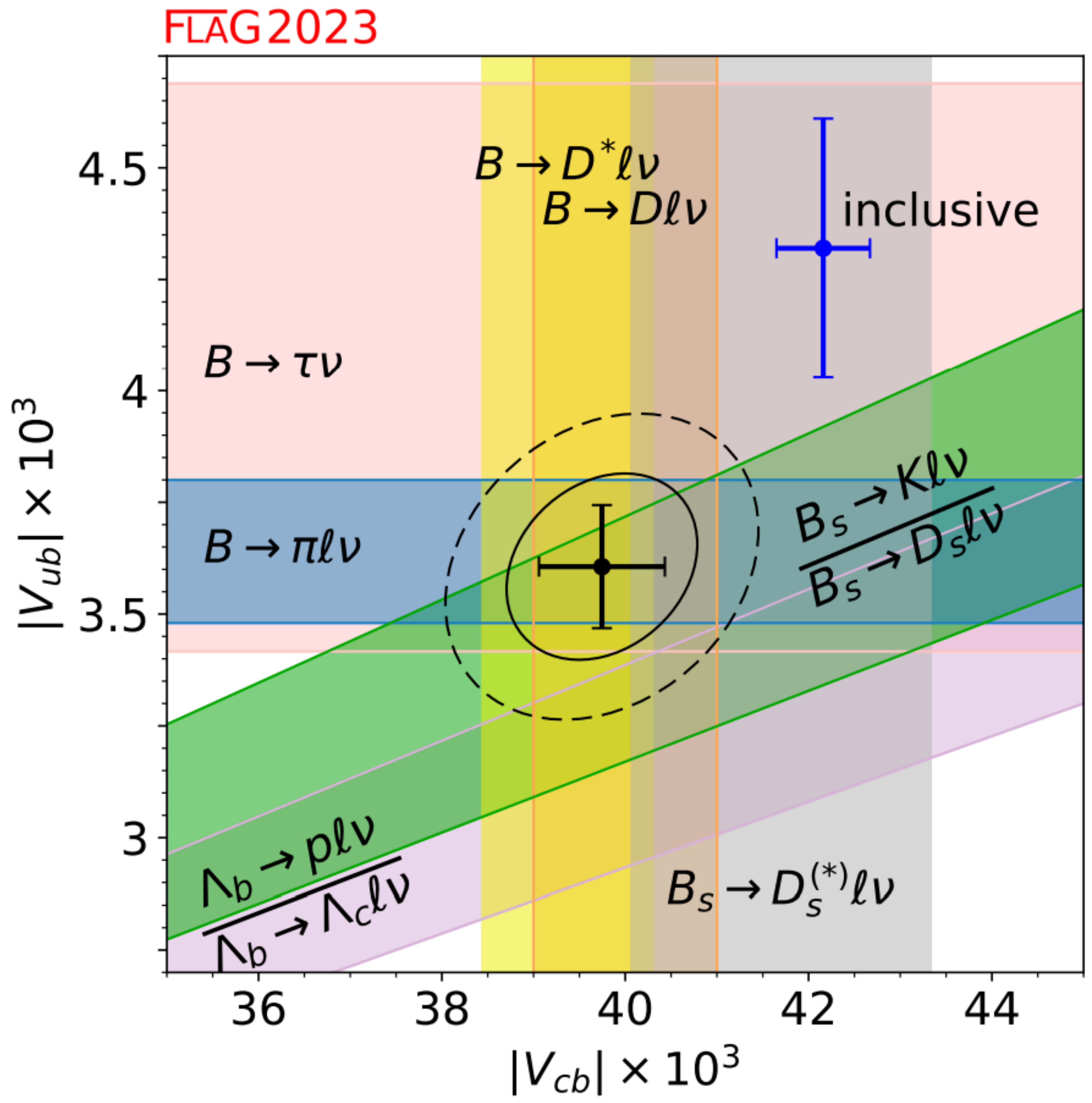
Spectator Effects in heavy hadron decays from HQET

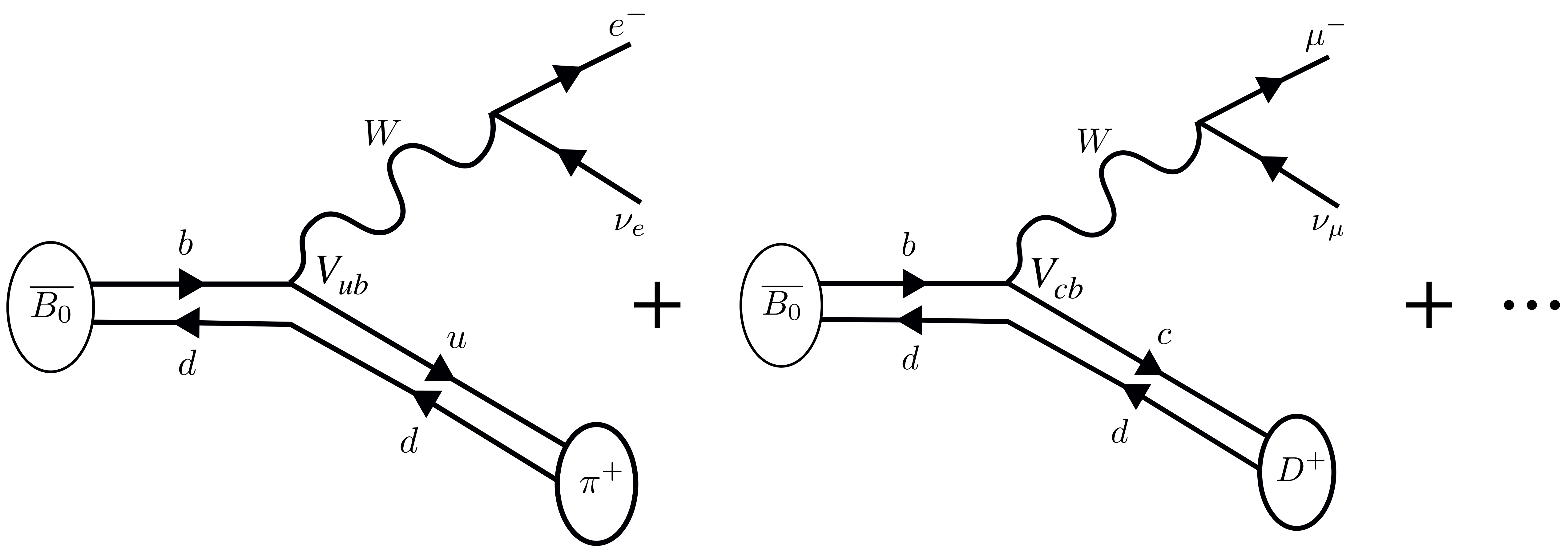
Joshua Lin

with
Will Detmold
Stefan Meinel

Inclusive/exclusive tensions



- Exclusive measurements (shaded bands) from a single decay channel

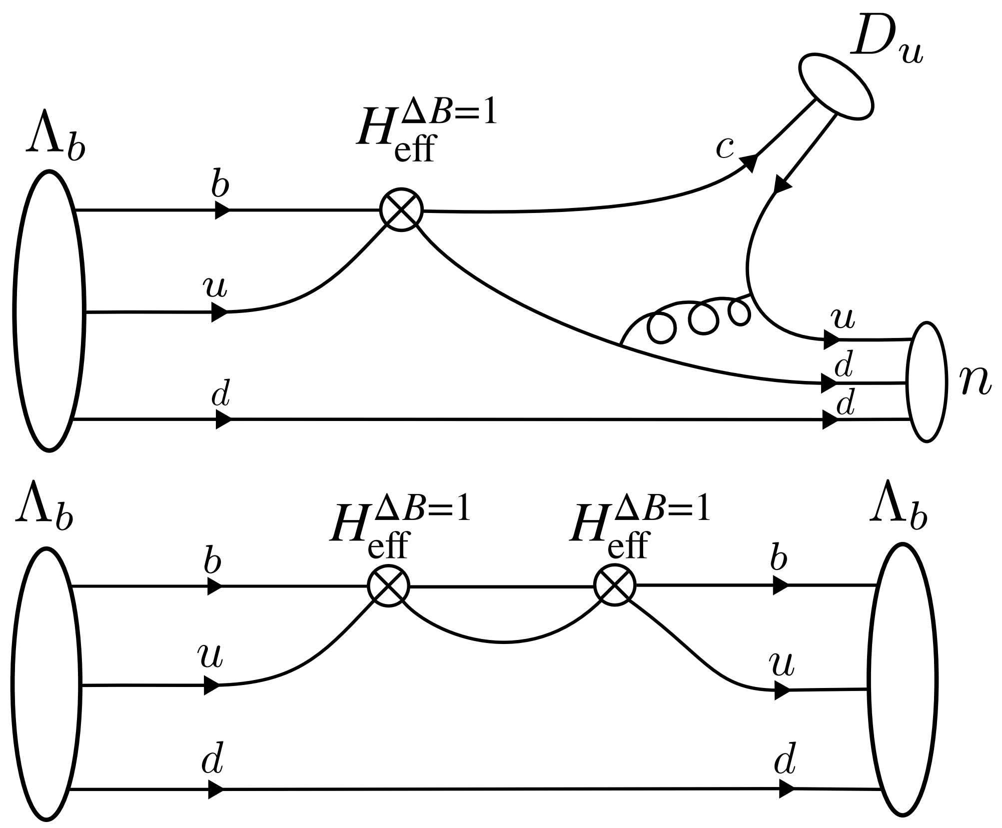


- Inclusive measurement from summing over all (semileptonic) channels. This is quite difficult theoretically!
- Persistent 3σ-tension.

$$H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \left[V_{cb}^* V_{us} (C_1 Q_1 + C_2 Q_2) + \cdots V_{cb}^* Q_l^c + \cdots \right]$$

↓ Optical Theorem

$$\Gamma(B \to X) = \frac{1}{2m_B} \langle B | \hat{T} | B \rangle = \frac{1}{2m_B} \langle B | \mathrm{Im}\, i \int d^4x T \left(H_{\text{eff}}^{\Delta B=1}(x) H_{\text{eff}}^{\Delta B=1}(0) \right) | B \rangle$$



Theory predictions for Inclusive Lifetimes

$$H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}}\left[V_{cb}^{*}V_{us}(C_1Q_1 + C_2Q_2) + \cdots V_{cb}^{*}Q_l^{c} + \cdots\right]$$

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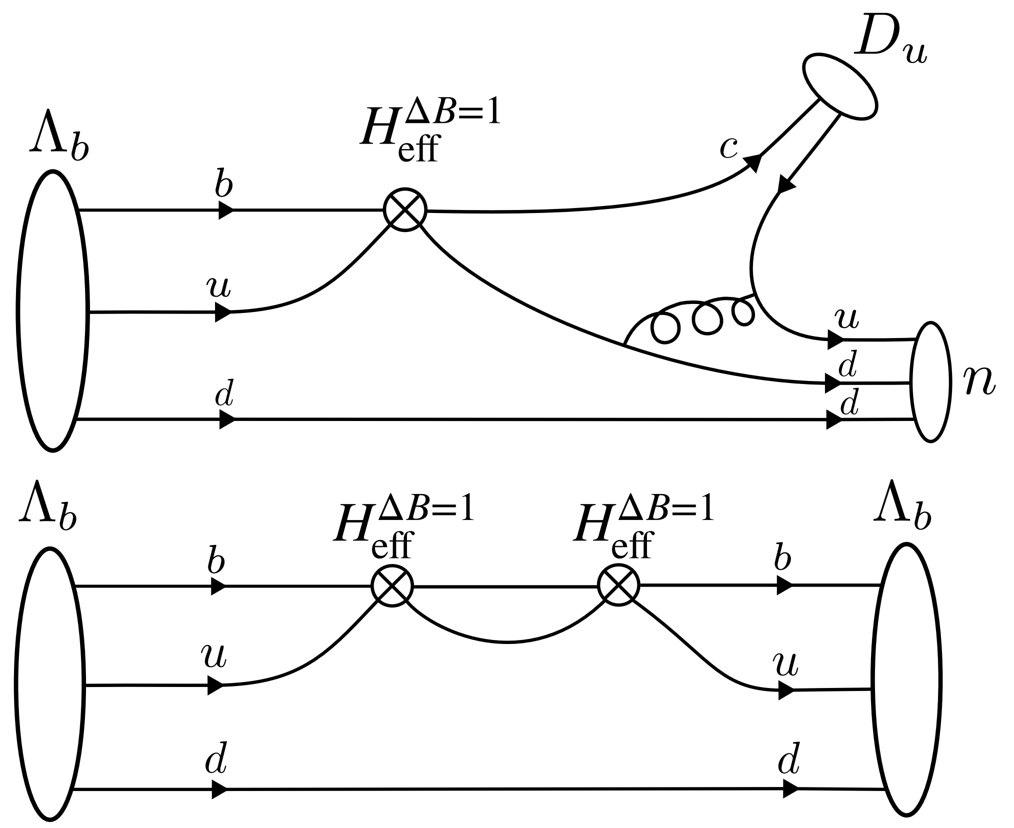
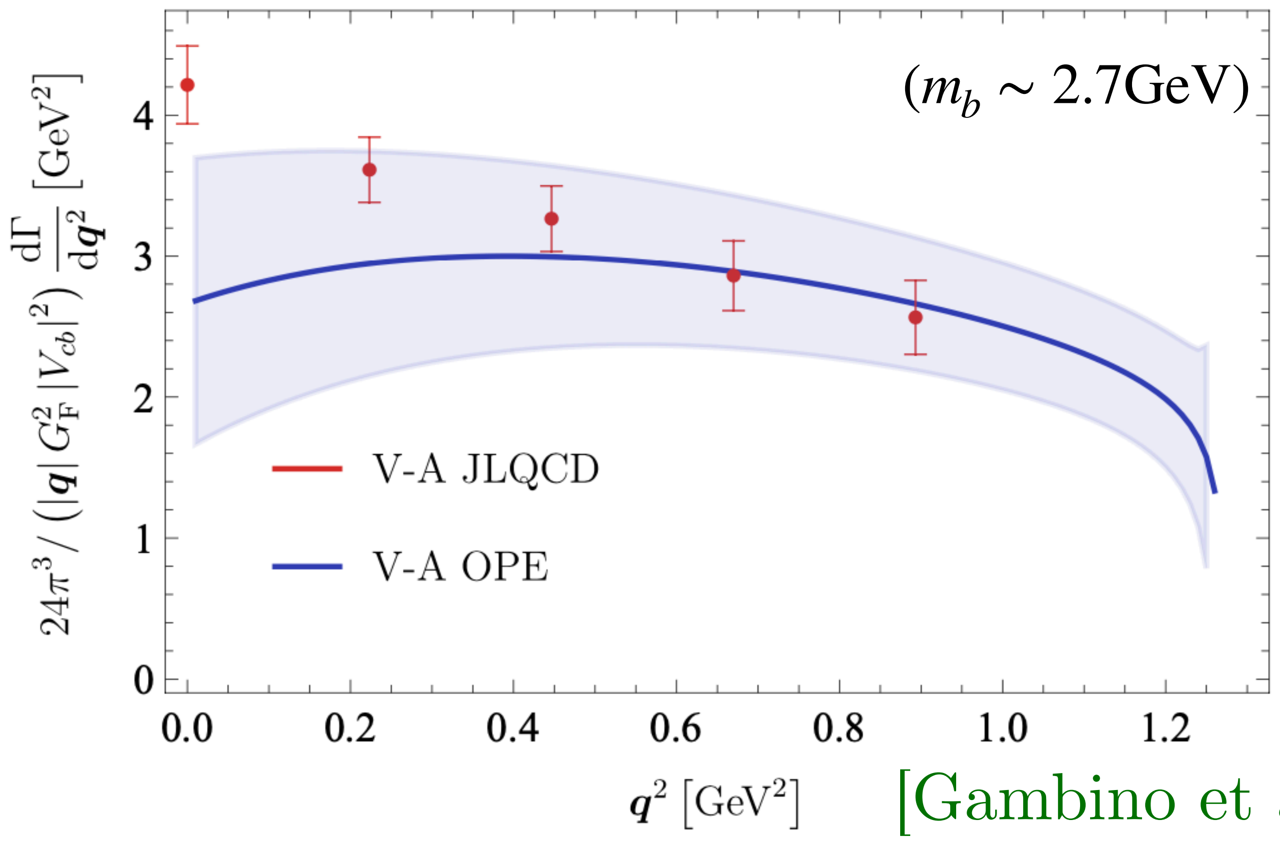


Spectral Reconstruction methods



[Gambino et al 2203.11762]

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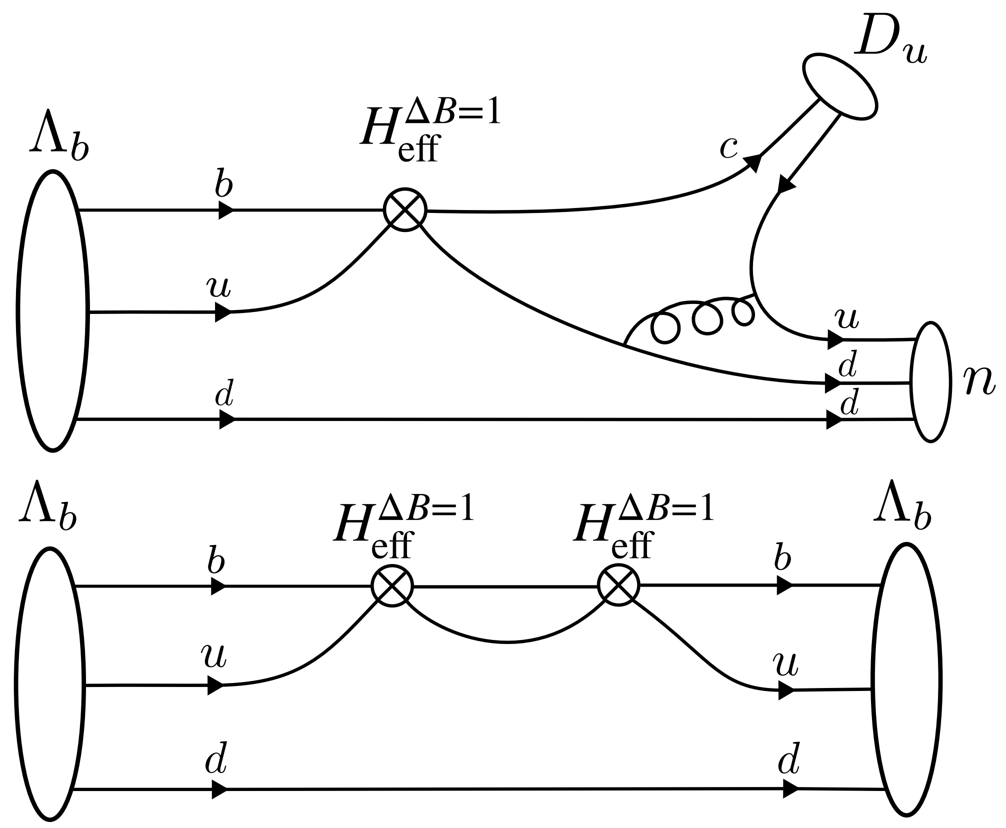

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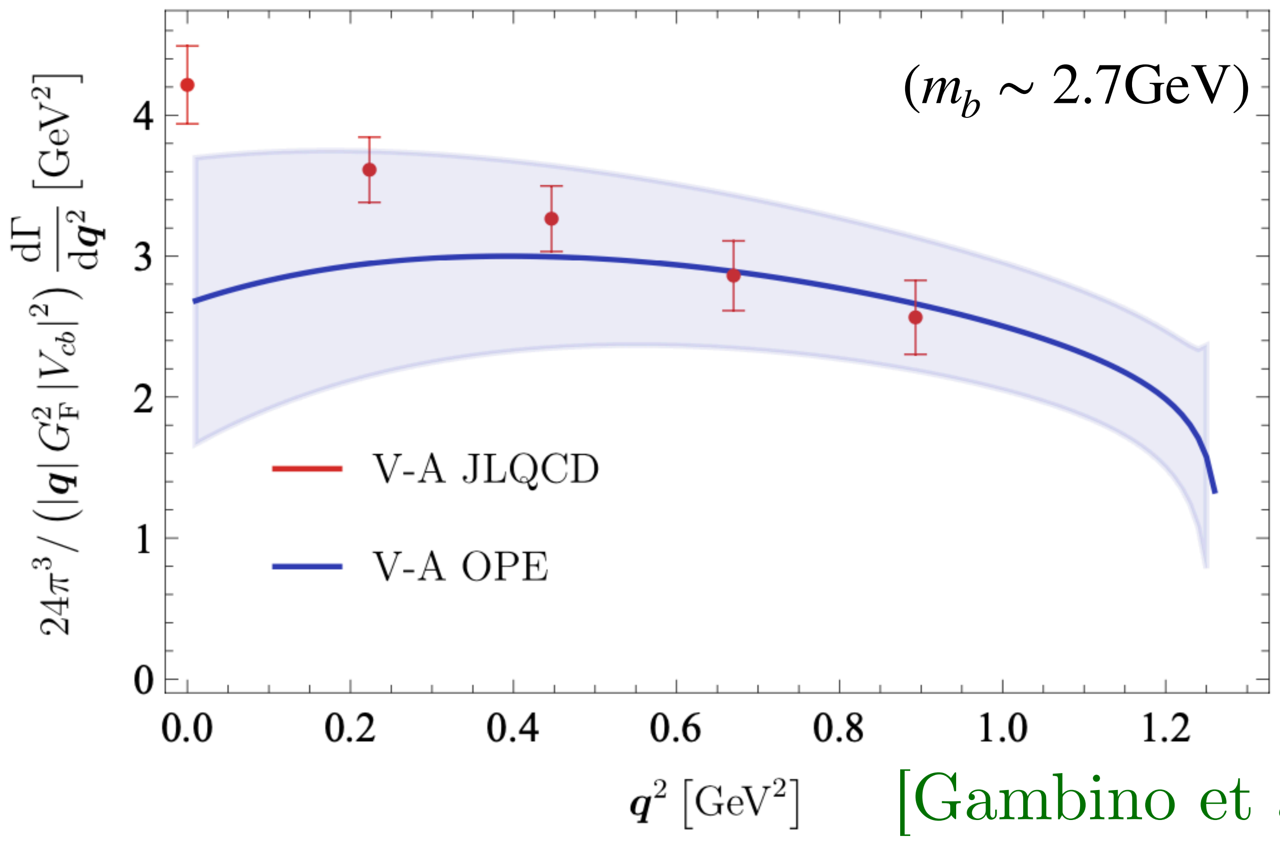

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Operator Product Expansion

$$\hat{T} = \frac{G_F^2|V_{cb}|^2 m_b^5}{192\pi^3}\left[c^{(3)}\bar{b}b + \frac{1}{m_b^2}c^{(5)}g_s\bar{b}\sigma^{\mu\nu}G_{\mu\nu}b + \frac{1}{m_b^3}\sum_k c_k^{(6)}O_k^{(6)} + O(1/m_b^4)\right]$$

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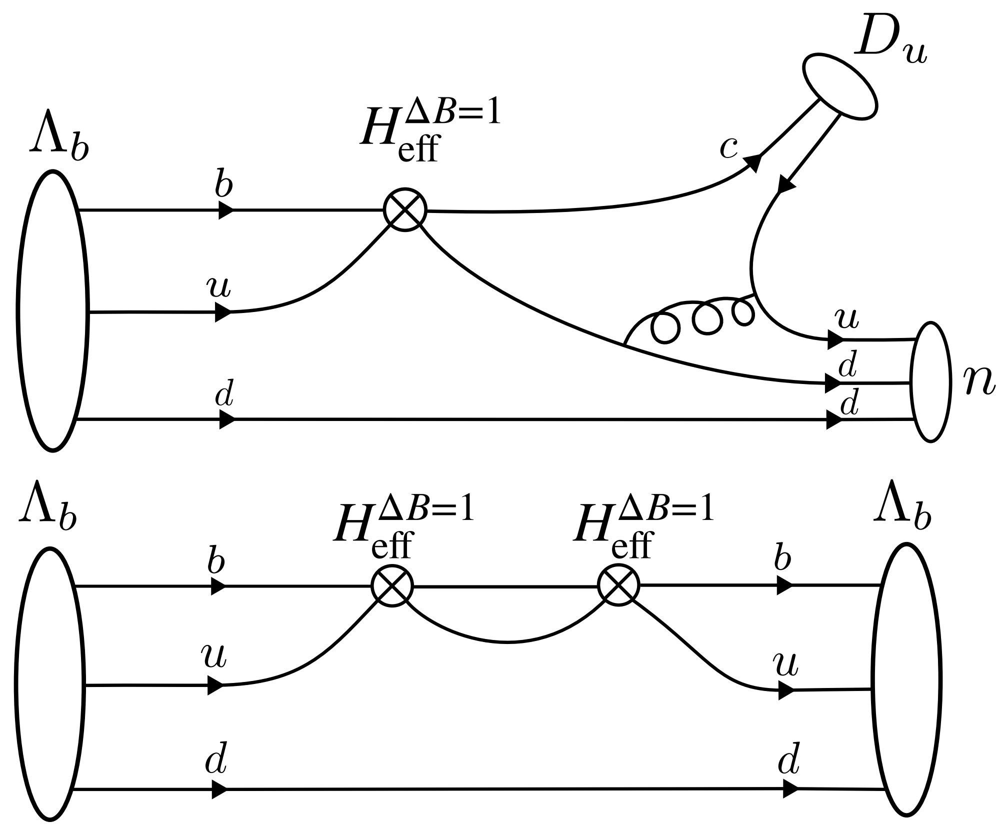

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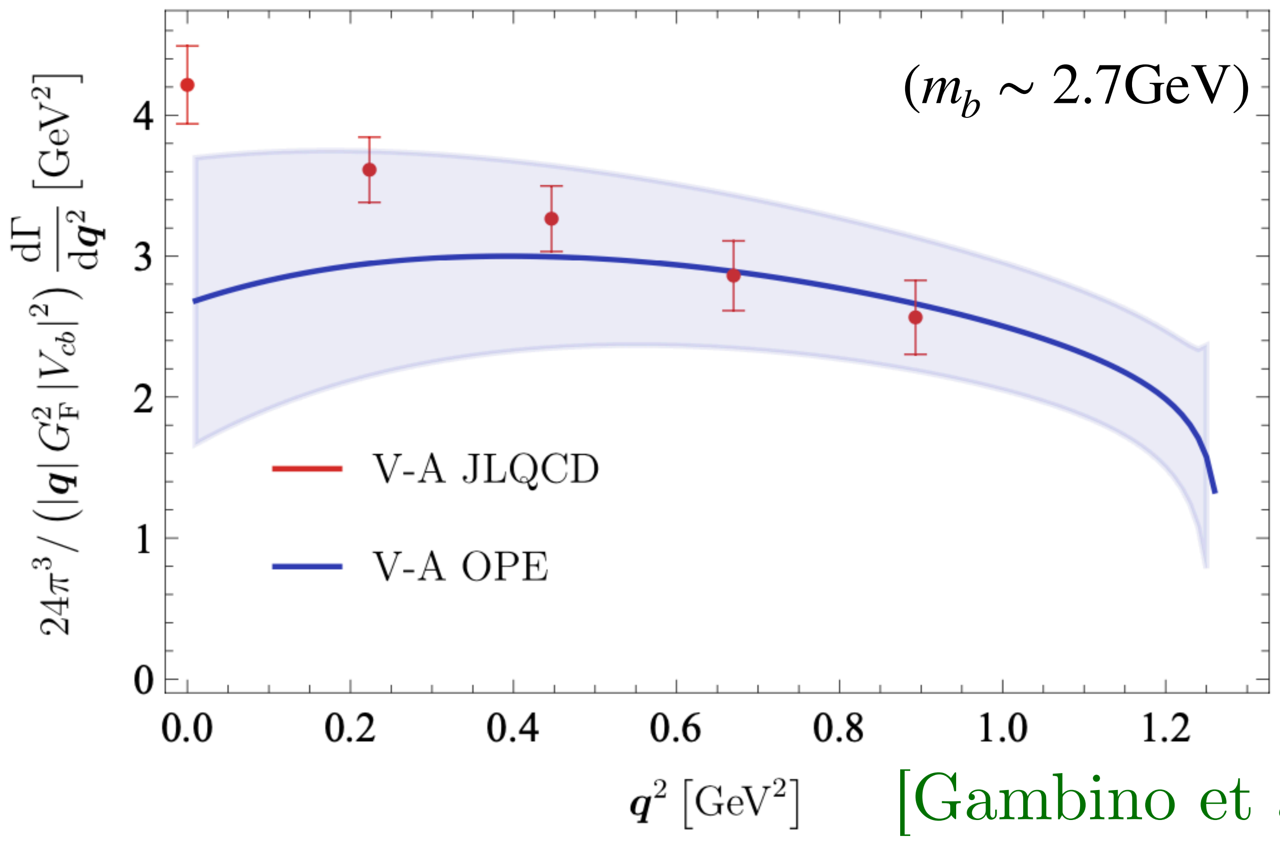

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Gradient Flow Schemes

[Black et al, 2310.18059]

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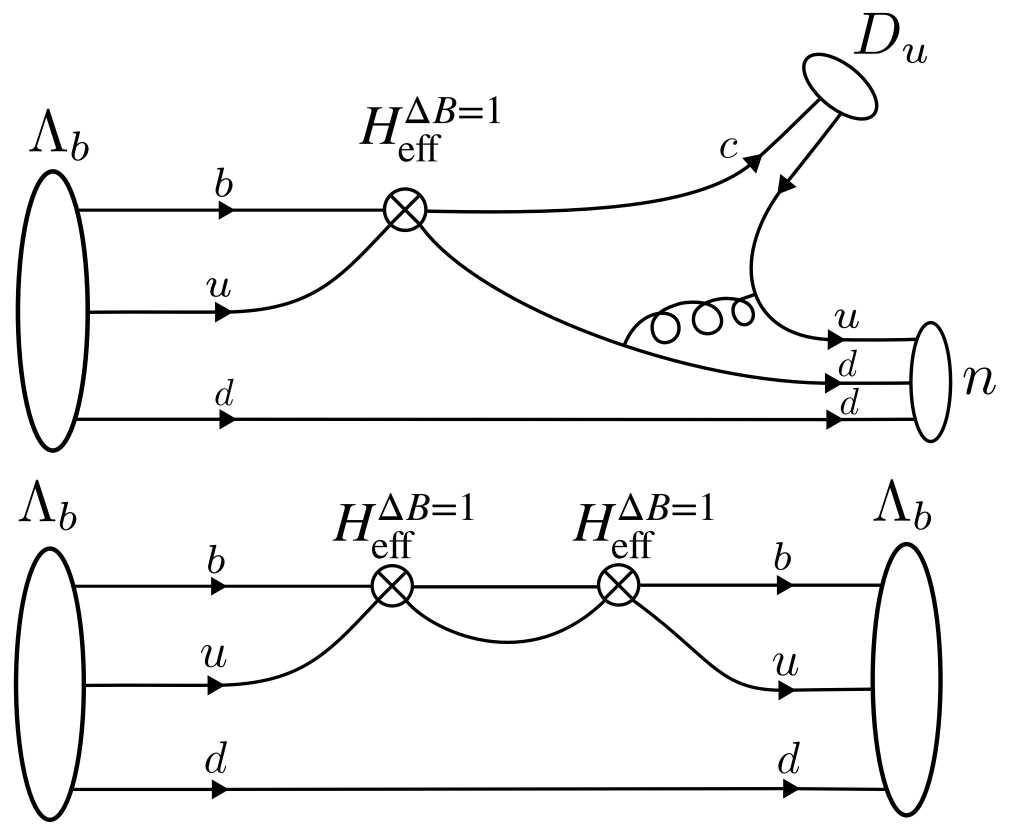

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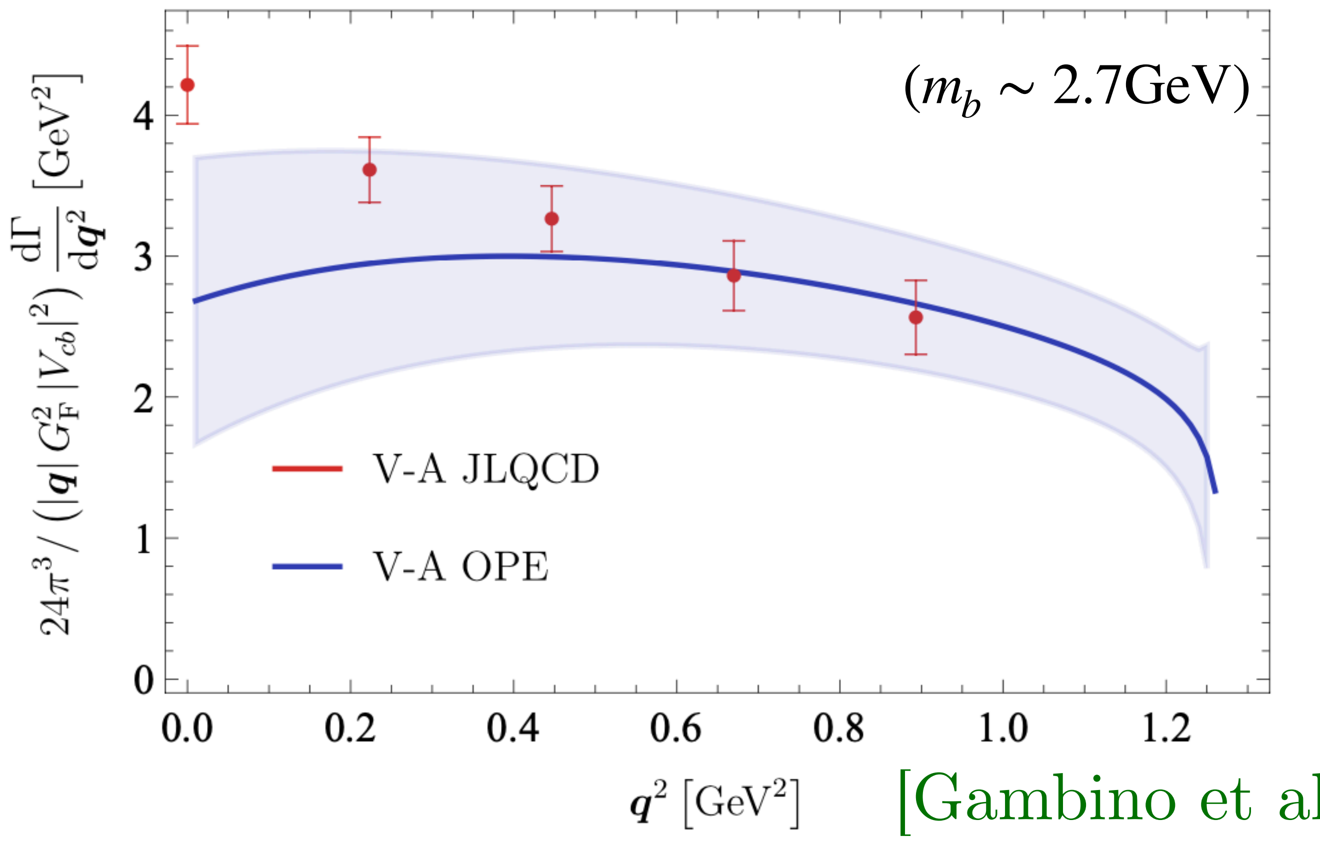

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Fits to physical parameters as functions of m_b
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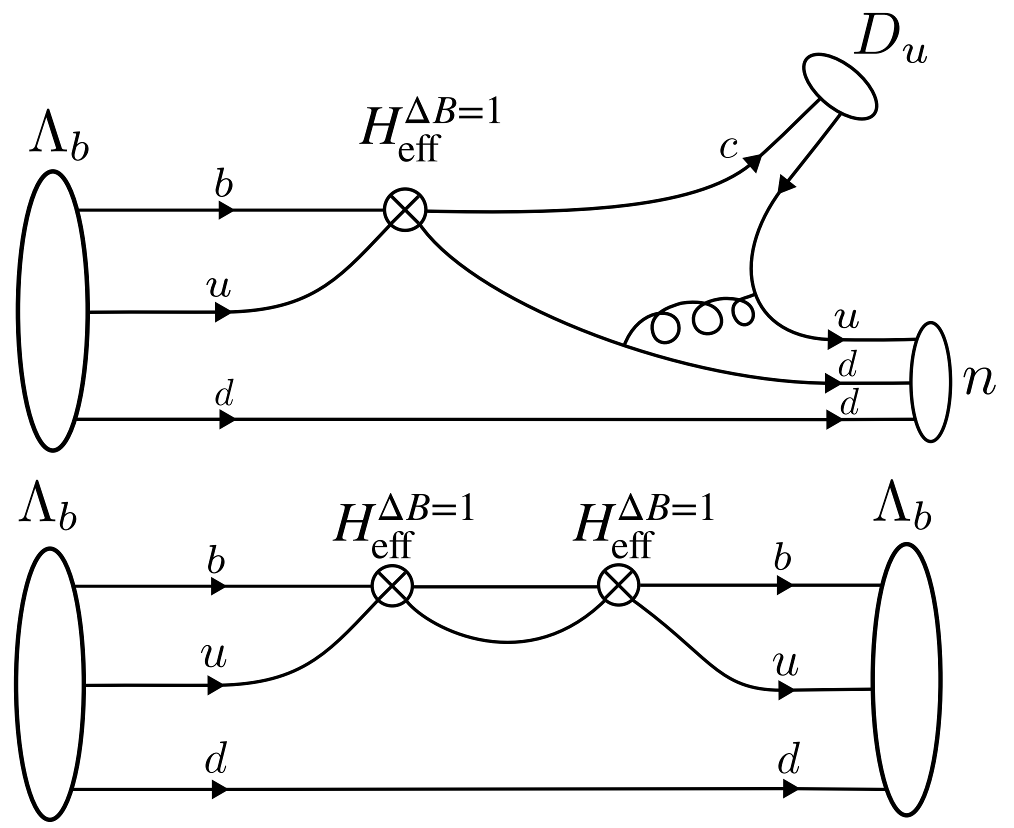

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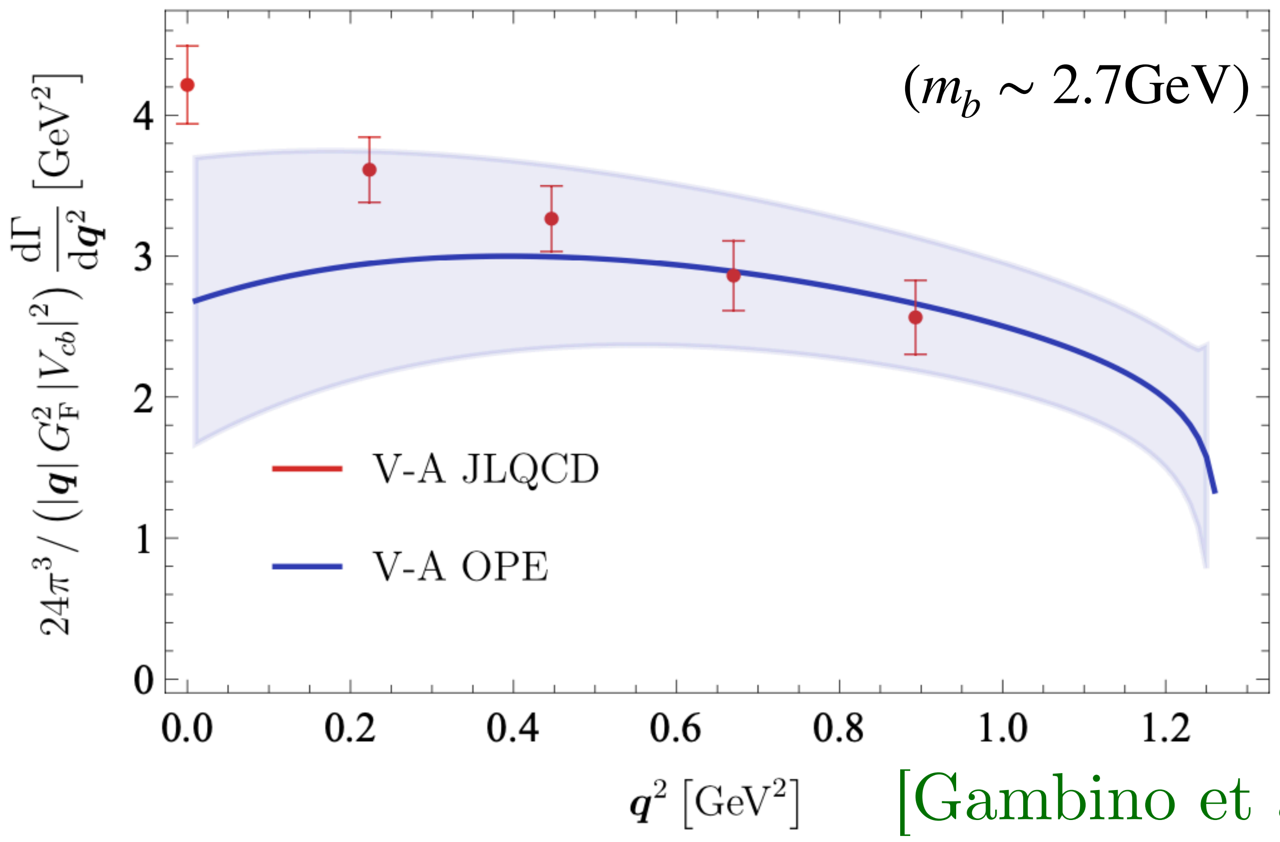

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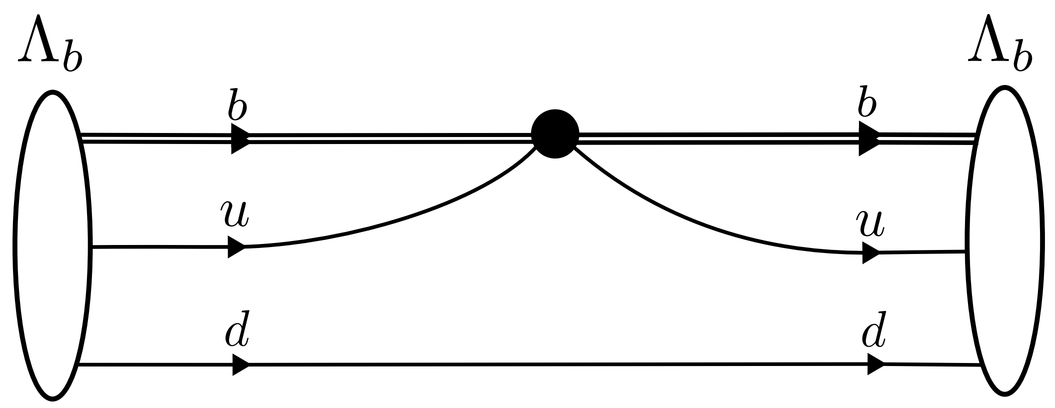
Heavy Quark Effective Theory

Now!



[Pierro, Sachrajda 9805028]

Why now?

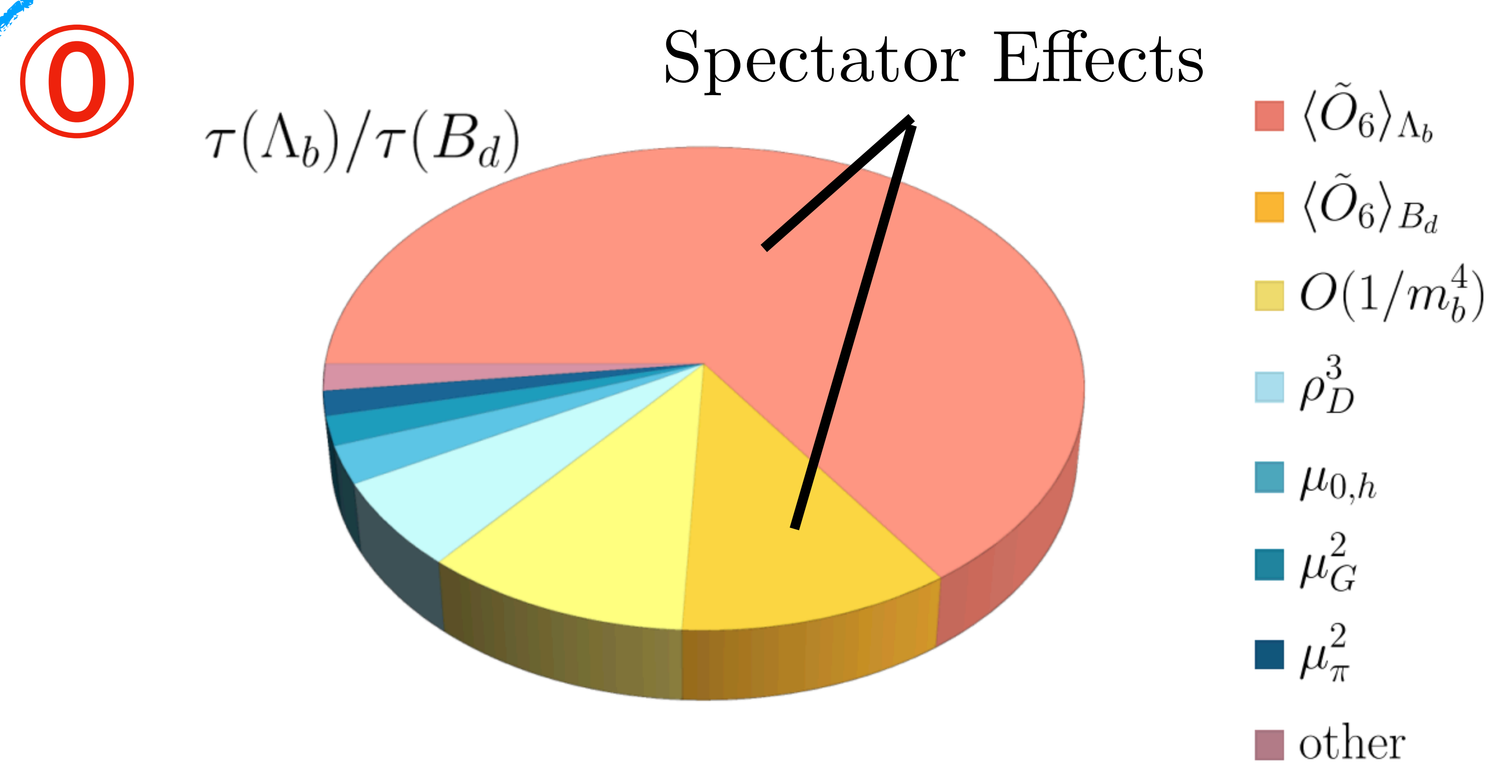


Fig. 9 Composition of the theoretical error in the HQE prediction of the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$.

"Here, clearly, future lattice QCD evaluations of these [dim-6] matrix elements would be highly desirable."

[J. Albrecht et al, 2402.04224]

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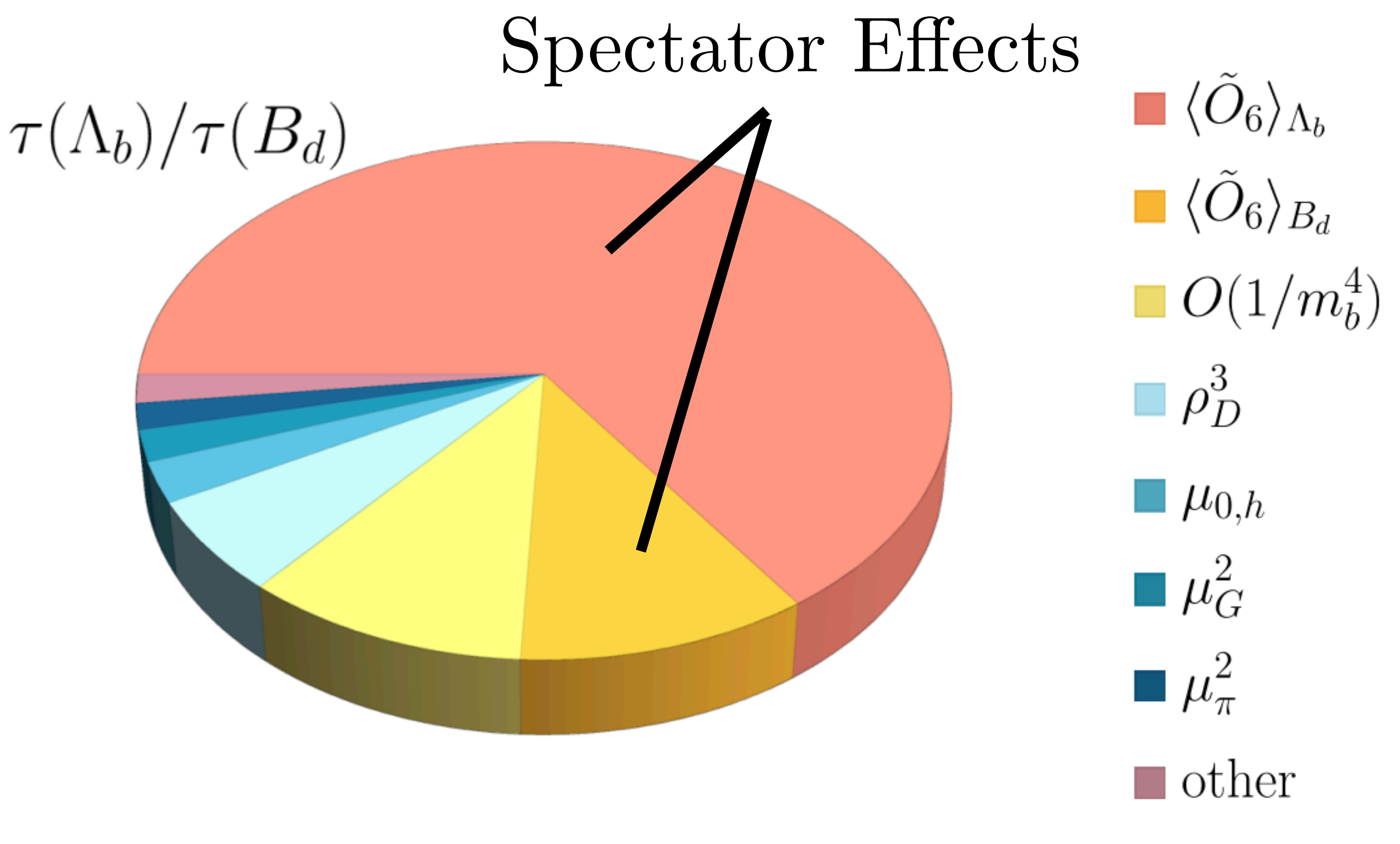
Why now?

⓪



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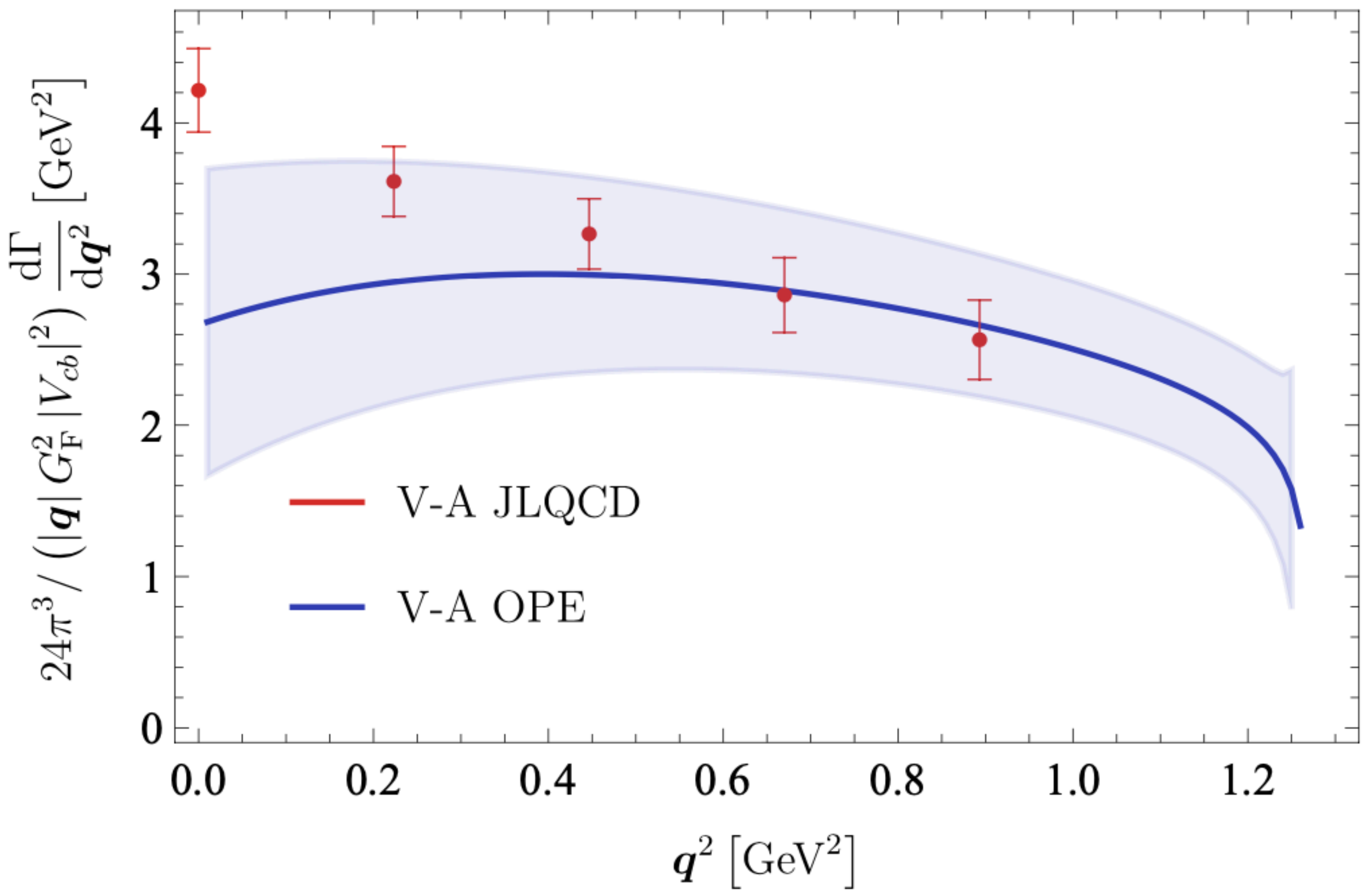

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[Gambino et al 2203.11762]

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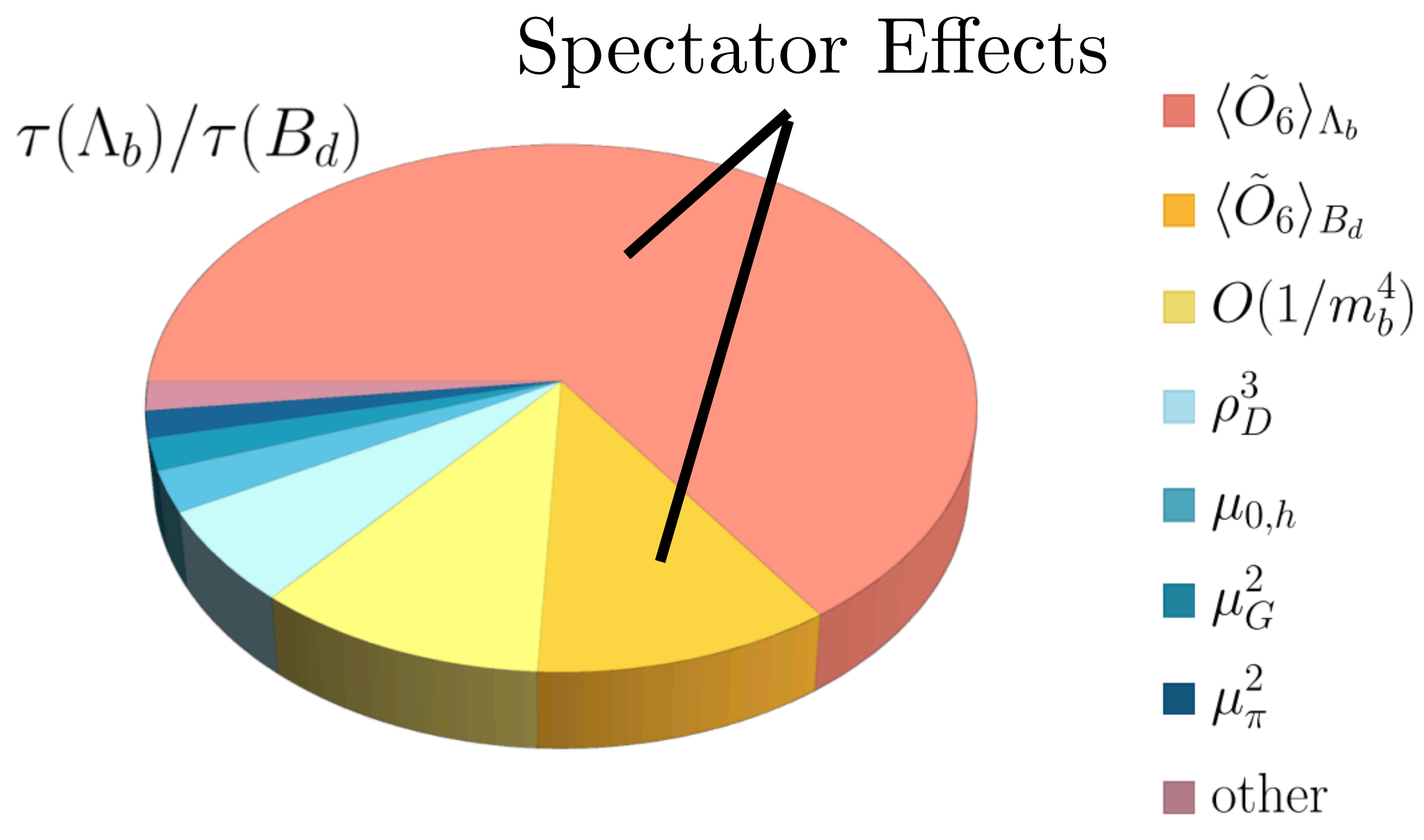

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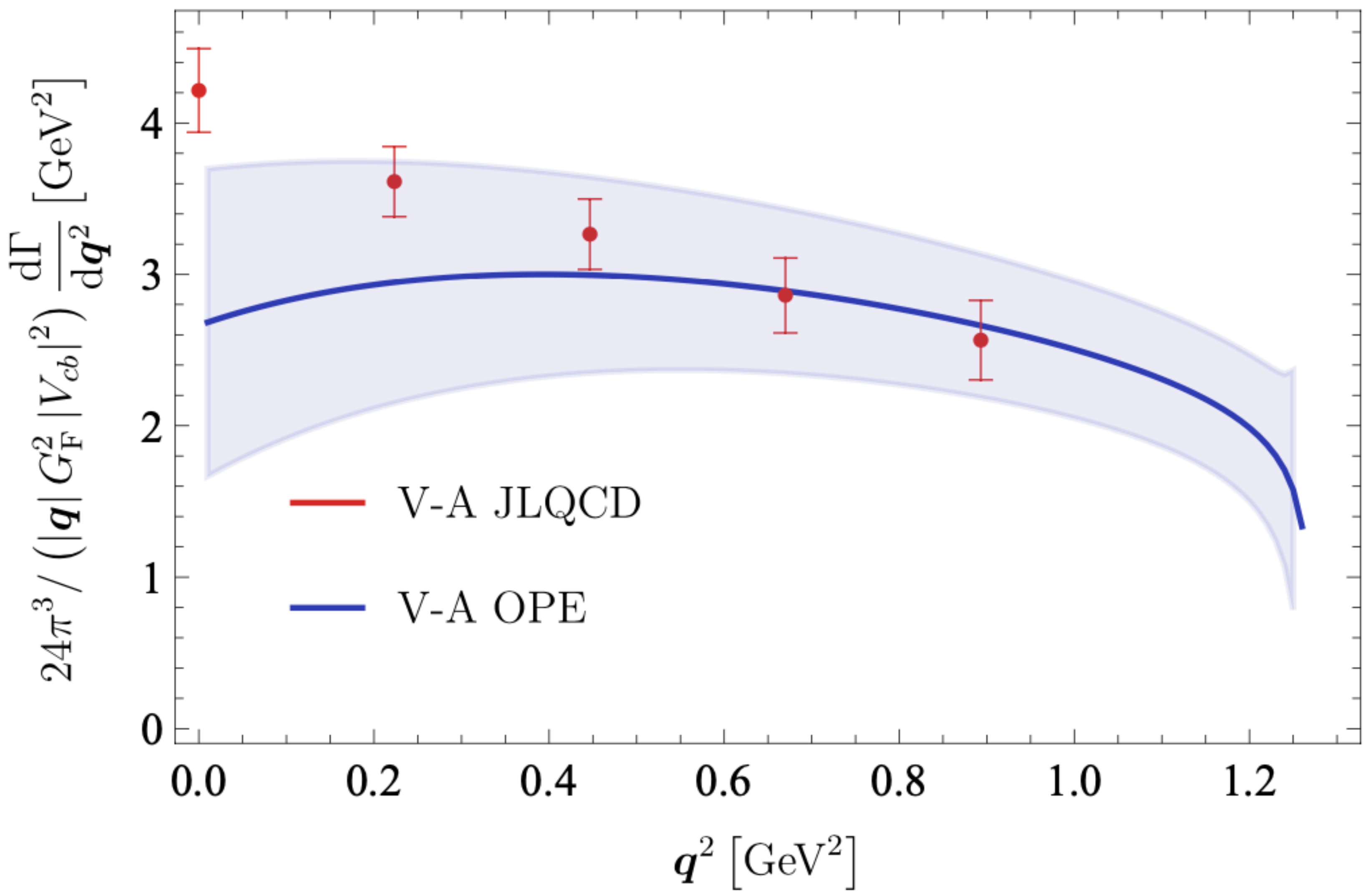


For newer methods (new systematics) it's good to compare against time-tested methods

[Gambino et al 2203.11762]

② Experimental measurements of the inclusive decay rates are performed by fits to OPE matrix elements

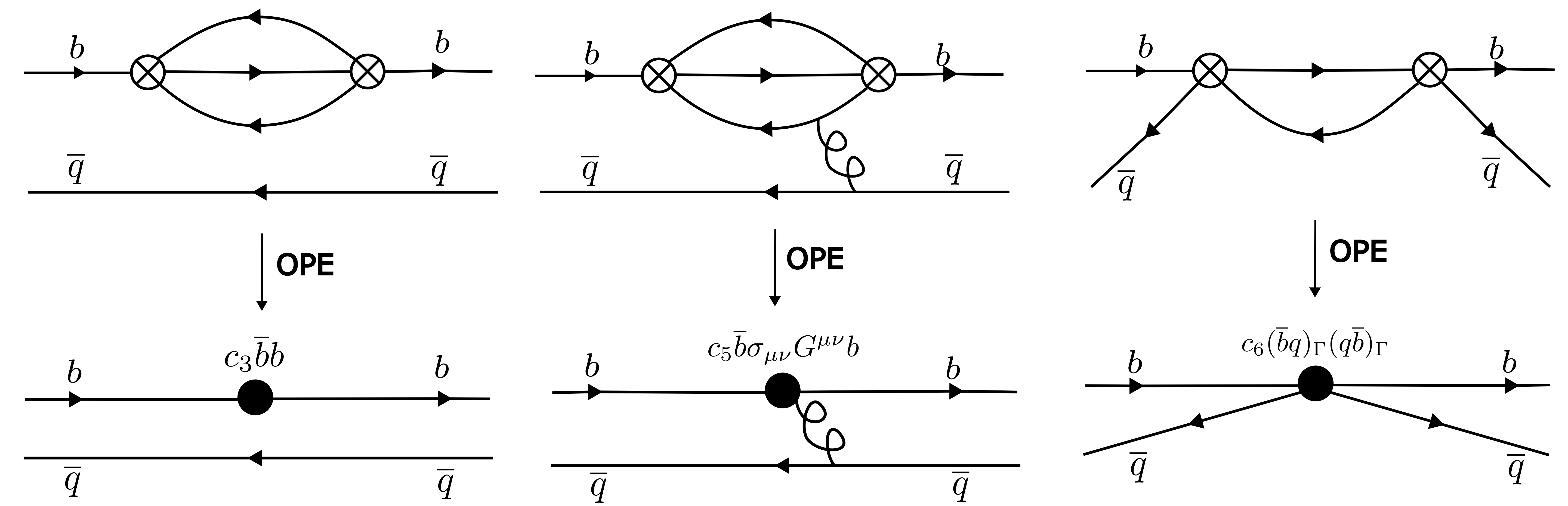
[HFLAV, 1909.12524]

Spectator Effects

[Review: Neubert Sachrajda, hep-ph/9603202]

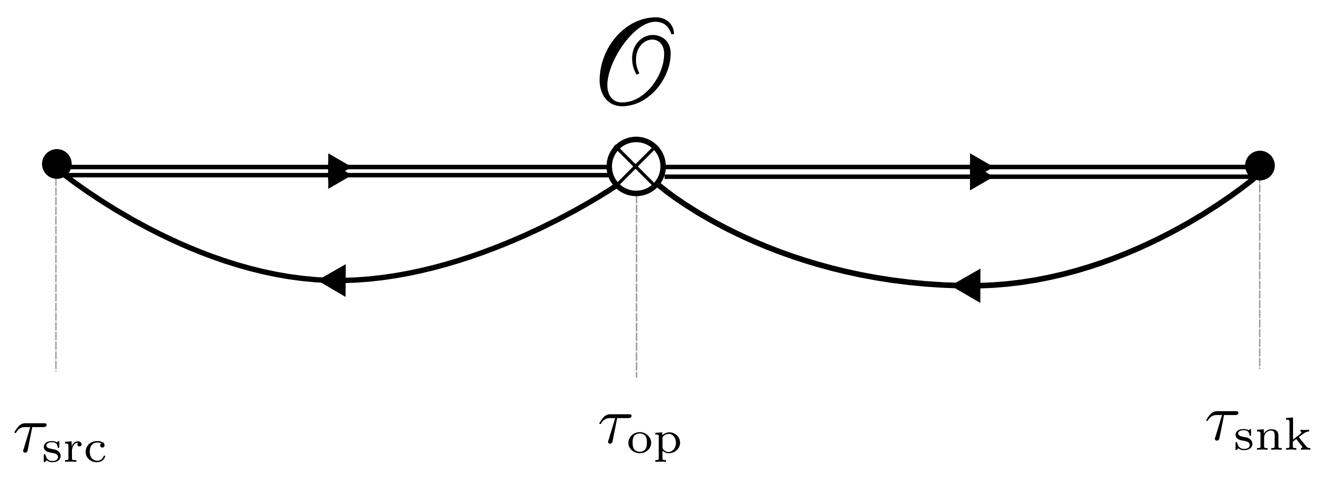
$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3} V_{cb}^2 \left[c_3 \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} + O\left(\frac{1}{m_b^3}\right)\right) + 2c_5 \left(\frac{\mu_G^2}{m_b^2} + O\left(\frac{1}{m_b^3}\right)\right) + c_6 \left(\frac{1}{m_b^3} \frac{\langle B | (\overline{b_+} q)_\Gamma (\overline{q} b_+)_\Gamma | B \rangle}{M_B} + \cdots \right) \right]$$

- μ_π^2, μ_G^2 from masses of hadrons, [I.I. Bigi, Th. Mannel, N. Uraltsev, hep-ph/1105.4574]

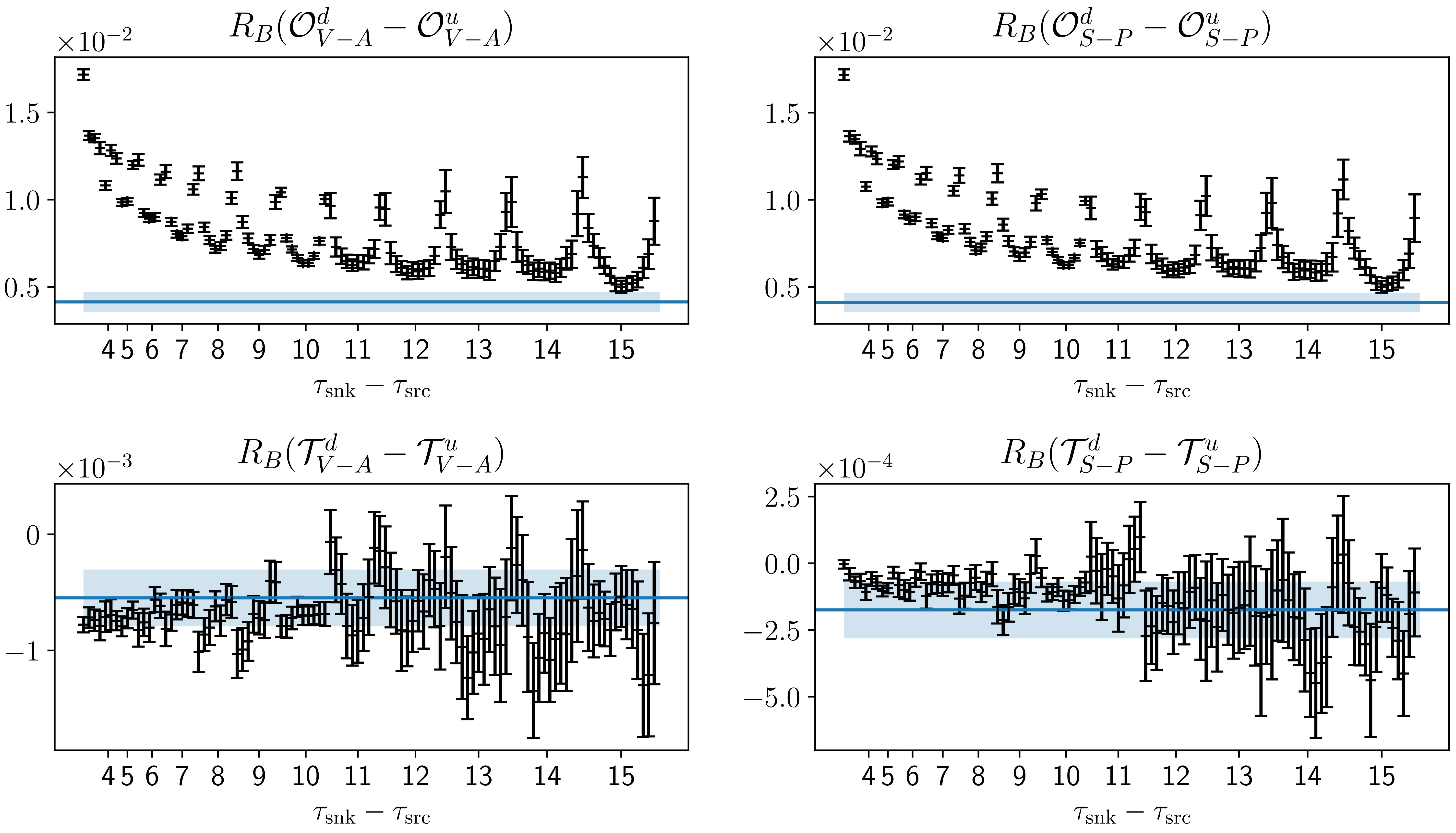


$$O_1 = \left(\overline{b_+}\gamma_\mu P_L q\right)\left(\overline{q}\gamma_\mu P_L b_+\right)$$
$$O_2 = \left(\overline{b_+} P_L q\right)\left(\overline{q} P_R b_+\right)$$
$$O_3 = \left(\overline{b_+} T_a \gamma_\mu P_L q\right)\left(\overline{q} T_a \gamma^\mu P_L b_+\right)$$
$$O_4 = \left(\overline{b_+} T_a P_L q\right)\left(\overline{q} T_a P_R b_+\right)$$

Bare matrix element fits



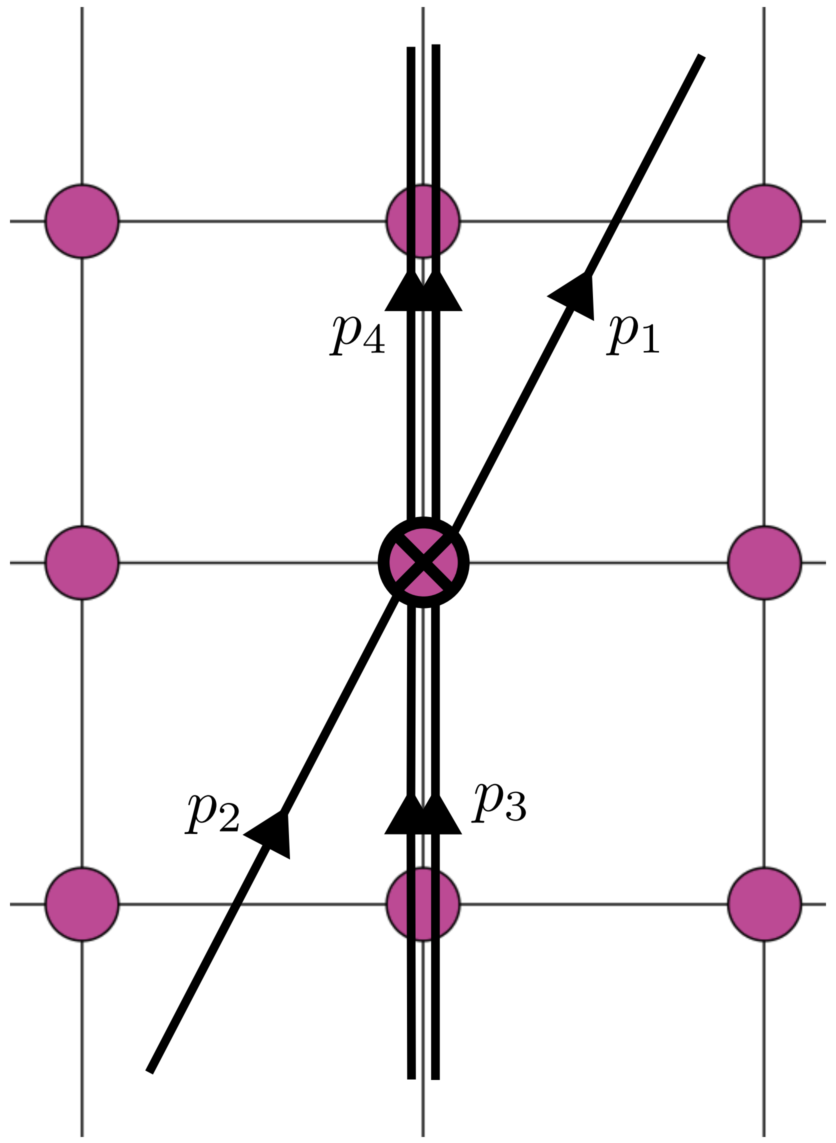
- Correlation functions computed for Wilson flow times $a^{-2}t \in \{0.5, 1.0, 2.0\}$
- Coupled two-point and three-point fits on various fitting ranges, AIC to choose how many excited states to include on each fitting range



[J.Lin, W. Detmold, S. Meinel: 2212.09275]

Renormalizing Lattice-HQET

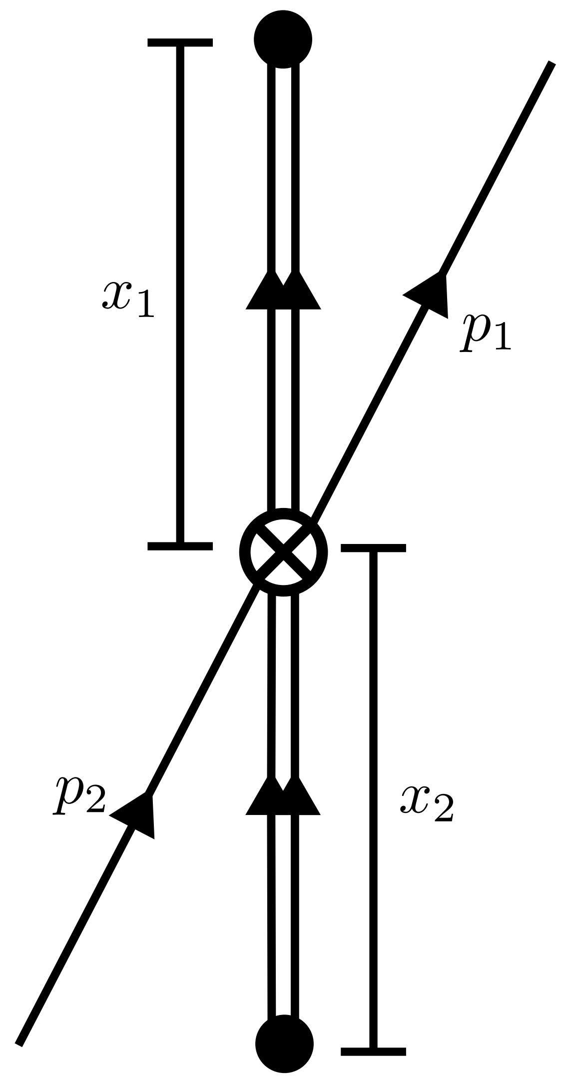
Lattice Perturbation Theory



- [Ishikawa et al, 1101.1072]
  Potentially poor convergence properties

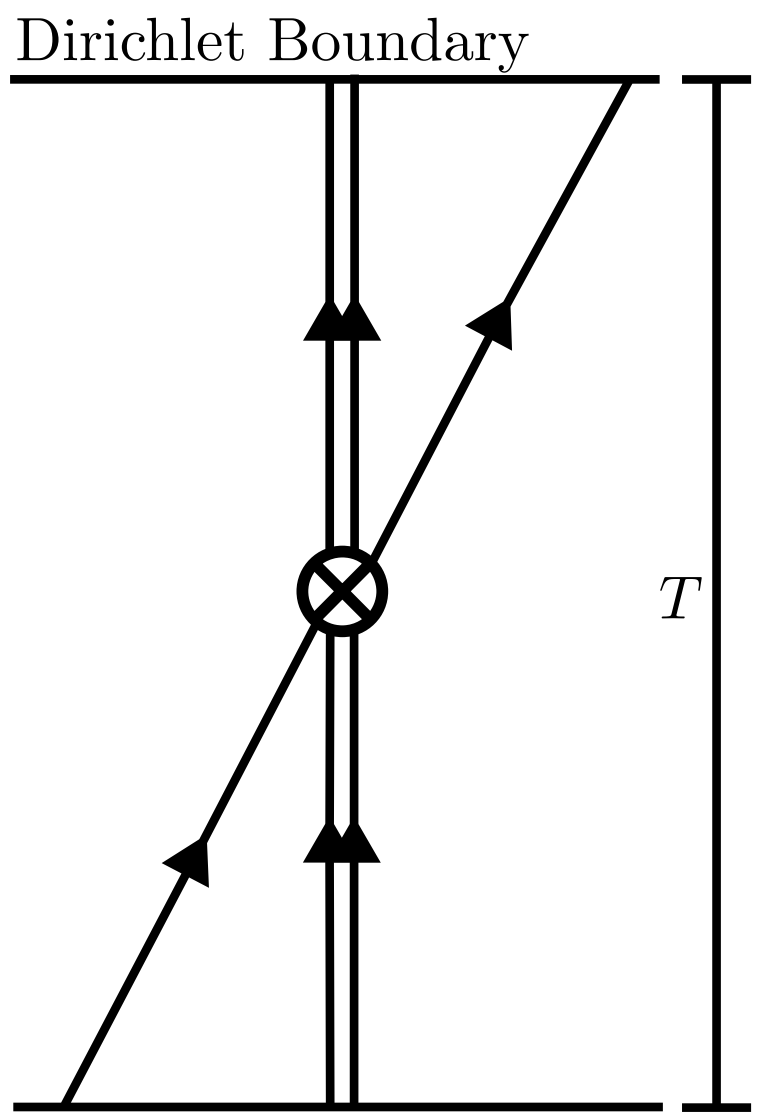
Nonperturbative Renormalization

RI-xMOM



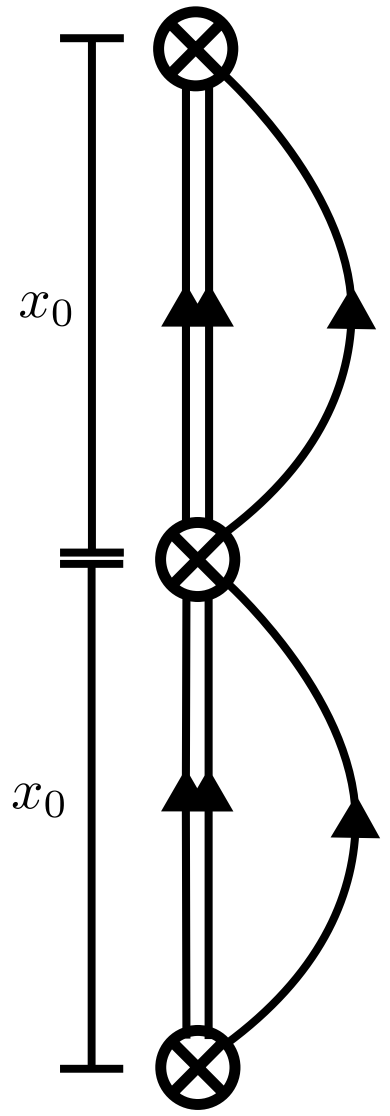
- Long history for light 4q operators (RIMOM)
- Gauge-fixed

Schrodinger-Functional



- Requires additional ensembles + step scaling

X-Space



- 4q HQET operators
  [Lin et al, 2404.16191]
- 4q light operators
  [Constantinou et al, 2406.08065]

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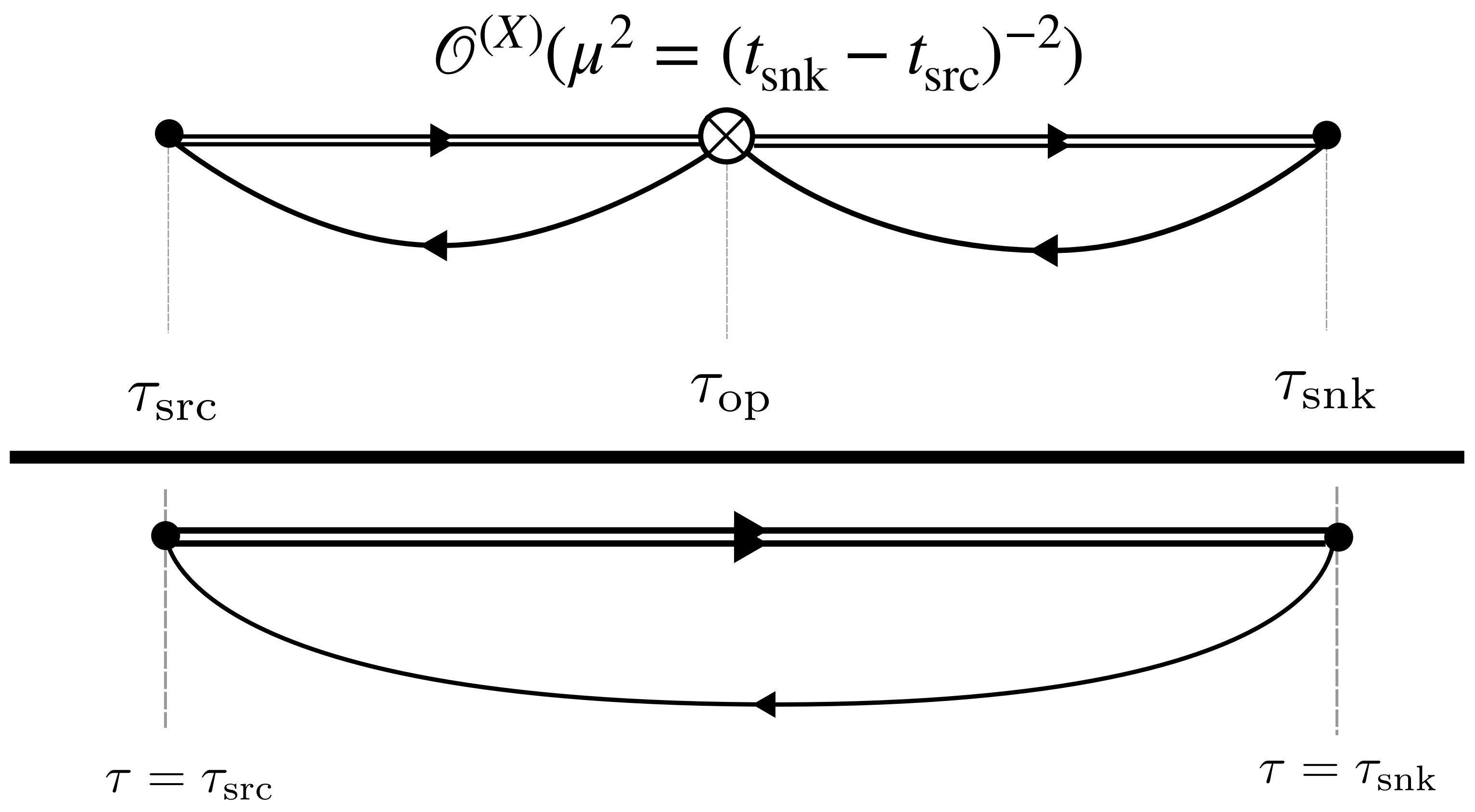
X-space schemes



= Tree level value

$$C^{(\overline{\rm MS};X)}_{ij,n\in\{1,2,3,4\}} := \sum_k Z^{(\overline{\rm MS})}_{ik} (Z^{(X)})^{-1}_{kj,n\in\{1,2,3,4\}} = \mathbb{1} +$$

$$\alpha_S(\mu) \begin{pmatrix} \frac{\log(\beta)}{\pi} + \frac{4\pi}{9} + \frac{8}{3\pi} & 0 & -\frac{3\log(\beta)}{4\pi} + \pi - \frac{3}{8\pi} & -\frac{5}{4\pi} \\ 0 & \frac{\log(\beta)}{\pi} + \frac{4\pi}{9} + \frac{8}{3\pi} & -\frac{1}{16\pi} & -\frac{3\log(\beta)}{4\pi} + \pi - \frac{7}{8\pi} \\ -\frac{\log(\beta)}{6\pi} + \frac{2\pi}{27} - \frac{5}{18\pi} & -\frac{1}{9\pi} & \frac{\log(\beta)}{8\pi} + \frac{7\pi}{18} + \frac{5}{8\pi} & -\frac{1}{12\pi} \\ -\frac{1}{36\pi} & -\frac{\log(\beta)}{6\pi} + \frac{2\pi}{27} - \frac{5}{18\pi} & -\frac{3}{16\pi} & \frac{\log(\beta)}{8\pi} + \frac{7\pi}{18} + \frac{23}{24\pi} \end{pmatrix}$$

$$\left(\beta = e^{2\gamma_E}\frac{\mu^2 t^2}{16}\right)$$

[J.Lin, W. Detmold, S. Meinel: 2404.16191]

- Due to the mixing between the four different operators, four nondegenerate sets of source/sink interpolators are needed to determine the entire mixing matrix.
- Different choices of source/sink sets lead to different renormalization conditions.

n	J_n	K_n
1	H_f^-	H_f^-
2	$H_{f,i}^{*-}$	$H_{f,i}^{*-}$
3	Λ_1	$\Sigma_{2,0}$
4	$\Sigma_{1,\alpha}$	$\Sigma_{1,\alpha}$

$$SU(2)_h \begin{cases} H_f^-(0^-) : \bar{q}_f \gamma_5 Q \\ H_{f,i}^{*-}(1^-) : \bar{q}_f \gamma_i Q \end{cases}$$

$$\Lambda_1 \left(\tfrac{1}{2}^+\right) : \epsilon^{abc}[q^{aT}\tau^A C\gamma_5 q^b]Q^c,$$

$$SU(2)_h \begin{cases} \Sigma_{1,\alpha}\left(\tfrac{1}{2}^+\right) : \epsilon^{abc}[q^{aT}\tau^S_\alpha C\gamma_i q^b]\gamma_i\gamma_5 Q^c, \\ \Sigma^*_{1,\alpha,i}\left(\tfrac{3}{2}^+\right) : \epsilon^{abc}[q^{aT}\tau^S_\alpha C\gamma_i q^b]Q^c - \tfrac{1}{3}\epsilon^{abc}[q^{aT}\tau_A C\gamma_j q^b]\gamma_i\gamma_j Q^c, \end{cases}$$

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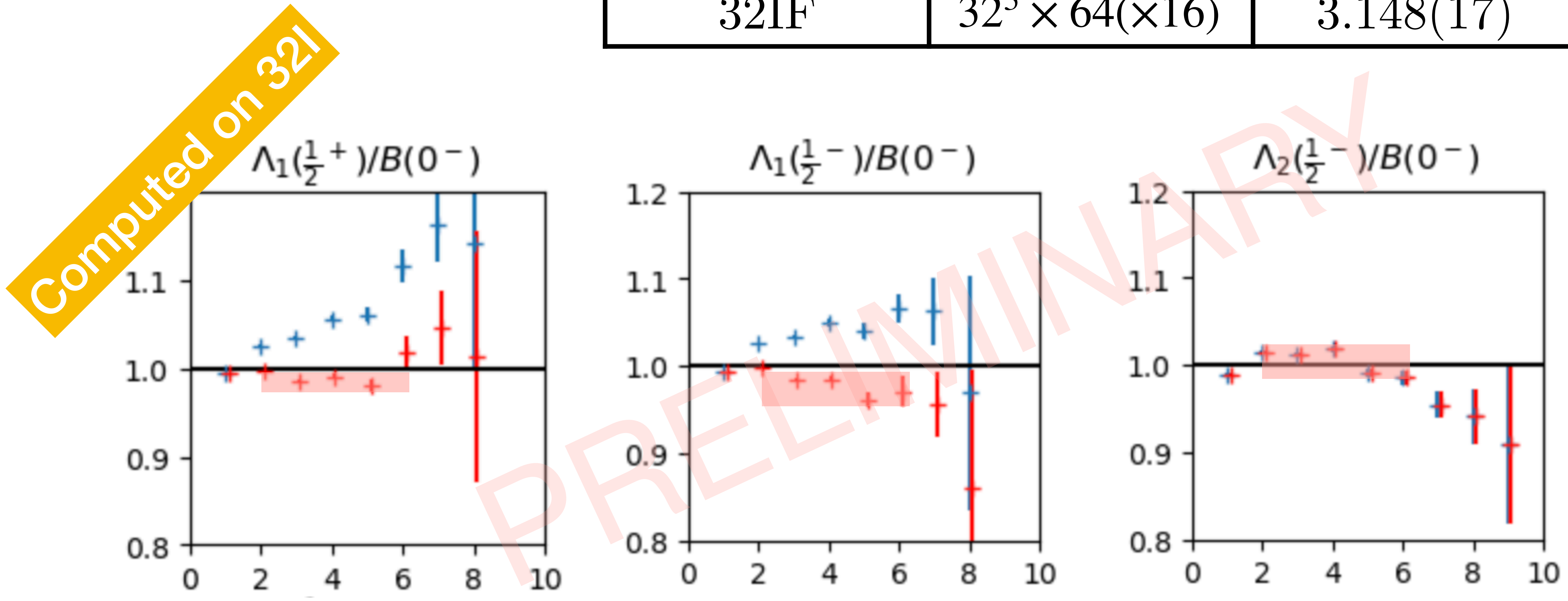
Window Problem (I) : Two-point

- Window Problem where $\sqrt{x^2} \ll \Lambda_{\text{QCD}}^{-1}$ is required to control perturbation-theory errors appearing in the perturbative matching to MS, and $a \ll \sqrt{x^2}$ is required to control discretisation artifacts.

- RBC/UKQCD 2+1f Domain Wall Fermion, Iwasaki action configurations were used [hep-lat/1411.7017],

Name	Volume	1/a (GeV)	$am_{u,d}, am_s$	N
24I	$24^3 \times 64(\times 16)$	1.785(5)	0.005,0.04	228
32I	$32^3 \times 64(\times 16)$	2.383(9)	0.004,0.03	239
32IF	$32^3 \times 64(\times 16)$	3.148(17)	0.0047,0.0186	97

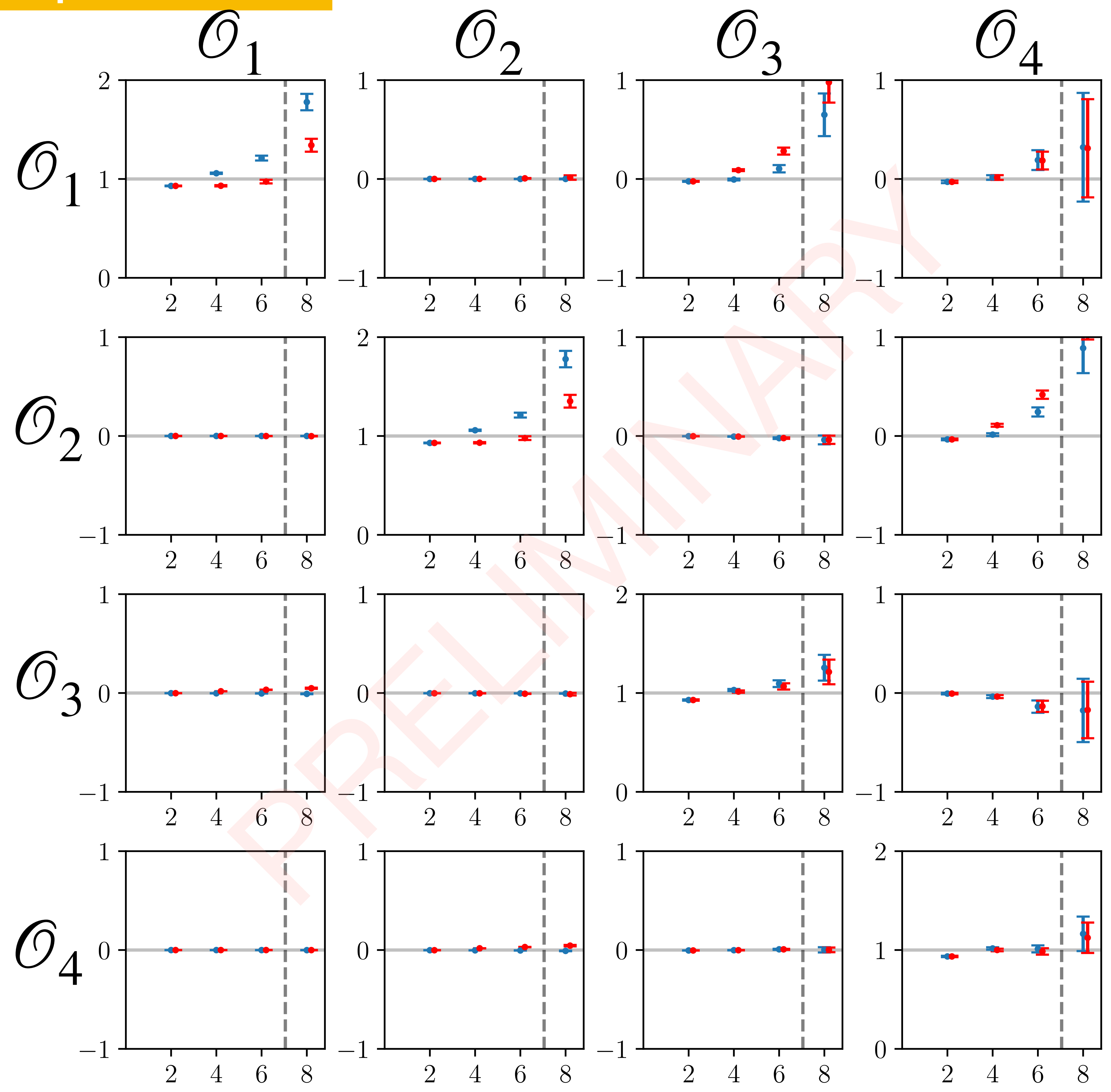


- Ratios of two-point functions cancel the static quark self-energy divergence. Bare ratios of Z-factors shown in blue

- One-loop run values are shown in red, with approximate plateaus for separations $2 \le t \le 6$ shaded

Window Problem (II) : Four quark operators

Computed on 32I



- Blue datapoints : $Z_{ij}(\mu^2 = (t_{\text{snk}} - t_{\text{src}})^{-2})$
- Red datapoints : Renormalization constants RG-run to a common scale using $O(\alpha_S)$ anomalous-dimension.
- Dashed vertical line : $\Lambda_{\text{QCD}}^{-1}$, where perturbation theory should break down.

- As $\Delta t \to 0$, $Z \to$ identity matrix.
- Some non-plateau behaviour in off-diagonal renormalization constants
  - May require $O(\alpha_S^2)$-anomalous dimensions, or $O(a)$-improvement of lattice operators

Outlook

- ☑ Matching between X-space and MS computed for spectator-effect four-quark operators and B-meson mixing four-quark operators in Heavy Quark Effective Field Theory.

- ☑ Window problem investigated, approximate plateaus found.

Bare matrix elements fitted at fixed lattice wilson-flow time → Renormalized in X-space scheme Matched to $\overline{\text{MS}}$-scheme → RG-run to $\mu = m_b$

- ☐ Quantification of errors:
  - ☐ Continuum Limit, chiral extrapolations
  - ☐ $O(a)$ mixing with dimension-7 operators, $O(\alpha_S^2)$-anomalous dimensions

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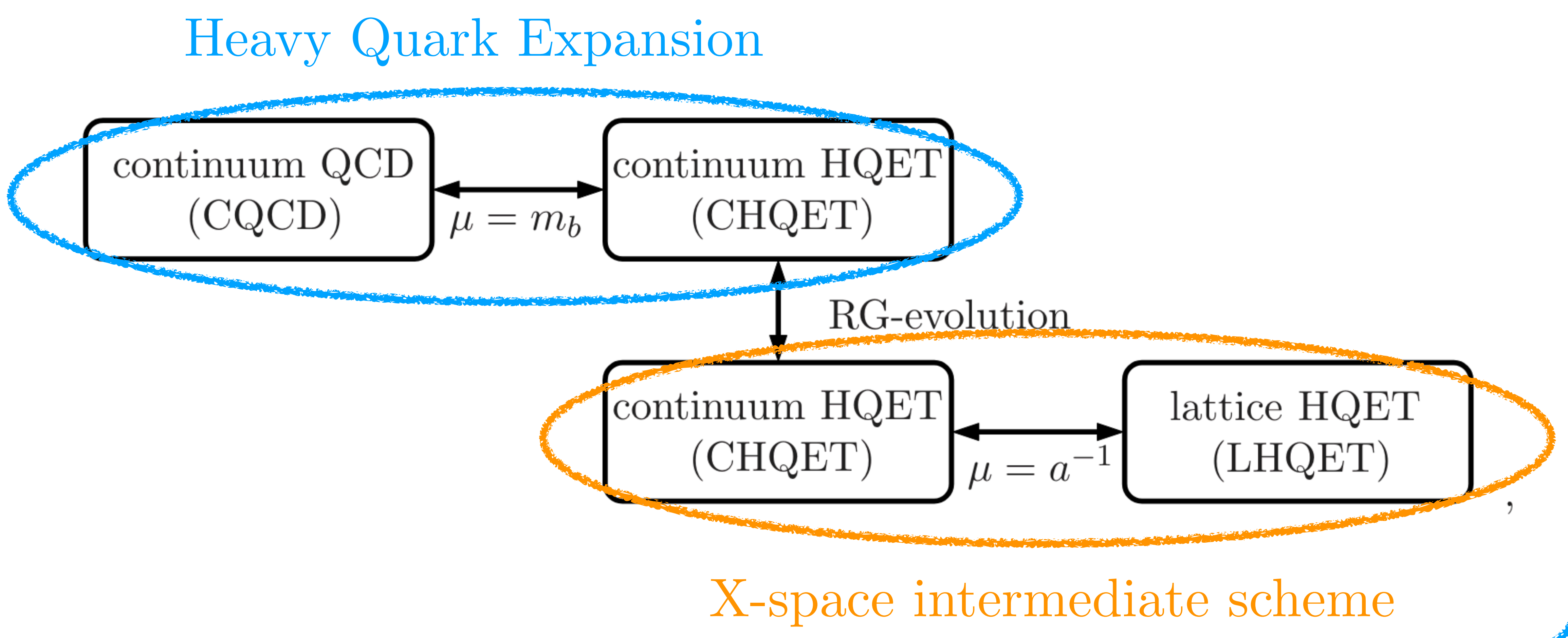
Position-Space Renormalization Scheme

- [Ishikawa et al, 1101.1072] Two-step matching procedure



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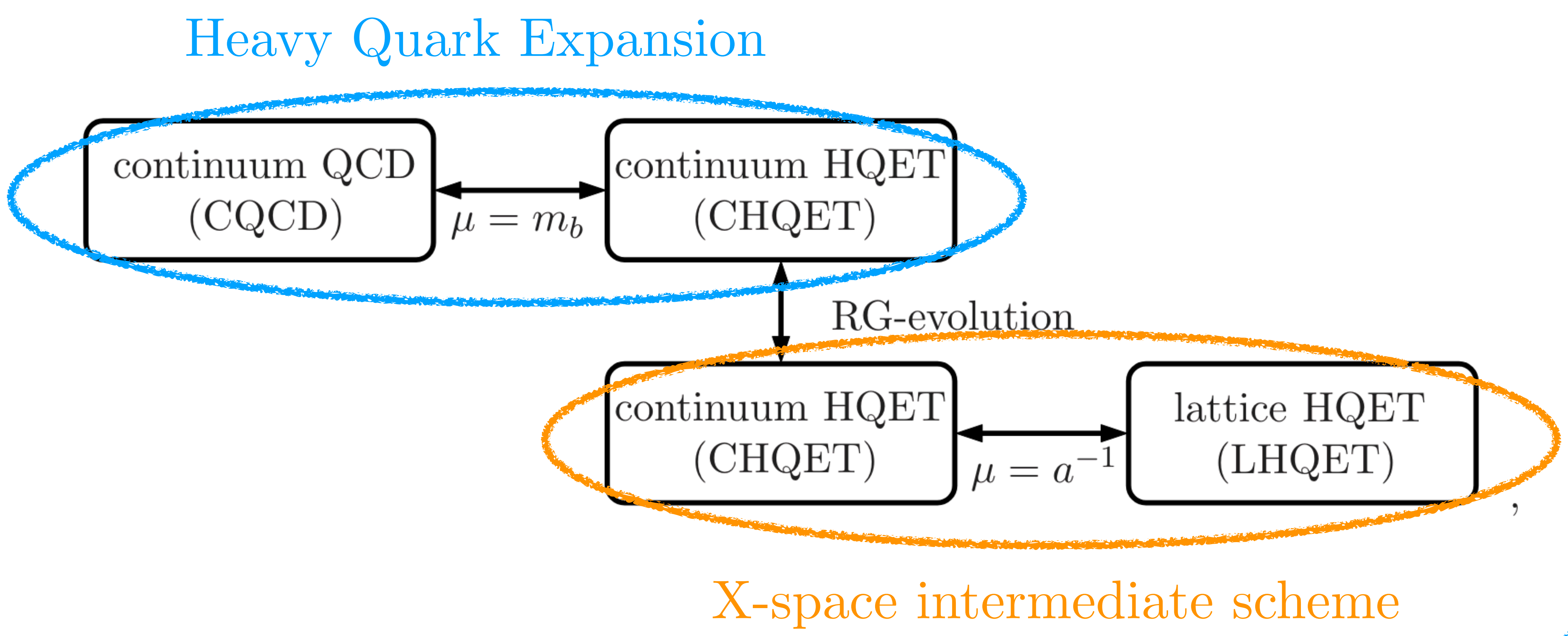
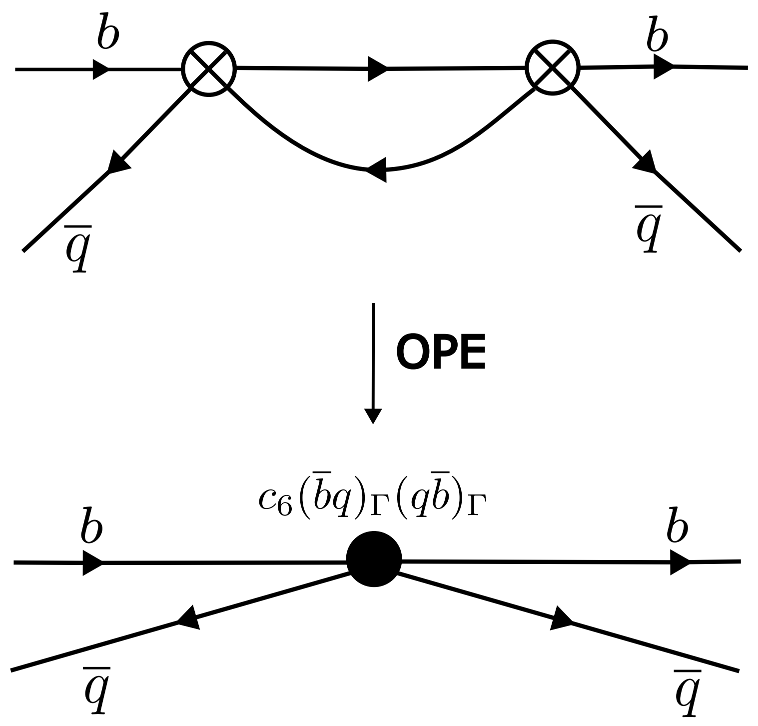


- Dominant $O(1/m_b^3)$ contribution come from phase-space enhanced Spectator Effects where light quarks contribute



$$O_1 = \left(\overline{b_+}\gamma_\mu P_L q\right)\left(\bar{q}\gamma_\mu P_L b_+\right)$$
$$O_2 = \left(\overline{b_+}P_L q\right)\left(\bar{q}P_R b_+\right)$$
$$O_3 = \left(\overline{b_+}T_a\gamma_\mu P_L q\right)\left(\bar{q}T_a\gamma^\mu P_L b_+\right)$$
$$O_4 = \left(\overline{b_+}T_a P_L q\right)\left(\bar{q}T_a P_R b_+\right)$$

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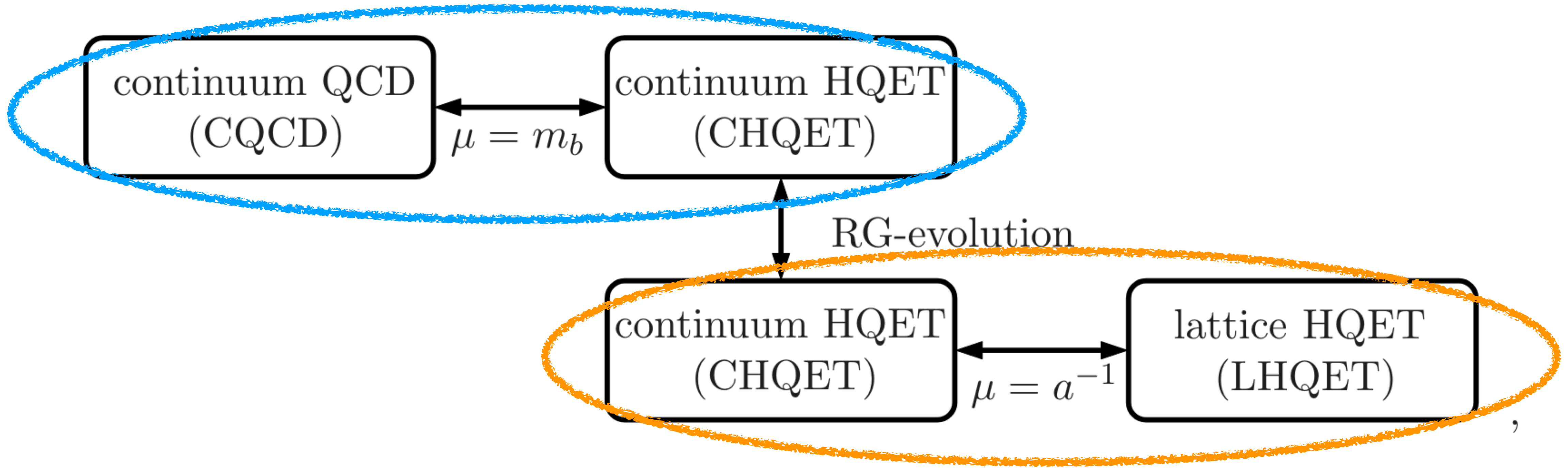
Position-Space Renormalization Scheme

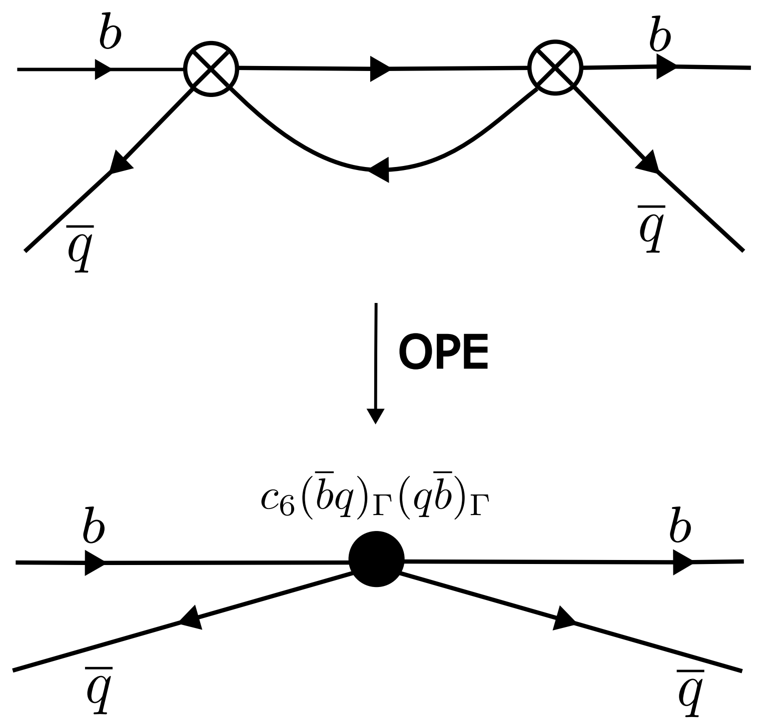
- [Ishikawa et al, 1101.1072] Two-step matching procedure

Heavy Quark Expansion

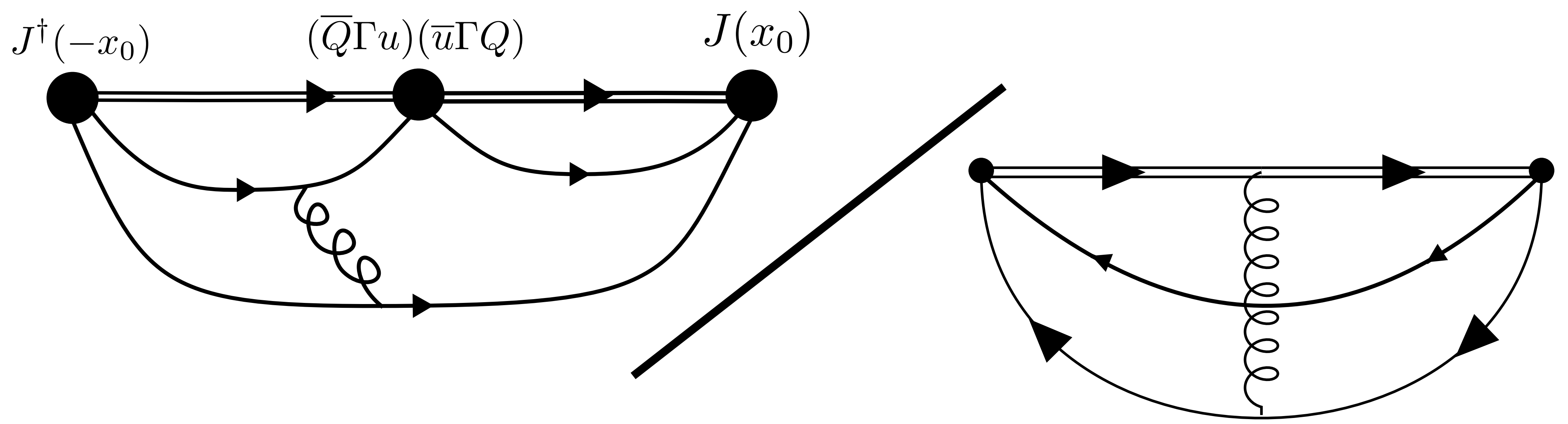


X-space intermediate scheme

- Dominant $O(1/m_b^3)$ contribution come from phase-space enhanced Spectator Effects where light quarks contribute



$$O_1 = \left(\overline{b_+}\gamma_\mu P_L q\right)\left(\overline{q}\gamma_\mu P_L b_+\right)$$
$$O_2 = \left(\overline{b_+}P_L q\right)\left(\overline{q}P_R b_+\right)$$
$$O_3 = \left(\overline{b_+}T_a\gamma_\mu P_L q\right)\left(\overline{q}T_a\gamma^\mu P_L b_+\right)$$
$$O_4 = \left(\overline{b_+}T_a P_L q\right)\left(\overline{q}T_a P_R b_+\right)$$



- The static action suffers from a power-divergent mixing of the kinetic operator $\overline{h}D_0h$ with a mass term $O(\alpha_S)\overline{h}h/a$
- This is cancelled in appropriate ratios!

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Rough schematic of the calculation

b-Hadron Lifetimes

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EFT MEs

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Bare MEs

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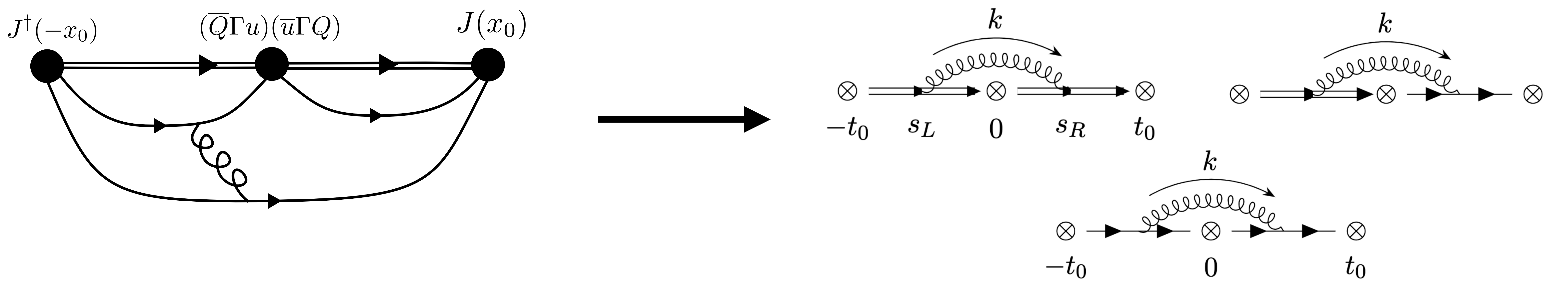
LQCD ensemble

Source-sink pairs: $(H_f^-, H_f^-), (H_{f,i}^{*-}, H_{f,i}^{*-}), (\Lambda_1, \Sigma_{2,0}), (\Lambda_2, \Sigma_{1,0}), (\Sigma_{1,\alpha}, \Sigma_{1,\alpha}), (\Sigma_{2,\alpha}, \Sigma_{2,\alpha}), (\Sigma^*_{1,\alpha,i}, \Sigma^*_{1,\alpha,i}), (\Sigma^*_{2,\alpha,i}, \Sigma^*_{2,\alpha,i})$



$$\int \frac{d^dp_L}{(2\pi)^d}\frac{d^dp_R}{(2\pi)^d}\frac{d^dk}{(2\pi)^d}\frac{e^{ip_Lx_L-ip_Rx_R}\, p_R^\alpha(p_R-k)^\beta(p_L-k)^\rho p_L^\delta}{(-p_L^2)(-(p_L-k)^2)(-(p_R-k)^2)(-p_R^2)(-k^2)}$$

$$\int \frac{d^dp_L}{(2\pi)^d}\frac{d^dp_R}{(2\pi)^d}\frac{d^dk}{(2\pi)^d}\frac{e^{ix_Lp_L}e^{-ix_Rp_R}\, p_R^\alpha(p_R-k)^\beta}{(-p_R^2)(-(p_R-k)^2)(-k^2)(v\cdot(p_L-k))(v\cdot p_L)}$$

→ Integration by parts

$$\int d^dk\left(\frac{\partial}{\partial k_\mu}\cdot p_\mu\right)\circ f(k) = 0$$

→ Two-loop master integrals

[J.Lin, W. Detmold, S. Meinel: 2404.16191]

Master integrals and blocks

* Master Integrals look like:

$$T_{LL}(x_L, x_R; n_1, n_2, n_3) = \int \frac{d^d p_L d^d p_R}{(2\pi)^{2d}} \frac{e^{ip_L x_L} e^{-ip_R x_R}}{(-p_L^2)^{n_1}(-p_R^2)^{n_2}(-(p_L - p_R)^2)^{n_3}}$$

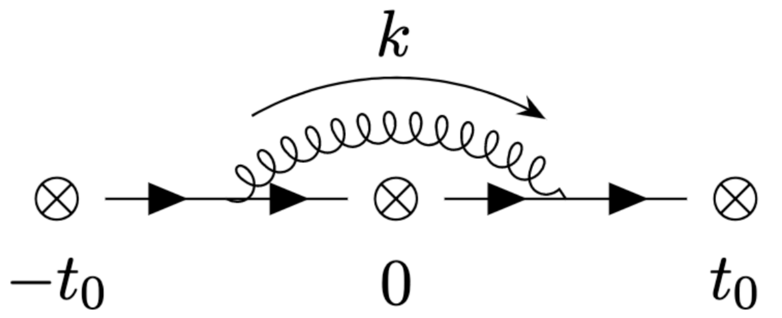
(agrees with [M.Costa et al. hep-lat/2102.00858])

$$= \frac{-\Gamma(\frac{d}{2} - n_1)\Gamma(d - n_1 - n_2 - n_3)}{\Gamma(n_2)\Gamma(n_3)\Gamma(\frac{d}{2})4^{n_1+n_2+n_3}\pi^d}(-x_R^2)^{-d+n_1+n_2+n_3} \int_0^1 \mathrm{d}x(1 - x_1)^{-\frac{d}{2}+n_1+n_2-1} x_1^{-\frac{d}{2}+n_1+n_3-1} {}_2F_1\left(\frac{d}{2} - n_1, d - n_1 - n_2 - n_3, \frac{d}{2}, \frac{-(x_L - x_1 x_R)^2}{x_1(1 - x_1)x_R^2}\right)$$

$$T_{HH}(x_L, x_R; n_1, n_2, n_3) = \int \frac{d^d p_L d^d p_R}{(2\pi)^{2d}} \frac{e^{ip_L x_L} e^{-ip_R x_R}}{(v \cdot p_L)^{n_1}(v \cdot p_R)^{n_2}(-(p_L - p_R)^2)^{n_3}}$$

$$= \frac{1}{4^{n_3}\pi^{\frac{d}{2}}} \frac{\Gamma(\frac{d}{2} - n_3)}{\Gamma(n_3)\Gamma(n_1 + n_2)} {}_2F_1\left(n_1, d - 2n_3, n_1 + n_2, \frac{v \cdot (x_L - x_R)}{v \cdot x_L}\right)(-iv \cdot (x_R - x_L))^{-1+n_1+n_2}(-(v \cdot x_L)^2)^{-\frac{d}{2}+n_3}\delta_\perp(v \cdot (x_L - x_R))\theta(v \cdot (xR - xL) > 0)$$

* Building blocks look like:



$$= g^2(\mu^2)^{\frac{4-d}{2}}(-t_0^2)^{8-\frac{7d}{4}}\left[\left(\frac{-1}{32\pi^6\epsilon} + \frac{10\log 2 - 3\log\pi - 3\gamma_E}{64\pi^6}\right)(\not{v})(\not{v}) + \left(\frac{-1}{32\pi^6\epsilon} + \frac{3 + 4\log 2 - 6\log\pi - 6\gamma_E}{128\pi^6}\right)(\gamma_\mu)(\gamma_\mu) + \left(\frac{1}{128\pi^6\epsilon} + \frac{-1 - 2\log 2 + 3\log\pi + 3\gamma_E}{256\pi^6\epsilon}\right)(\gamma_\alpha\gamma_\beta\not{v})(\gamma_\alpha\gamma_\beta\not{v})\right]$$

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Evanescents

b-Hadron Lifetimes

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EFT MEs

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Bare MEs

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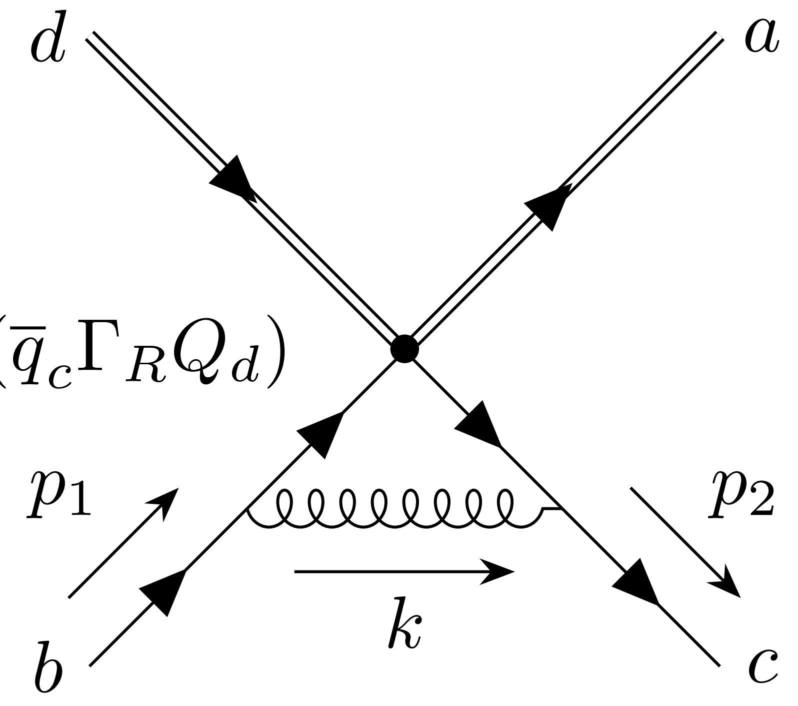
LQCD ensemble

- In any dimensional-regularisation scheme, you have to contend with evanescent operators:

$(\overline{Q}_a\Gamma_L q_b)(\overline{q}_c\Gamma_R Q_d)$



$$= \int \frac{d^dk}{(2\pi)^d}\frac{-i}{k^2+i\epsilon}\left(\frac{1+\not{v}}{2}\Gamma_L\frac{i}{(\not{p}-\not{k})+i\epsilon}(ig\gamma_\mu T^A)\right)_{ab} \times \left((ig\gamma_\mu T^A)\frac{i}{(\not{p}-\not{k})+i\epsilon}\Gamma_R\frac{1+\not{v}}{2}\right)_{cd}$$

$$= \frac{\alpha_S}{8\pi\epsilon}\left(\frac{1+\not{v}}{2}\Gamma_L\gamma^\mu\gamma^\nu T^A\right)_{ab}\left(T^A\gamma^\nu\gamma^\mu\Gamma_R\frac{1+\not{v}}{2}\right)_{cd} + O(\epsilon^0)$$

- These are operators that vanish at d=4, but still have finite contribution to any renormalisation scheme

$$E_1 := (\overline{Q}\gamma_\mu P_L\gamma_\alpha\gamma_\beta q)(\overline{q}\gamma_\beta\gamma_\alpha\gamma_\mu P_L Q) - 4O_1,$$
$$E_2 := (\overline{Q}P_L\gamma_\alpha\gamma_\beta q)(\overline{q}\gamma_\beta\gamma_\alpha P_R Q) - 4O_2,$$
$$E_3 := (\overline{Q}\gamma_\mu P_L\gamma_\alpha\gamma_\beta T^A q)(\overline{q}\gamma_\beta\gamma_\alpha\gamma_\mu P_L T^A Q) - 4O_3,$$
$$E_4 := (\overline{Q}P_L\gamma_\alpha\gamma_\beta T^A q)(\overline{q}\gamma_\beta\gamma_\alpha P_R T^A Q) - 4O_4\,.$$

- For regularisation independent schemes, you need to match evanescent-subtracted operators (as continuum evanescent operators have no analogue in lattice discretisations)

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Matching Coefficients

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b-Hadron Lifetimes

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EFT MEs

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Bare MEs

↕

LQCD ensemble

Four-quark operators determining spectator effects in inclusive lifetimes

$$\left(\beta := e^{2\gamma_E}\frac{\mu^2 t^2}{16}\right)$$

$$O_1^f := (\overline{Q}\gamma_\mu P_L q_f)(\overline{q}_f \gamma_\mu P_L Q),$$
$$O_2^f := (\overline{Q} P_L q_f)(\overline{q}_f P_R Q),$$
$$O_3^f := (\overline{Q}\gamma_\mu P_L T^A q_f)(\overline{q}_f \gamma_\mu P_L T^A Q),$$
$$O_4^f := (\overline{Q} P_L T^A q_f)(\overline{q}_f P_R T^A Q),$$

$$C^{(\overline{\mathrm{MS}};X)}_{ij,n\in\{1,2,3,4\}} := \sum_k Z^{(\overline{\mathrm{MS}})}_{ik} (Z^{(X)})^{-1}_{kj,n\in\{1,2,3,4\}} = \mathbb{1} + \alpha_S(\mu) \begin{pmatrix} \frac{\log(\beta)}{\pi} + \frac{4\pi}{9} + \frac{8}{3\pi} & 0 & -\frac{3\log(\beta)}{4\pi} + \pi - \frac{3}{8\pi} & -\frac{5}{4\pi} \\ 0 & \frac{\log(\beta)}{\pi} + \frac{4\pi}{9} + \frac{8}{3\pi} & -\frac{1}{16\pi} & -\frac{3\log(\beta)}{4\pi} + \pi - \frac{7}{8\pi} \\ -\frac{\log(\beta)}{6\pi} + \frac{2\pi}{27} - \frac{5}{18\pi} & -\frac{1}{9\pi} & \frac{\log(\beta)}{8\pi} + \frac{7\pi}{18} + \frac{5}{8\pi} & -\frac{1}{12\pi} \\ -\frac{1}{36\pi} & -\frac{\log(\beta)}{6\pi} + \frac{2\pi}{27} - \frac{5}{18\pi} & -\frac{3}{16\pi} & \frac{\log(\beta)}{8\pi} + \frac{7\pi}{18} + \frac{23}{24\pi} \end{pmatrix}$$

Four-quark operators determining B-mixing

$$O_1 := (\overline{Q}_+ P_L q)(\overline{Q}_- P_L q),$$
$$O_2 := (\overline{Q}_+ P_L T^A q)(\overline{Q}_- P_L T^A q),$$
$$O_3 := (\overline{Q}_+ P_L q)(\overline{Q}_- P_R q),$$
$$O_4 := (\overline{Q}_+ P_L T^A q)(\overline{Q}_- P_R T^A q)\,.$$

$$C^{(\overline{\mathrm{MS}},X)}_{\{O_1,O_2\}} = \mathbb{1} + \frac{\alpha_S}{\pi} \begin{pmatrix} \frac{7\log(\beta)}{9} + \frac{4\pi^2}{9} + \frac{23}{9} & \frac{2\log(\beta)}{3} - \frac{\pi^2}{3} + \frac{4}{3} \\ \frac{4\log(\beta)}{27} - \frac{2\pi^2}{27} + \frac{8}{27} & \frac{5\log(\beta)}{9} + \frac{5\pi^2}{9} + \frac{19}{9} \end{pmatrix}$$

$$C^{(\overline{\mathrm{MS}},X)}_{\{O_3,O_4\}} = \mathbb{1} + \frac{\alpha_S}{\pi} \begin{pmatrix} \log(\beta) + \frac{4\pi^2}{9} + \frac{25}{9} & \frac{3\log(\beta)}{4} - \frac{\pi^2}{3} + \frac{7}{6} \\ \frac{\log(\beta)}{6} - \frac{2\pi^2}{27} + \frac{7}{27} & \frac{3\log(\beta)}{4} + \frac{5\pi^2}{9} + \frac{43}{18} \end{pmatrix}$$

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