# The Four Gluon Vertex from Lattice QCD

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# Motivation

QCD Dynamics at a fundamental level

importance of higher order corrections (two loop contributions to gluon DSE, etc.)

Test our understanding of QCD Green functions (ghost dominance at IR)

Effective charge $$\frac{g^2}{4\,\pi}\;\Gamma^{(4g)}(p^2)\;p^2 D(p^2)$$

Landau gauge

(pure gauge) Lattice point of view

# Lattice Approach

Importance Sampling **allows to access** the **gluon field** $A^a_\mu(x)$

$$\mathcal{G}^{(n)}(x_1, \ldots, x_n) = \langle 0 | \, T \, \left( A^{a_1}_{\mu_1}(x_1) \cdots A^{a_n}_{\mu_n}(x_n) \right) \, | 0 \rangle$$

Larger n: → Noisy — Disconnected parts

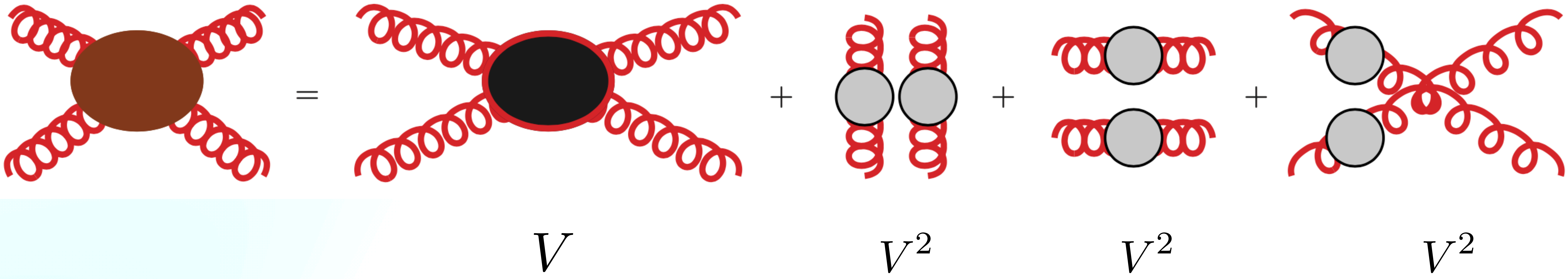
Noisy → Larger sets of configurations

Disconnected parts → kinematical configurations or Combinations of components

# 4-gluon correlation function

$$\langle\, A_{\mu_1}(p_1)\; A_{\mu_2}(p_2)\; A_{\mu_3}(p_3)\; A_{\mu_4}(p_4)\,\rangle =$$

$$= \int \mathcal{D}A\; A_{\mu_1}(p_1)\; A_{\mu_2}(p_2)\; A_{\mu_3}(p_3)\; A_{\mu_4}(p_4)\; e^{-S}$$



# 4-gluon correlation function

$\vec{p}_i \propto \vec{p}$

Single momentum scale

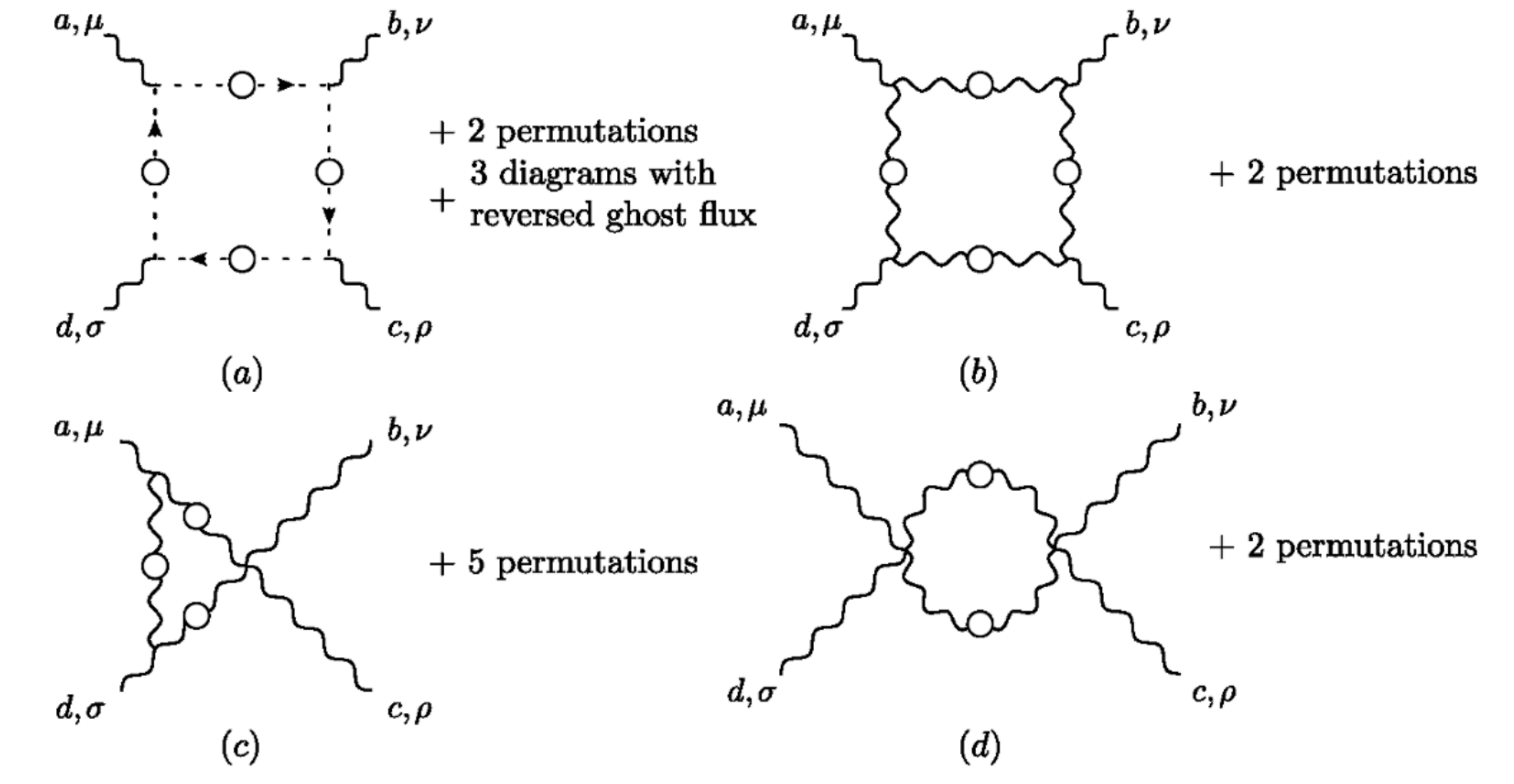
In Landau gauge simplifies the tensor analysis

Contributions only from the tensors proportional to $\delta_{\mu\nu}$

# Recent Continuum Calculations

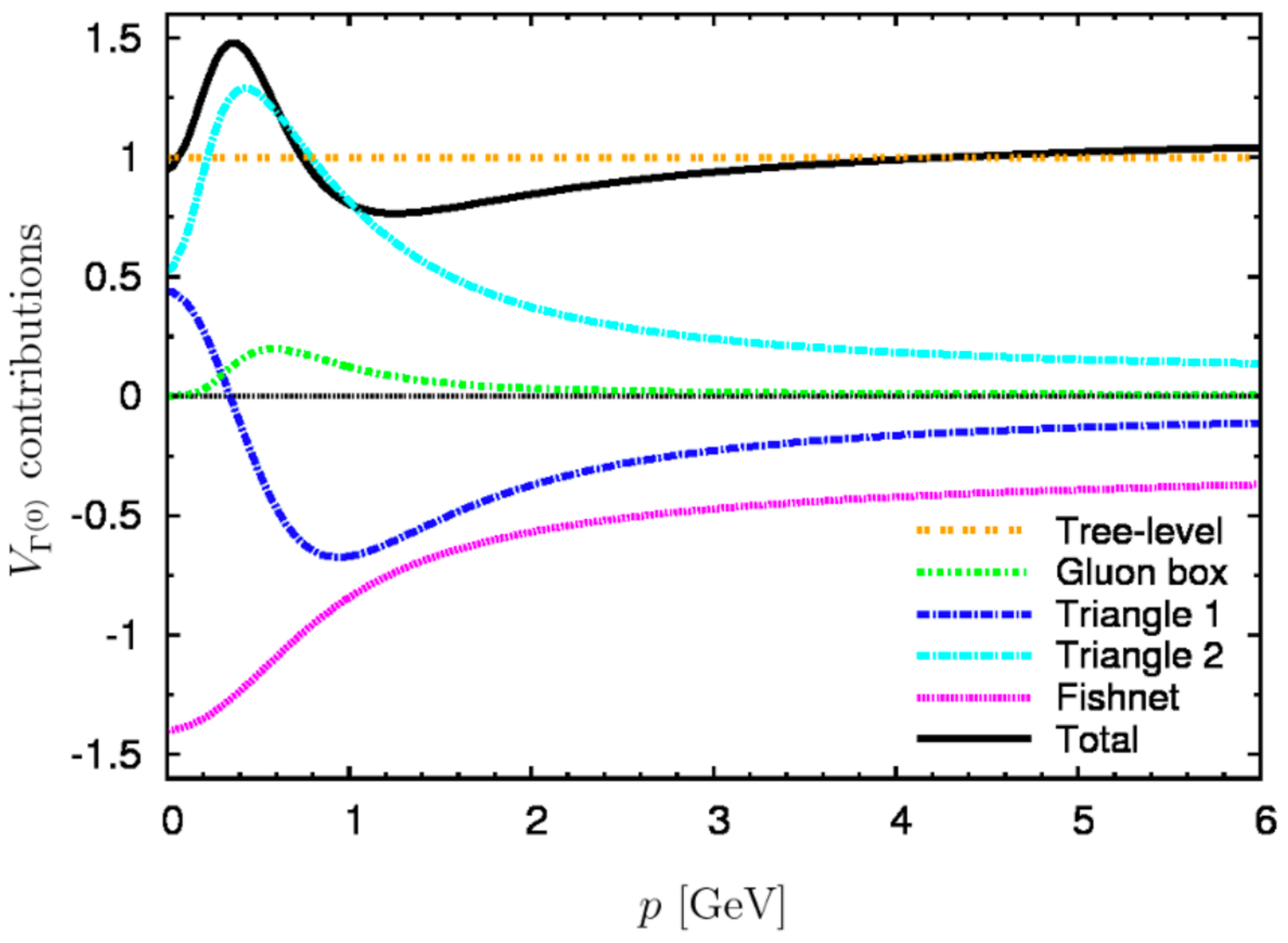
C Kellermann, C S Fischer, Phys. Rev. D **78** (2008) 025015
D Binosi, D Ibañez, J Papavassiliou, JHEP **1409** (2014) 059
A K Cyrol, M Q Huber, L von Smekal, Eur Phys J C **75** (2015) 102
A C Aguilar, M N Ferreira, J Papavassiliou, L R Santos, Eur Phys J C **84** (2024), 676
N Barrios, P De Fabritiis, M Peláez, Phys Rev **109** (2024) L091502

D Binosi, D Ibañez, J Papavassiliou, *JHEP* **1409** (2014) 059

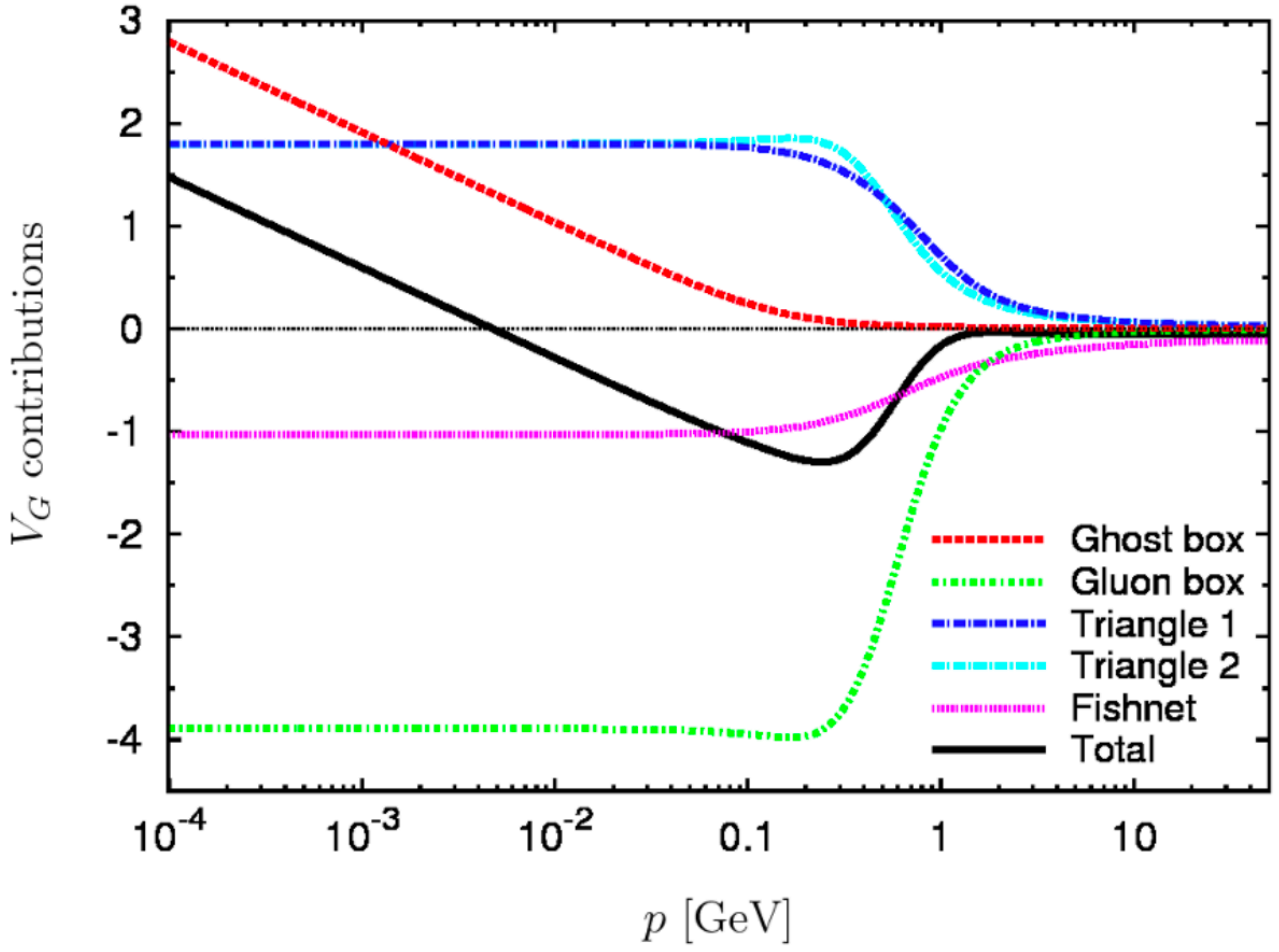



$$\Gamma^{abcd}_{\mu\nu\rho\sigma}(p,p,p,-3p)\Big|_{gg} = V_{\Gamma^{(0)}}(p^2)\Gamma^{abcd(0)}_{\mu\nu\rho\sigma} + V_G(p^2)G^{abcd}_{\mu\nu\rho\sigma},$$

$$G^{abcd}_{\mu\nu\rho\sigma} = (\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})\underbrace{(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho})}_{R_{\mu\nu\rho\sigma}}$$

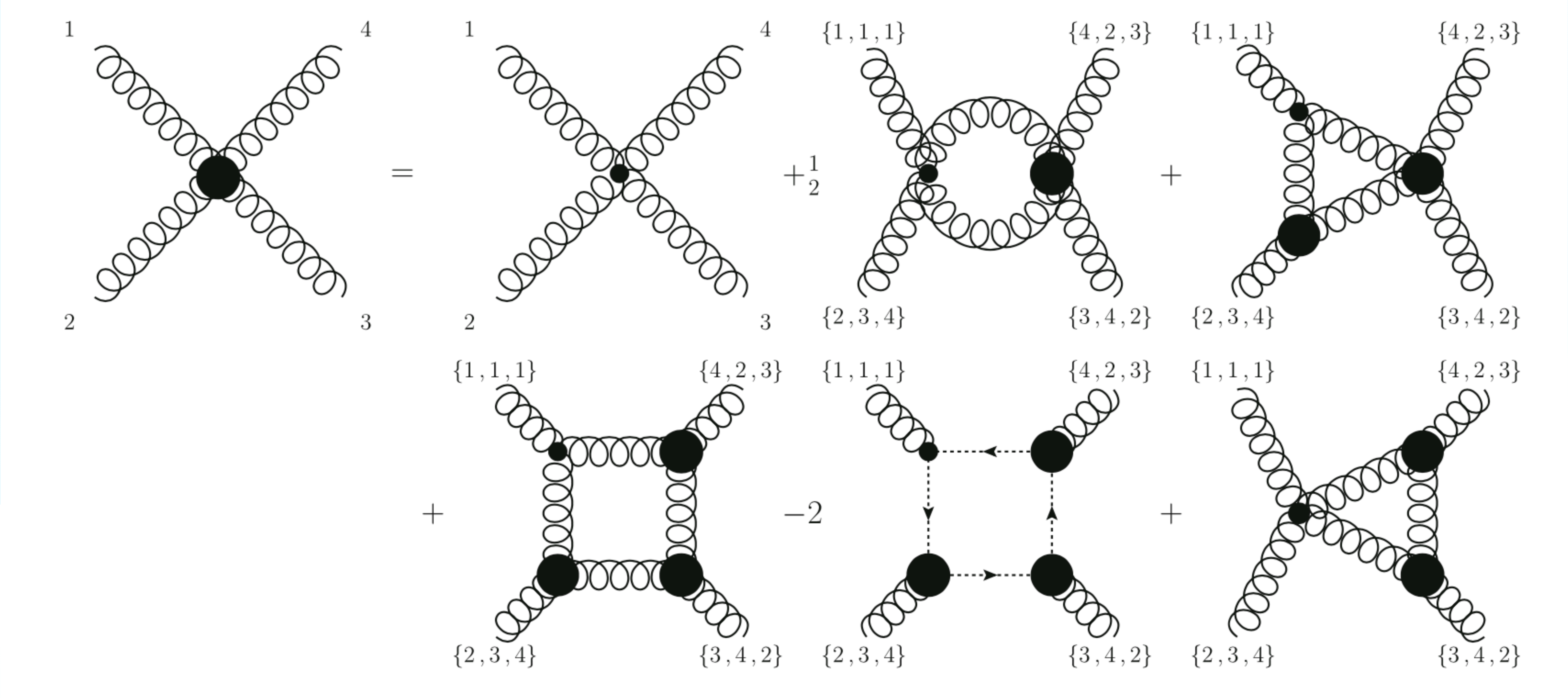


$V_{\Gamma^{(0)}}$ contributions
1.5
1
0.5
0
-0.5
-1
-1.5
0
1
2
3
4
5
6
$p$ [GeV]
Tree-level
Gluon box
Triangle 1
Triangle 2
Fishnet
Total

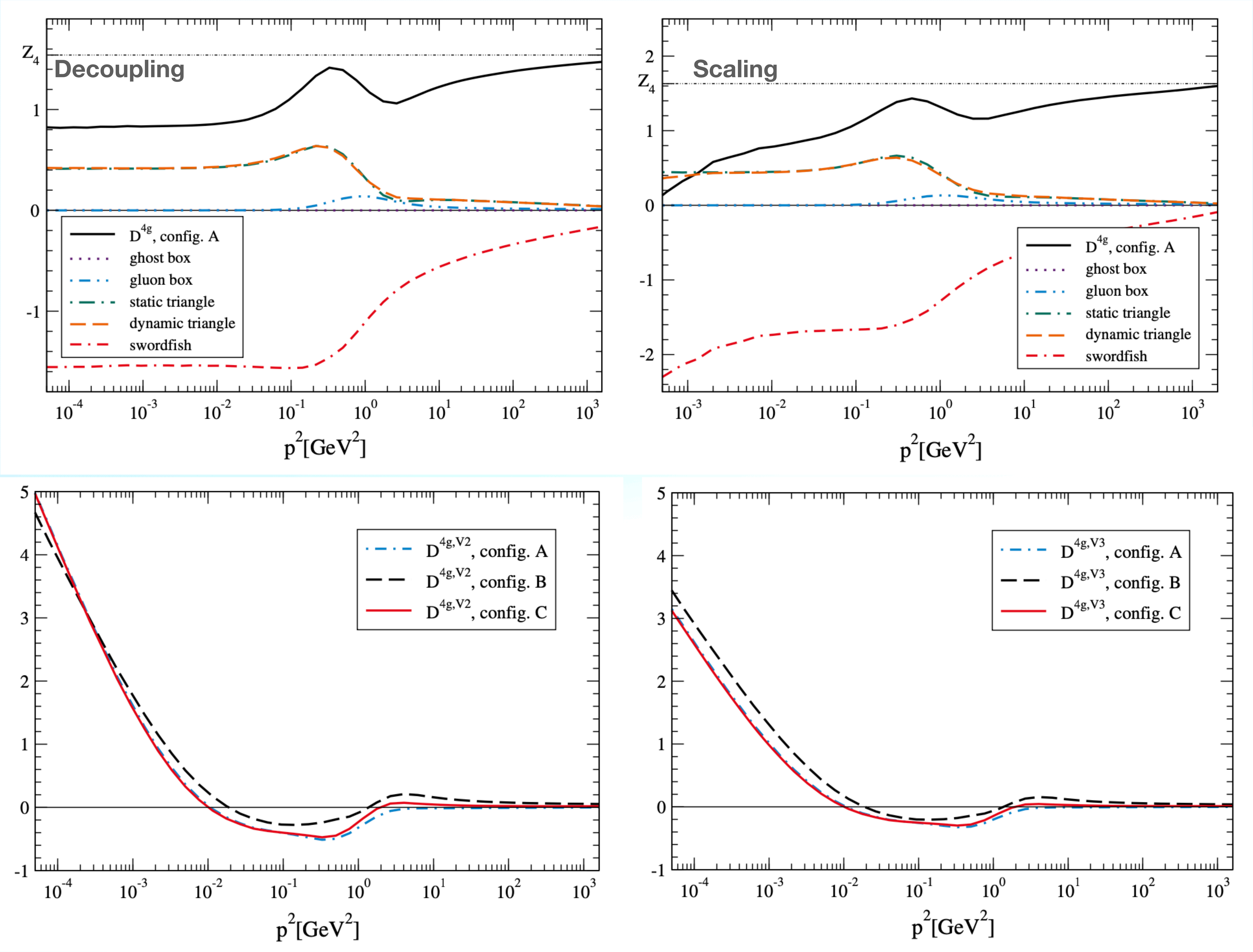



$V_G$ contributions
3
2
1
0
-1
-2
-3
-4
$10^{-4}$
$10^{-3}$
$10^{-2}$
0.1
1
10
$p$ [GeV]
Ghost box
Gluon box
Triangle 1
Triangle 2
Fishnet
Total

# A K Cyrol, M Q Huber, L von Smekal, *Eur Phys J C* **75** *(2015) 102*




**IR dominant (primitively divergent diagrams)**
**One-loop contributions (disregard contributions with five-point diagrams and ghost-gluon five-point functions)**
**Take tree level tensor structure for three-gluon and four-gluon diagrams**



Decoupling
Scaling
$Z_4$
$D^{4g}$, config. A
ghost box
gluon box
static triangle
dynamic triangle
swordfish
$D^{4g,V2}$, config. A
$D^{4g,V2}$, config. B
$D^{4g,V2}$, config. C
$D^{4g,V3}$, config. A
$D^{4g,V3}$, config. B
$D^{4g,V3}$, config. C
$p^2$[GeV$^2$]

## For **tensor analysis** see:

J A Gracey, Phys Rev D90, 025011 (2014) arXiv: 1406.1618
G Eichmann, C S Fischer, W Heupel, Phys Rev D92, 056006 (2015) arXiv: 1505.06336

$$\widetilde{\Gamma}^{(0)\,abcd}_{\mu\nu\eta\zeta} = f_{abr}f_{cdr}\left(\delta_{\mu\eta}\delta_{\nu\zeta} - \delta_{\mu\zeta}\delta_{\nu\eta}\right) + f_{acr}f_{bdr}\left(\delta_{\mu\nu}\delta_{\eta\zeta} - \delta_{\mu\zeta}\delta_{\nu\eta}\right) + f_{adr}f_{bcr}\left(\delta_{\mu\nu}\delta_{\eta\zeta} - \delta_{\mu\eta}\delta_{\nu\zeta}\right)$$

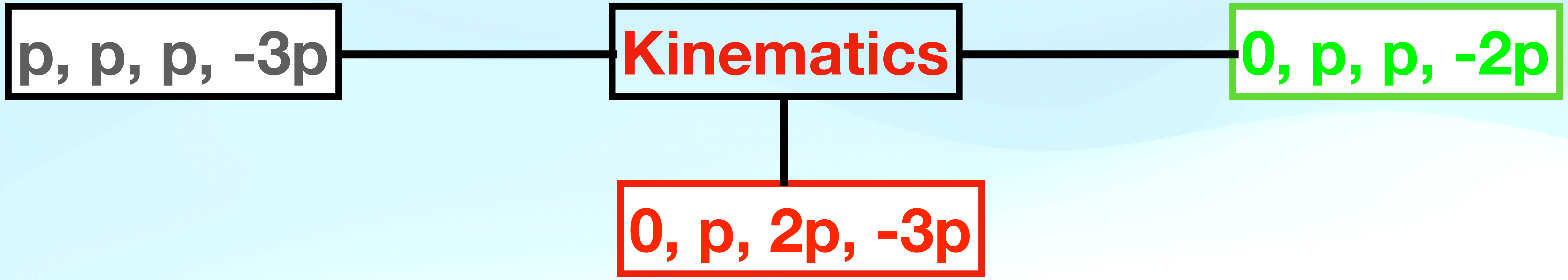
$$\widetilde{\Gamma}^{(1)\,abcd}_{\mu\nu\eta\zeta} = d_{abr}d_{cdr}\left(g_{\mu\eta}g_{\nu\zeta} + g_{\mu\zeta}g_{\nu\eta}\right) + d_{acr}d_{bdr}\left(g_{\mu\zeta}g_{\nu\eta} + g_{\mu\nu}g_{\eta\zeta}\right) + d_{adr}d_{bcr}\left(g_{\mu\nu}g_{\eta\zeta} + g_{\mu\eta}g_{\nu\zeta}\right)$$

$$\widetilde{\Gamma}^{(2)\,abcd}_{\mu\nu\eta\zeta} = \left(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right)\left(\delta_{\mu\nu}\delta_{\eta\zeta} + \delta_{\mu\eta}\delta_{\nu\zeta} + \delta_{\mu\zeta}\delta_{\nu\eta}\right)$$

$$\widetilde{\Gamma} = F(p^2)\,\widetilde{\Gamma}^{(0)} + G(p^2)\,\widetilde{\Gamma}^{(1)} + H(p^2)\,\widetilde{\Gamma}^{(2)}$$

**Not an orthogonal basis**

$$\mathcal{G}^{(4)} = \widetilde{\Gamma}\left(P^{\perp}(p)D(p^2)\right)^3\left(P^{\perp}(3p)D(9p^2)\right)$$




Measure the three form factors

$F^{(0)}(p^2)$ $F^{(1)}(p^2)$ $F^{(2)}(p^2)$

**Tree Level Tensor**

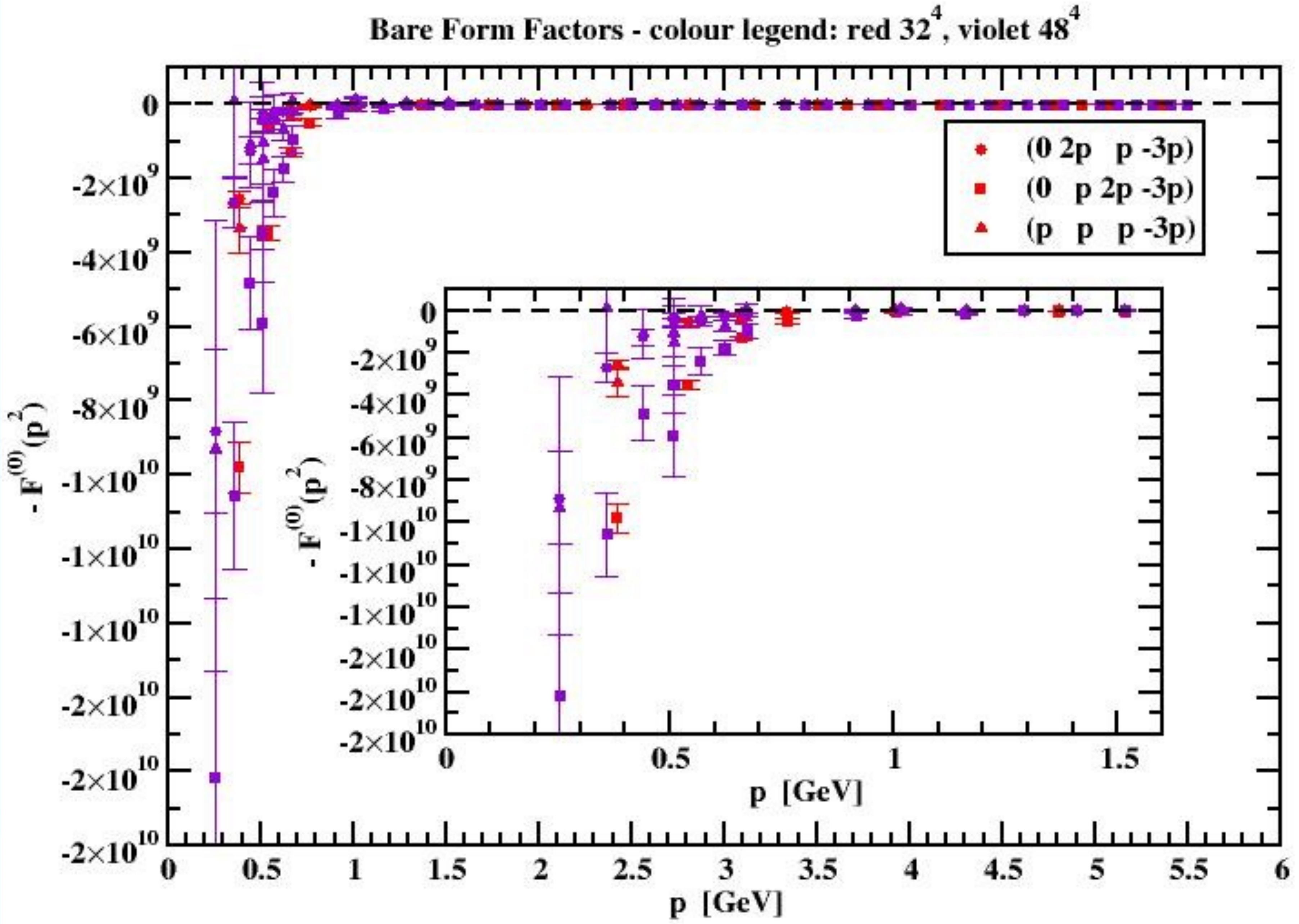
$\beta = 6.0 \qquad a = 0.102 \text{ fm} \qquad a^{-1} = 1.943 \text{ GeV}$

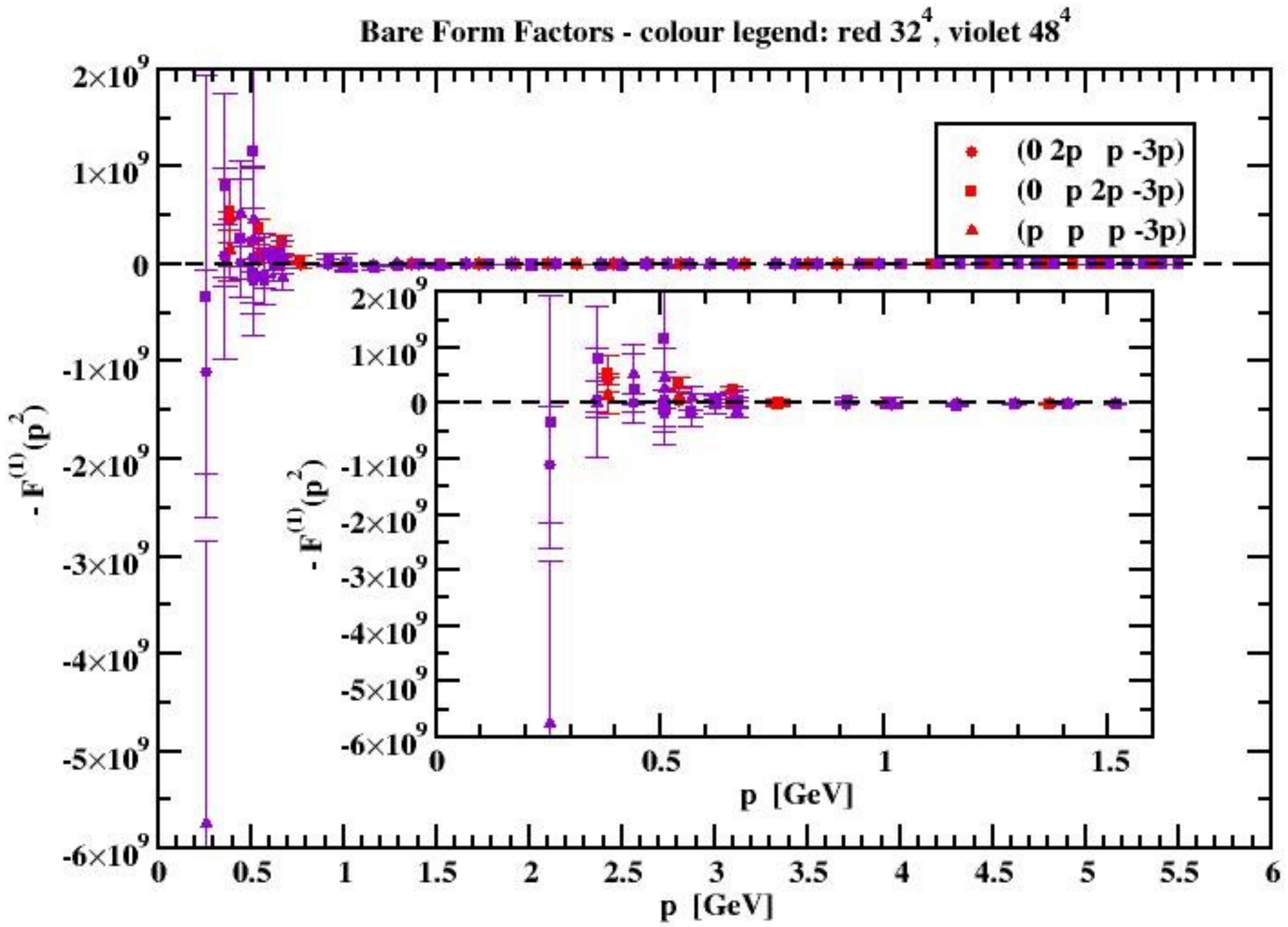
| Lattice | Configs | $p_{min}$ |
|---|---|---|
| $32^4$ | 9038 | 381 MeV |
| $48^4$ | 9035 | 254 MeV |

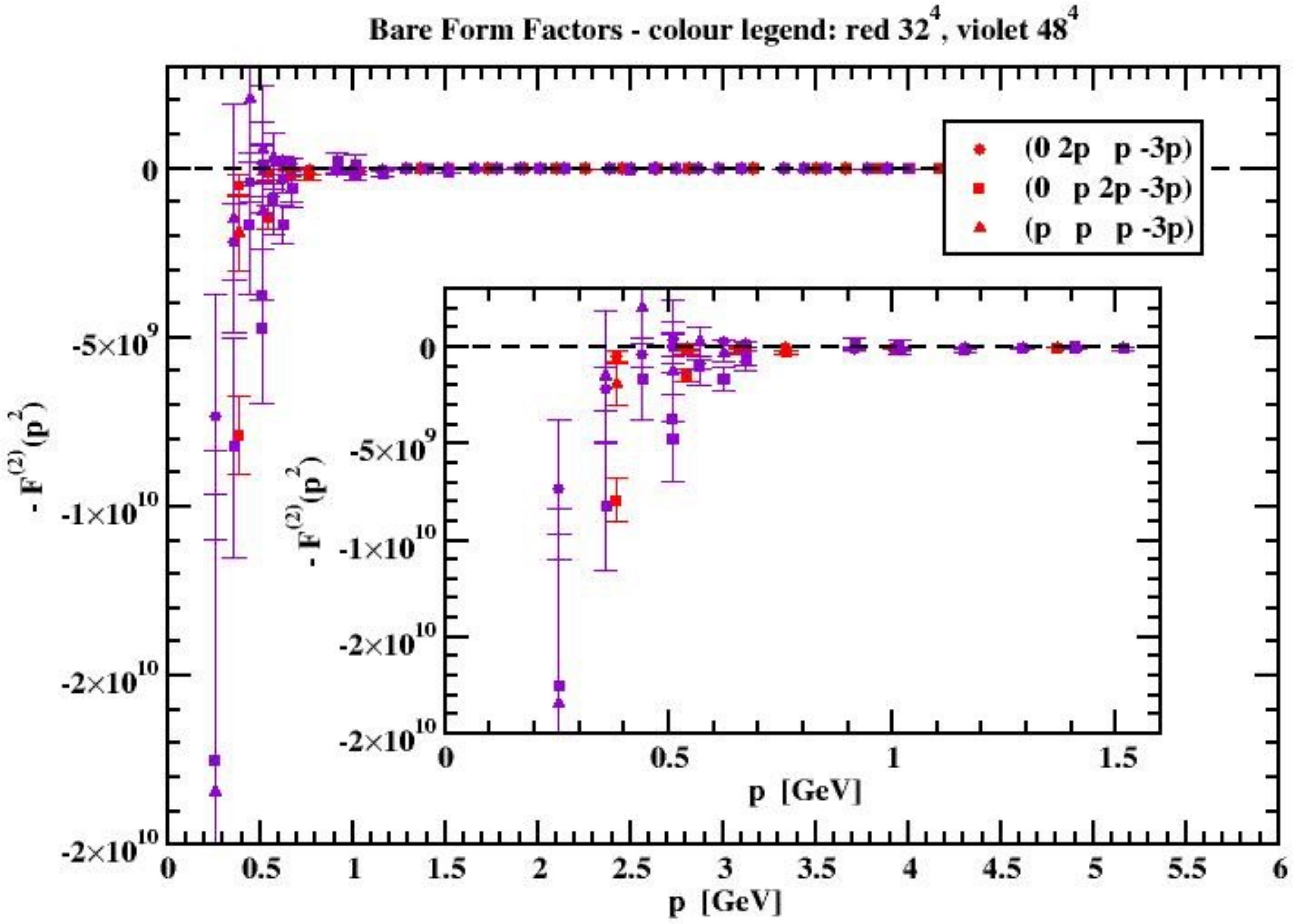
**Averaged over equivalent momenta, including the negative momenta !**

(1 1 1 1)

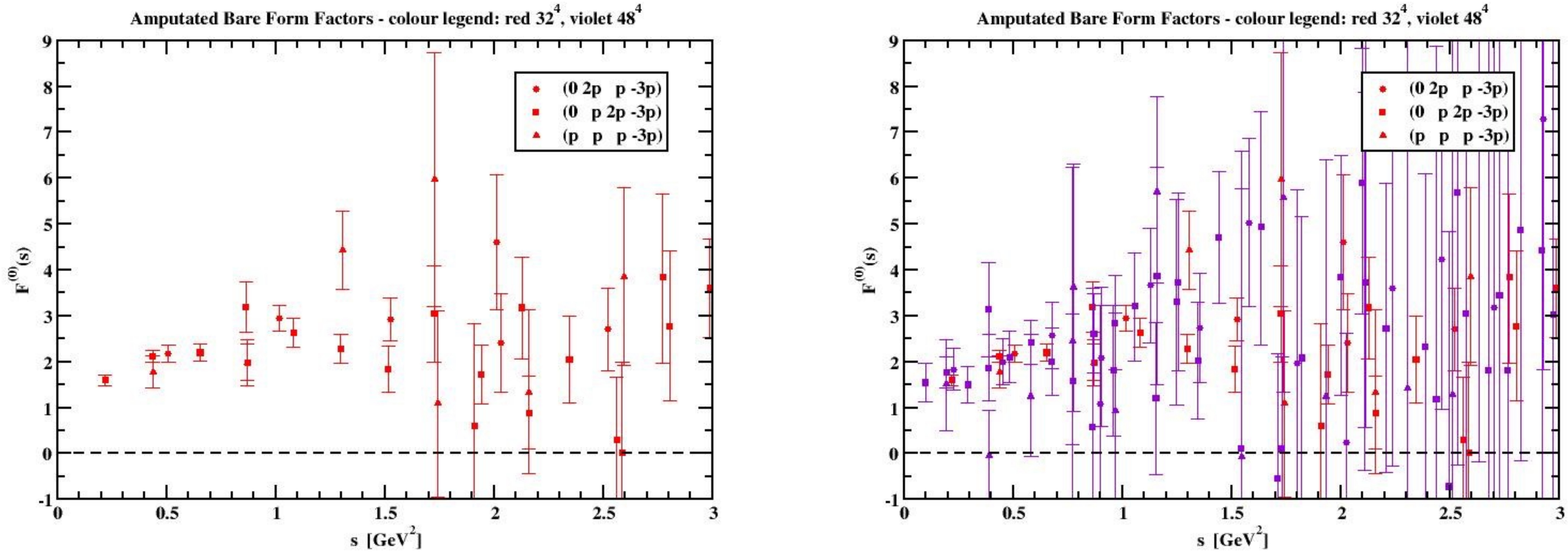
(-1 1 1 1) (1 -1 1 1) (1 1 -1 1) (1 1 1 -1)
(-1 -1 1 1) (-1 1 -1 1) (-1 1 1 -1) (1 -1 -1 1) (1 -1 1 -1) (1 1 -1 -1)
(1 -1 -1 -1) (-1 1 -1 -1) (-1 -1 1 -1) (-1 -1 -1 1)
(-1 -1 -1 -1)


Bare Form Factors - colour legend: red 32^4, violet 48^4
(0 2p p -3p)
(0 p 2p -3p)
(p p p -3p)
-F^(0)(p^2)
p [GeV]


Bare Form Factors - colour legend: red 32^4, violet 48^4
(0 2p p -3p)
(0 p 2p -3p)
(p p p -3p)
-F^(1)(p^2)
p [GeV]

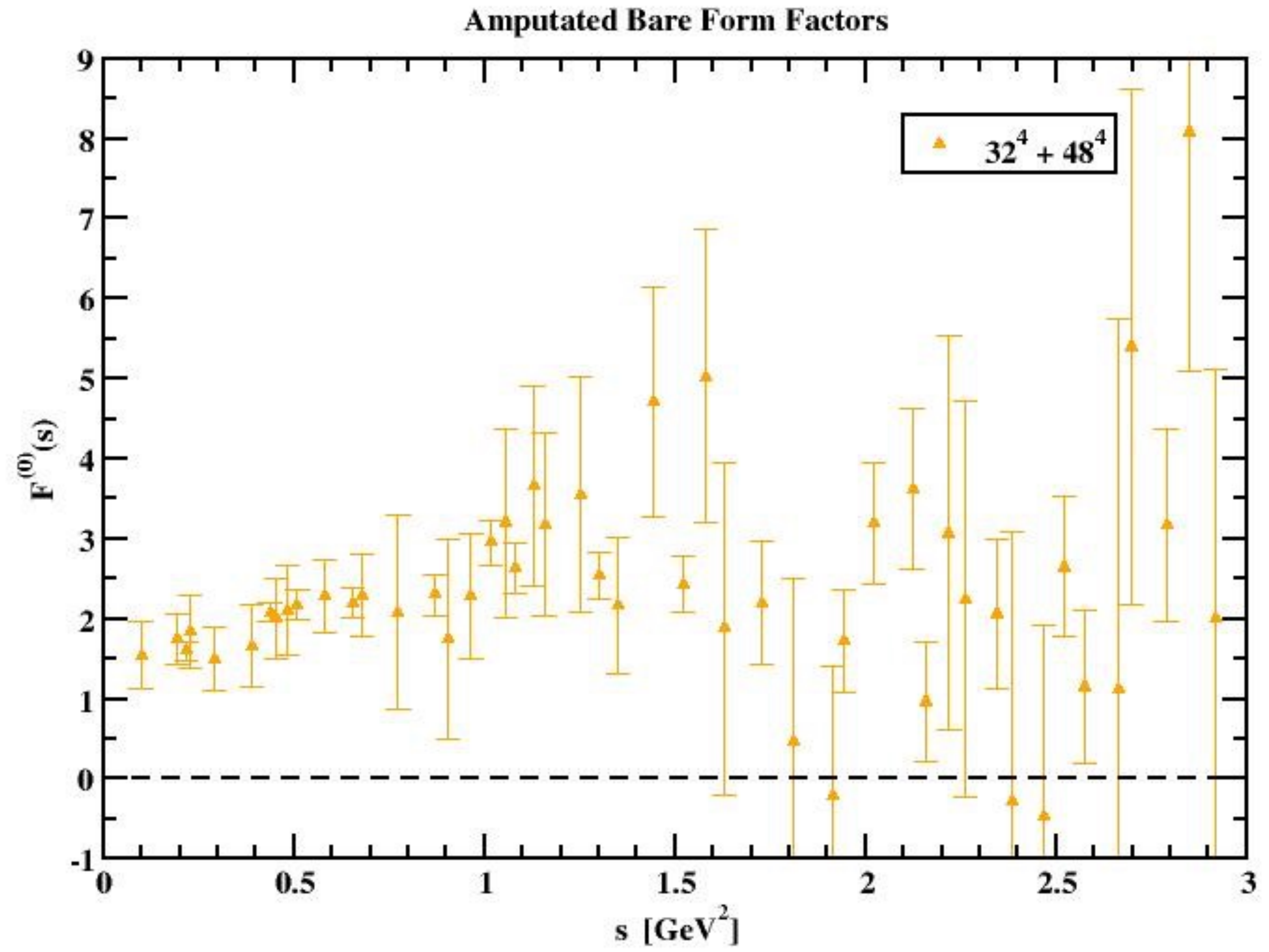

Bare Form Factors - colour legend: red 32^4, violet 48^4
(0 2p p -3p)
(0 p 2p -3p)
(p p p -3p)
-F^(2)(p^2)
p [GeV]

# Bare Amputated Green Function

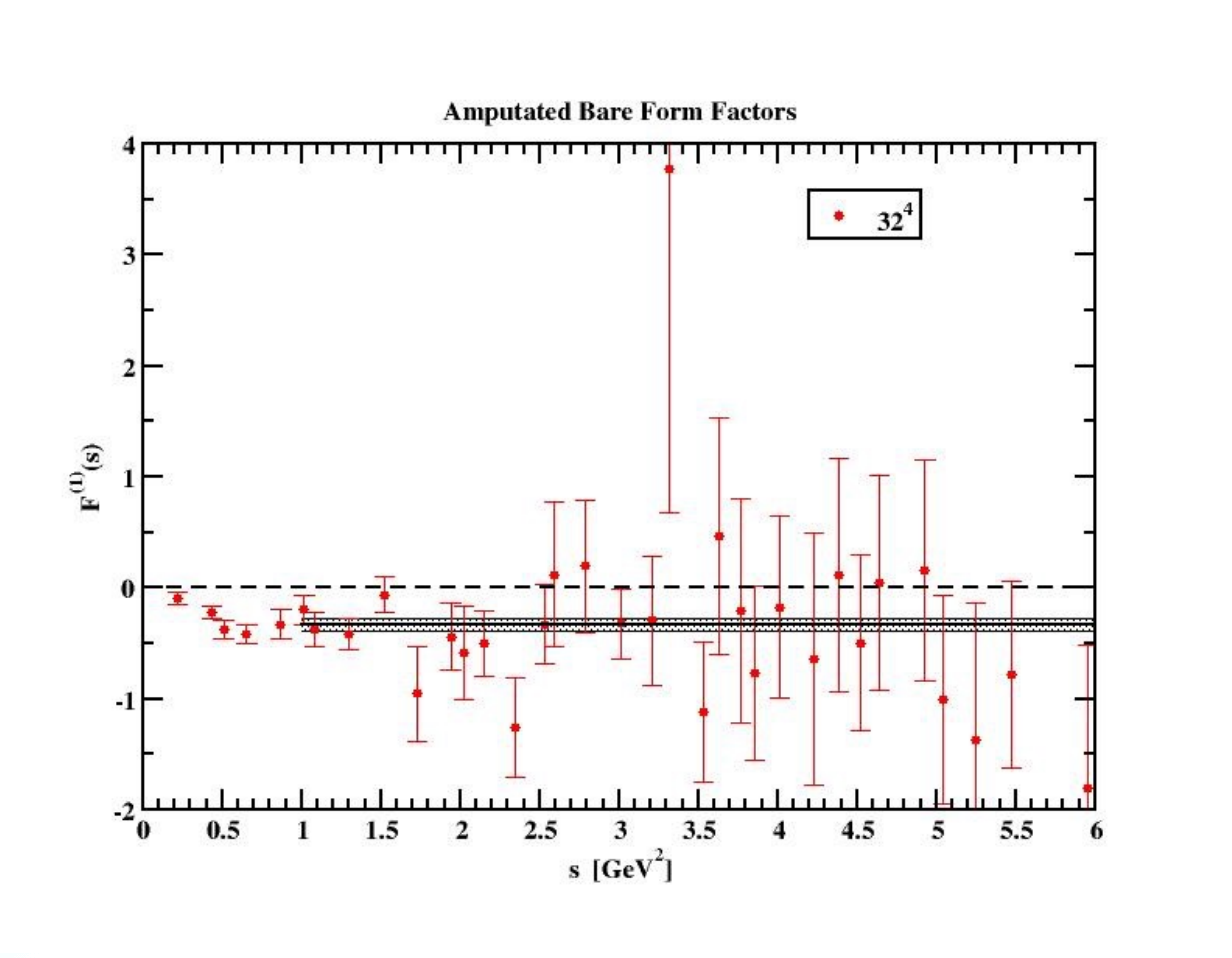



$$s = \frac{1}{4}\sum_i p_i^2$$

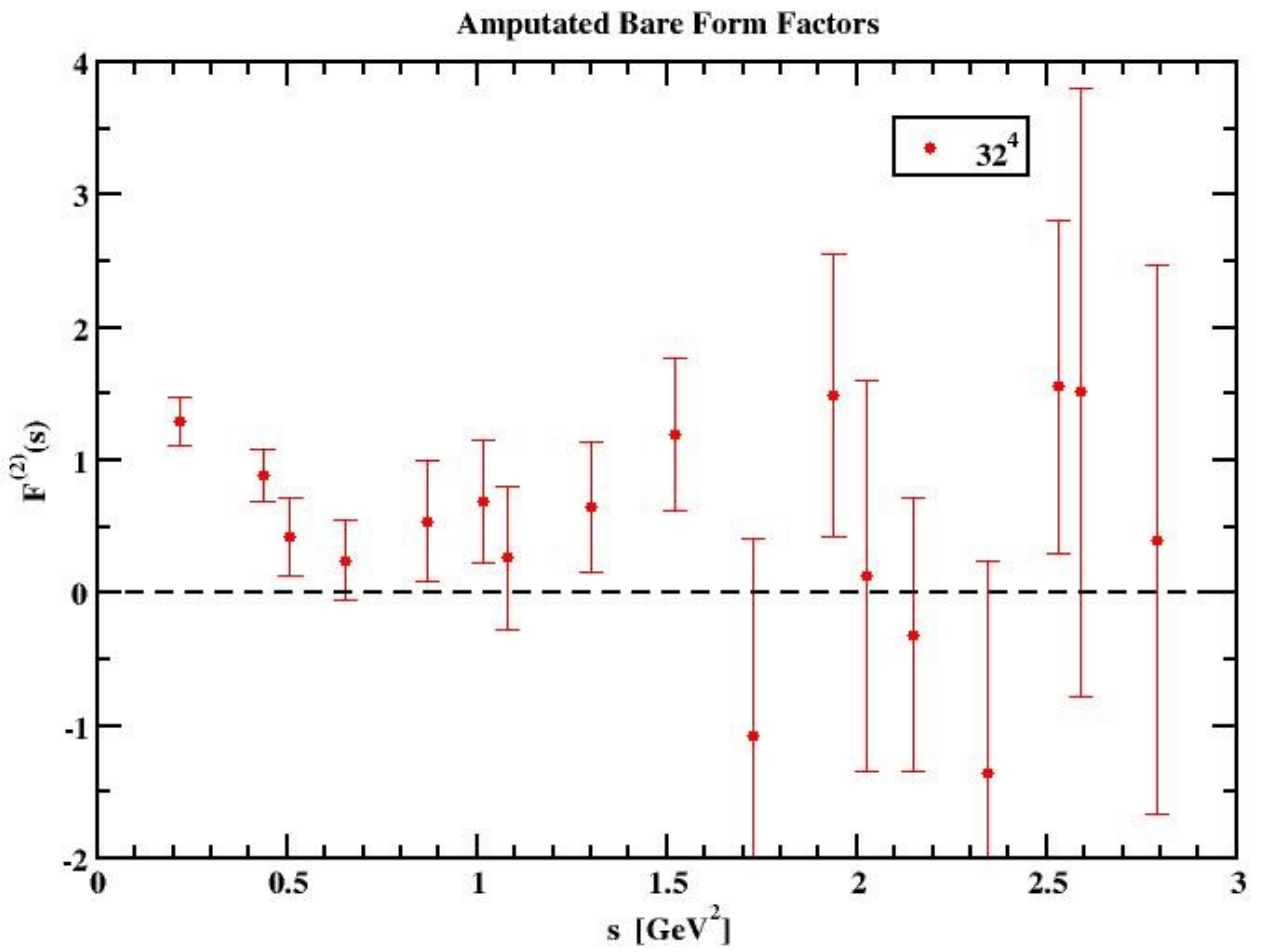
# Bare Amputated Green Function



# Bare Amputated Green Function



# Bare Amputated Green Function



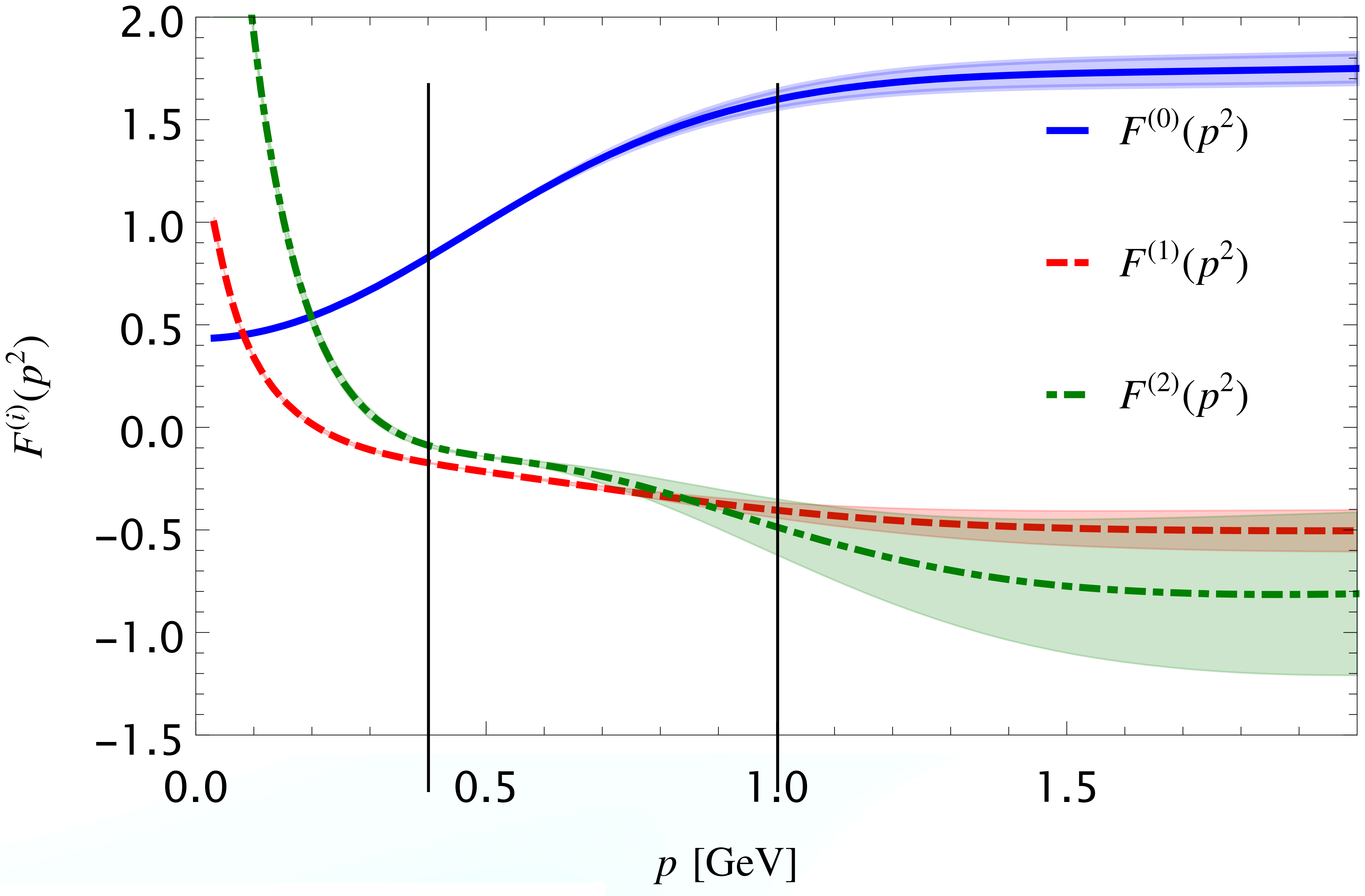
# Continuum Calculations

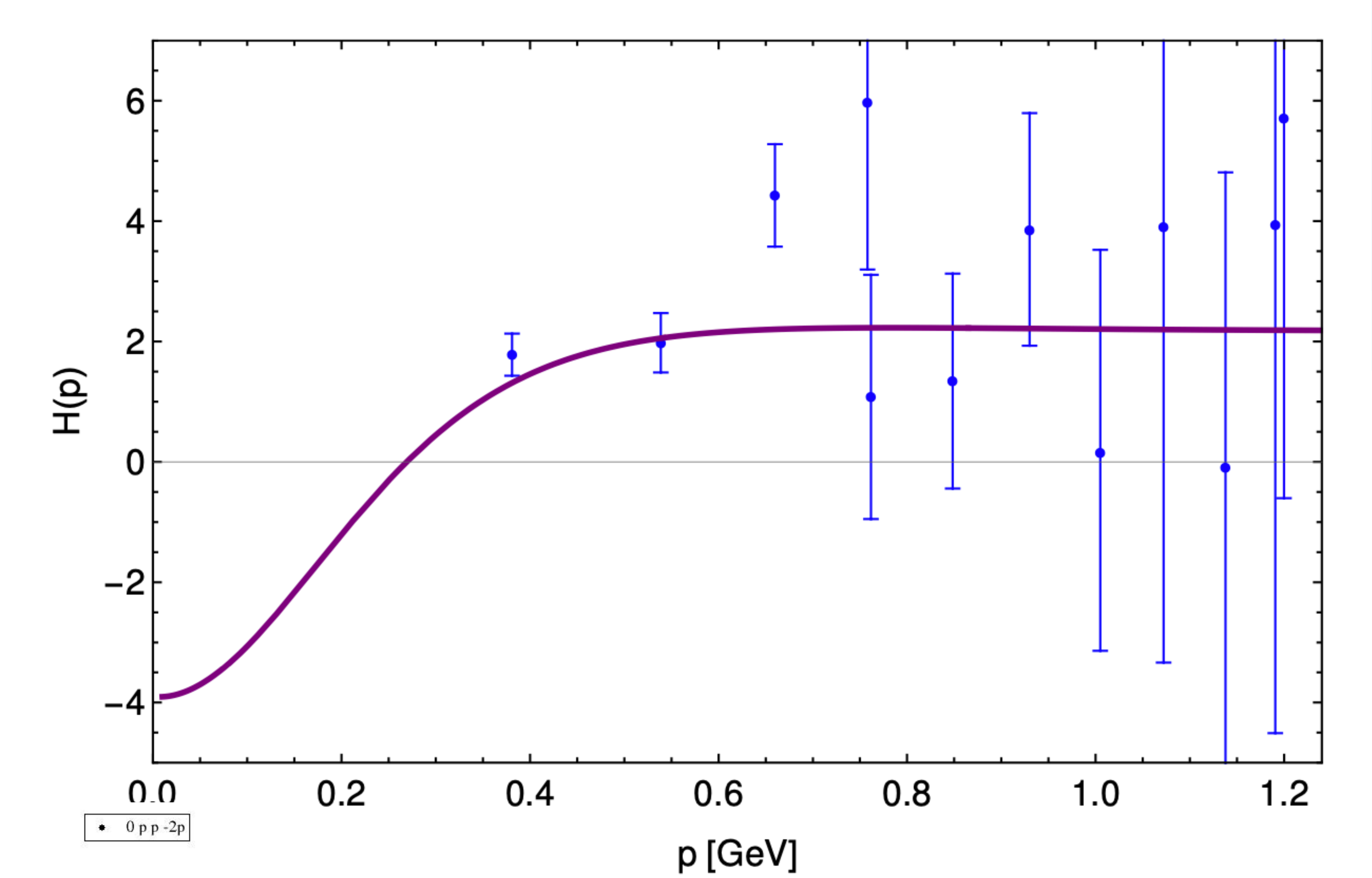
**One loop truncated Dyson-Schwinger equation by**

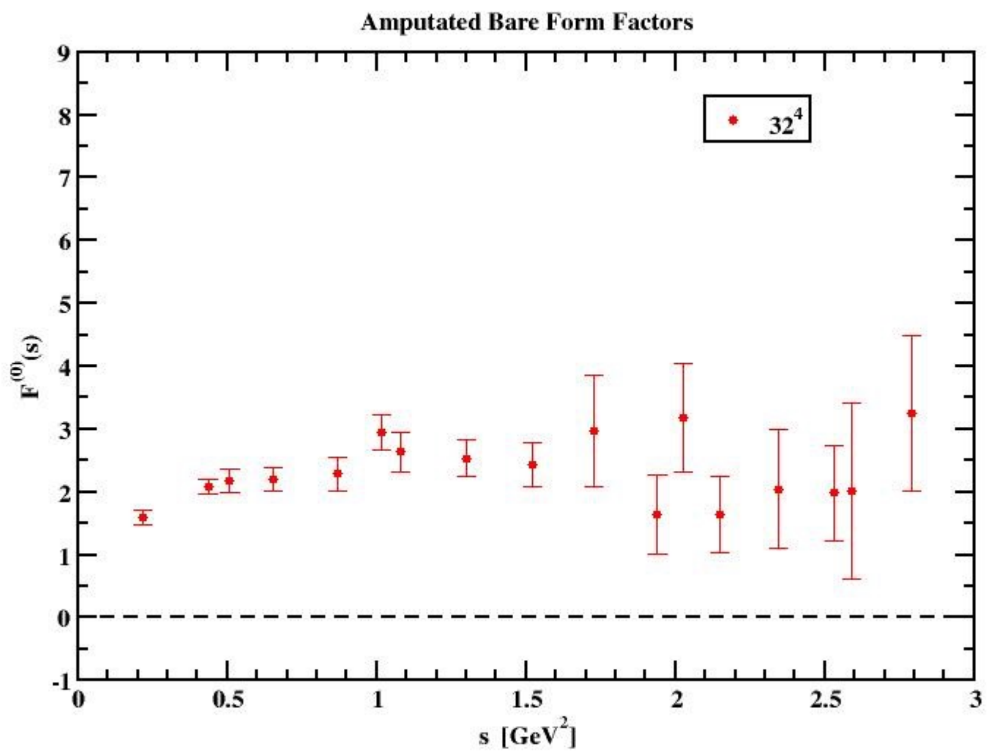
**A C Aguilar, M N Ferreira, J Papavassiliou, L R Santos, Eur Phys J C 84 (2024) 676**

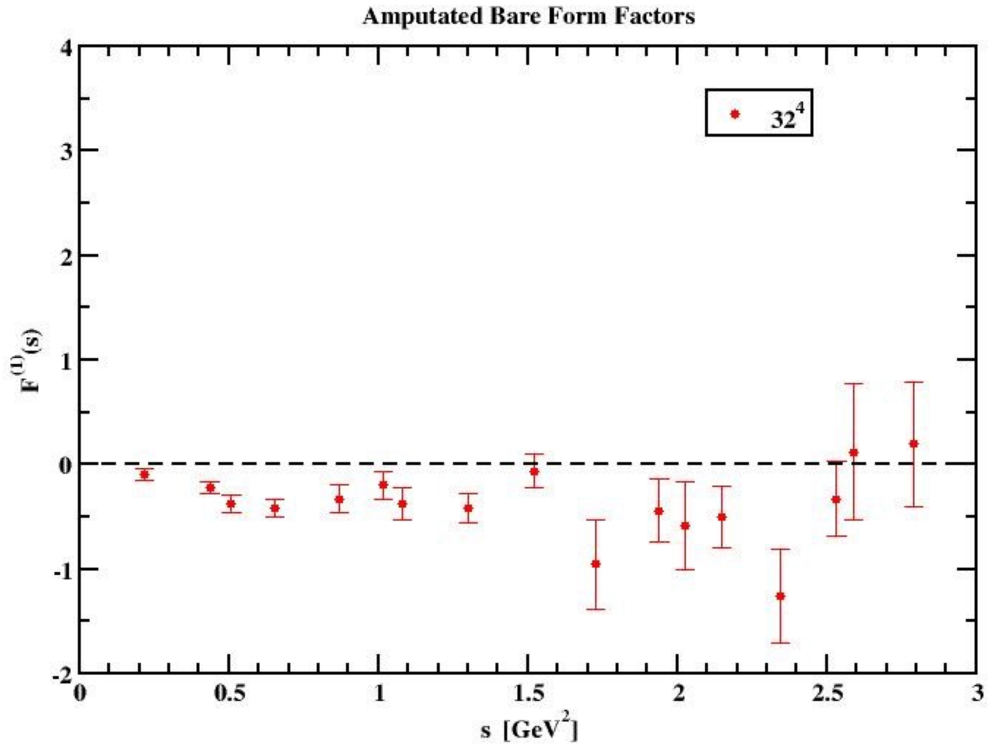
**Curci-Ferrari model (massive QCD)**

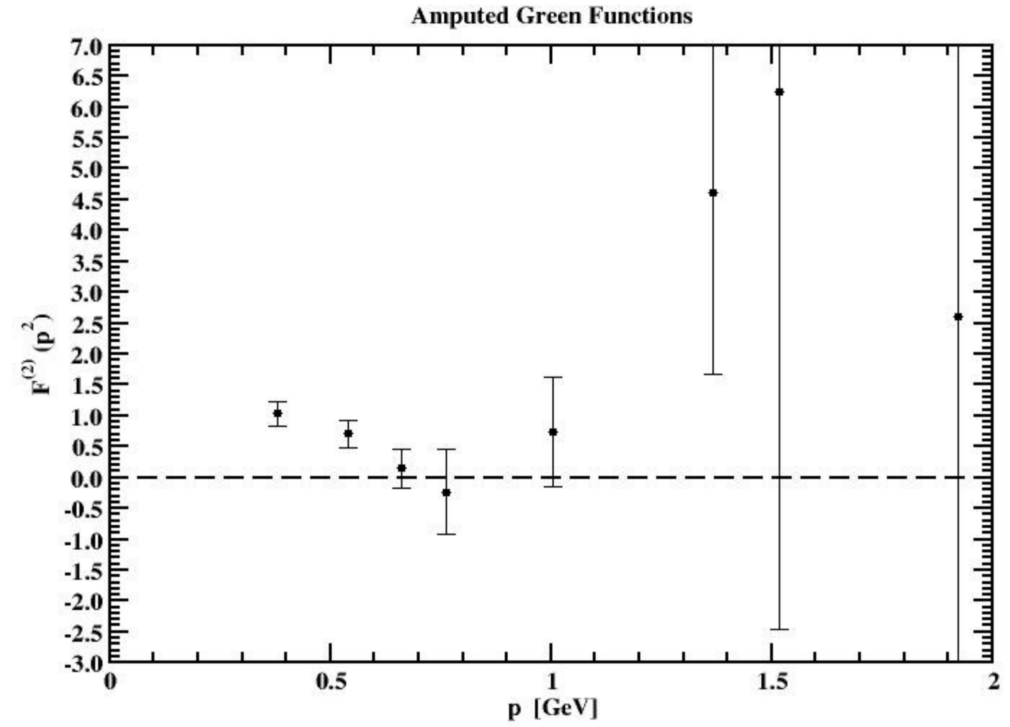
**N Barrios, P De Fabritiis, M Peláez, Phys Rev D 109 (2024) L091502**



# Summary and Conclusions

- gluon correlation functions up to 4-external legs are possible with standard lattice methods (call for large statistical ensembles)
- Dominated by tree level tensor structure but not only
- Good (at least) qualitative agreement between Lattice and continuum approaches
- Look at other kinematics and extend collaboration to get a better picture
- Need large statistics or methods to reduce variance

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