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Lattice 2024

Topological Data Analysis of Monopole Currents in U(1) Lattice Gauge Theory

Biagio Lucini
Swansea University

Jeff Giansiracusa
Durham University

Next talk Fri 12:35

Biagio Lucini
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Previous talk Fri 11:55

Jeff Giansiracusa
Durham University

Topological Data Analysis

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Topological Data Analysis

Extract topological information from data

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Topological Data Analysis

Extract topological information from data

By computing the **homology** of a
differential complex
associated to the data

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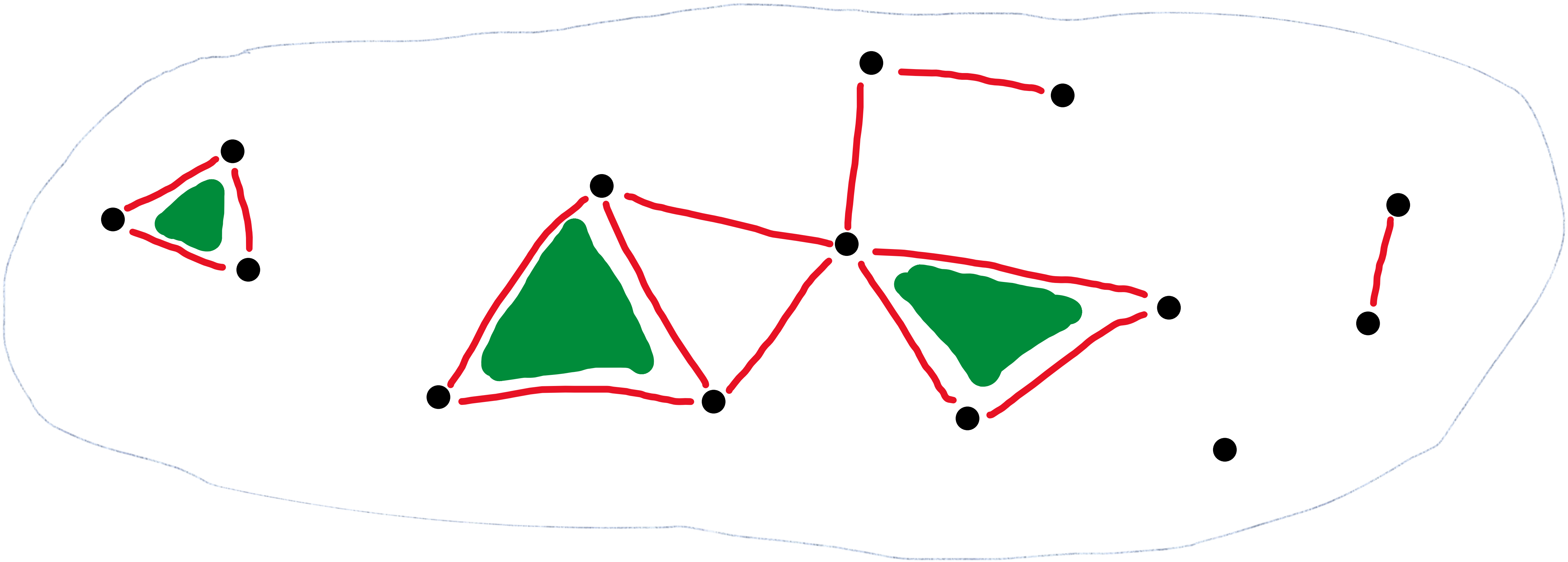
By computing the **homology** of a *differential complex* associated to the data

e.g. a simplicial complex



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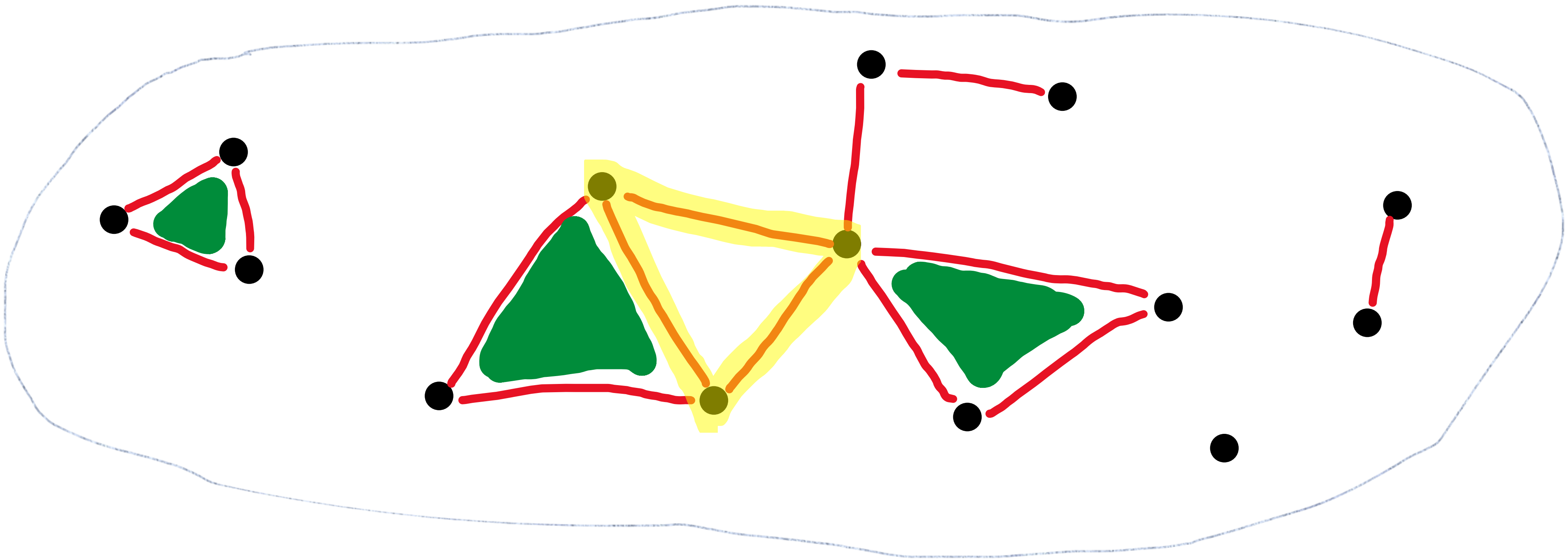

Cycle which is **not** a boundary

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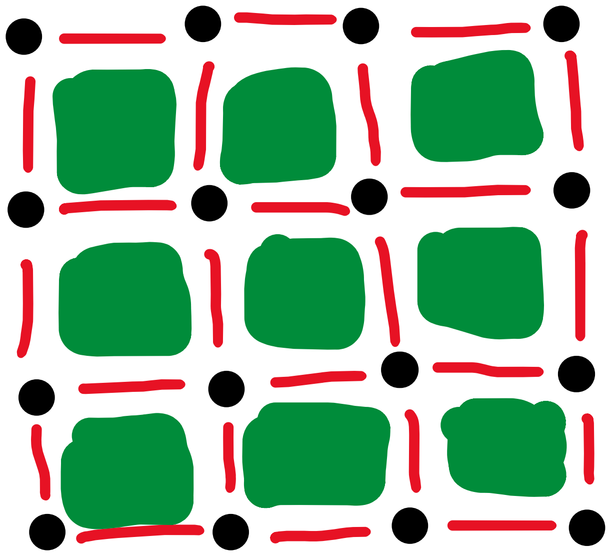
A 4-dimensional lattice has a natural **cubical complex**



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Motivation

Want to use TDA to study confinement mechanisms in LGTs

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Motivation

Want to use TDA to study
confinement mechanisms in LGTs

1. Can we

use TDA to construct robust observables

that quantitatively characterise the deconfinement phase transition?

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Motivation

1. Can we

use TDA to construct robust observables

that quantitatively characterise the deconfinement phase transition?

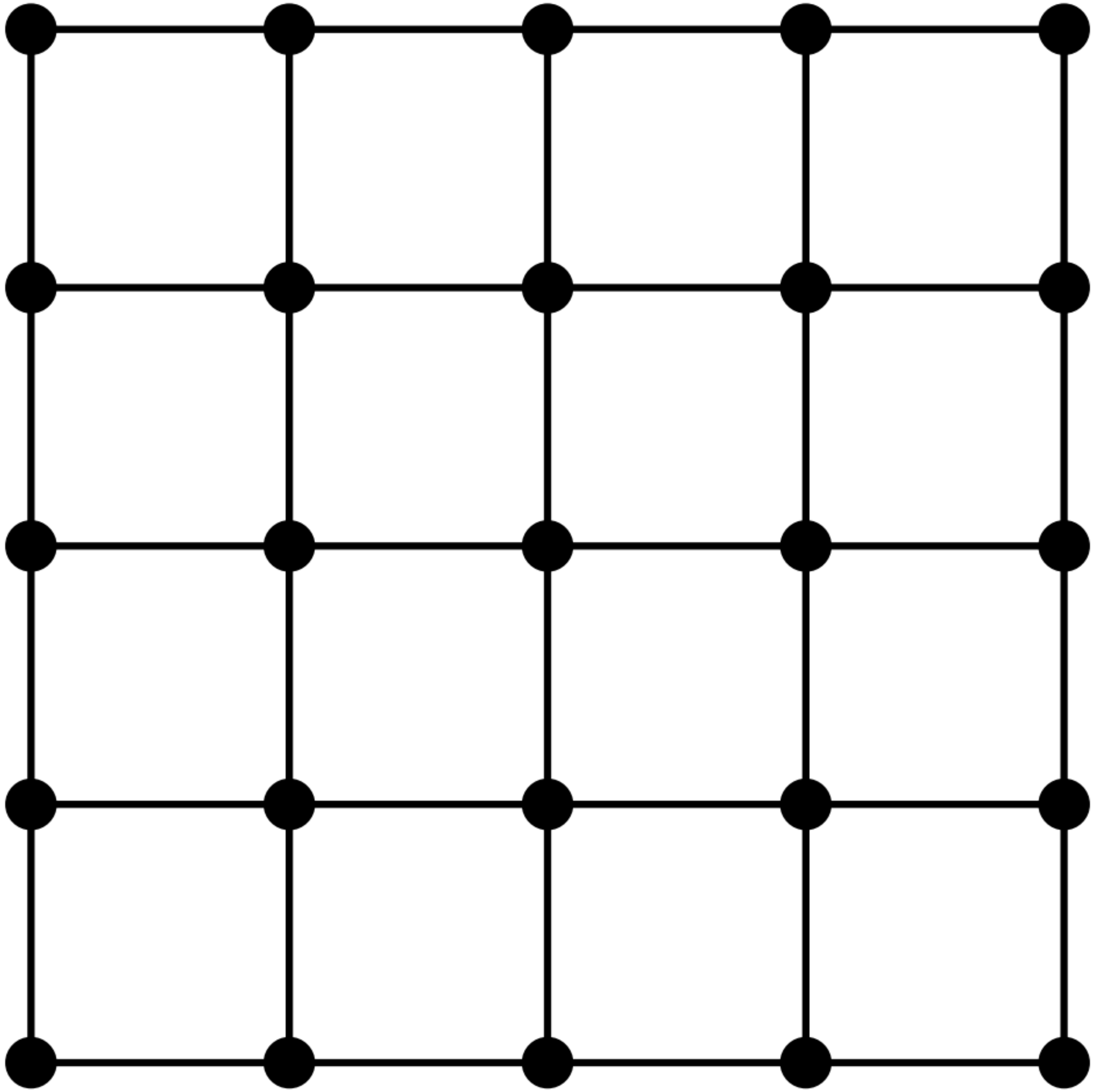
2. Can we

use TDA to elucidate the structures formed by topological defects

in a configuration?

Nice Toy Model:

U(1) Lattice Gauge Theory



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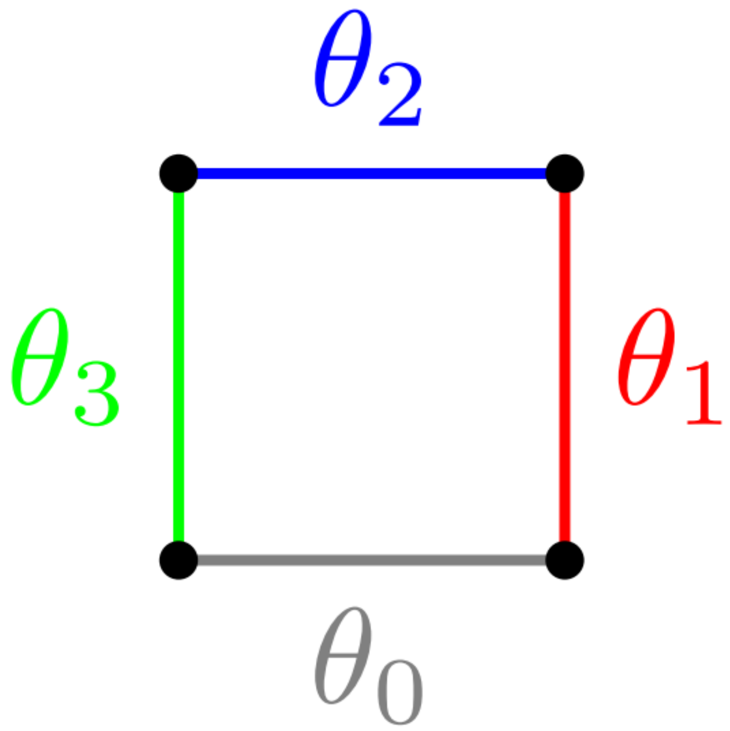
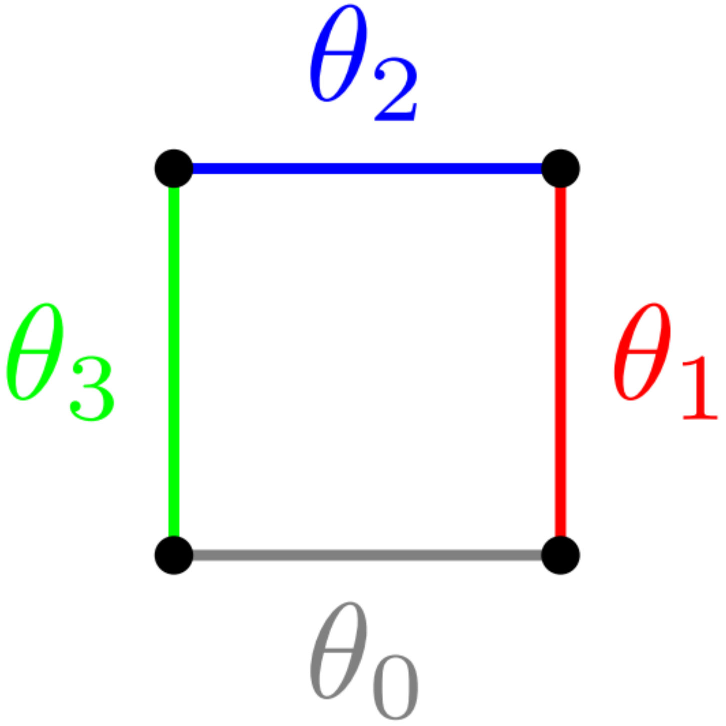


$$\theta_i \in (-\pi, \pi]$$



$$\theta_i \in (-\pi, \pi]$$

$$\theta_p \equiv \theta_0 + \theta_1 - \theta_2 - \theta_3$$

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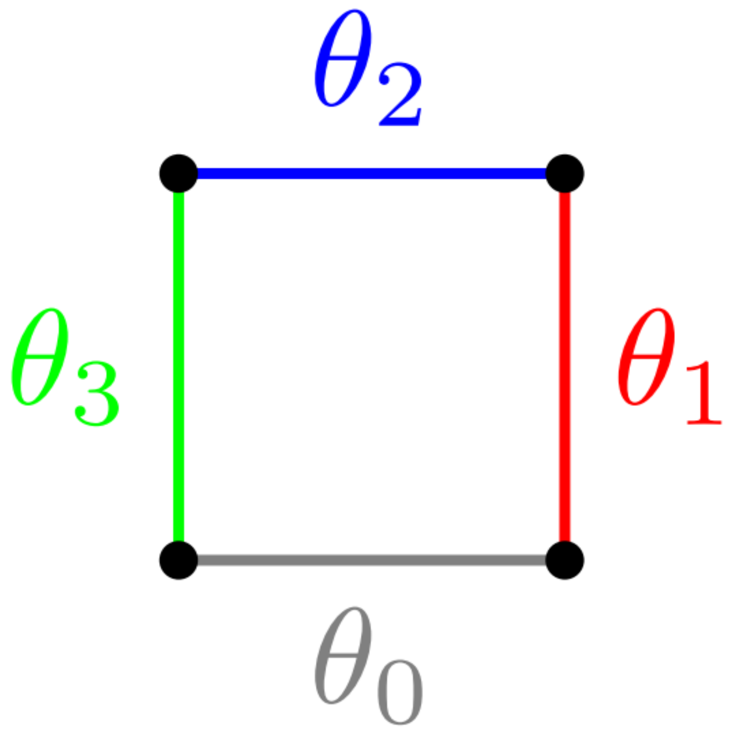

$$\theta_i \in (-\pi, \pi]$$

$$\theta_p \equiv \theta_0 + \theta_1 - \theta_2 - \theta_3$$
$$\in (-4\pi, 4\pi]$$

Corresponds with **magnetic flux** through a plaquette

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Given **Wilson action**

$$S = \beta \sum_p (1 - \cos\theta_p)$$

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S has minima periodic in 2π

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Given **Wilson action**

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$$\theta_p \in (-4\pi, 4\pi]$$

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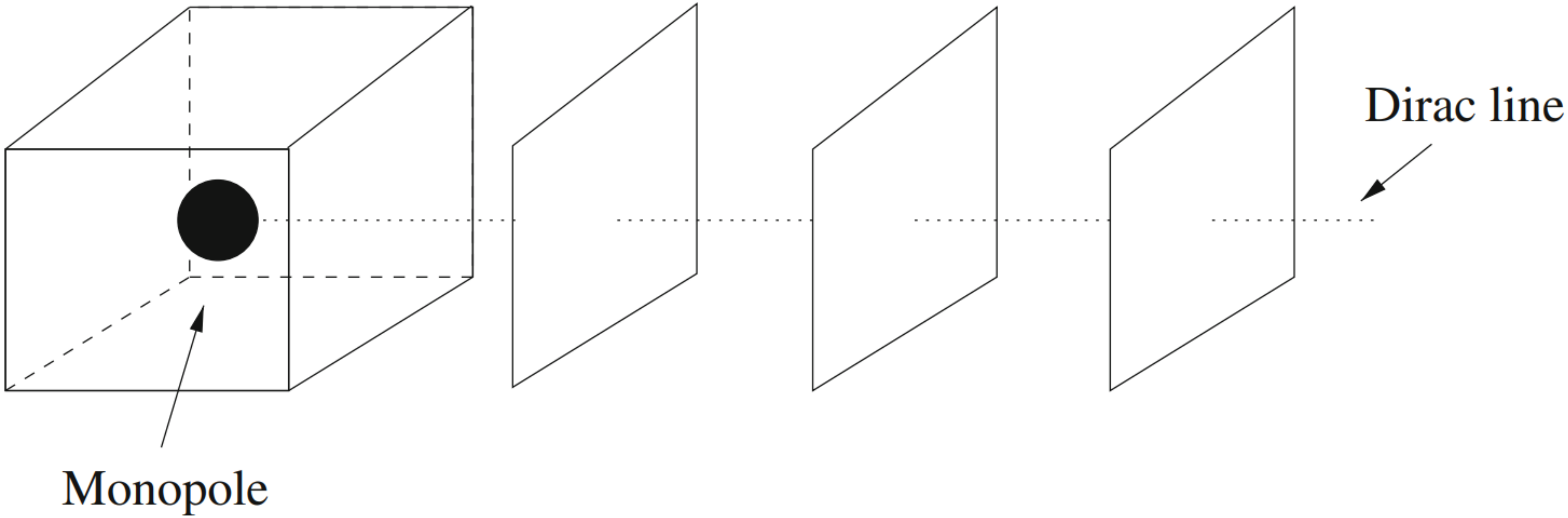
Defines stable objects of 2π flux called **“Dirac strings”**



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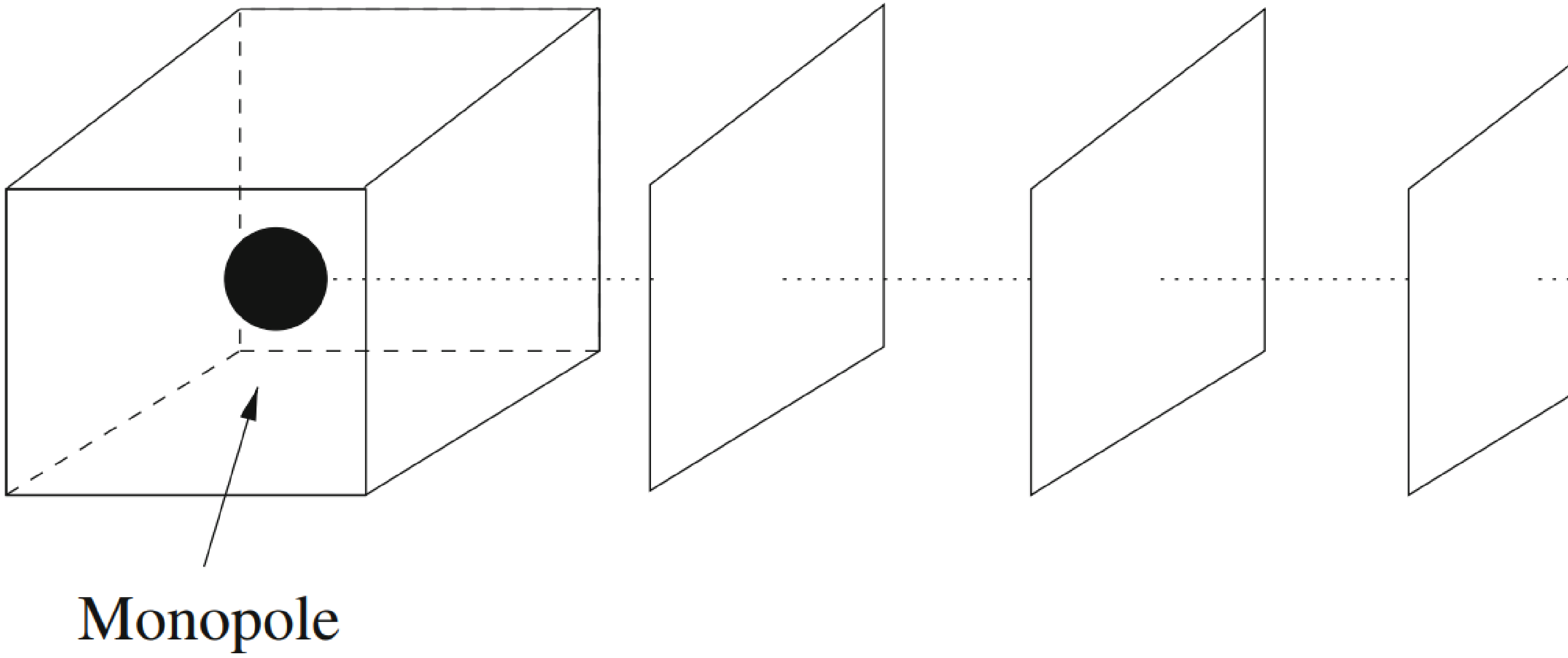
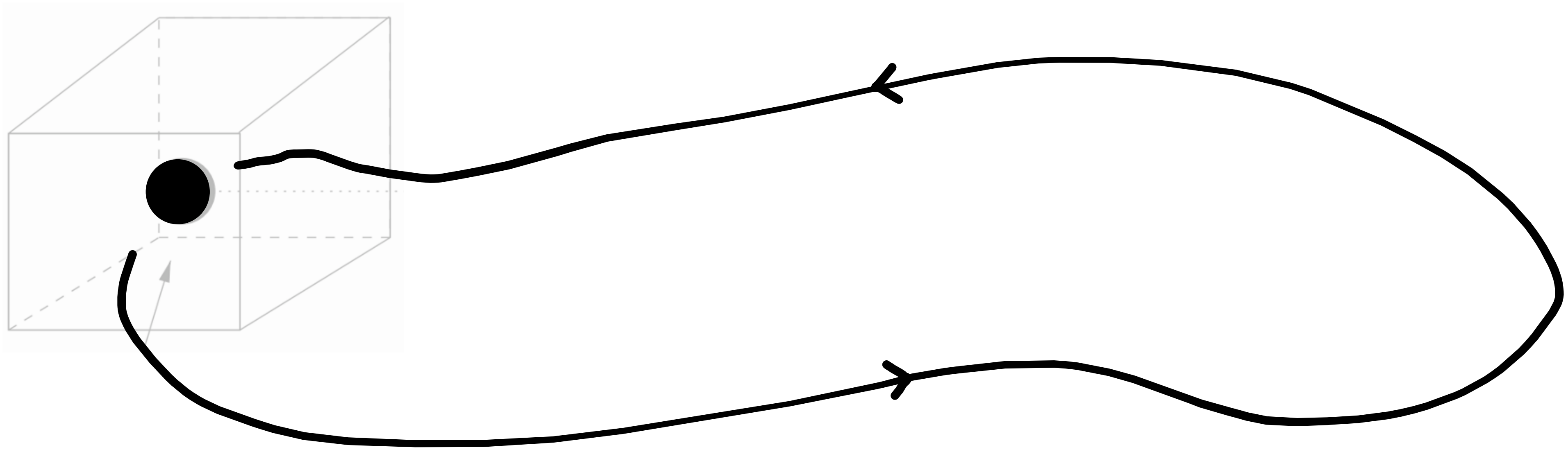
Endpoint of a Dirac string is a source of magnetic field



T. DeGrand and D. Toussaint, Phys. Rev. D 22 (1980) 2478
In 3-dim, a particle-like topological defect is detected by measuring the flux emanating from a unit 3-cube

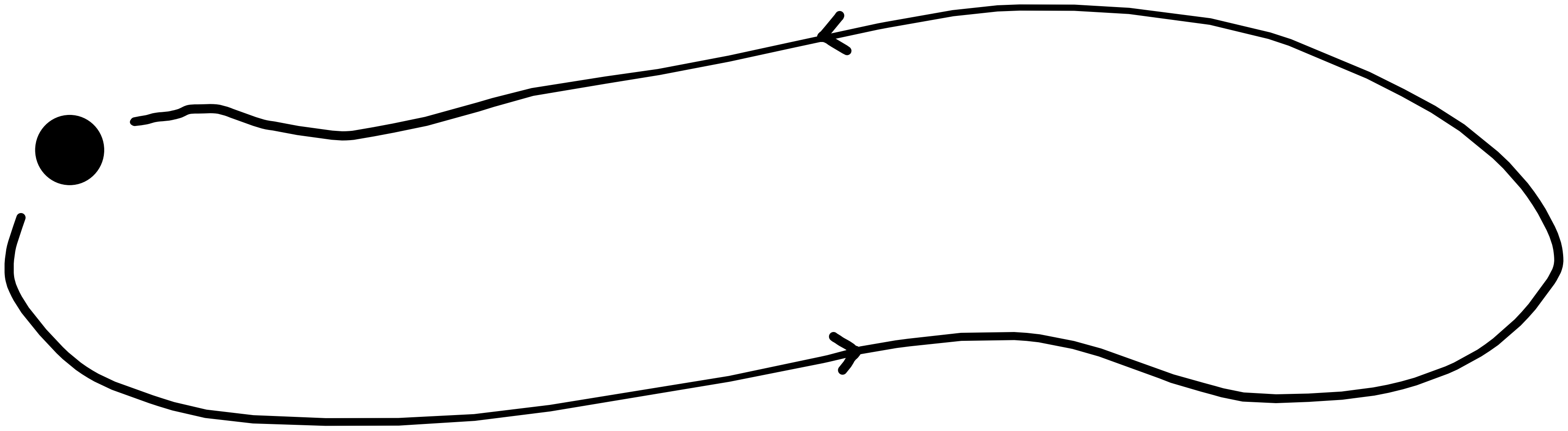
In **4 dimensions**

Monopoles sweep out closed curves



monopole current

Represents creation-annihilation pair of propagating monopole-anti-monopole



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In 4 dimensions

There exists a
first-order phase transition

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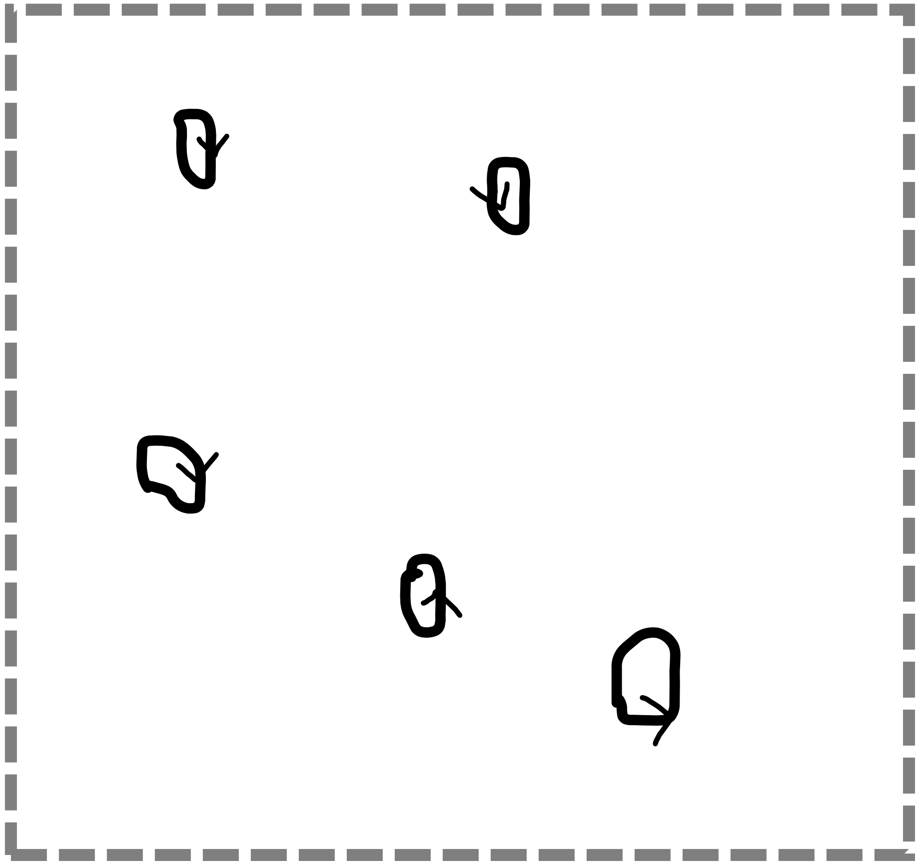
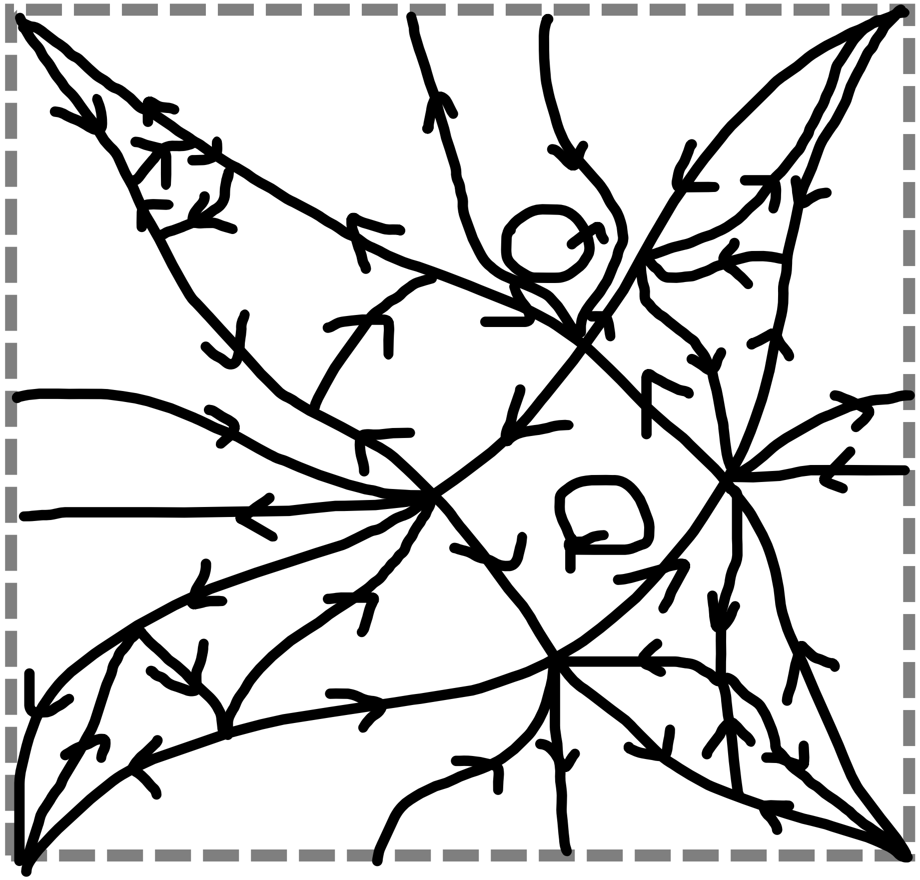


Deconfined Phase



Confining Phase

Caveat: This schematic is a 2-dim analogue

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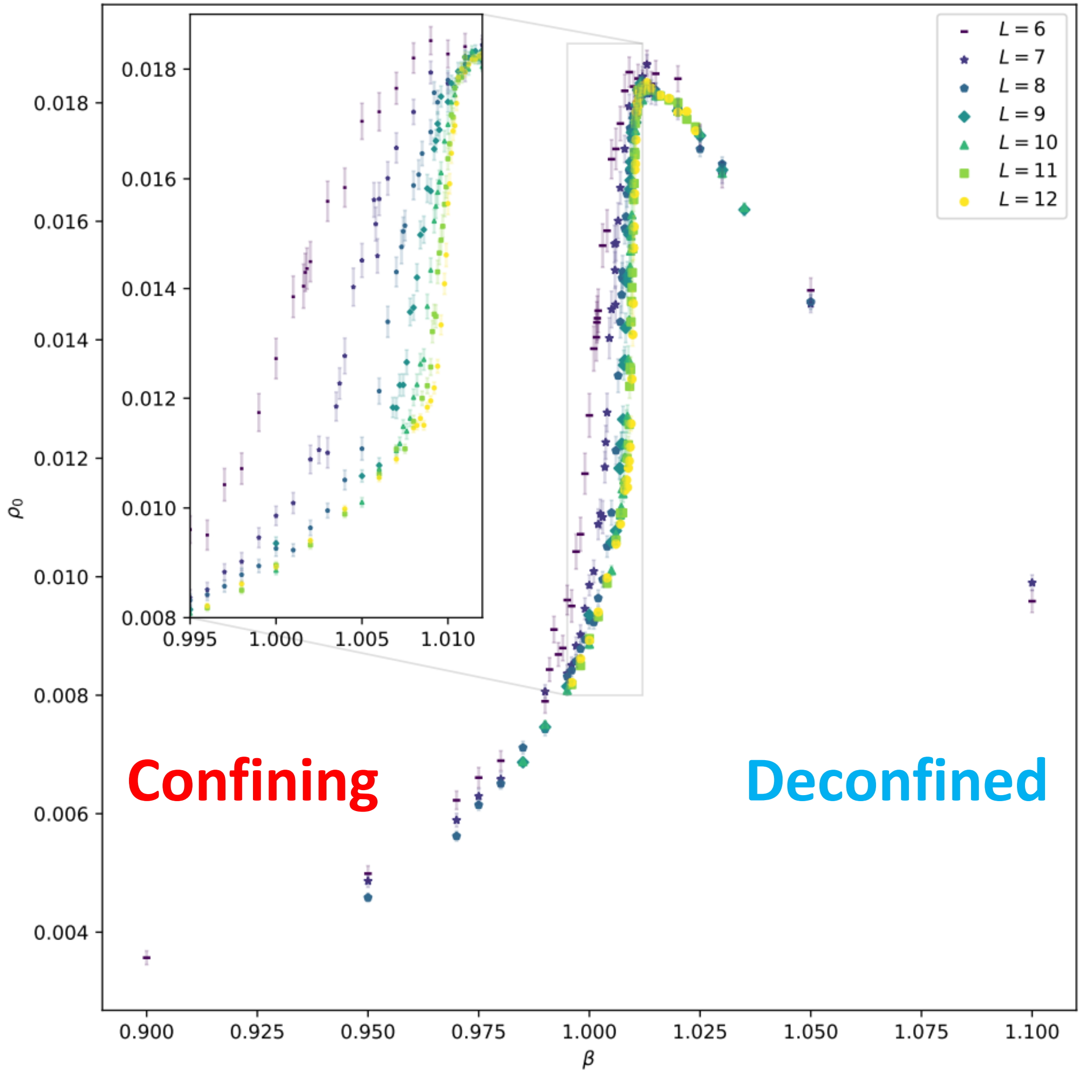
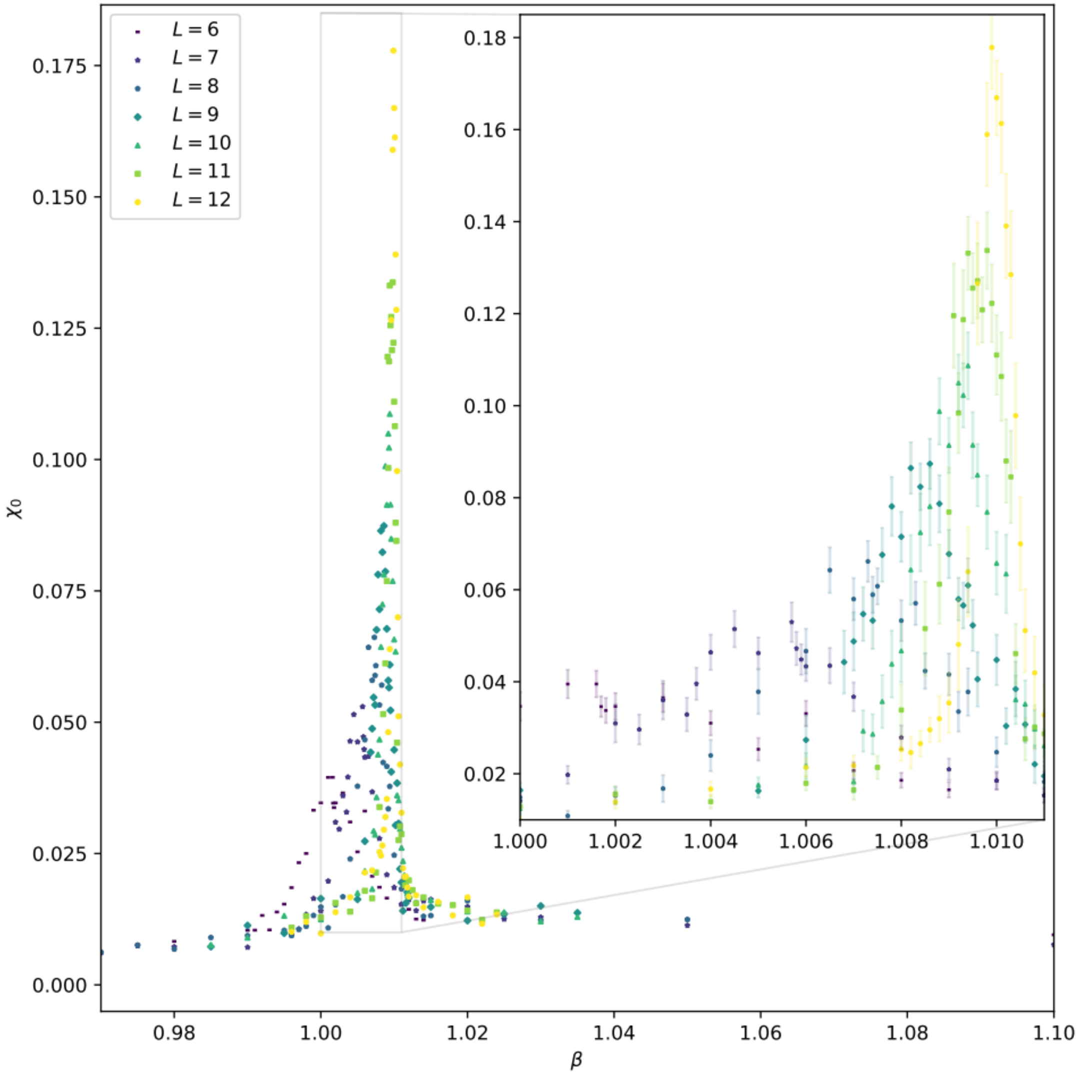
H_0(Current Network) = **Number of Components**

(divided by lattice volume)





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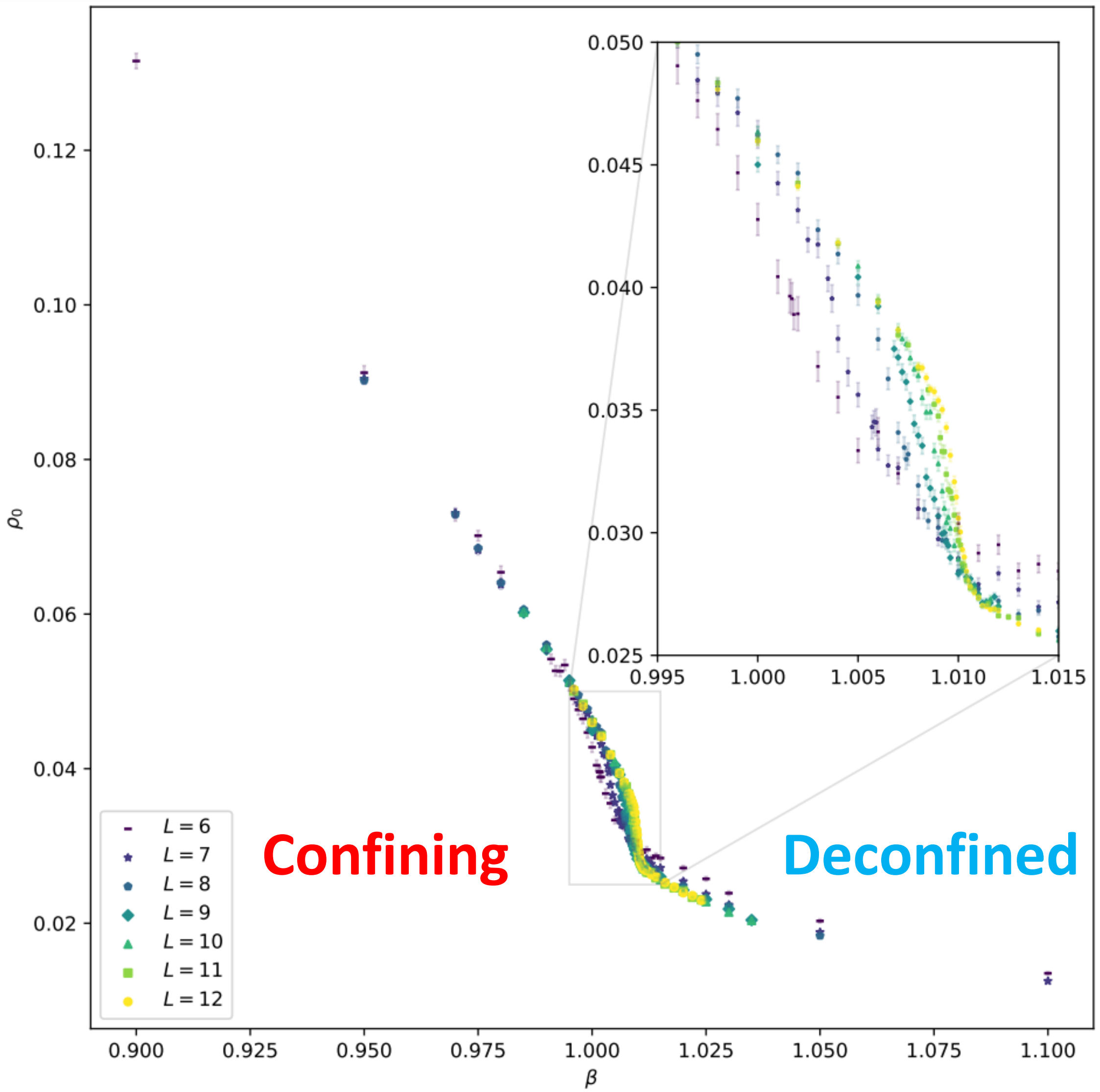
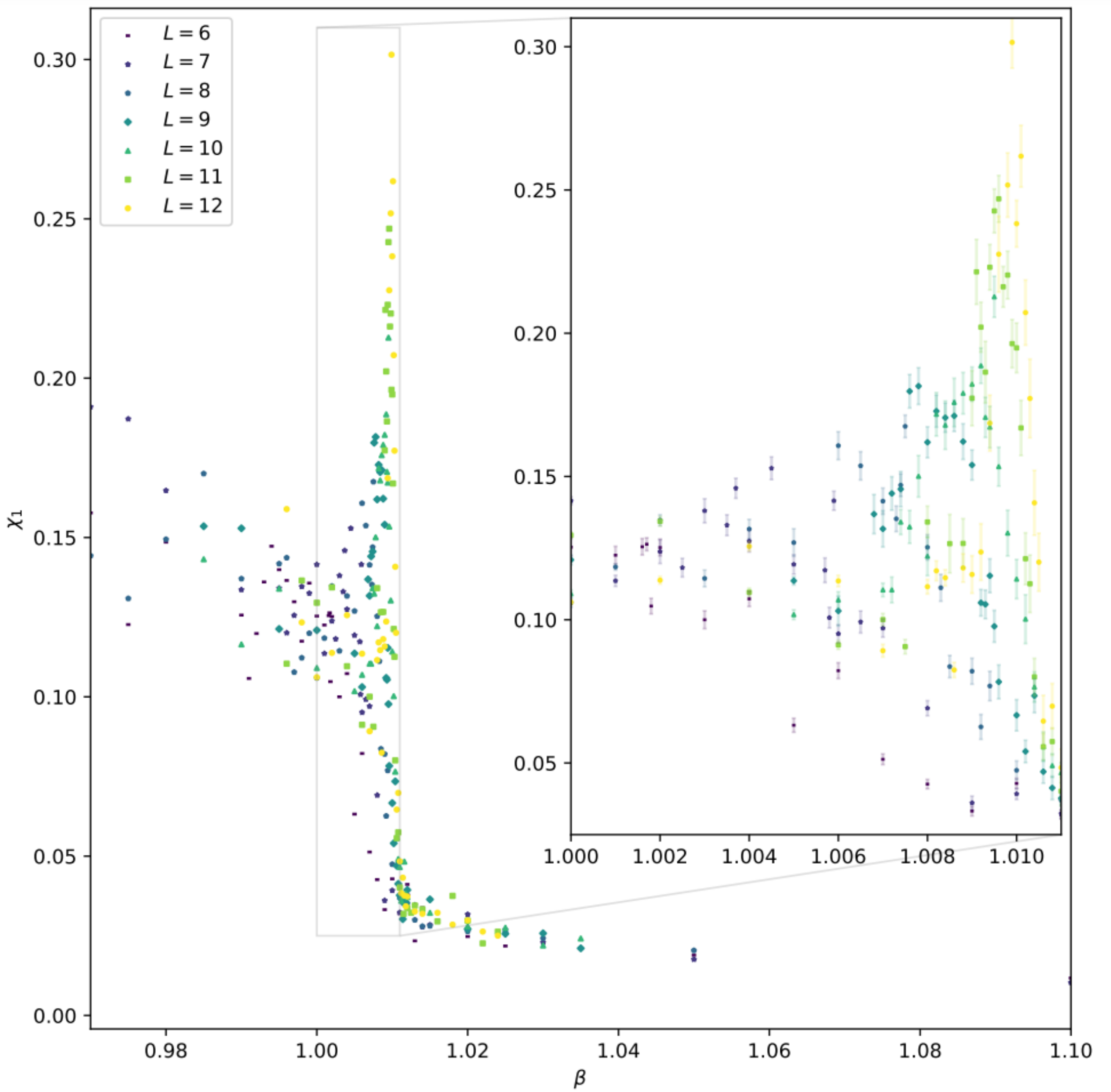
H_1(Current Network) = **Number of Loops**

(divided by lattice volume)





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Finite size scaling analysis

$$L \to \infty$$

E	ρ_0	ρ_1
1.01071(3)	1.01076(6)	1.01076(6)

X. Crean, J.Giansiracusa, and B. Lucini,
Topological Data Analysis of Monopole Current Networks in U(1) Lattice Gauge Theory (2024)
arXiv:2403.07739

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Design cubical filtration
to analyse

1. “size” of a monopole current loop

2. Network structures in the configuration

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Design cubical filtration
to analyse

1. “size” of a monopole current loop

2. Network structures in the configuration

Idea:
Expand volumes radially outwards from
1-dim monopole current strings

Like a ***balloon animal***

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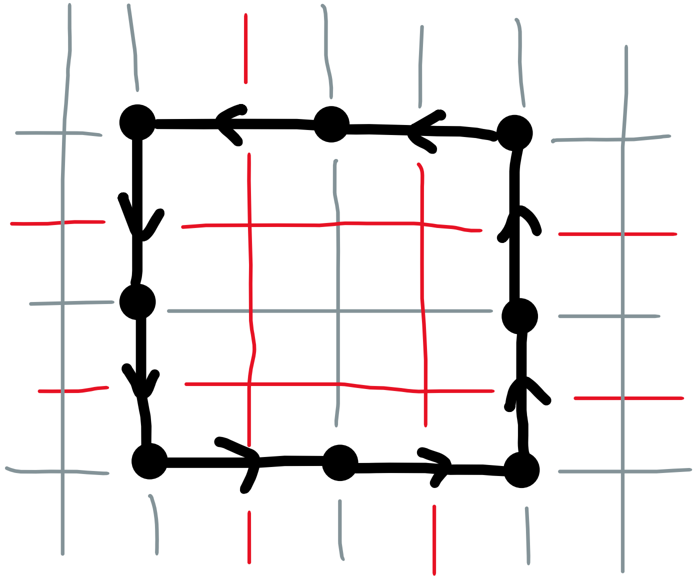
Filtration



t = 0

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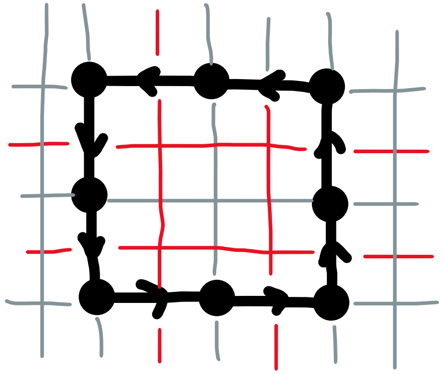
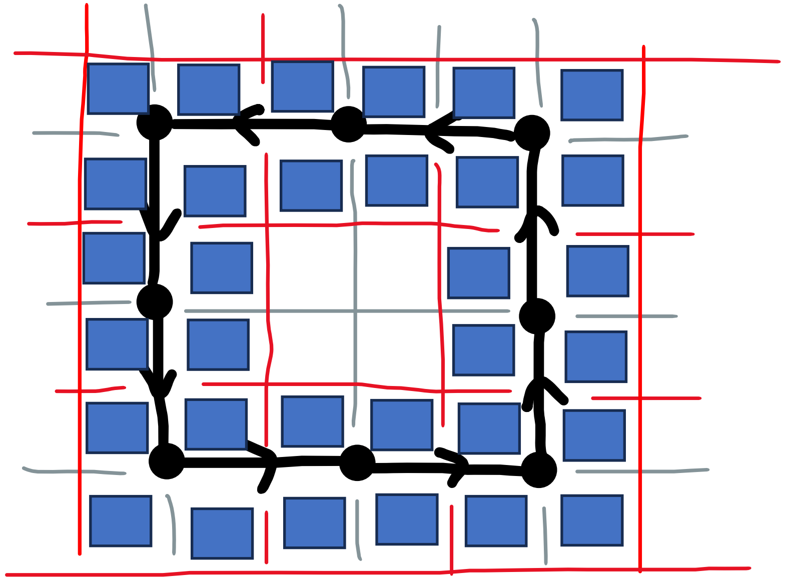
Filtration



t = 0



t = 1

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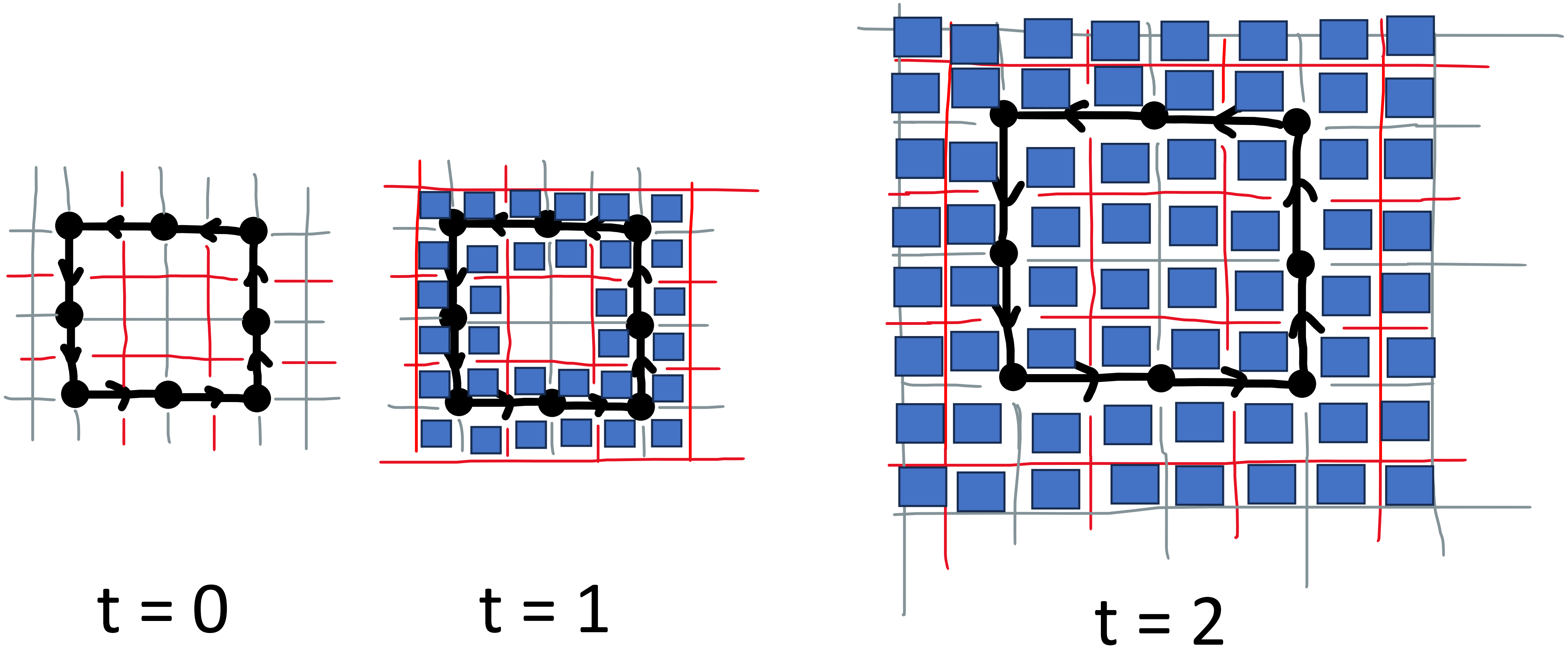
Filtration



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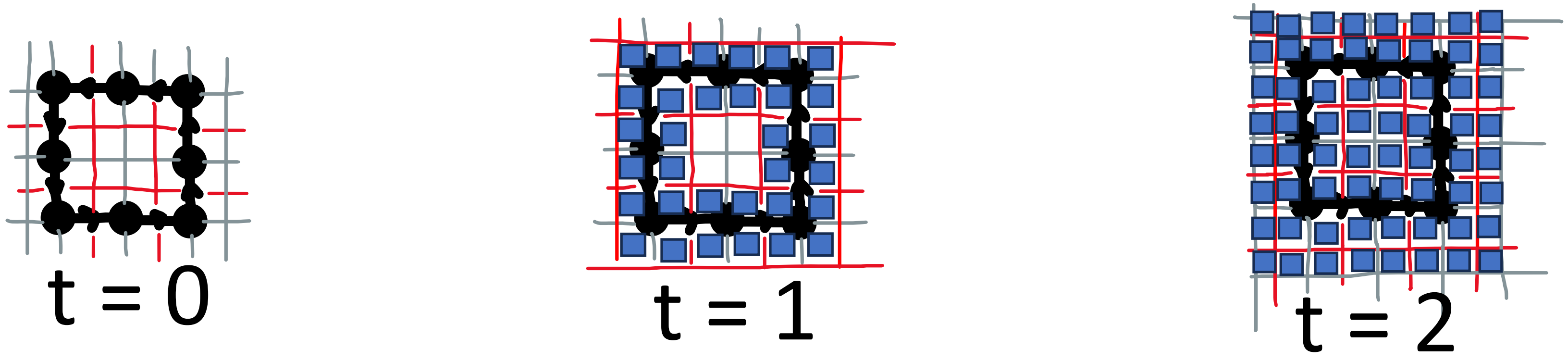

Using this filtration
can extract

the **critical temperature**
via ML classifier

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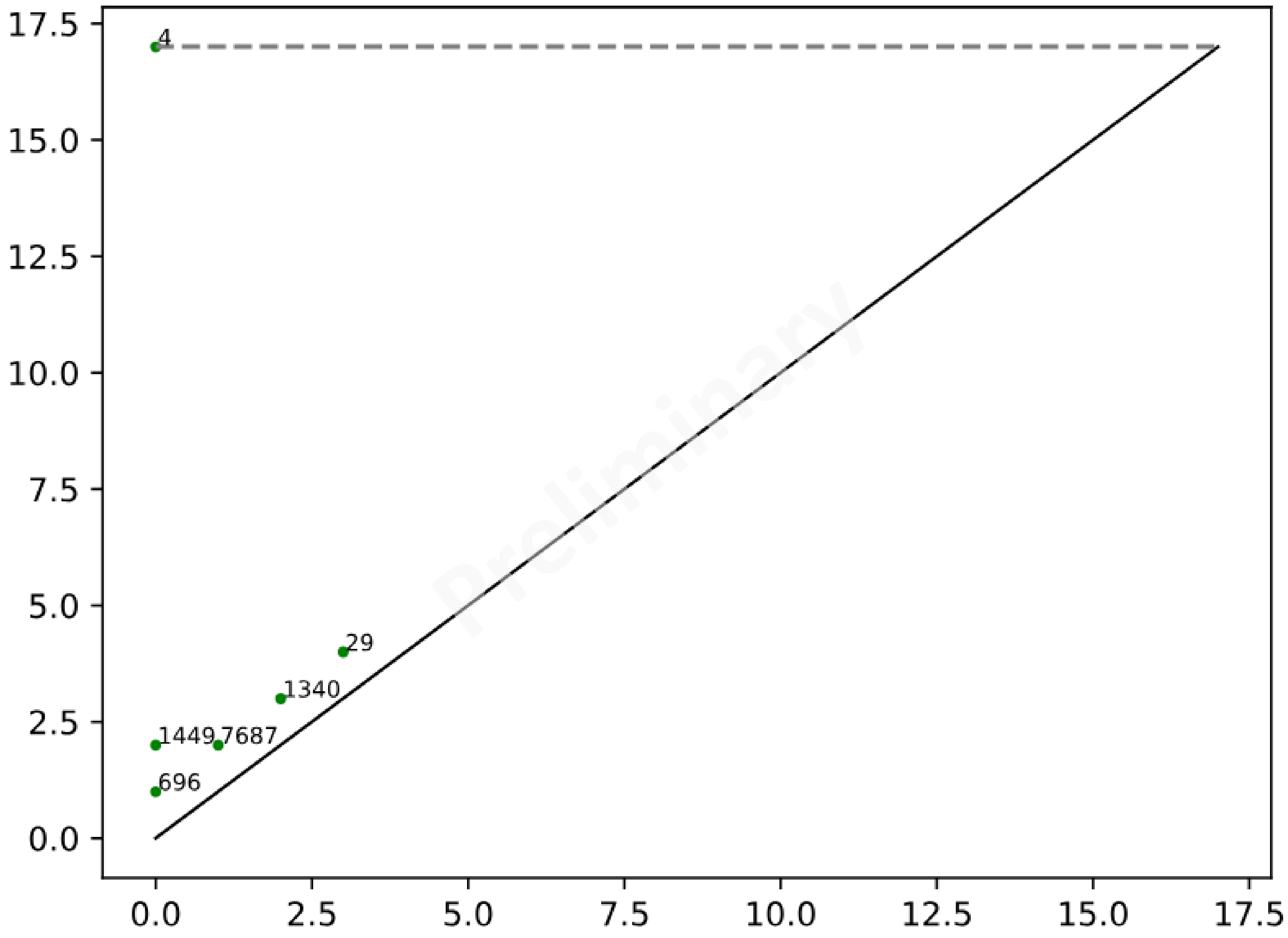
Computing persistent homology outputs **persistence diagram**

where each topological feature is given
1. a birth time b
2. a death time d

these are plotted as points (b,d)

Confined phase

with percolating current network



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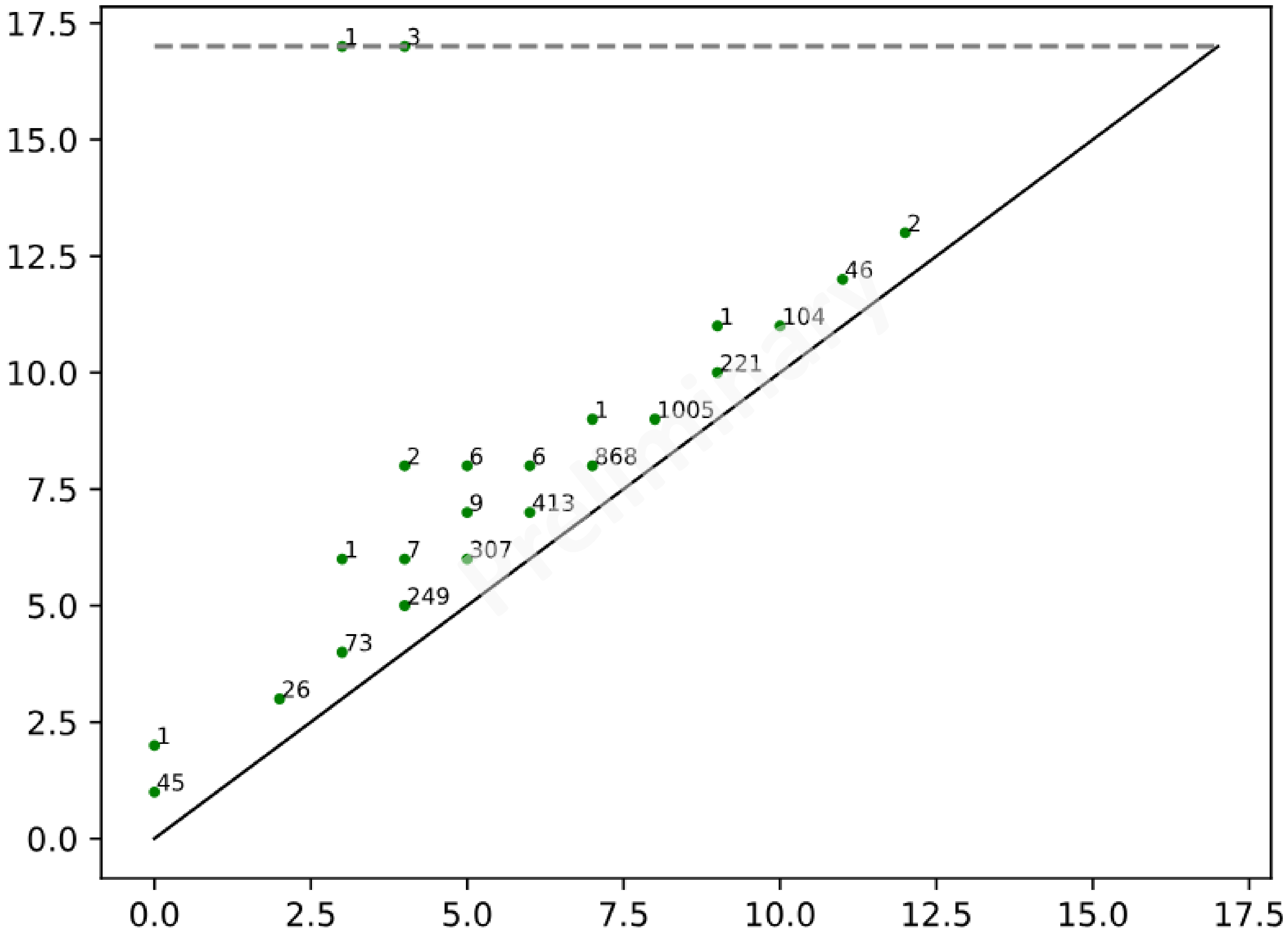
Deconfined phase

with small independent networks



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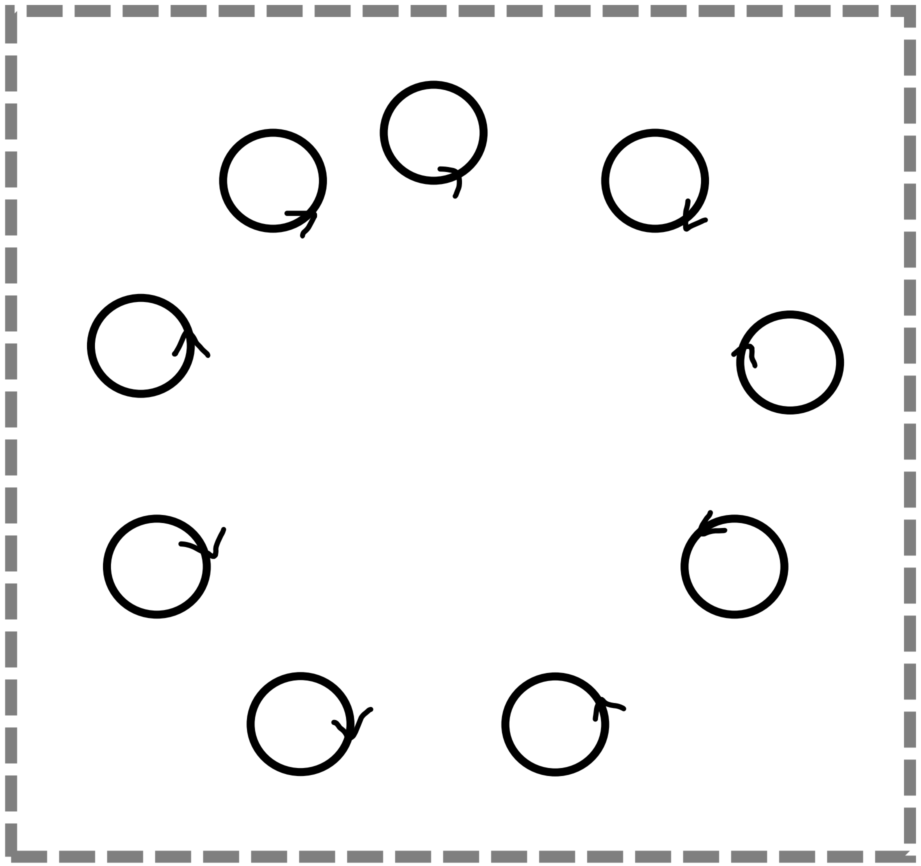
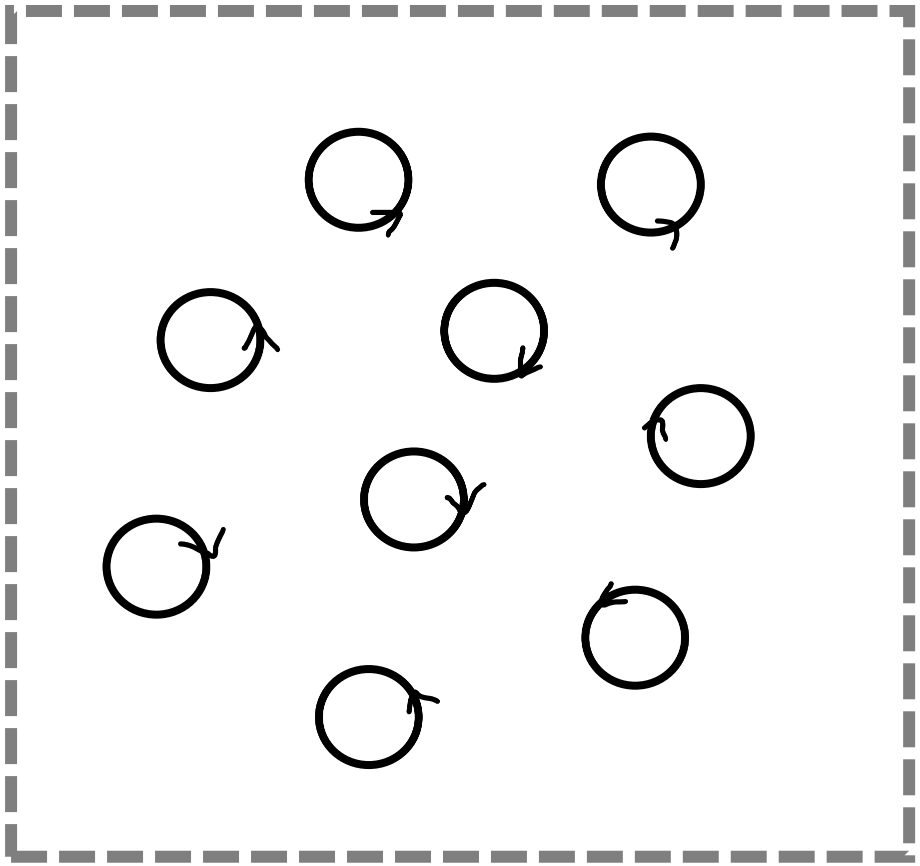
Do networks form large scale topological structures?





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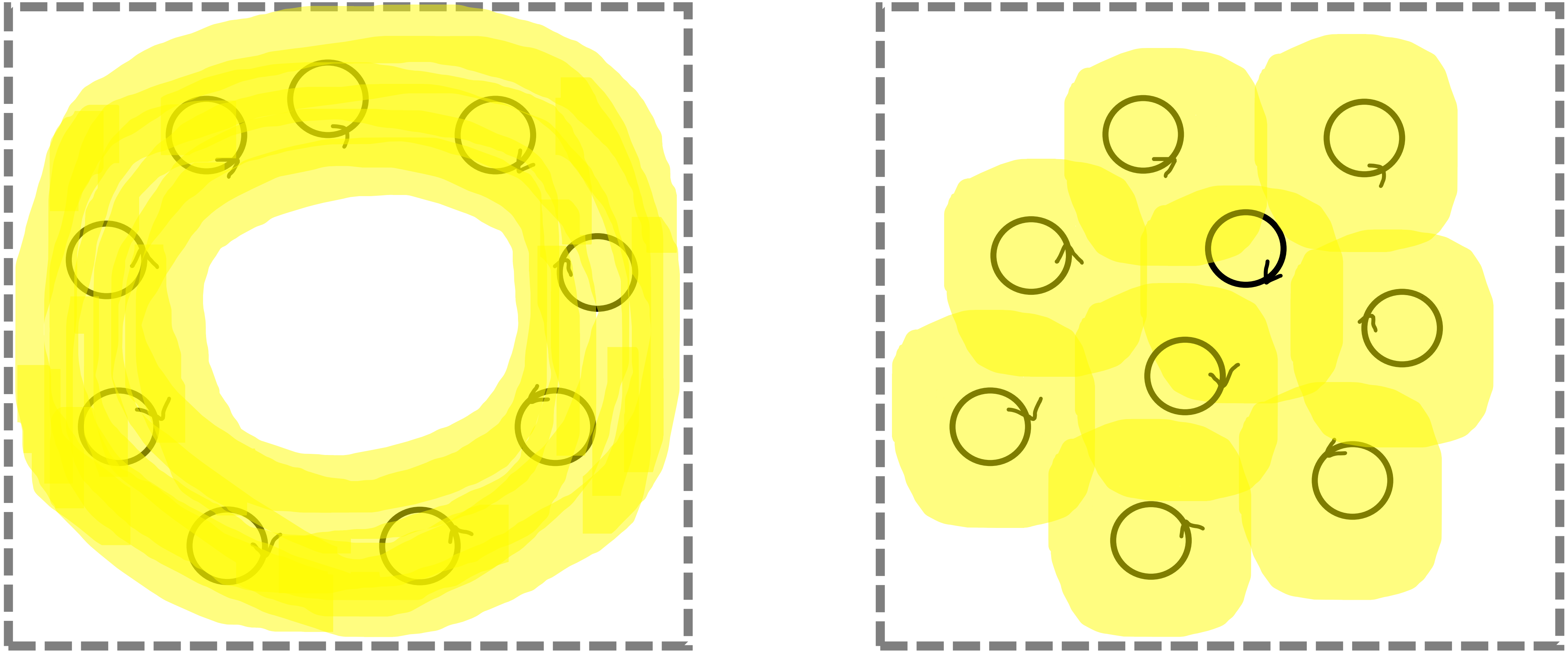

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Compare with random uniform density of loops

(Paper in preparation)

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Thanks for listening

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Next talk Fri 12:35

B. Lucini – Abelian monopoles in *SU(3)*