Gluon Collins-Soper Kernel from lattice QCD

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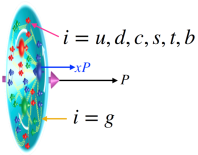
3D hadron structure: from PDF to TMD PDF

- Parton distribution function (PDF): $f_{i/h}(x)$



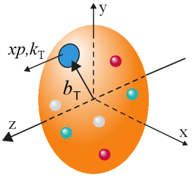
- probability of finding a parton i in hadron h carrying momentum fraction $x \to$ longitudinal

- Transverse-momentum-dependent PDF (TMD PDF):

$$f_{i/h}(x, \vec{k}_\mathrm{T}), \quad \text{or coordinate-space} \quad f_{i/h}(x, \vec{b}_\mathrm{T}) = \int \mathrm{d}^2 \vec{k}_\mathrm{T}\, e^{i\vec{k}_\mathrm{T}\cdot\vec{b}_\mathrm{T}} f_i(x, \vec{k}_\mathrm{T})$$

- probability of finding parton i with fraction x and transverse momentum $\vec{k}_\mathrm{T}$ $\to$ longitudinal and transverse

(or the Fourier conjugate $\vec{b}_\mathrm{T}$)

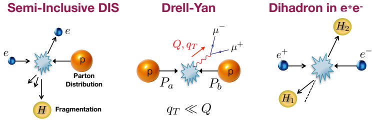


$\Rightarrow$ Rich hadron 3D internal structure in TMD PDFs!

Collins-Soper kernel

- TMD PDFs can be determined in various processes

need ability to relate different energy scales



- Evolution of TMD PDFs:

1. UV renormalization scale μ 2. rapidity scale ζ

The evolution kernels are universal (independent of external hadron h)

$$f_{i/h}(x, b_{\mathrm{T}}, \mu, \zeta) = f_{i/h}(x, b_{\mathrm{T}}, \mu_0, \zeta_0)$$

$$\times \exp\left[\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \underline{\gamma_\mu^i(\mu', \zeta_0)}\right] \exp\left[\frac{1}{2}\underline{\gamma_\zeta^i(\mu, b_{\mathrm{T}})} \ln \frac{\zeta}{\zeta_0}\right]$$

UV anomalous dimension rapidity anomalous dimension (Collins-Soper kernel)

- UV anomalous dimension γ_μ^i is perturbative as long as scales are large

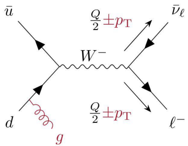
But CS kernel γ_ζ^i is always nonperturbative for $b_{\mathrm{T}} \gtrsim \Lambda_{\mathrm{QCD}}^{-1}$

(even if the evolution variables μ, ζ are perturbative)

W boson mass

- CS kernel also required as input into measurements of several observables

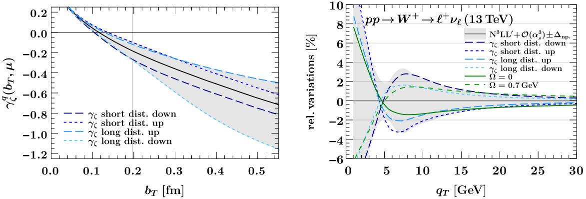
E.g. W boson mass extracted from $p\bar{p} \to W^- \to l^- \nu_l$



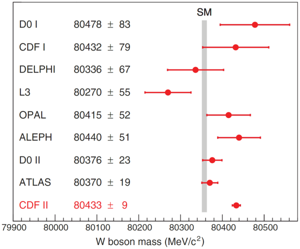
- Need robust understanding of all QCD theory

especially non-perturbative QCD effects

Variations in CS kernel $\Rightarrow$ % variations in $d\sigma/dq_T$



figures from Johannes Michel, MIT

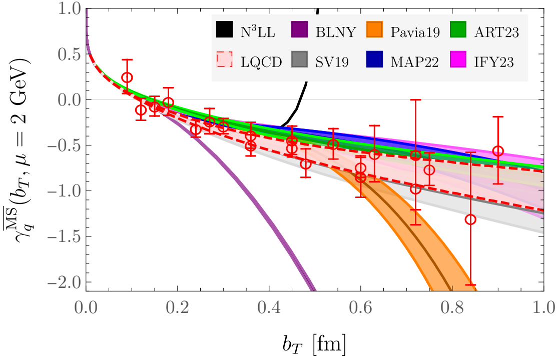


CDF II, Science 2022

- Distribution shape is sensitive to CS kernel, measurement of M_W affected

Quark Collins-Soper kernel

- Our group's LQCD calculation of quark CS kernel:



Avkhadiev, Shanahan, Wagman, Zhao, PRD 108 (2023) 11, 114505
PRL 132 (2024) 23, 231901

- First such calculation with systematic control of quark mass, operator mixing, and discretization effects

- Model-dependence in pheno. parameterizations is significant
lattice results are precise enough to discriminate between different models

What about gluon CS kernel?

- Experimentally:

lack of data for gluon TMDs. But can expect in the near future from EIC

- Theoretically:

- perturbative region: 1-loop result is

$$\gamma_\zeta(\mu, b_T) = -\frac{\alpha_s}{\pi} \ln \frac{b_T^2 \mu^2}{4e^{-2\gamma_E}} \times \begin{cases} C_F, & \text{quark} \\ C_A, & \text{gluon} \end{cases} + \mathcal{O}(\alpha_s^2)$$

only differ by a group theory factor (C_A v.s. C_F), almost the same as quark

- non-perturbative region: nobody knows!

- This work: extend our calculation to the gluon CS kernel

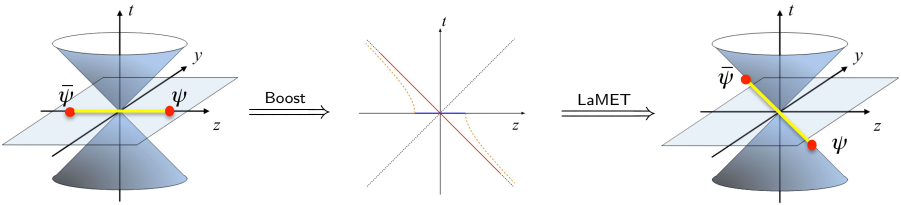
it will be the first lattice prediction for future experiments

LaMET

- LQCD can not directly access parton physics defined on light-cone
- Large-Momentum Effective Theory (LaMET): Provides a framework to

link Euclidean equal-time correlation functions to light-cone one

X. Ji, PRL 110 (2013), SCPMA57 (2014)



Quasi: equal-time

Physical: light-cone

- Quasi distribution calculable on lattice, with same IR physics as light-cone

Differences in UV accounted for by perturbative matching

Quasi-TMDs

- Quasi-TMDs can be related to light-cone TMDs via LaMET

Ebert, Schindler, Stewart & Zhao, JHEP 04, 178 (2022)

Quasi-TMDs   pert. matching   light-cone TMDs   CS kernel

$$\frac{\tilde{f}(x, b_\mathrm{T}, \mu, P^z)}{\sqrt{S_r(b_\mathrm{T}, \mu)}} = H(\mu, xP^z) f(x, b_\mathrm{T}, \mu, \zeta) \exp\left[\frac{1}{2}\gamma_\zeta(b_\mathrm{T}, \mu) \ln\frac{(2xP^z)^2}{\zeta}\right]$$

Soft factor

$$+ \mathcal{O}\left[\frac{1}{(xP^z b_T)^2}, \frac{M^2}{(xP^z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(xP^z)^2} + (x \to 1 - x)\right]$$

- CS kernel extracted from ratio with different momenta P_1 and P_2

$$\gamma_\zeta(b_T, \mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\tilde{f}(x, b_\mathrm{T}, \mu, P_1^z)}{\tilde{f}(x, b_\mathrm{T}, \mu, P_2^z)}\right] + \delta\gamma_\zeta(x, \mu, P_1^z, P_2^z) + \text{p.c.}$$

with $\delta\gamma_\zeta(x, \mu, P_1^z, P_2^z)$ compute from pert. matching kernel

- Power corrections need to be under control $\to$ x away from 0 and 1

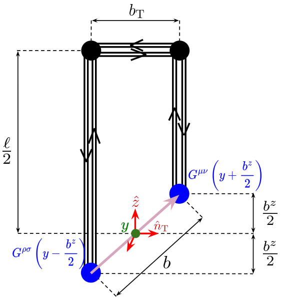
Staple-shaped Operator

- Operators for the gluon quasi-TMDs

$$\mathcal{O}_g^{\mu\nu,\rho\sigma}(b) = G^{\mu\nu}\left(\frac{b}{2}\right) W_{\sqsupset}^{\text{adj}}(b, l) G^{\rho\sigma}\left(-\frac{b}{2}\right)$$

- For the unpolarized case, four operators are

multiplicatively renormalizable



$$\mathcal{O}_g^{(1)} = \mathcal{O}_g^{0i,0i}, \quad \mathcal{O}_g^{(2)} = \mathcal{O}_g^{3i,3i}$$

$$\mathcal{O}_g^{(3)} = \frac{1}{2}(\mathcal{O}_g^{0i,3i} + \mathcal{O}_g^{3i,0i}), \quad \mathcal{O}_g^{(4)} = \mathcal{O}_g^{3\mu,3\mu}$$

Zhu et al, JHEP 02, 114 (2023)

⇒ renormalization cancelled in the ratio

- Symmetry properties: by Hermiticity and translation invariance

$$\mathcal{O}_g^{\mu\nu,\rho\nu}(b) = [\mathcal{O}_g^{\mu\nu,\rho\nu}(b)]^\dagger = \mathcal{O}_g^{\rho\nu,\mu\nu}(-b)$$

⇒ These operators are real and symmetric under $b \to -b$

Quasi-TMDs

- Two observables can be used to compute the CS kernel on lattice

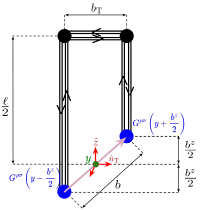
- Quasi-beam functions from 2pt and 3pt functions

$$\tilde{B}(b^z, b_\mathrm{T}, \ell, P^z) = \langle h(P^z)|\mathcal{O}(b_\mu, 0, \ell)|h(P^z)\rangle$$

- Quasi-TMD wavefunctions (WFs) from 2pt functions

$$\tilde{\psi}(b^z, b_\mathrm{T}, \ell, P^z) = \langle 0|\mathcal{O}(b_\mu, -P^z, \ell)|h(P^z)\rangle$$

Staple-shaped operator $\mathcal{O}$



- For quark CS kernel, quasi-TMD WFs are used

- lower computational cost for 2pts

- For gluon CS kernel, we prefer quasi-beam functions

- No quark disconnected contractions

- 3pts can be computed by correlating 2pts with gluon operator

Workflow

CS kernel from ratio:

$$\gamma_\zeta(b_T,\mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\int \frac{\mathrm{d}b^z}{2\pi}\, e^{ixP_1^z b^z} N(P_1^z) \lim_{\ell\to\infty} \tilde{B}(b^z, b_T, \ell, P_1^z)\tilde{\Delta}^S(b_T, l)}{\int \frac{\mathrm{d}b^z}{2\pi}\, e^{ixP_2^z b^z} N(P_2^z) \lim_{\ell\to\infty} \tilde{B}(b^z, b_T, \ell, P_2^z)\tilde{\Delta}^S(b_T, l)}\right]$$

$$+ \delta\gamma_\zeta(x, \mu, P_1^z, P_2^z) + \text{p.c.}$$

Quasi-soft factor $\tilde{\Delta}^S(b_T, l)$ is a Wilson loop to remove the linear divergence $\sim l + b_T$

1. Position-space MEs
   $\ell \to \infty$ extrapolation
2. x-space MEs
   integral range $b^z_{\max}$
3. Ratio of MEs
   with pert. matching

$\Longrightarrow$ CS kernel $\gamma_\zeta(b_T,\mu)$ ($x \sim 0.5$ with p.c. under control) $\Longrightarrow$ Repeat for each b_T

1-loop matching for gluon available in
Schindler, Stewart & Zhao, JHEP 08, 084 (2022)

Lattice setup

- Calculation carried out on a single MILC ensemble:

A. Bazavov et al. (MILC) PRD 87 (2013) 054505

$L^3 \times T = 48^3 \times 64,\ a = 0.12$ fm, $m_\pi = 148$ MeV

$N_{\text{cfg}} \times N_{\text{src}} \approx 470 \times 16$ (will be increased to $\sim 1000 \times 256 \rightarrow 30\times$ more)

- CS kernel is universal — independent of hadronic state

pion state is primary target (suppressed power corrections $M^2/(xP^z)^2$)

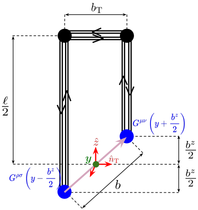
nucleon state will also be studied at the same time

- All multip. renormlizable operators calculated

11 values of $\ell \in [0.84, 3.48]$ fm to suppress finite-ℓ effect

4 values of $P^z = 0.86, 1.29, 1.72, 2.15$ GeV

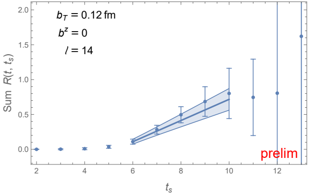
Results shown below are with pion and $O^{0i,0i}$ (most precise)



- Step 1. extract MEs from 3pts

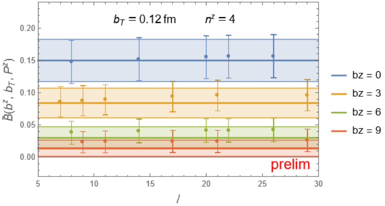
  summation method used

$$\sum_{t=1}^{t_s-1} R(t, t_s) = \mathrm{ME} \cdot t_s + c + \cdots$$

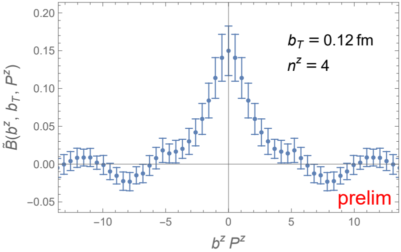


- Step 2. dependence on ℓ

  mild finite-ℓ effect with linear div. removed



- Step 3. MEs as function of $b^z P^z$



After averaging all staple orientations, MEs are numerically real and symmetric

Quark vs. Gluon

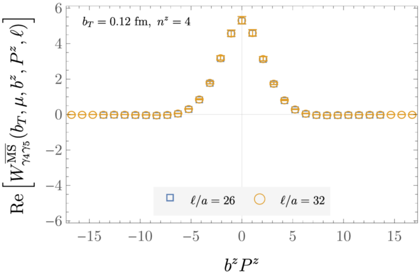
- Comparison of quark and gluon cases

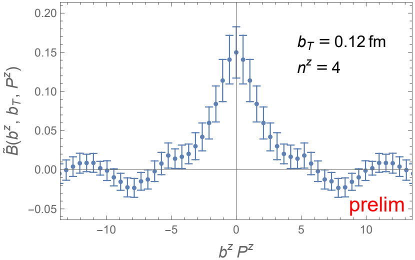
- $b_T = 0.12$ fm, $P^z = 4 \times \dfrac{2\pi}{L} = 0.86$ GeV

- same number of measurements $N_{\rm cfg} \times N_{\rm src} \sim 470 \times 16$

Quark: few % errors

Gluon: $20 - 30\%$ errors





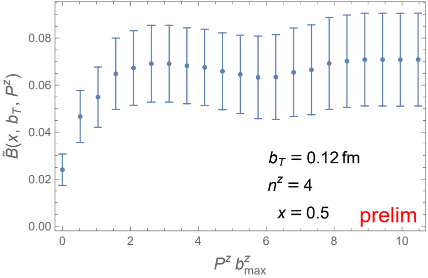
- An order of magnitude more stats are needed to achieve a similar precision

- Fourier transform to x-space

$$\tilde{B}(x, b_T, P^z) = \int_{|b^z|<b^z_{\max}} \frac{\mathrm{d}b^z}{2\pi} e^{ixP^z b^z} N(P^z)\tilde{B}(b^z, b_T, P^z)$$

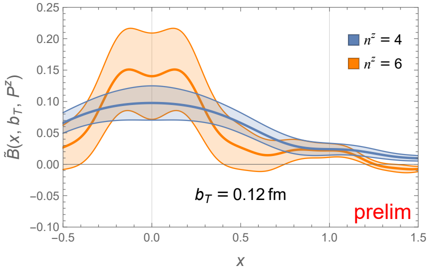
- Dependence on $b^z_{\max}$

Fourier transformation is saturated for $P^z b^z_{\max} \gtrsim 5$ with errors



- MEs as a function of x

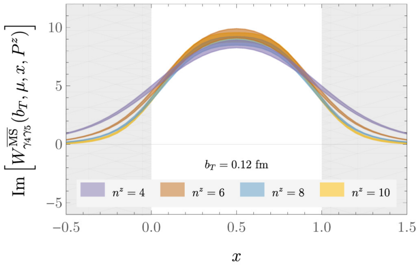
tails outside physical range $x \in [-1, 1]$ are reduced as P^z increases



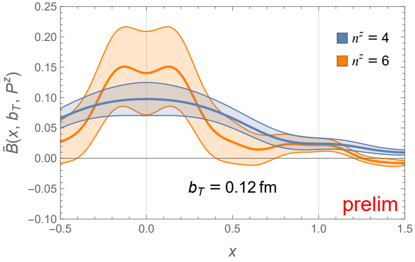
Quark vs. Gluon

- Quark and gluon x-space MEs have different symmetries

- Quark (with pion state)



- Gluon



Symmetric under $x \to 1 - x$

momentum fraction of two quarks

Symmetric under $x \to -x$

Gluons are their own anti-particles

- For gluon, even more stats are needed since the ratio is not taken around the peak, and unfortunately signal lost at $x \sim 0.5$ with current error
- But meaningful results will be achieved with $\sim 30\times$ more statistics

Conclusion and outlook

CS kernel from quark to gluon:

- In contrast to quark TMDs, gluon TMDs are almost unknown (both experiments and lattice QCD)
- Matrix elements have some different symmetry properties
- Current statistics suggests that meaningful results can be achieved with an order of magnitude more data — which is what we're doing ($N_{\mathrm{cfg}} \times N_{\mathrm{src}} \approx 500 \times 16 \to 1000 \times 256$)

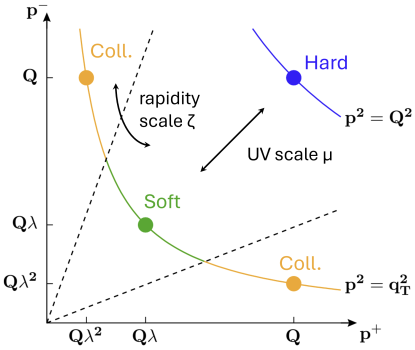
Thank you!

Backup Slides

Rapidity divergence

- Regulators such as dimensional regularization only regulate UV divergences

rapidity divergences arise in soft and collinear need a dedicated regulator



Ebert, Stewart & Zhao, JHEP 09, 037 (2019)

- A concrete example

$$I_{\text{div}} = \int \mathrm{d}p^+ \mathrm{d}p^- \frac{f(p^+p^-)}{(p^+p^-)^{1+\epsilon}} = \frac{1}{2}\int \frac{\mathrm{d}(p^-/p^+)}{p^-/p^+} \int \mathrm{d}(p^+p^-) \frac{f(p^+p^-)}{(p^+p^-)^{1+\epsilon}}$$