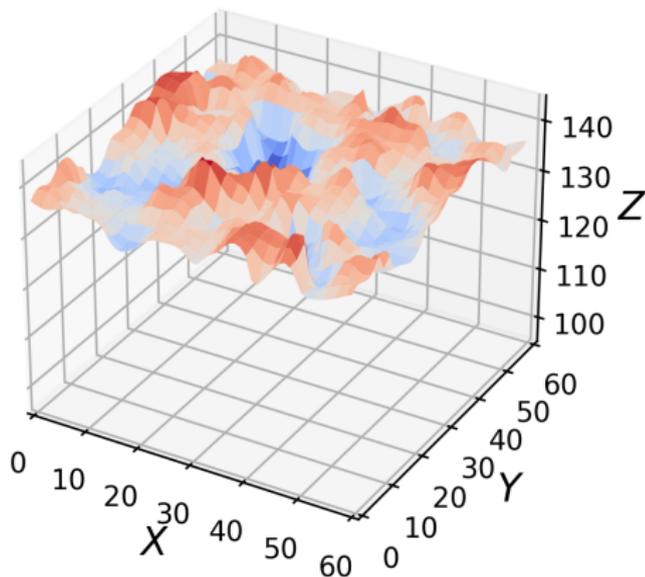


- ▶ Calculating the interface tension from the surface fluctuation

$$\langle z_n^2 \rangle = \frac{T}{4\pi^2 \sigma n^2}$$

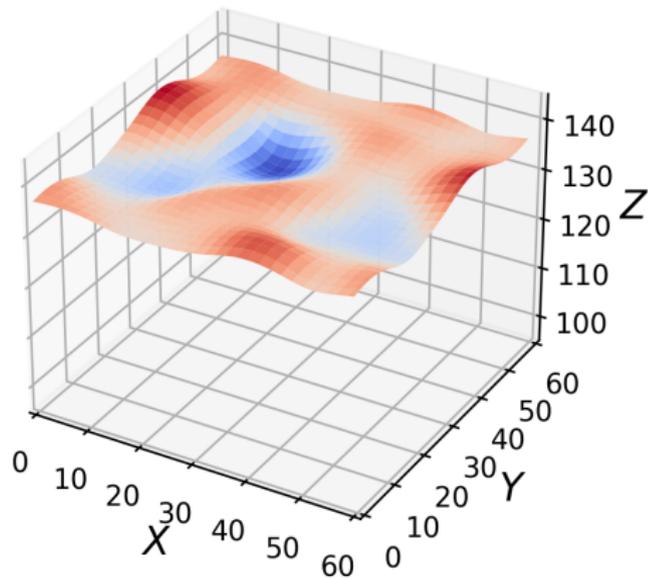
Smearing

- ▶ Smearing is necessary as the surfaces are too rough



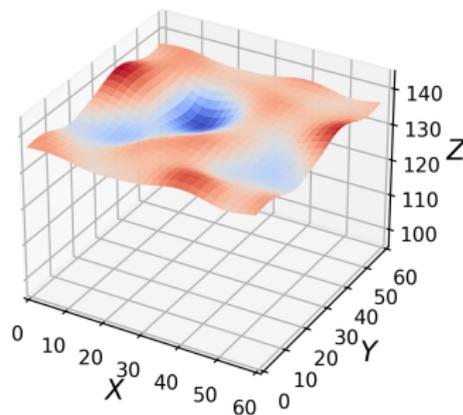
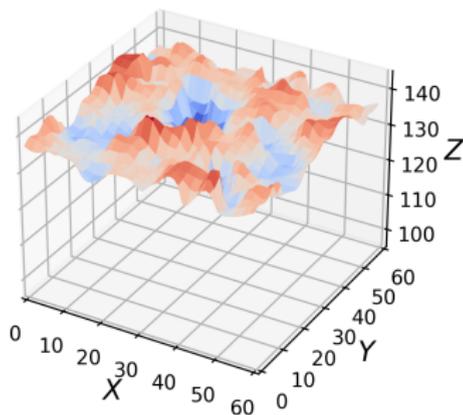
Smearing

- ▶ Eliminates the UV modes and noise



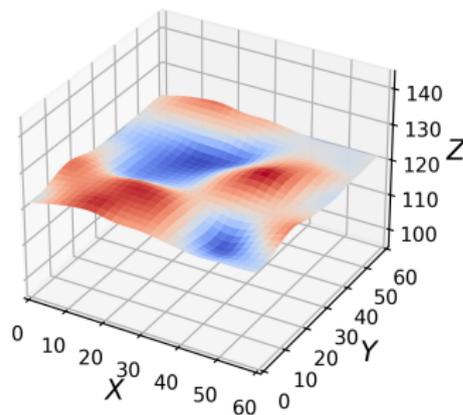
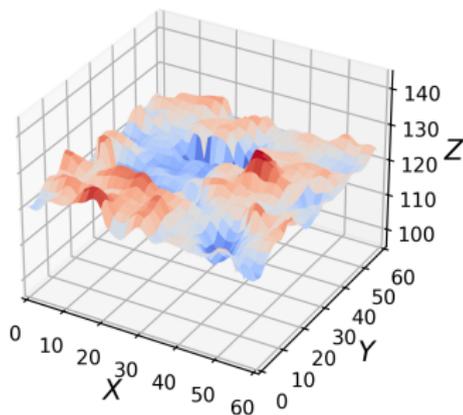
Smearing

- ▶ Long range structure preserved



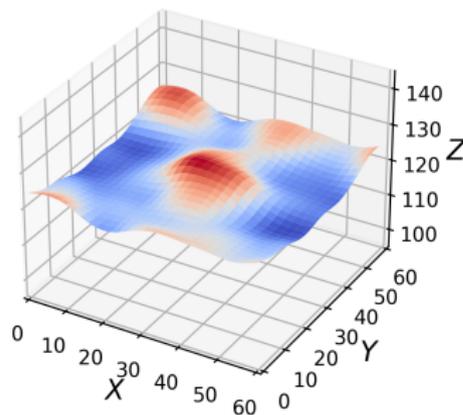
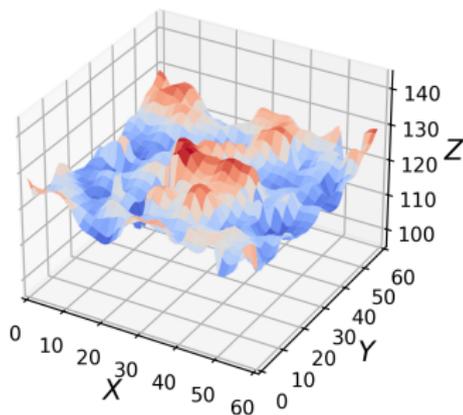
Smearing

- Effects of the number of smearing steps



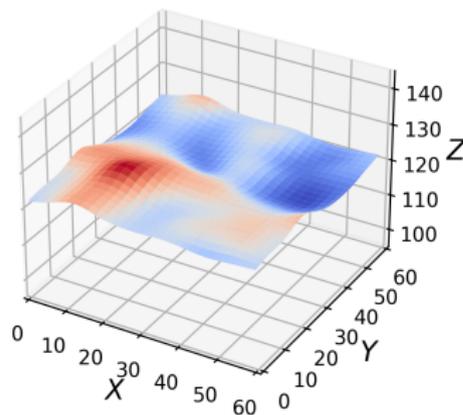
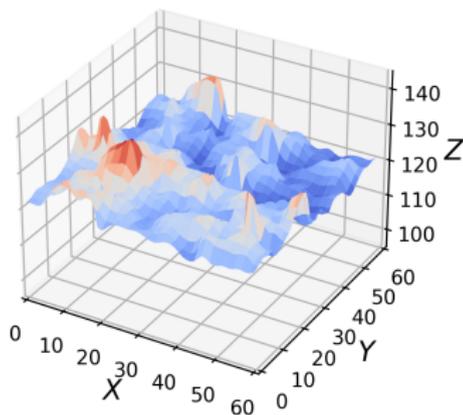
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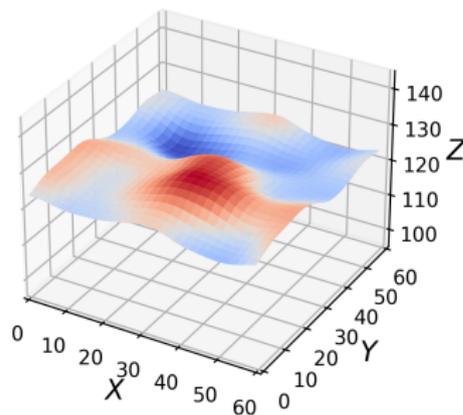
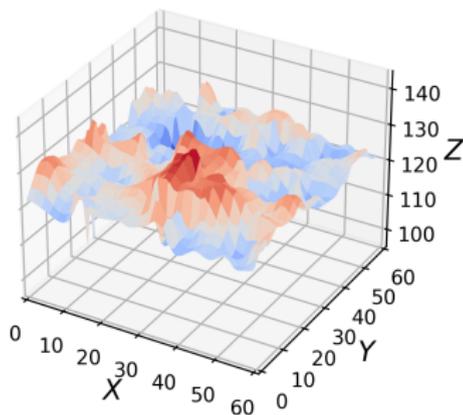
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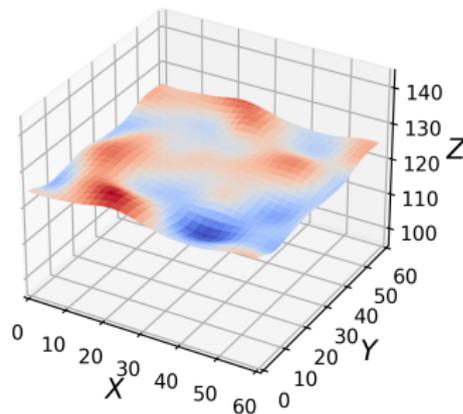
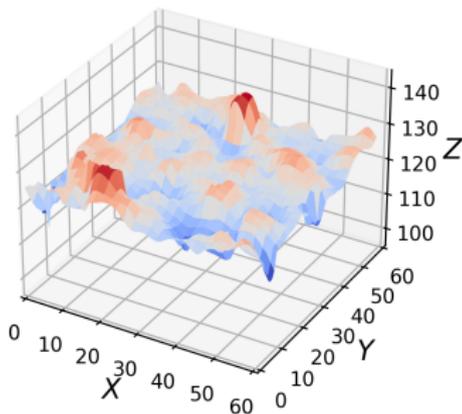
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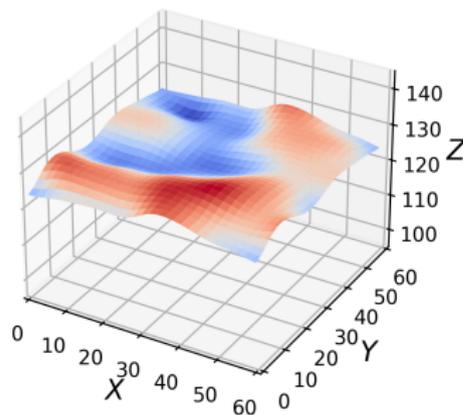
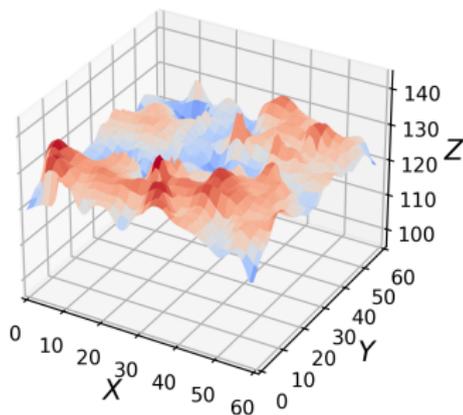
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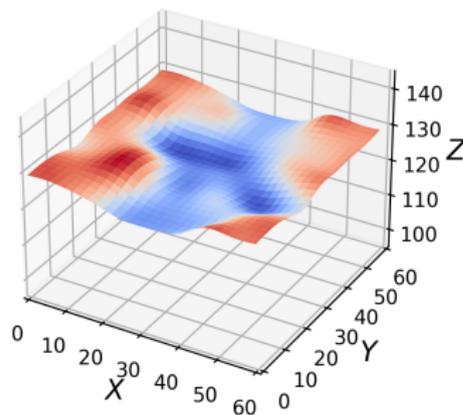
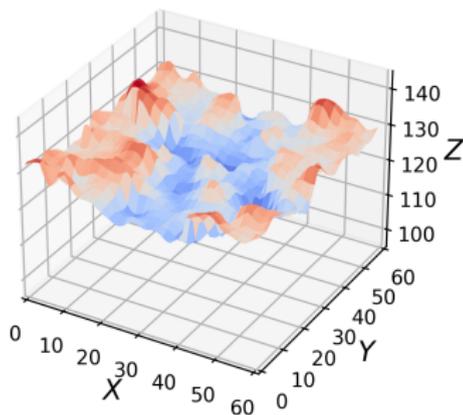
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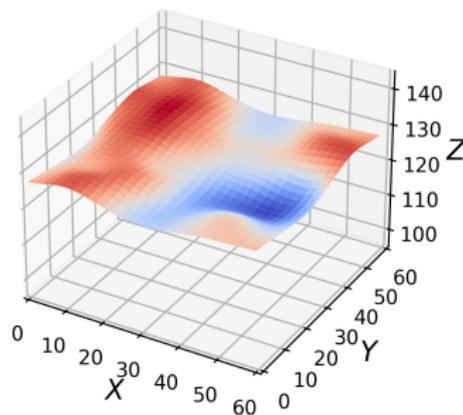
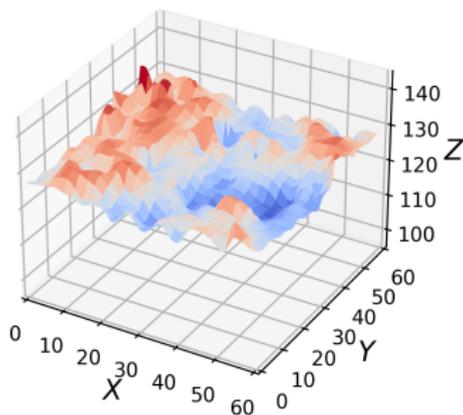
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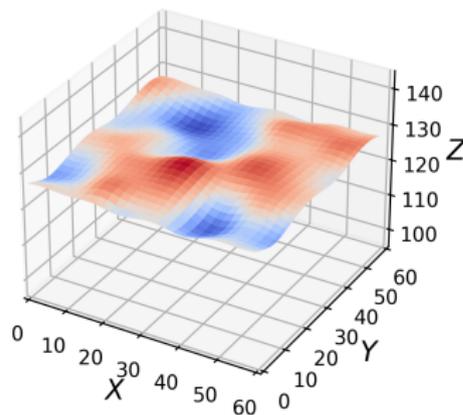
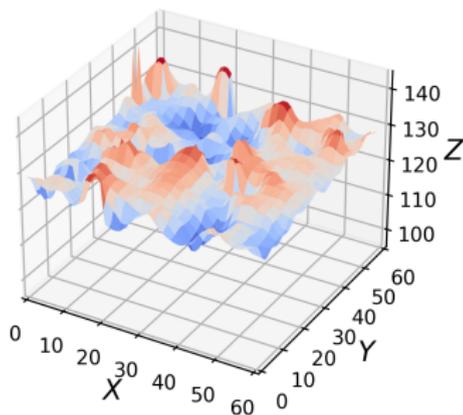
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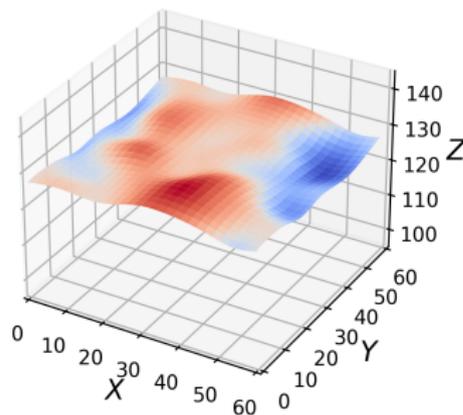
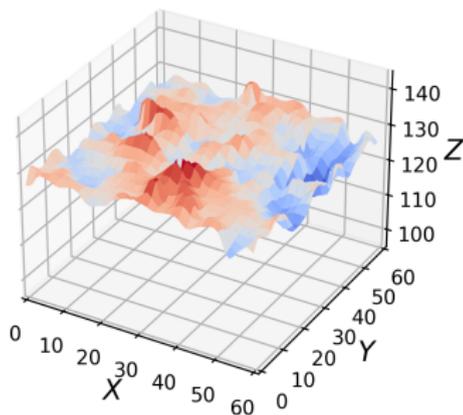
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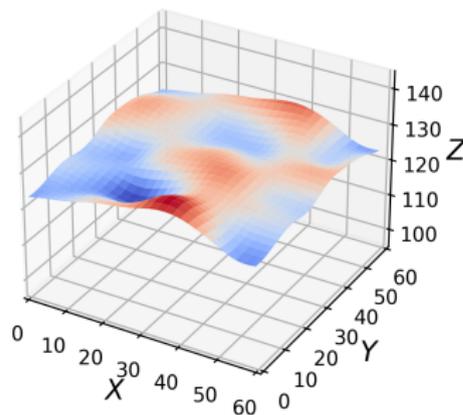
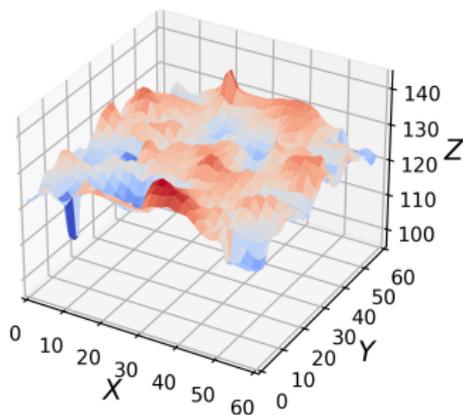
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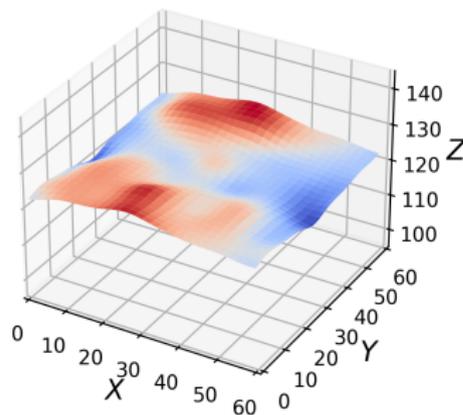
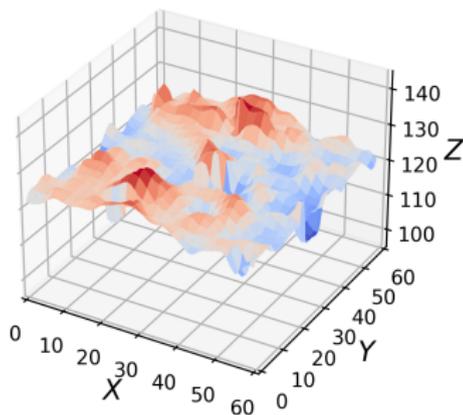
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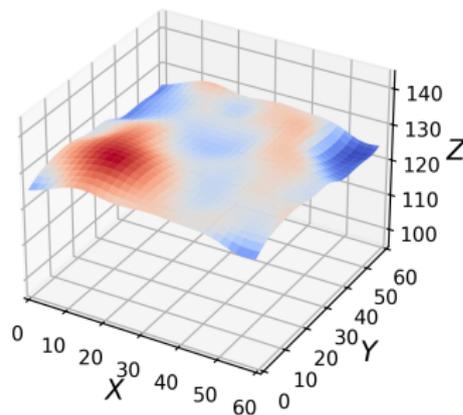
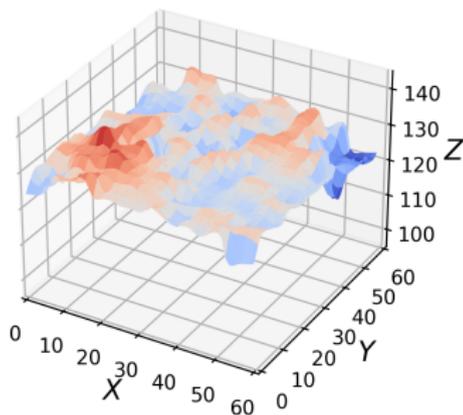
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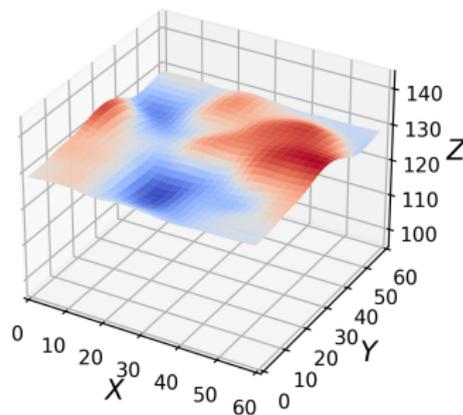
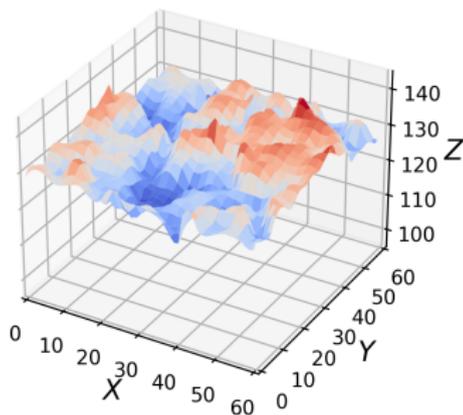
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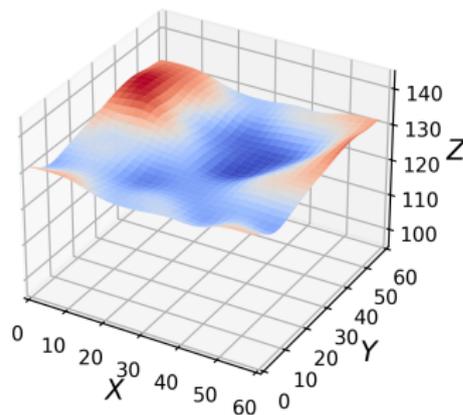
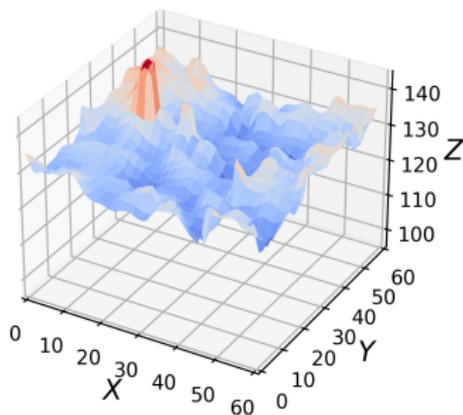
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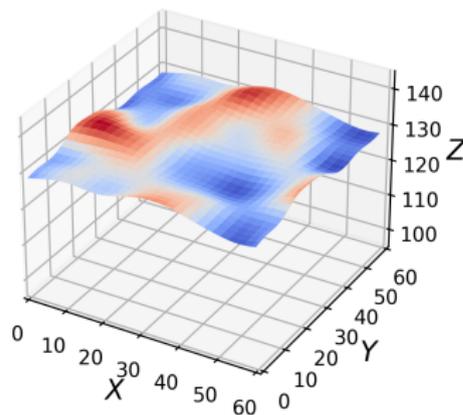
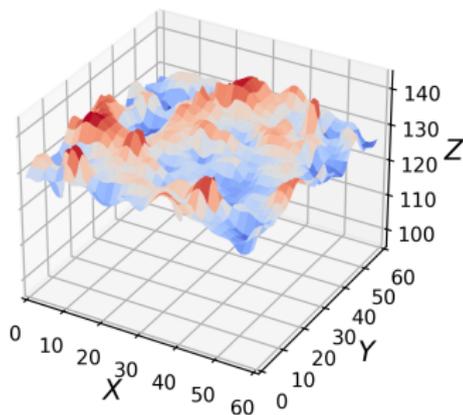
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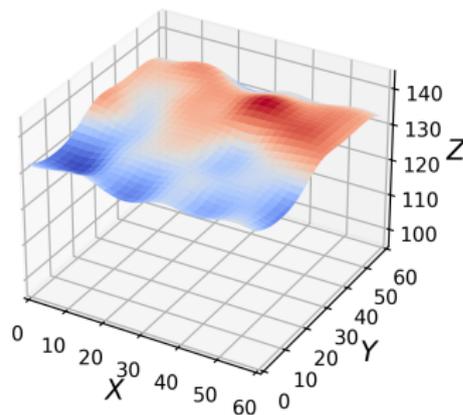
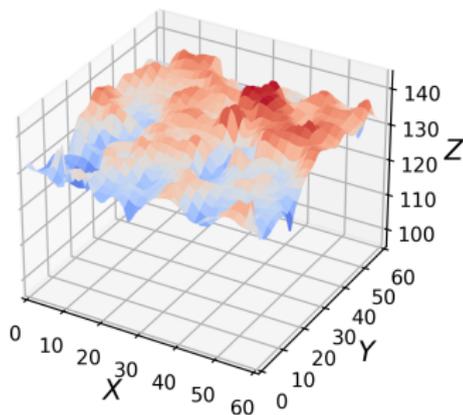
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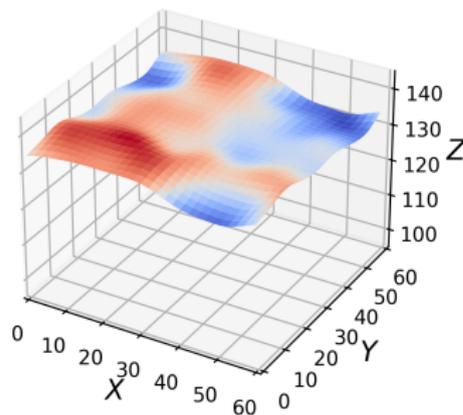
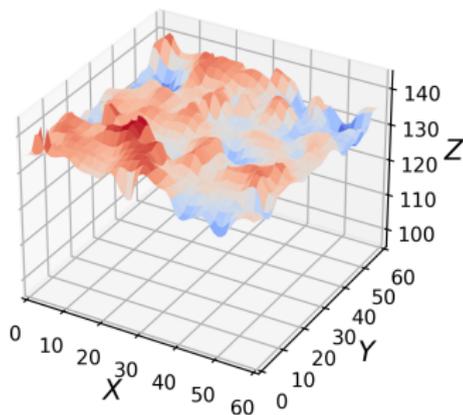
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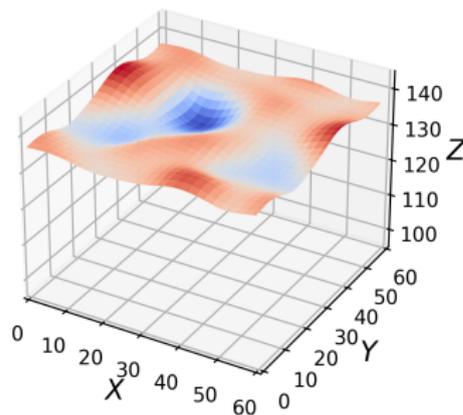
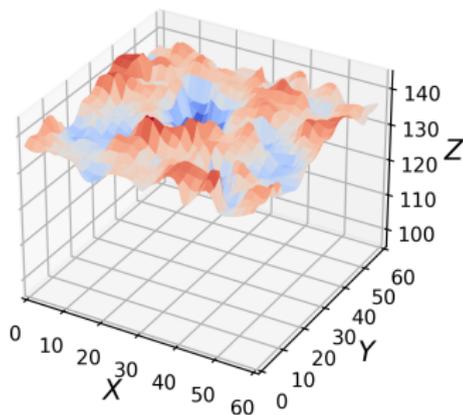
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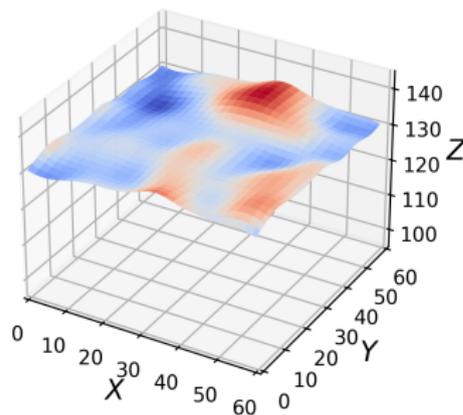
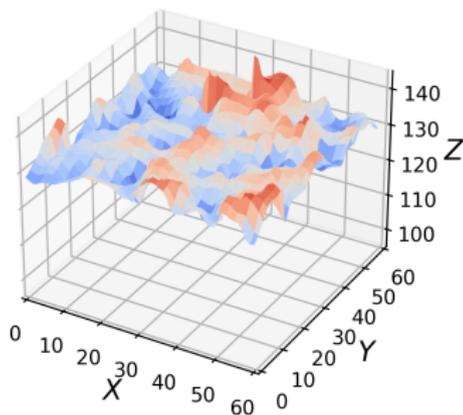
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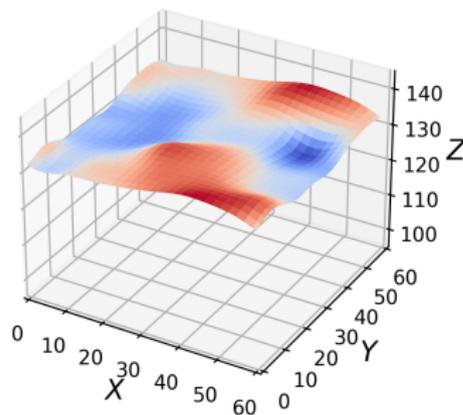
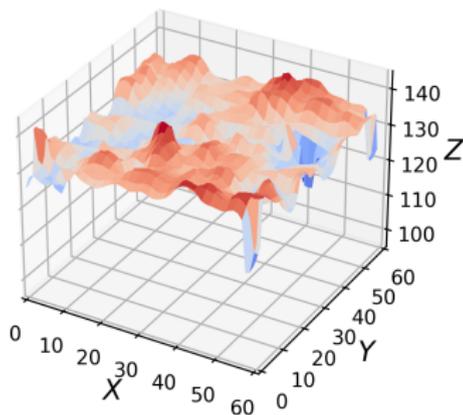
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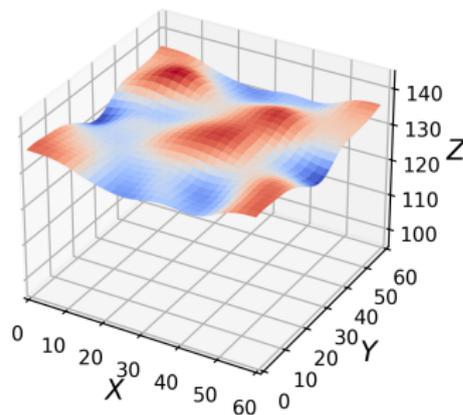
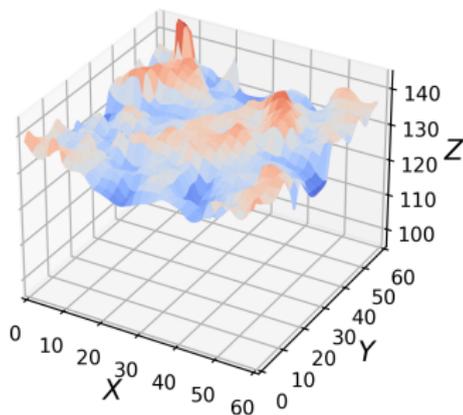
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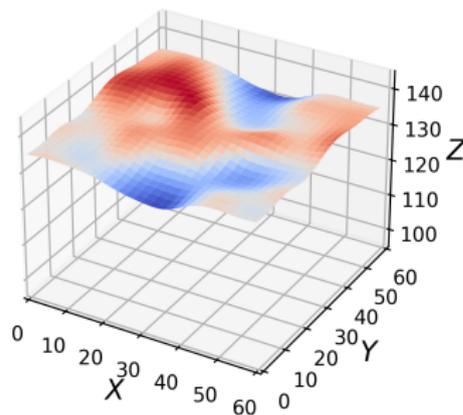
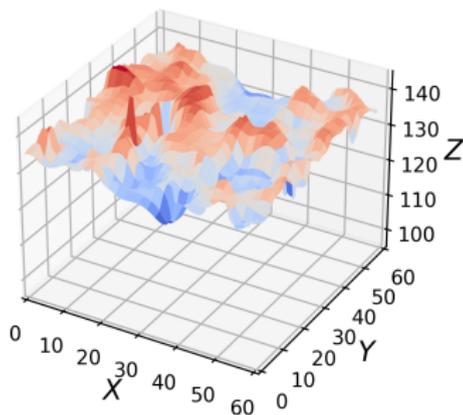
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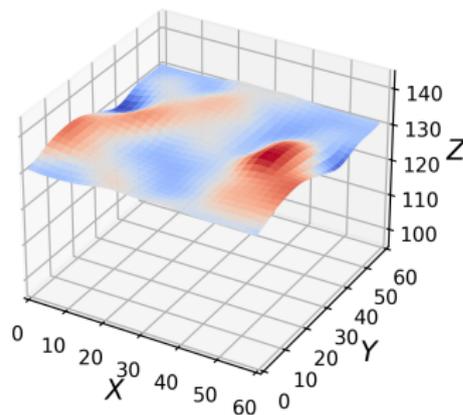
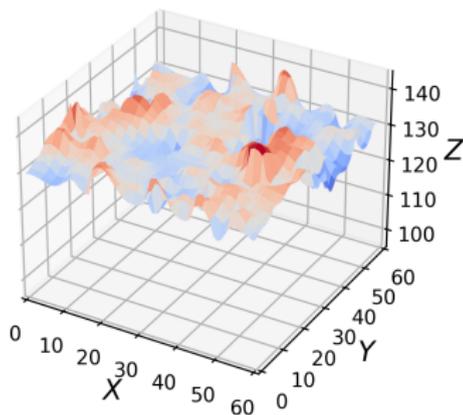
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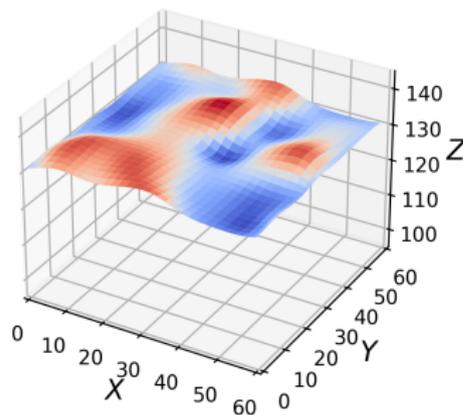
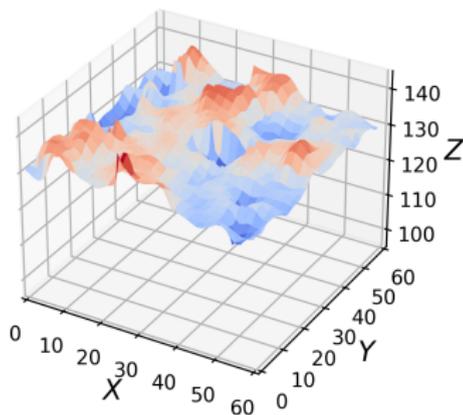
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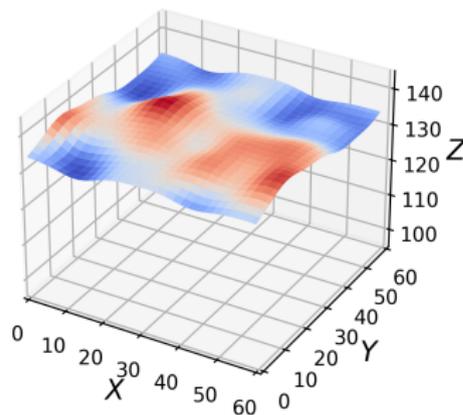
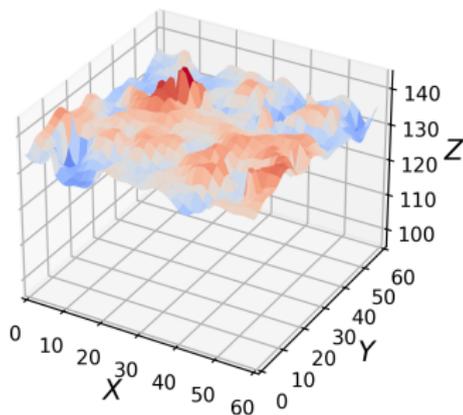
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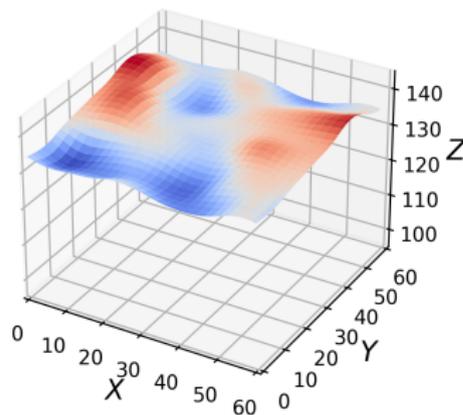
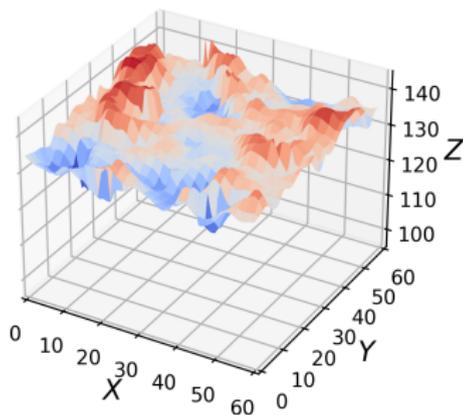
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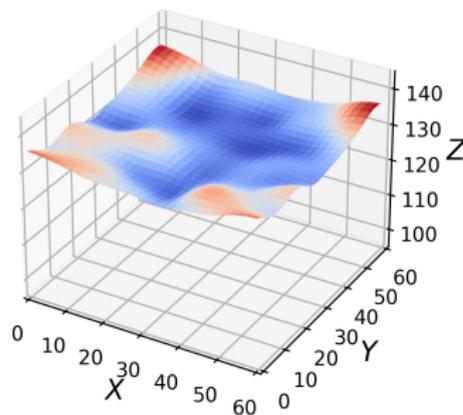
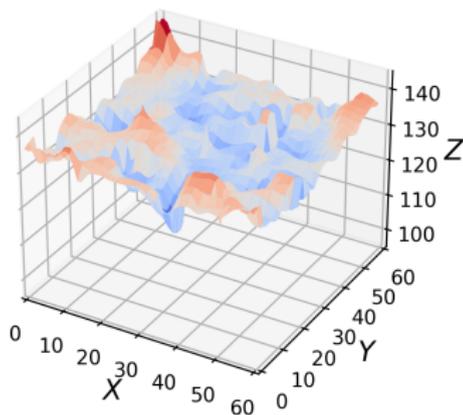
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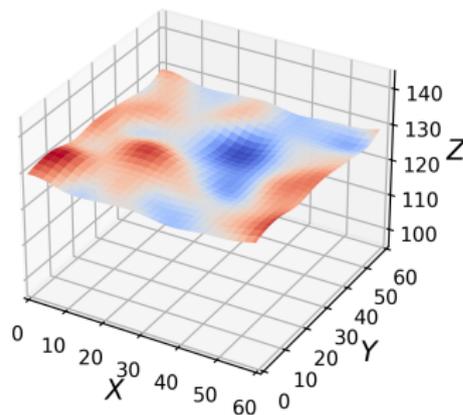
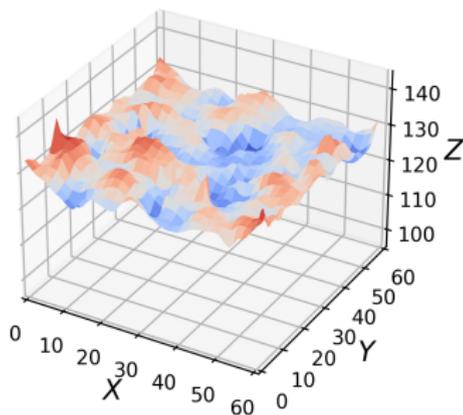
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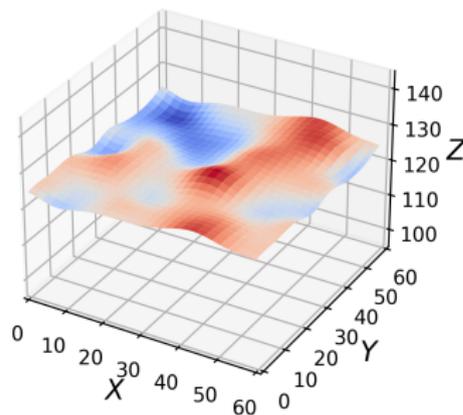
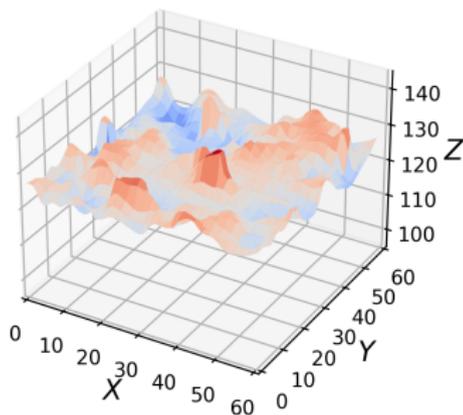
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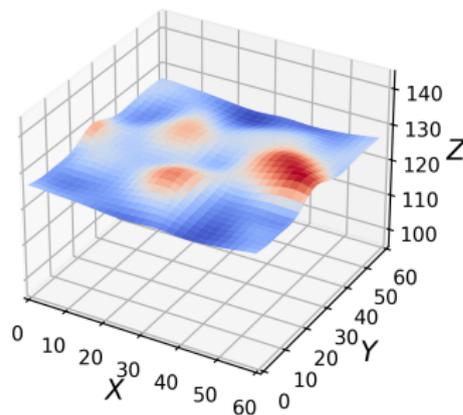
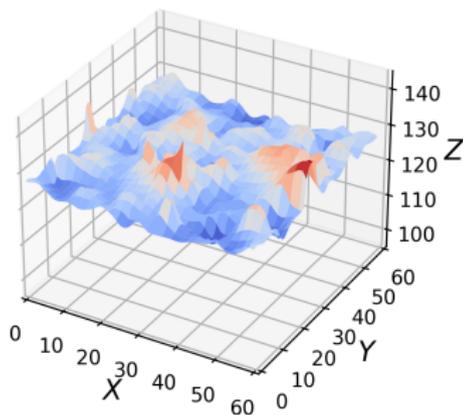
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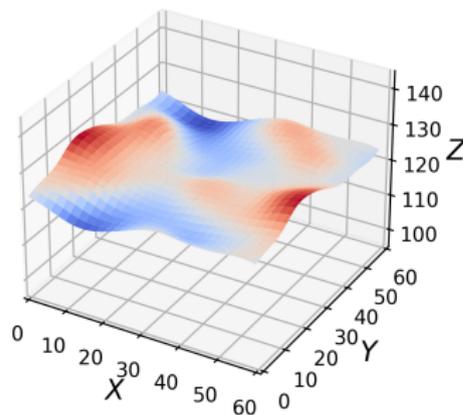
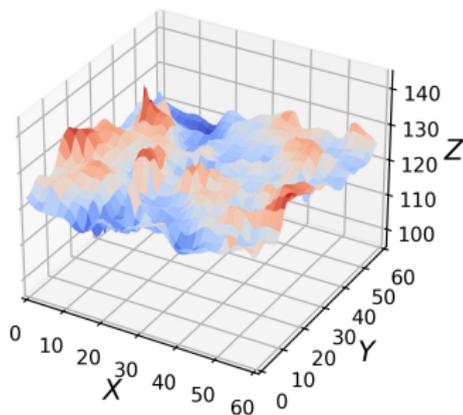
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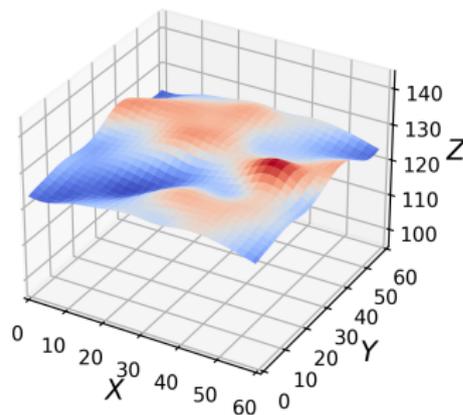
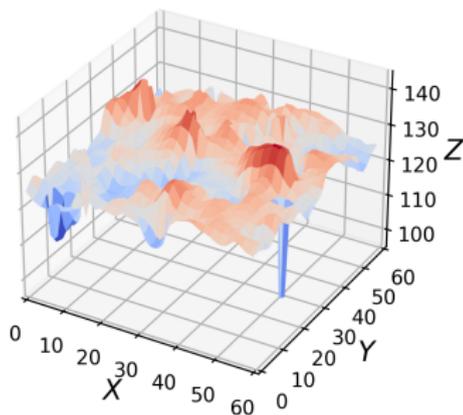
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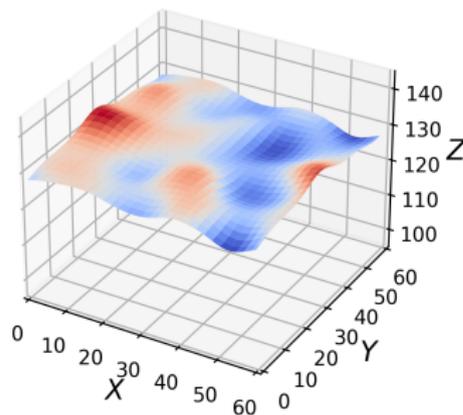
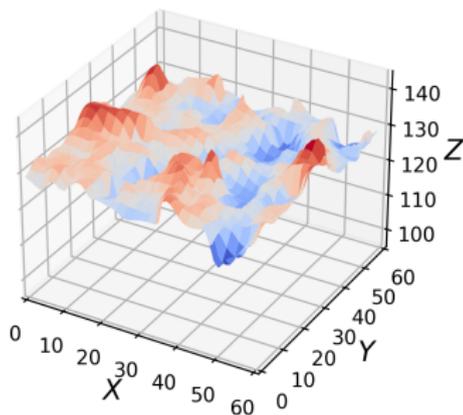
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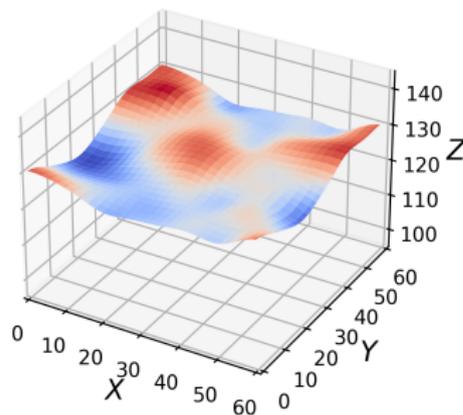
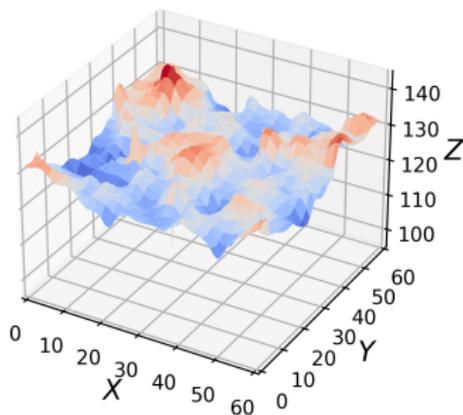
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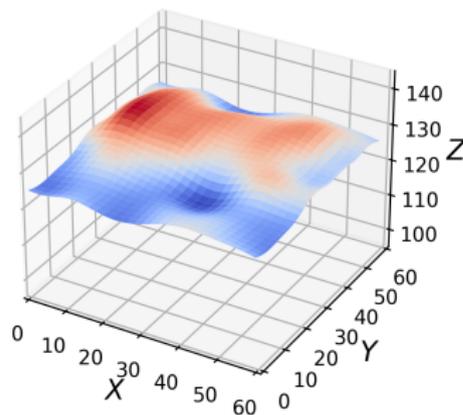
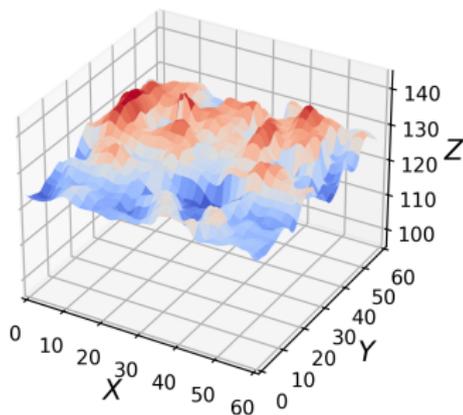
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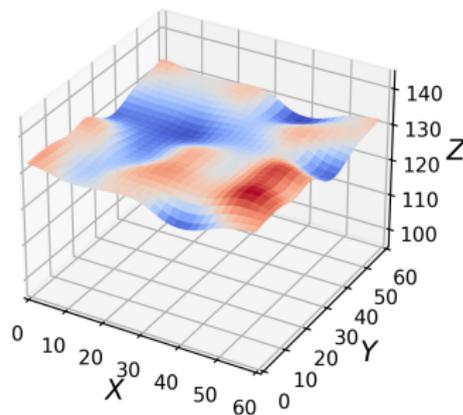
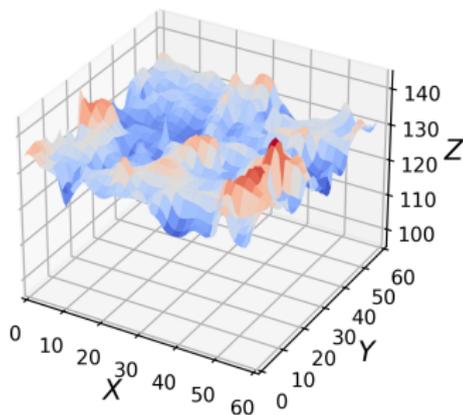
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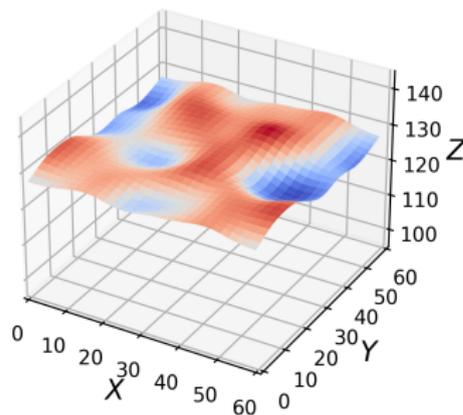
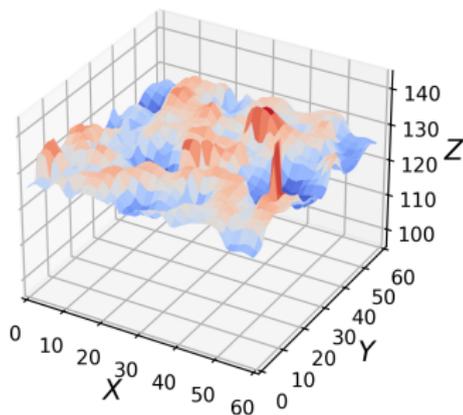
Smearing

- Effects of the number of smearing steps



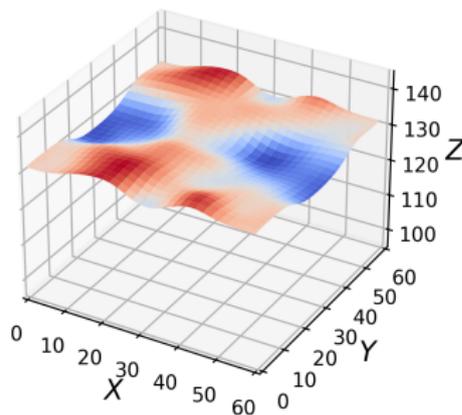
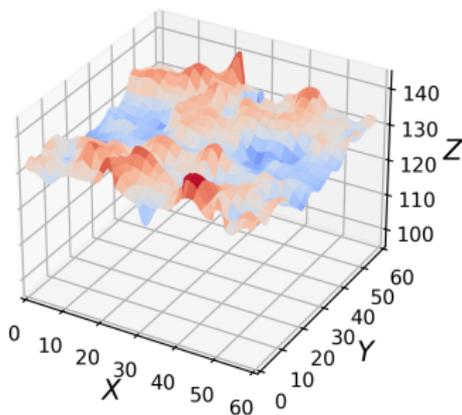
Smearing

- Effects of the number of smearing steps



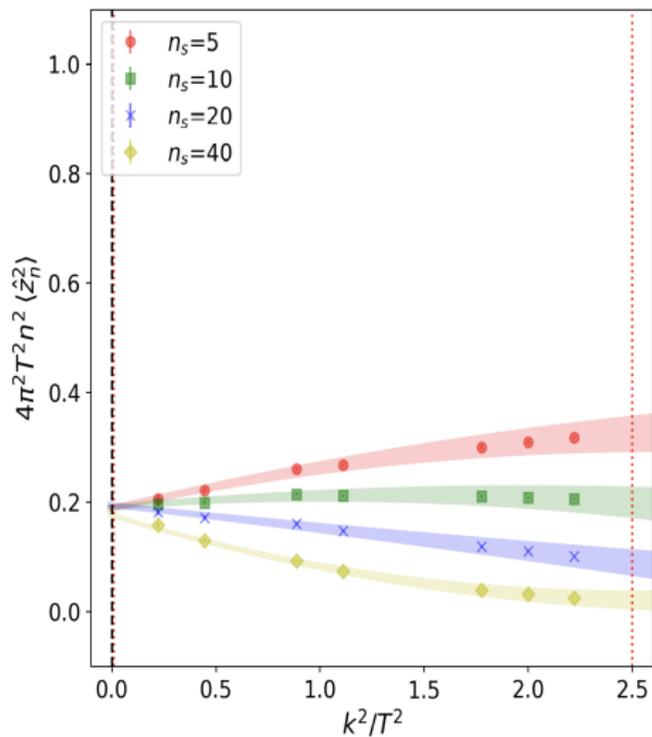
Smearing

- Effects of the number of smearing steps



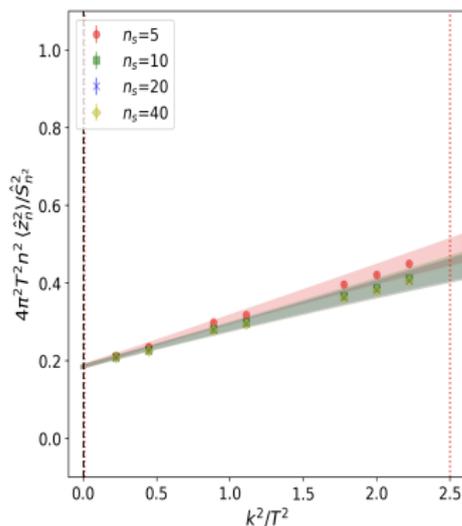
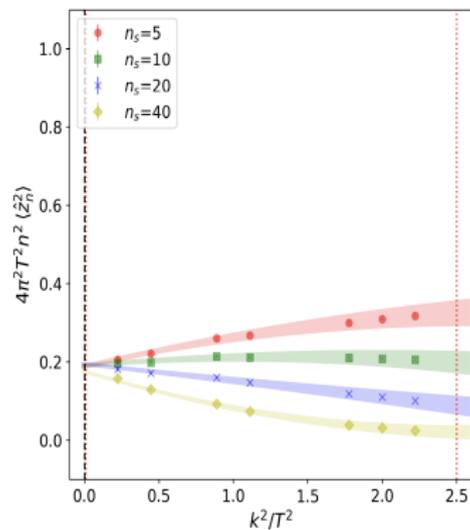
Smearing

► SU(16)



Smearing Correction

► Kernel correction



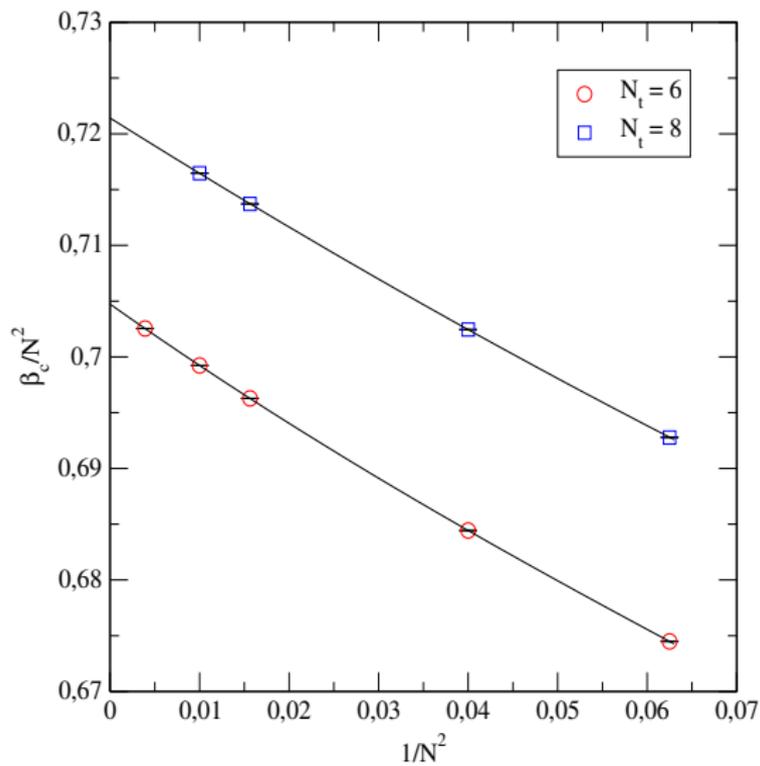
Results

► Critical couplings

N_c	N_t	β_c
4	6	10.7919
4	8	11.0844
5	6	17.1108
5	8	17.5612
8	6	44.5620
8	8	45.6778
10	6	69.9225
10	8	71.6475
16	6	179.8509

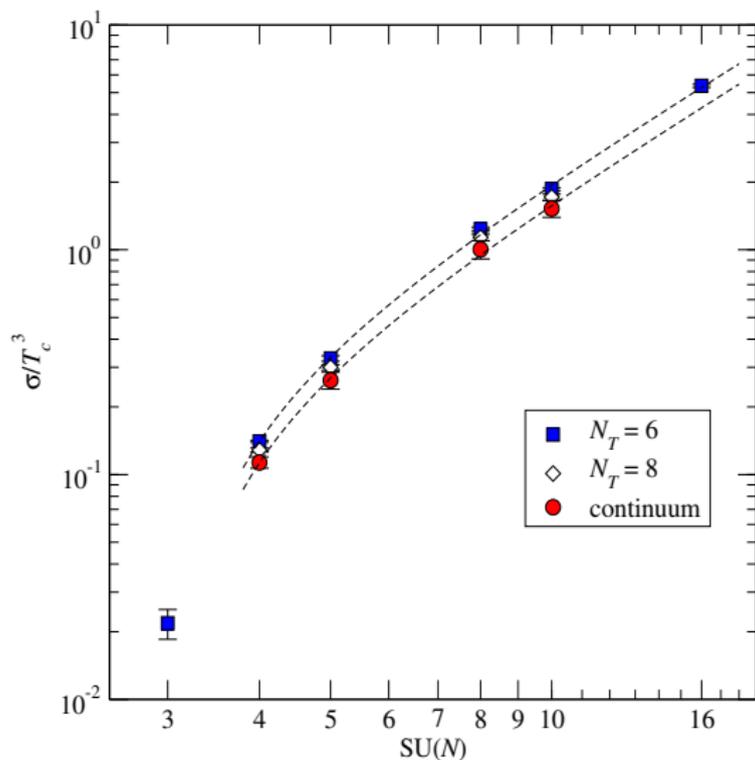
Results

► Critical couplings



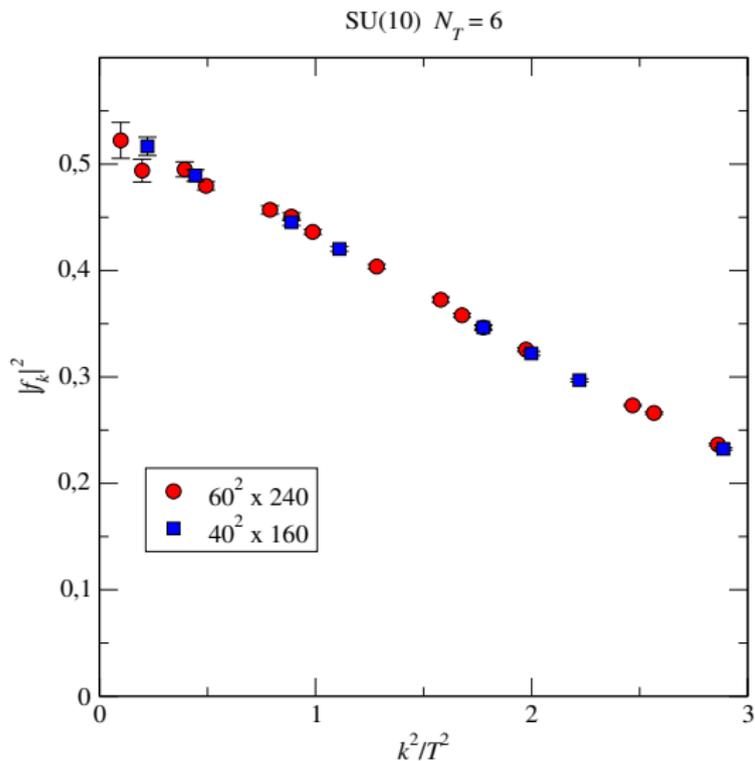
Surface Tension

- Quadratic fit: $c_0 + c_2 N^2$, $c_0 = -0.19(2)$, $c_2 = 0.0189(11)$



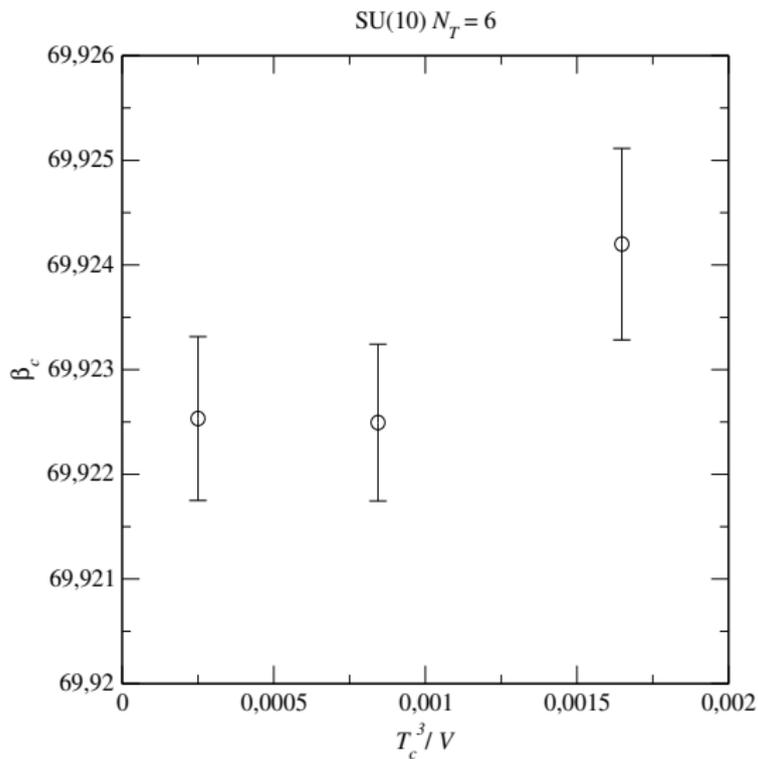
Volume effects

- ▶ Similar behaviour with different volumes



Volume effects

► Weak volume dependence



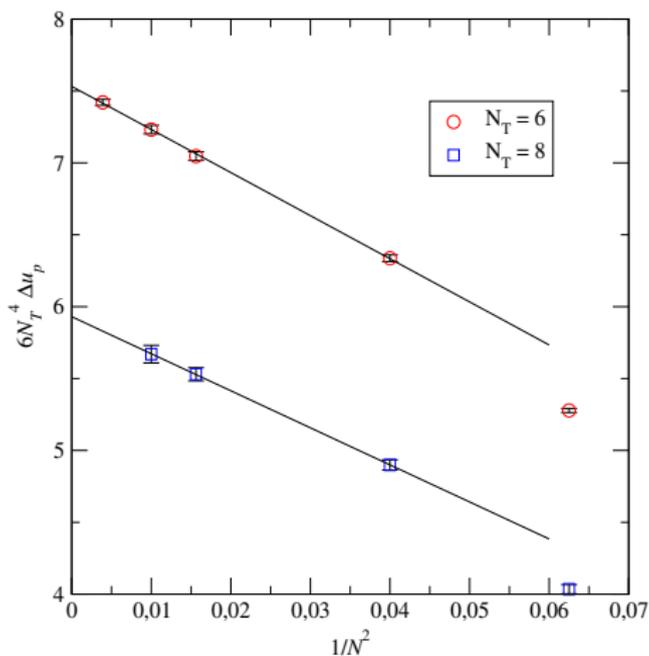
Plaquette Difference

N_c	N_t	β_c	$\langle u_c \rangle$	$\langle u_d \rangle$
4	6	10.7919	0.4380 (1.0303e-06)	0.4373 (1.5171e-06)
4	8	11.0844	0.4176 (6.6930e-07)	0.4175 (1.1014e-06)
5	6	17.1108	0.4469 (2.8967e-06)	0.4461 (8.1081e-07)
5	8	17.5612	0.4258 (6.6025e-07)	0.4256 (1.3036e-06)
8	6	44.5620	0.4564 (2.7630e-06)	0.4555 (2.7272e-06)
8	8	45.6778	0.4346 (1.3351e-06)	0.4344 (1.3022e-06)
10	6	69.9225	0.4586 (2.6406e-06)	0.4577 (2.6915e-06)
10	8	71.6475	0.4367 (1.7921e-06)	0.4364 (1.7316e-06)
16	6	179.8509	0.4609 (1.8135e-06)	0.4599 (1.9464e-06)

Plaquette Difference

$$\Delta u_{(N_T=6)} = 0.0009686(24) - 0.00385(11)/N^2$$

$$\Delta u_{(N_T=8)} = 0.0002413(24) - 0.00105(8)/N^2$$



Advantages

- ▶ Computation in polynomial time, enables huge volume simulations
- ▶ Surface is easily located, unaffected by probability density distribution
- ▶ More suitable for cases of stronger transition
- ▶ The use of mixed phase configurations result in absence of tunneling barriers that cause slowdown

Conclusion

- ▶ Traditional method works well with smaller systems and when the transition is not too strong
- ▶ Complementary method as larger systems are required to explore the continuum limit
- ▶ Lots of smearing required
- ▶ Presents as a consistent and reliable method for studying stronger transitions in general

References

- ▶ Classical Field Dynamics of the Electroweak Phase Transition - Moore and Turok (1996)
- ▶ Electroweak Bubble Nucleation, Nonperturbatively - Moore and Rummukainen (2000)
- ▶ Properties of the deconfining phase transition in $SU(N)$ gauge theories - Lucini, Teper, and Wenger (2005)
- ▶ $SU(N)$ gauge theories at deconfinement - Lucini, Rago, and Rinaldi (2012)

Outlook

- ▶ Prototype for a strongly coupled transition of strongly coupled physics
- ▶ Thin wall bubble nucleation rate computation
- ▶ Latent heat determination