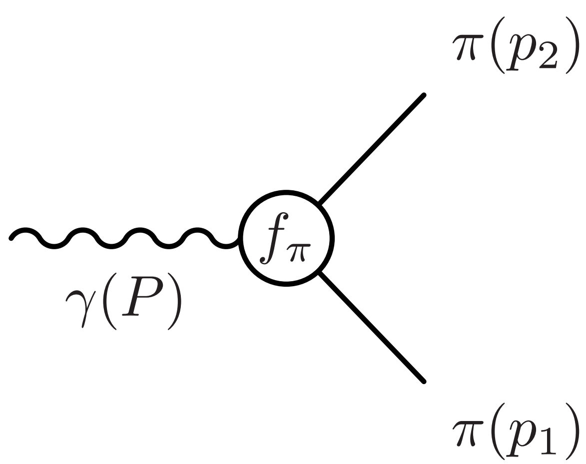
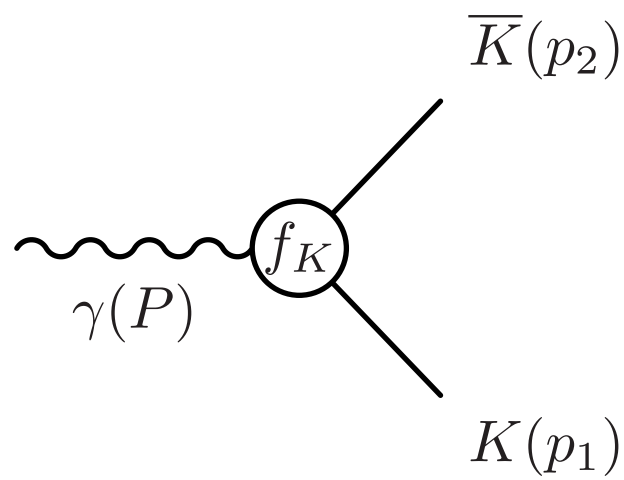
Timelike pseudoscalar form factors in a coupled channel





Felipe Ortega-Gama

William & Mary

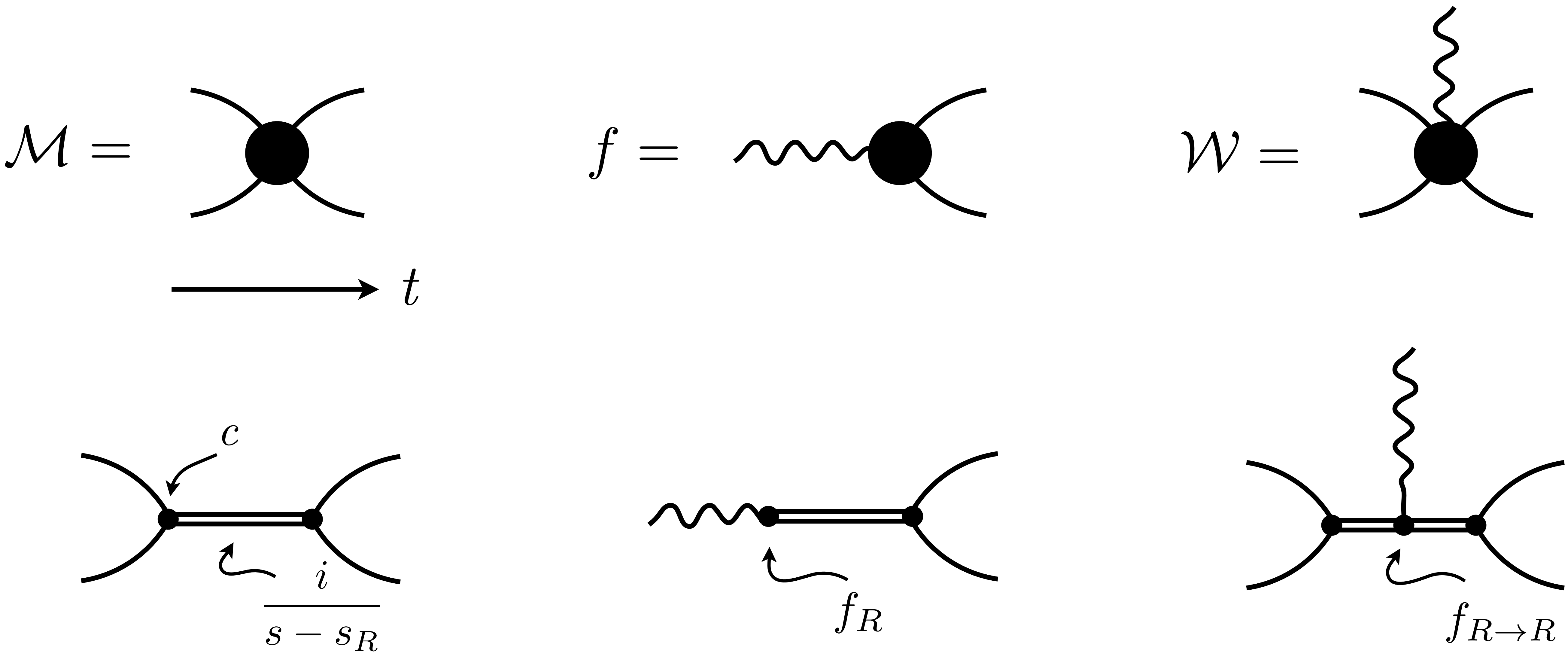
felortga@jlab.org

Jul - 30 - 2024

In collaboration with J. Dudek

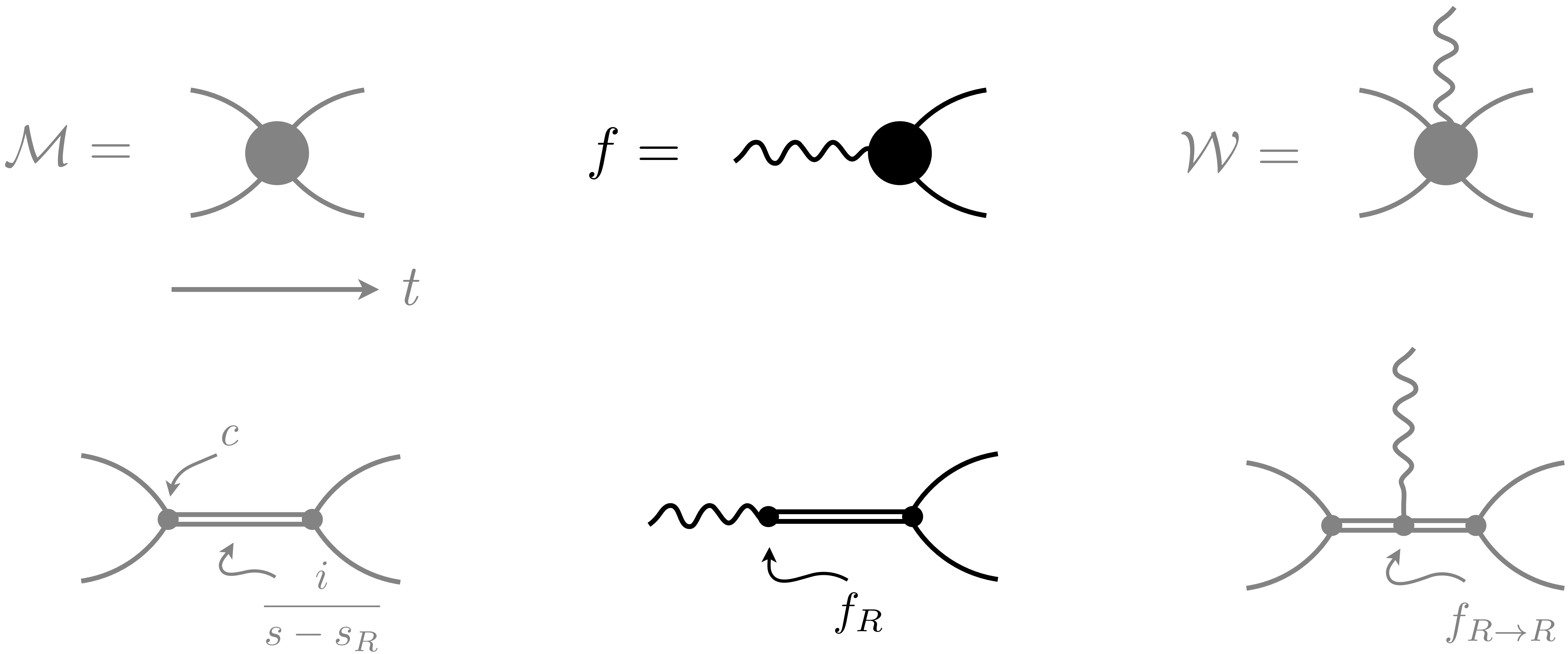
Resonance properties

$2 \to 2$ $\mathcal{J} \to 2$ $2 + \mathcal{J} \to 2$



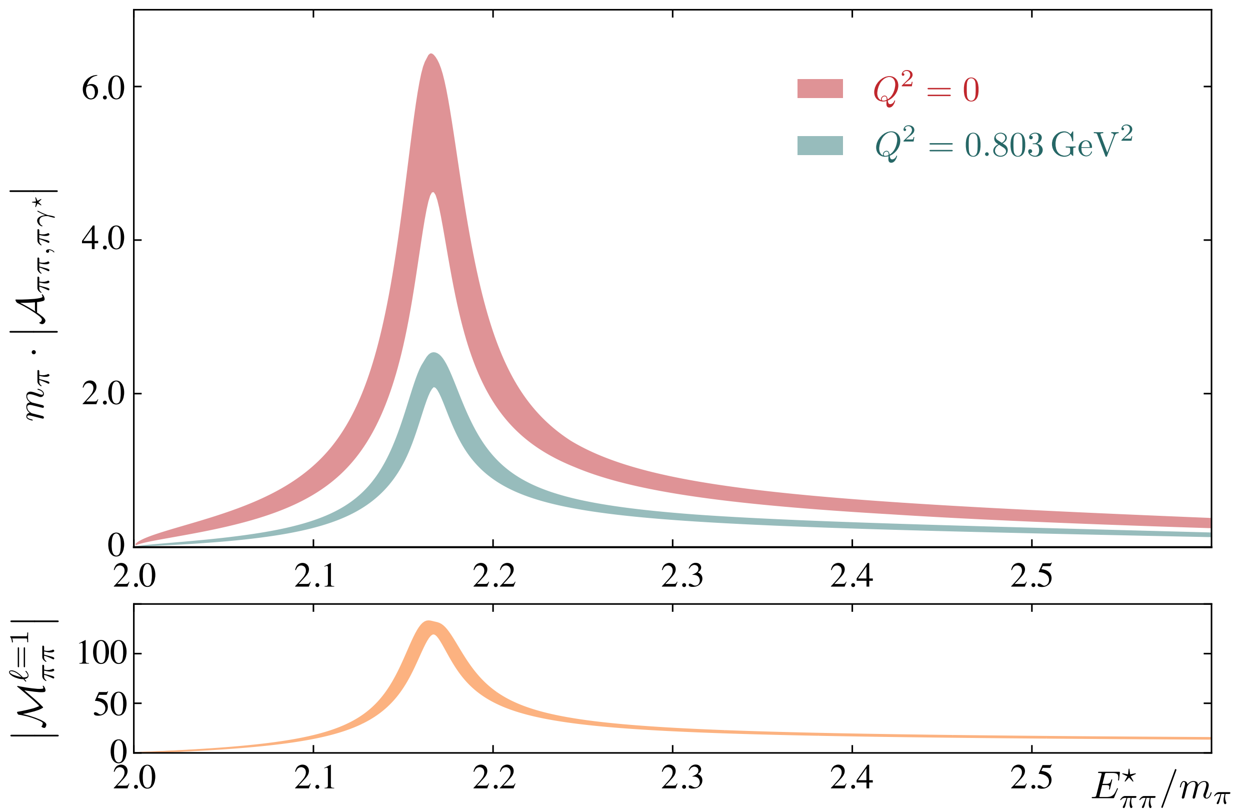
Resonance properties

$2 \to 2$ $\quad\quad$ $\mathcal{J} \to 2$ $\quad\quad$ $2 + \mathcal{J} \to 2$



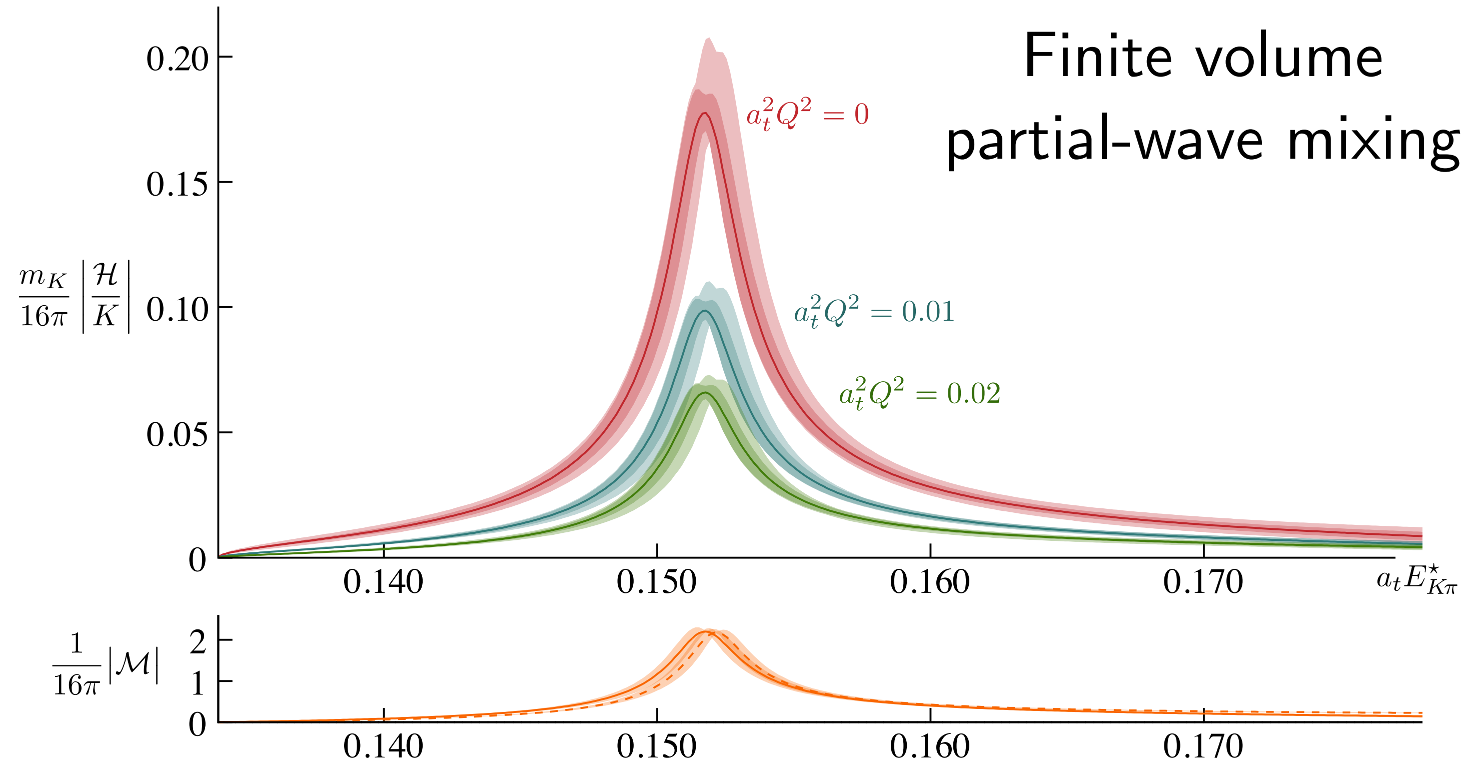
Production amplitudes from LQCD

$\gamma\pi \to \rho \to \pi\pi$



[PRD **93**, 114508 (2016)]

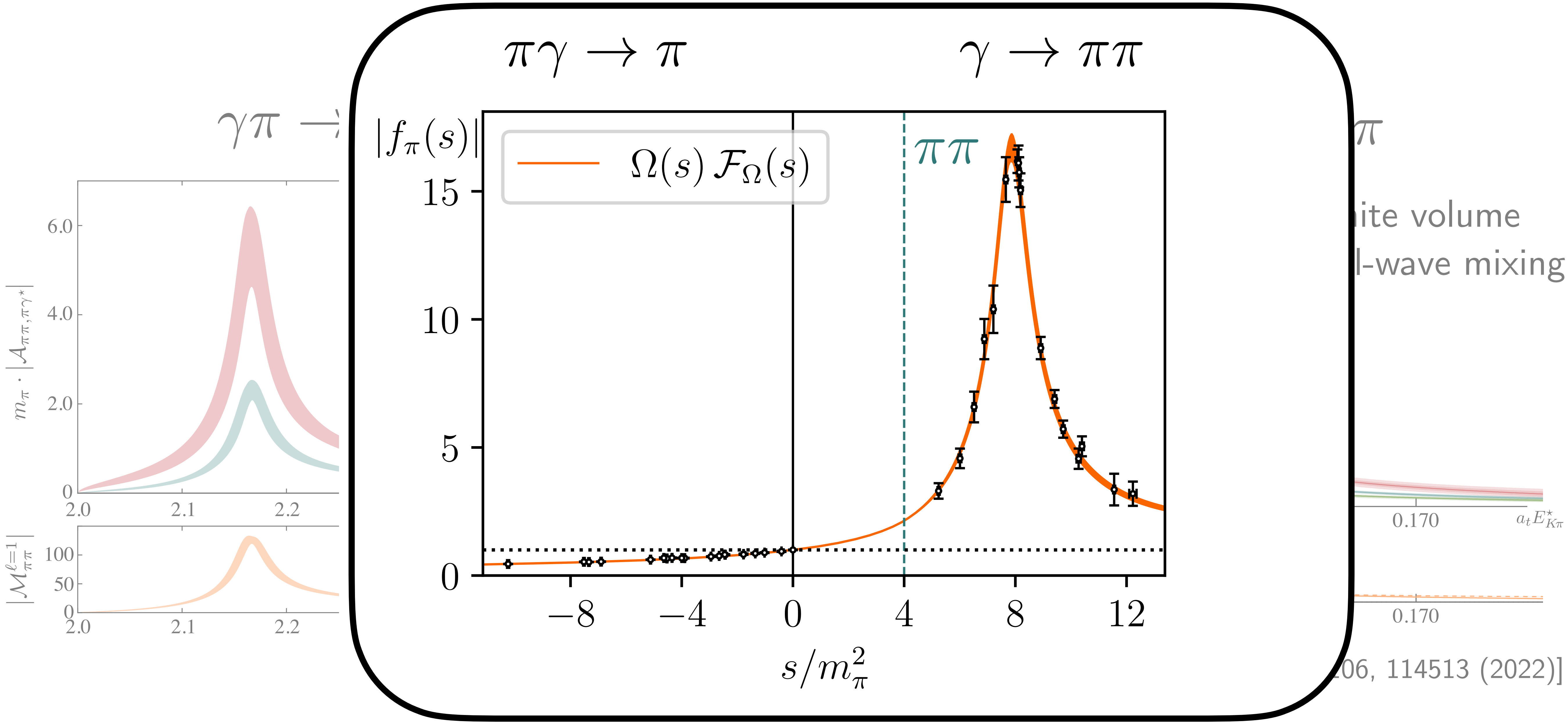
$\gamma K \to K^\star \to K\pi$



[PRD 106, 114513 (2022)]

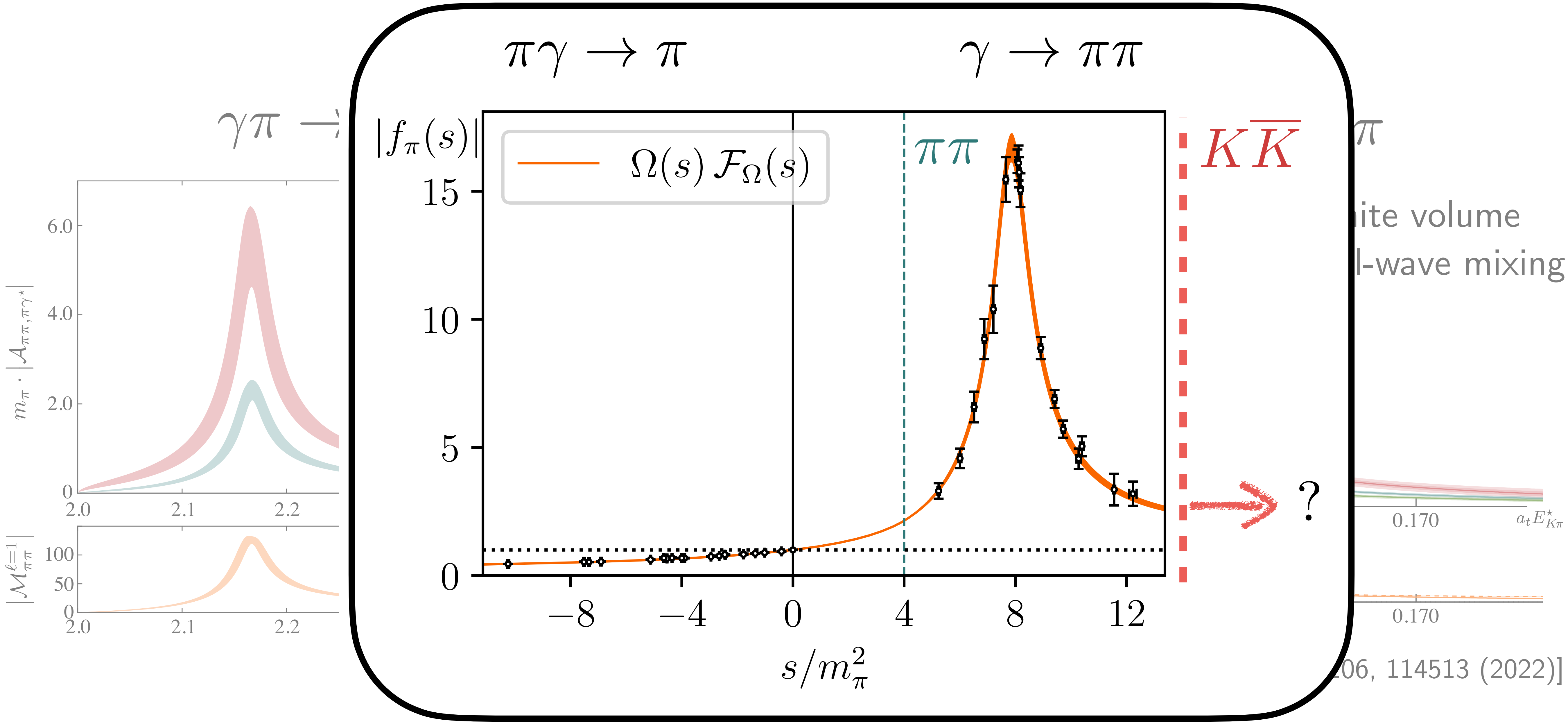
- Couplings to hadronic decay channels
- *Couplings to electroweak currents*

Production amplitudes from LQCD



- Couplings to hadronic decay channels
- *Couplings to electroweak currents*

Production amplitudes from LQCD

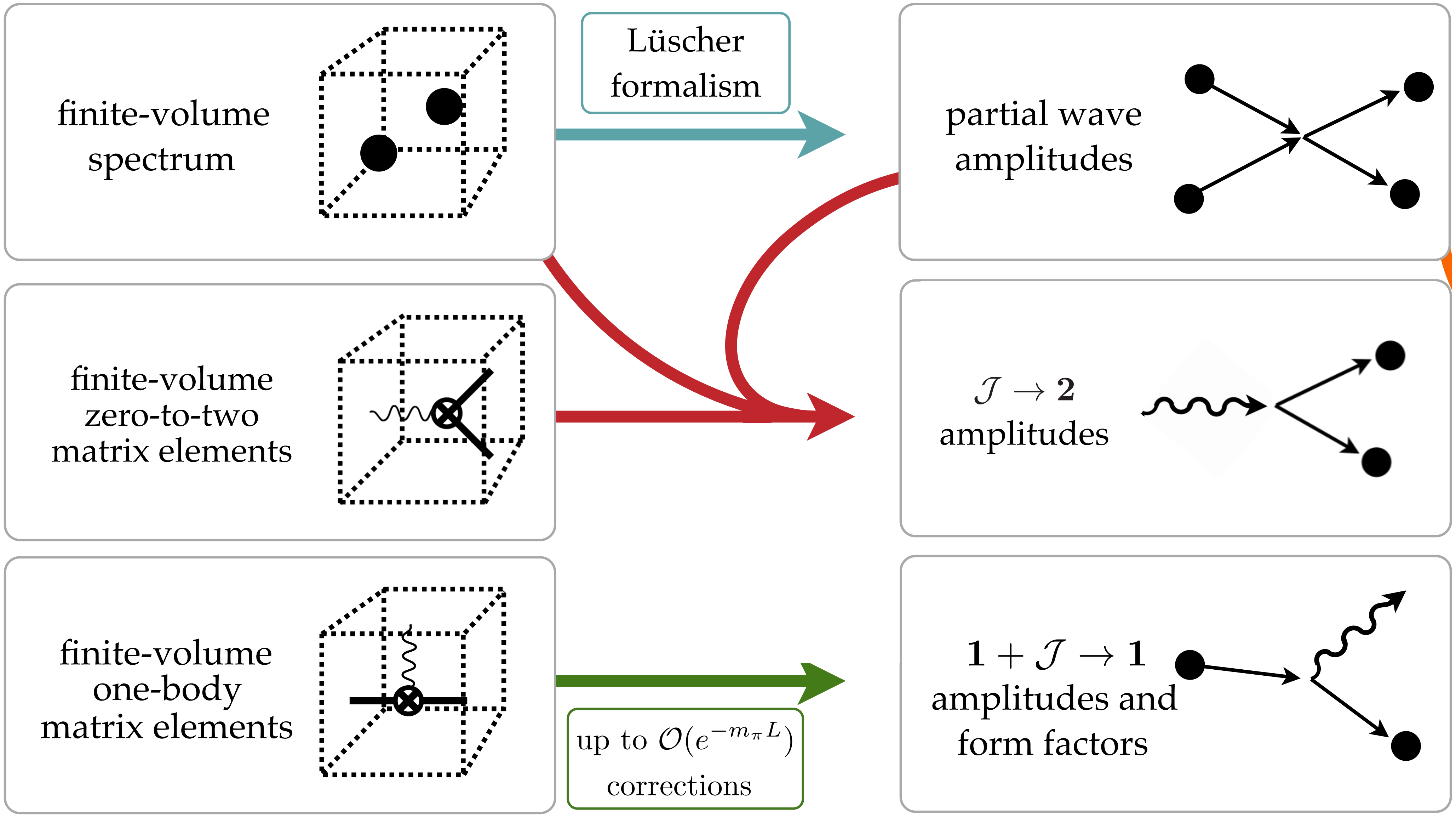


- Couplings to hadronic decay channels
- *Couplings to electroweak currents*

Form factors from LQCD

See also previous talk by N. Miller

$$\det(\mathcal{M}^{-1} + F) = 0$$

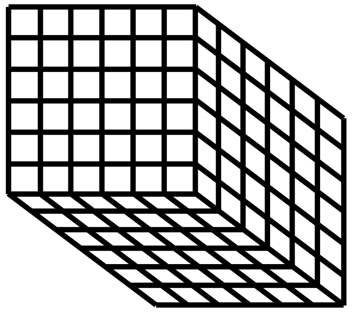


timelike

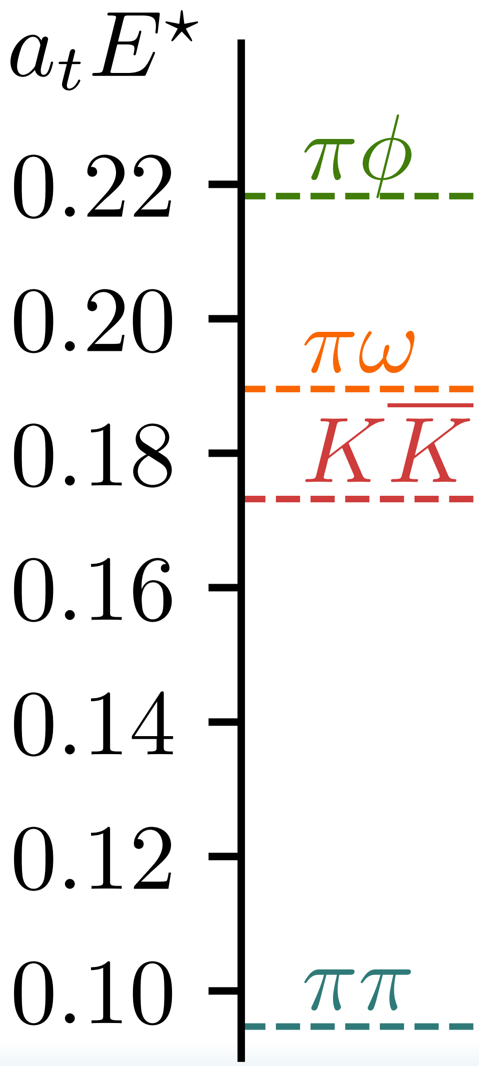
spacelike

Lattice parameters

	$a_t m$	m/MeV
π	0.0474	284
K	0.0866	519



$J^P(I^G)$=1⁻(1⁺)



$$L^3 \times T = 24^3 \times 256$$

$$m_\pi L \approx 4$$

$$a_t^{-1} \approx 6 \text{ GeV}$$

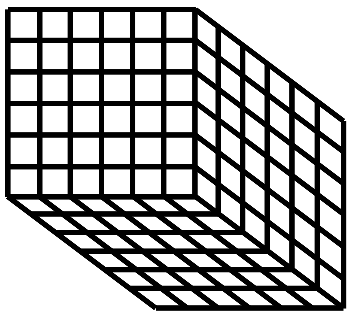
$$\xi = a_s/a_t = 3.455(6)$$

400 gauge configurations

Scattering $J^P(I^G)$=$1^-(1^+)$

$$C_{ij}(t) \equiv \left\langle O_i(t) O_j^\dagger(0) \right\rangle = \sum_{n=0} Z_i^{n*} Z_j^n e^{-E_n t}$$

- $q\bar{q}$ *like* (single-meson) operators ρ ρ_2 b_1 ...
- Two-meson *like* operators $\pi\pi$ $K\overline{K}$



Mixes J^P into cubic irreps

5 $|\mathbf{P}|$'s, 10 irreps

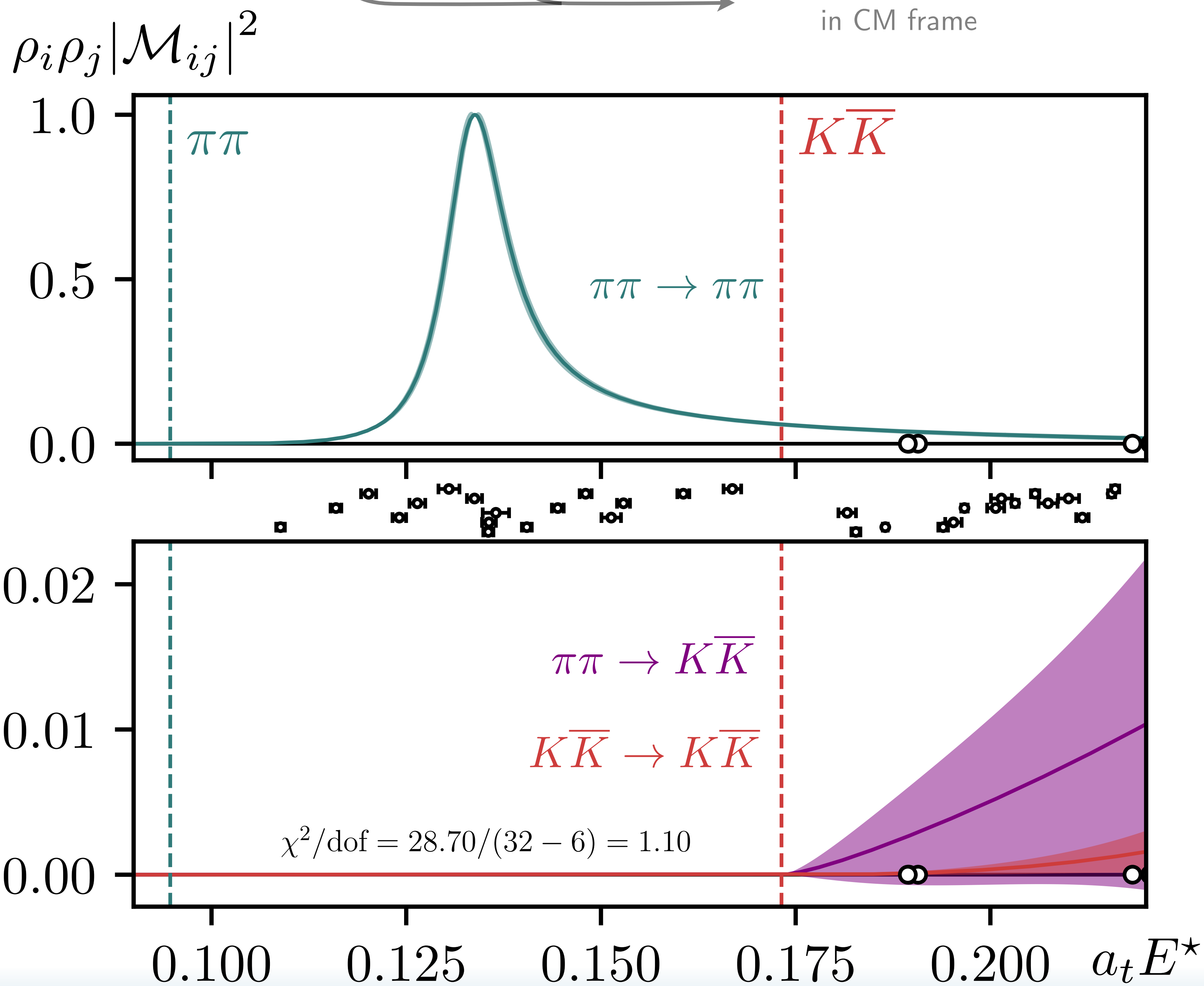
1. GEVP $\quad C_{ij}(t) v_j^n = C_{ij}(t_0) v_j^n \lambda_n(t - t_0)$
2. Fit eigenvalues $\quad \lambda_n(t - t_0) = e^{-E_n(t - t_0)}$
3. Lüscher Quantization Condition $\quad \det(\mathcal{M}^{-1} + F) = 0$

Scattering coupled channel fit

$$\mathcal{M}^{-1}_{ab} = \frac{1}{2k^\star_a} K^{-1}_{ab} \frac{1}{2k^\star_b} - i\rho_{\mathrm{CM},ab}$$

$$K_{ab} = \frac{g_a g_b}{-s + m_r^2} + \gamma_{ab}$$

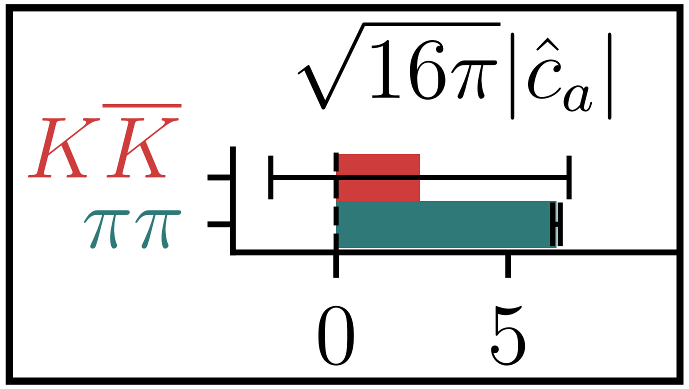
Pseudoscalar momentum in CM frame



ρ resonance

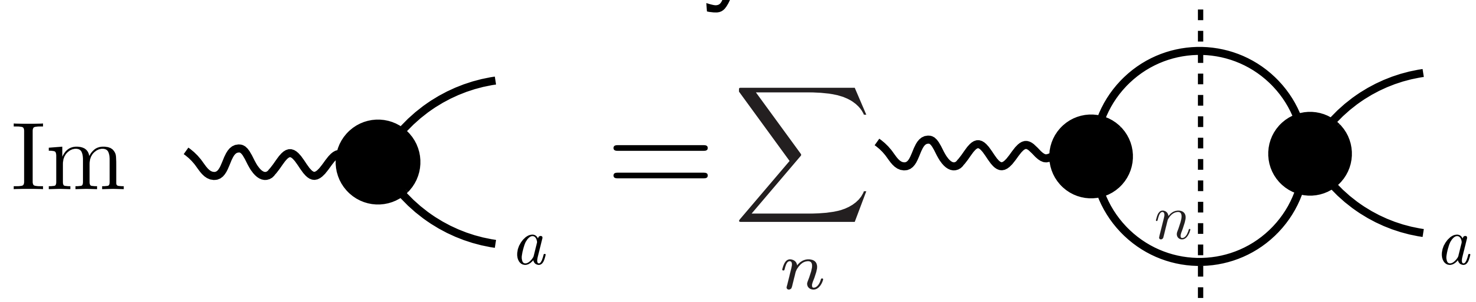
$$\mathcal{M}_{ab}(s \sim s_R) \sim \frac{c_a\, c_b}{s_R - s}$$

$$\mathrm{Re}(\sqrt{s_R}) = 797 \pm 2.6 \text{ MeV}$$
$$\mathrm{Im}(\sqrt{s_R})/2 = 28.5 \pm 1 \text{ MeV}$$



$$\hat{c}_a = \frac{c_a}{k^\star_a}$$

Unitarity of form factors

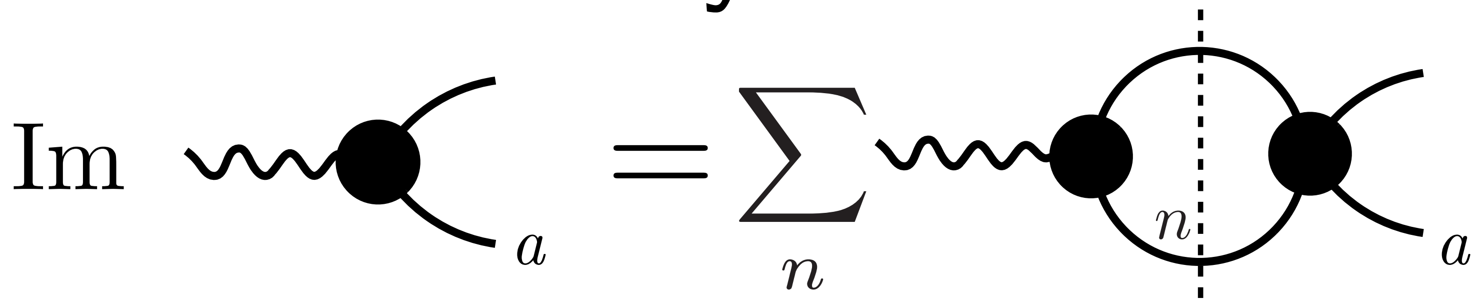


$$\mathrm{Im} f_a = \sum_n f_n \rho_n \mathcal{M}^*_{na}$$

"K-matrix" representation

$$\begin{pmatrix} f_\pi \\ f_K \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{\pi\pi\to\pi\pi} & \mathcal{M}_{\pi\pi\to K\overline{K}} \\ \mathcal{M}_{\pi\pi\to K\overline{K}} & \mathcal{M}_{K\overline{K}\to K\overline{K}} \end{pmatrix} \begin{pmatrix} \mathcal{A}_{\pi\pi} \\ \mathcal{A}_{K\overline{K}} \end{pmatrix}$$

Unitarity of form factors

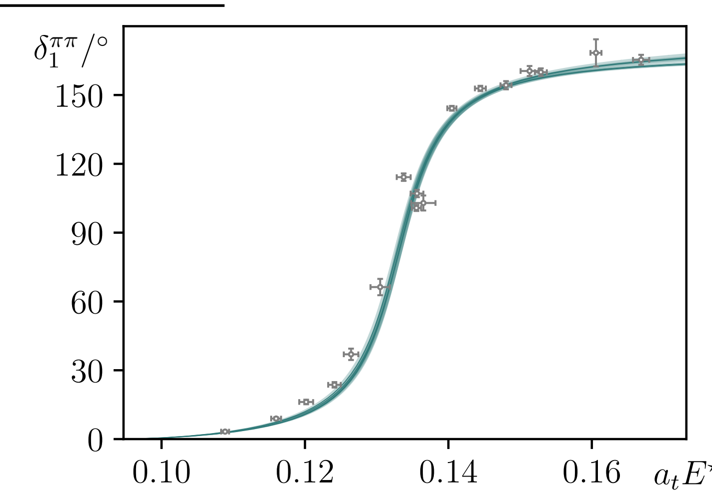


$$\mathrm{Im} f_a = \sum_n f_n \rho_n \mathcal{M}^*_{na}$$

"K-matrix" representation

$$\begin{pmatrix} f_\pi \\ f_K \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{\pi\pi\to\pi\pi} & \mathcal{M}_{\pi\pi\to K\overline{K}} \\ \mathcal{M}_{\pi\pi\to K\overline{K}} & \mathcal{M}_{K\overline{K}\to K\overline{K}} \end{pmatrix} \begin{pmatrix} \mathcal{A}_{\pi\pi} \\ \mathcal{A}_{K\overline{K}} \end{pmatrix}$$

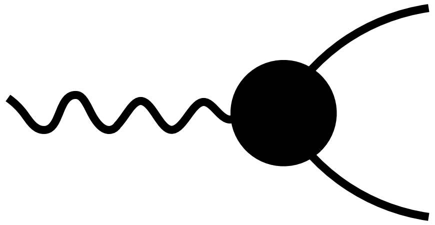
Elastic unitarity: $\mathcal{M}_{\pi\pi\to\pi\pi} = \dfrac{1}{\rho} \sin(\delta_1^{\pi\pi})\, e^{i\delta_1^{\pi\pi}}$



Omnès representation: $f_\pi = \Omega \times \mathcal{F}_\Omega$

$$\Omega(s) = \exp\left(\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\delta_1^{\pi\pi}(s')}{s'(s'-s)} \right)$$

Pair-production



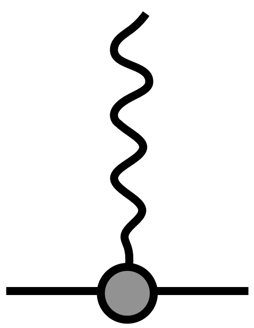
$$\lim_{t\to\infty} \frac{\left\langle \mathcal{J}(t)\Omega_n^\dagger(0)\right\rangle}{\left\langle \Omega_n(t)\Omega_n^\dagger(0)\right\rangle} \propto \mathcal{H}_L$$

✦ Optimized operators

$$\Omega_n \propto v_j^n \, O_j$$

1. Extract invariant:

$$\mathcal{H}_L = K\, \mathcal{F}_n^{(L)}$$



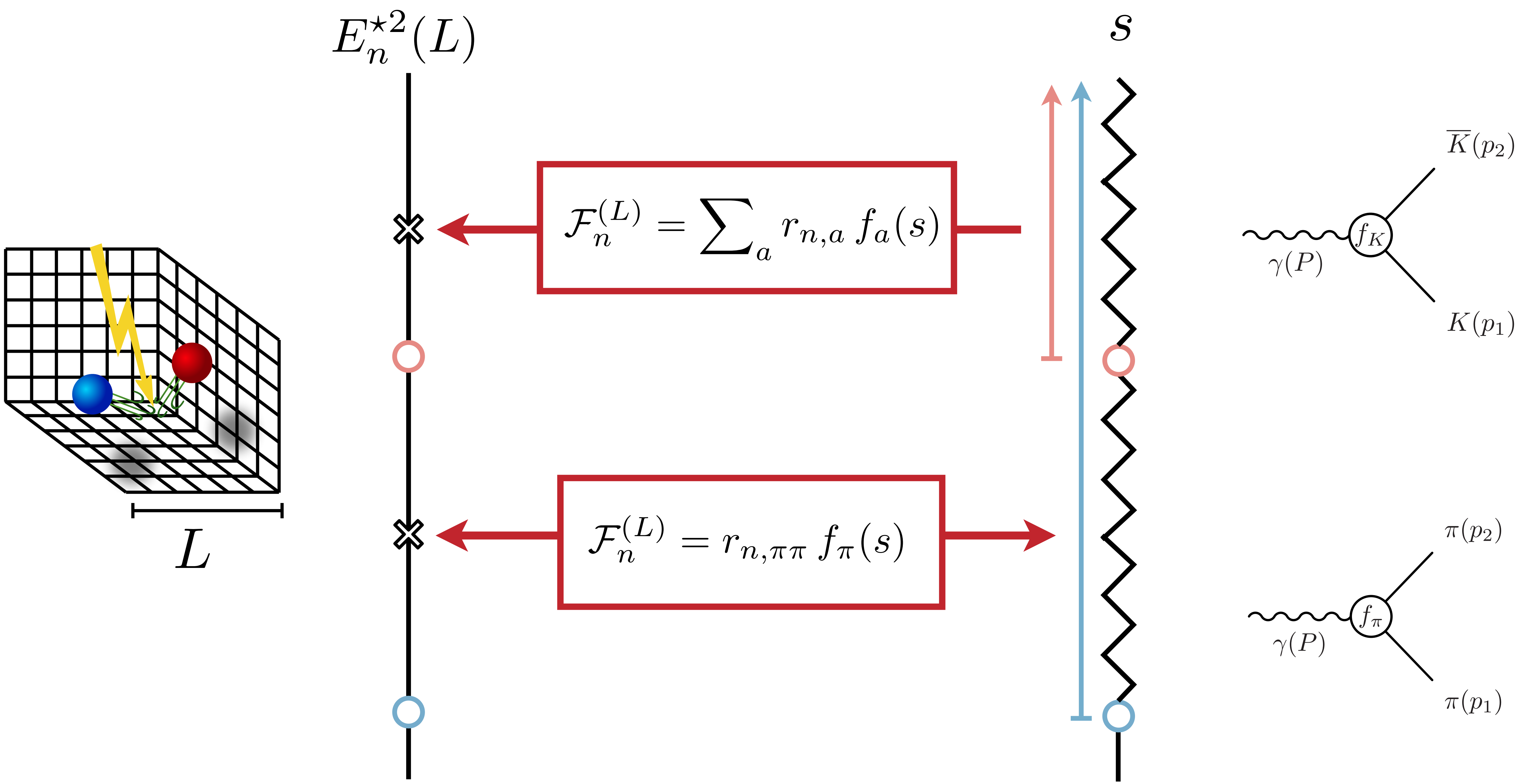
For renormalization of the current

2. FV correction:

At Lüscher energy

$$\frac{\partial}{\partial E^\star}\left(\mathcal{M}^{-1} + F\right) \Longrightarrow r_{n,a}\ ,\ \tilde{r}_{n,a}$$

Finite Volume correction: $r_{n,a}$



$$\langle 0|\mathcal{J}|E_n^\star\rangle_L \propto \mathcal{F}_n^{(L)} \qquad \langle 0|\mathcal{J}|P,a\rangle \propto f_a$$

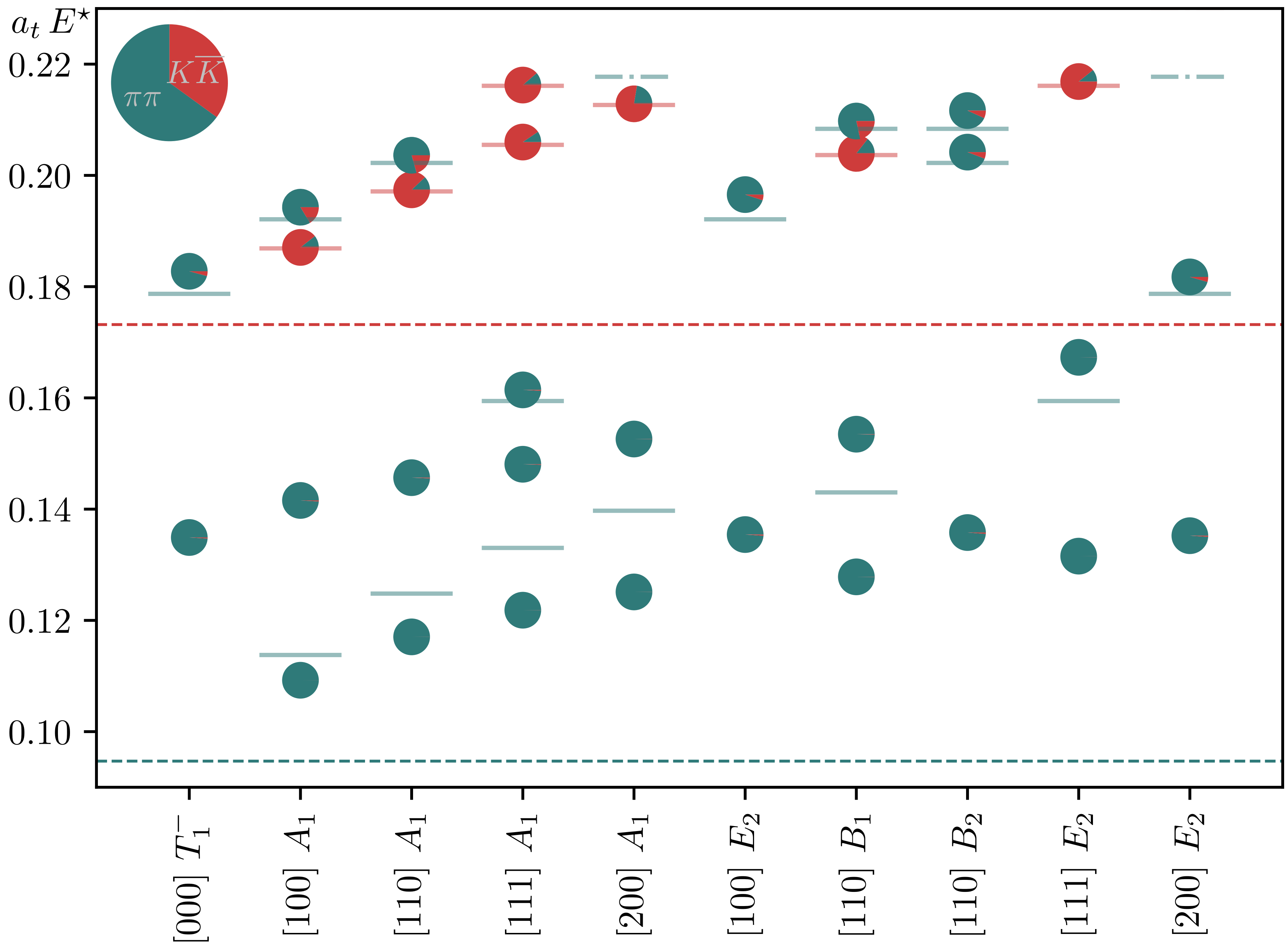
Finite volume correction (coupled channel)

✦ 25 $\pi\pi$-like levels

✦ 7 KK-like levels

$$\mathcal{F}^{(L)} \propto \sum_a v_{0,a} f_a$$

$v_{0,a}$



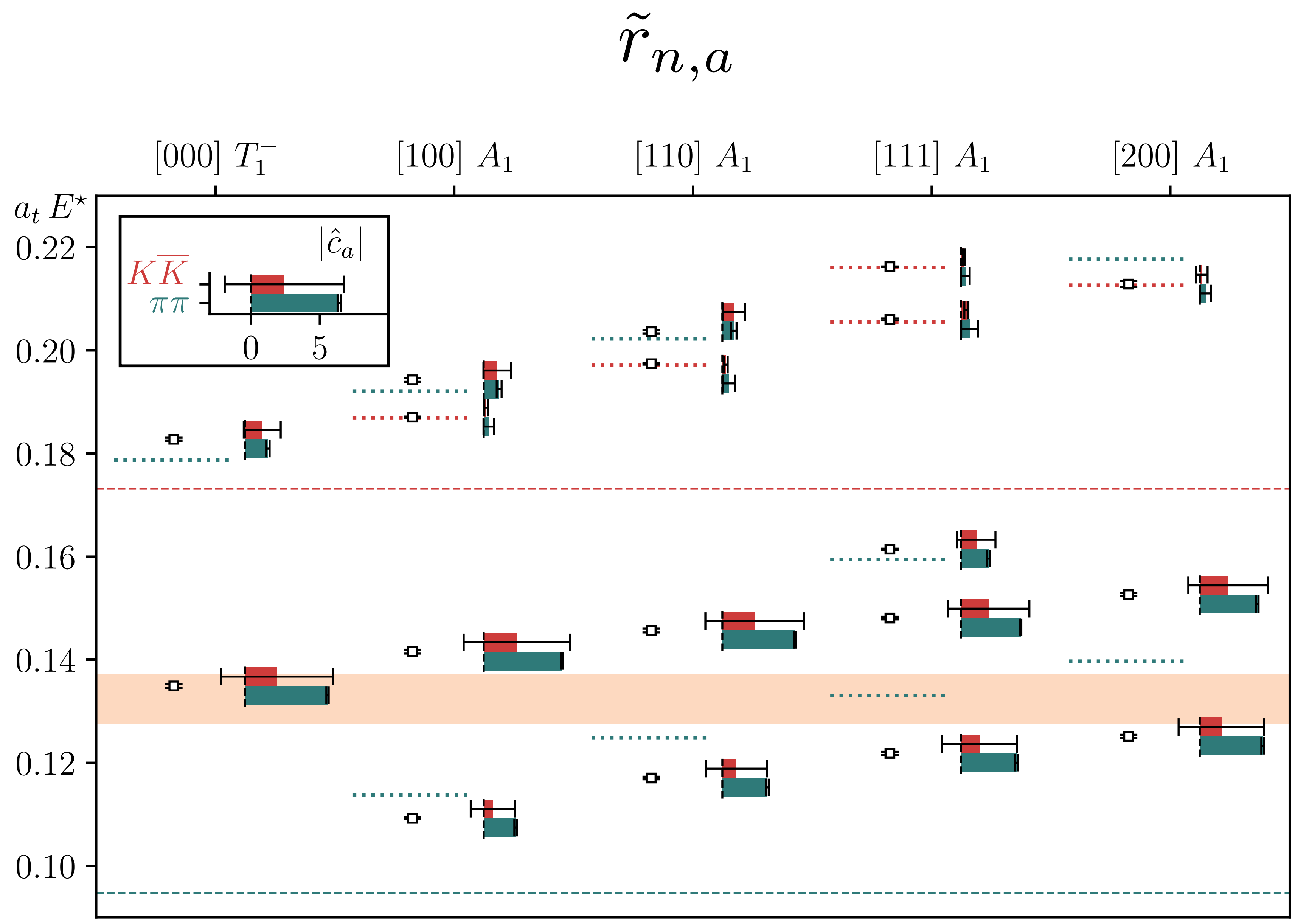
Finite volume correction (coupled channel)

In the narrow width limit $\tilde{r}_{\mathfrak{n},a} \approx \frac{c_a}{k_a^\star} = \hat{c}_a$

$$f_a = \frac{1}{k_a^\star} \mathcal{M}_{ab} \frac{1}{k_b^\star} \mathcal{F}_b$$

$$\mathcal{F}^{(L)} \propto \sum_a v_{0,a} f_a$$

$$\mathcal{F}_n^{(L)} = \sum_a \tilde{r}_{n,a} \mathcal{F}_a$$

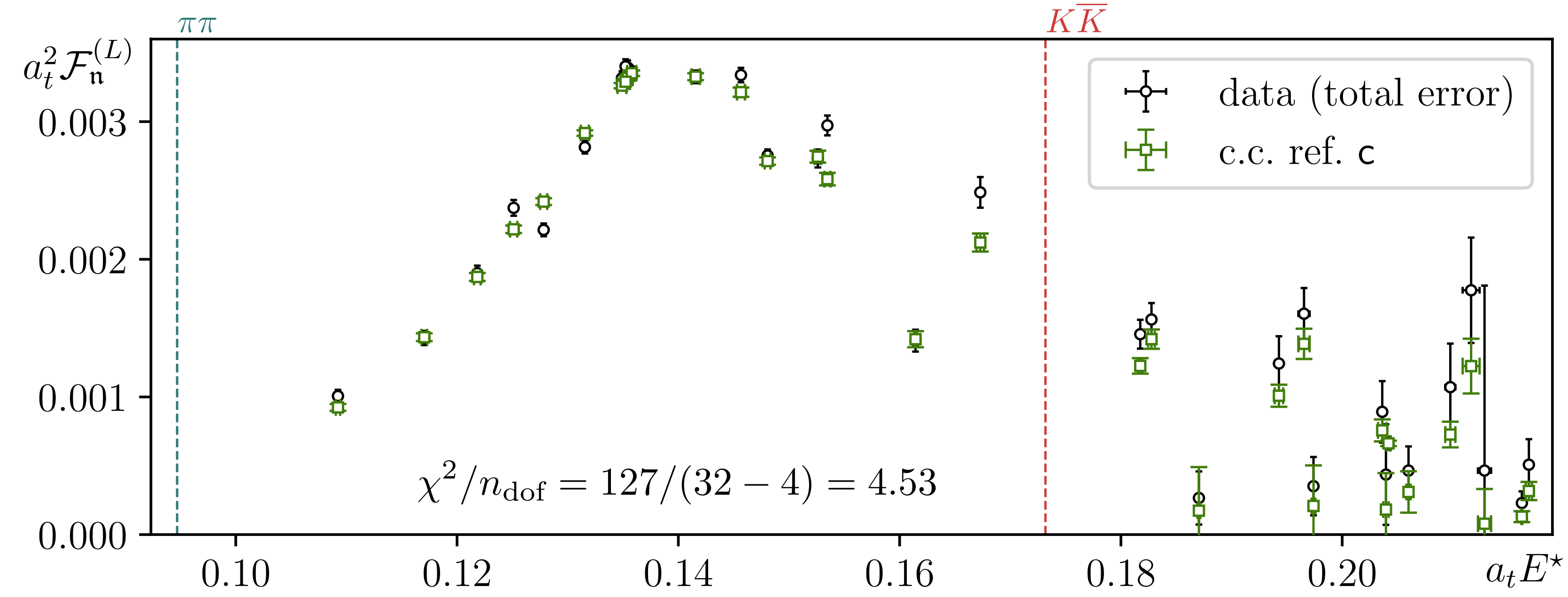


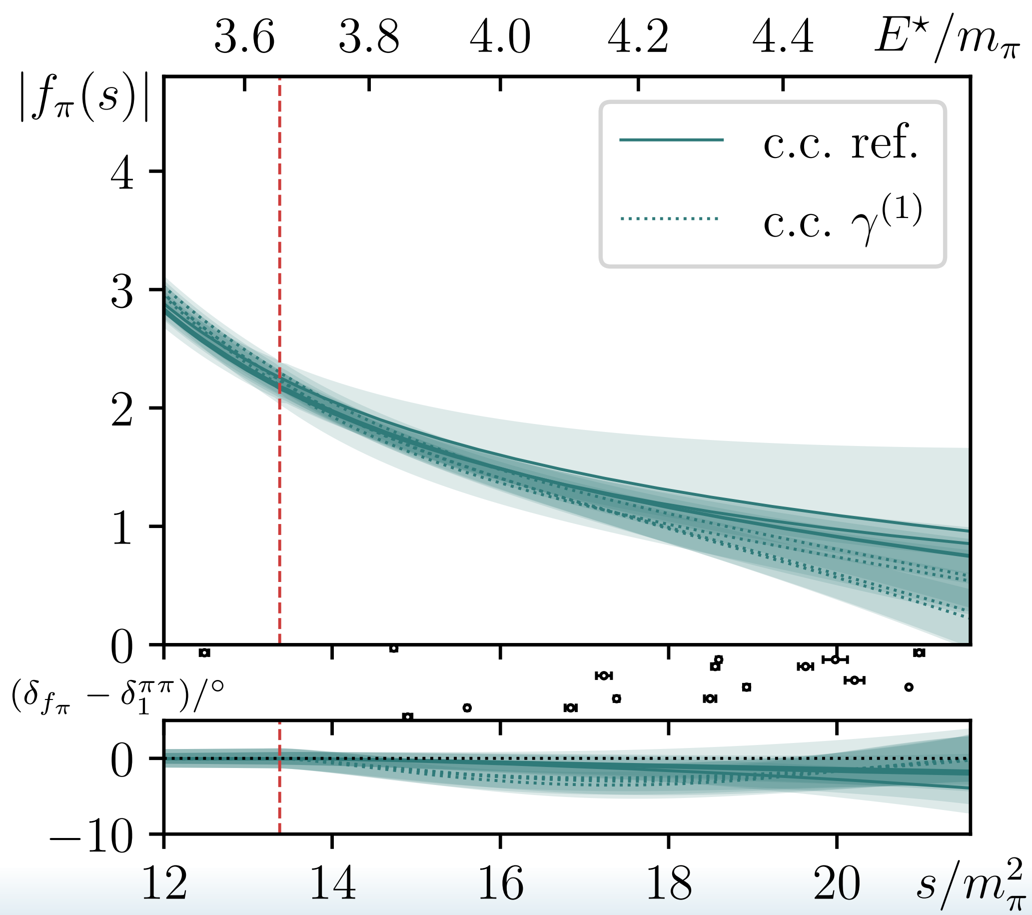
Coupled channel fit

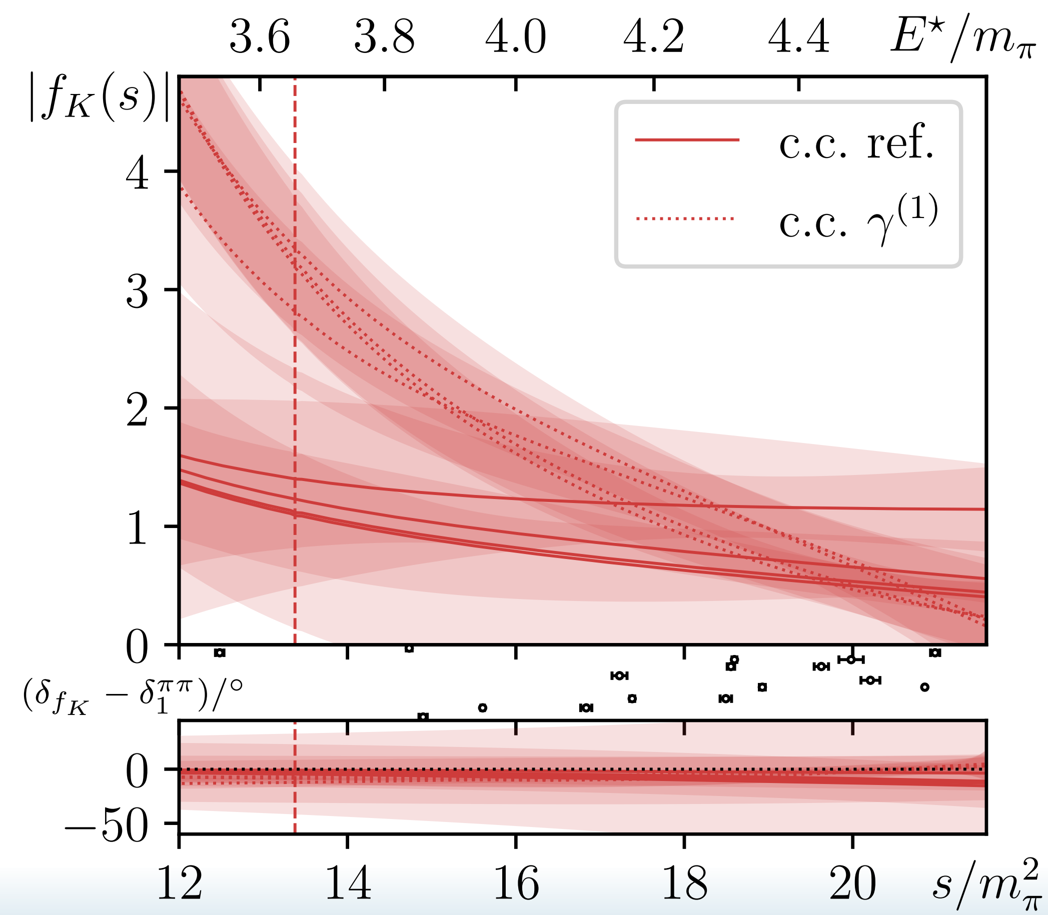
Model	$N_{\pi\pi}$	$N_{K\overline{K}}$
c	1	1

$$f_a = \frac{1}{k_a^\star}\mathcal{M}_{ab}\frac{1}{k_b^\star}\mathcal{F}_b$$

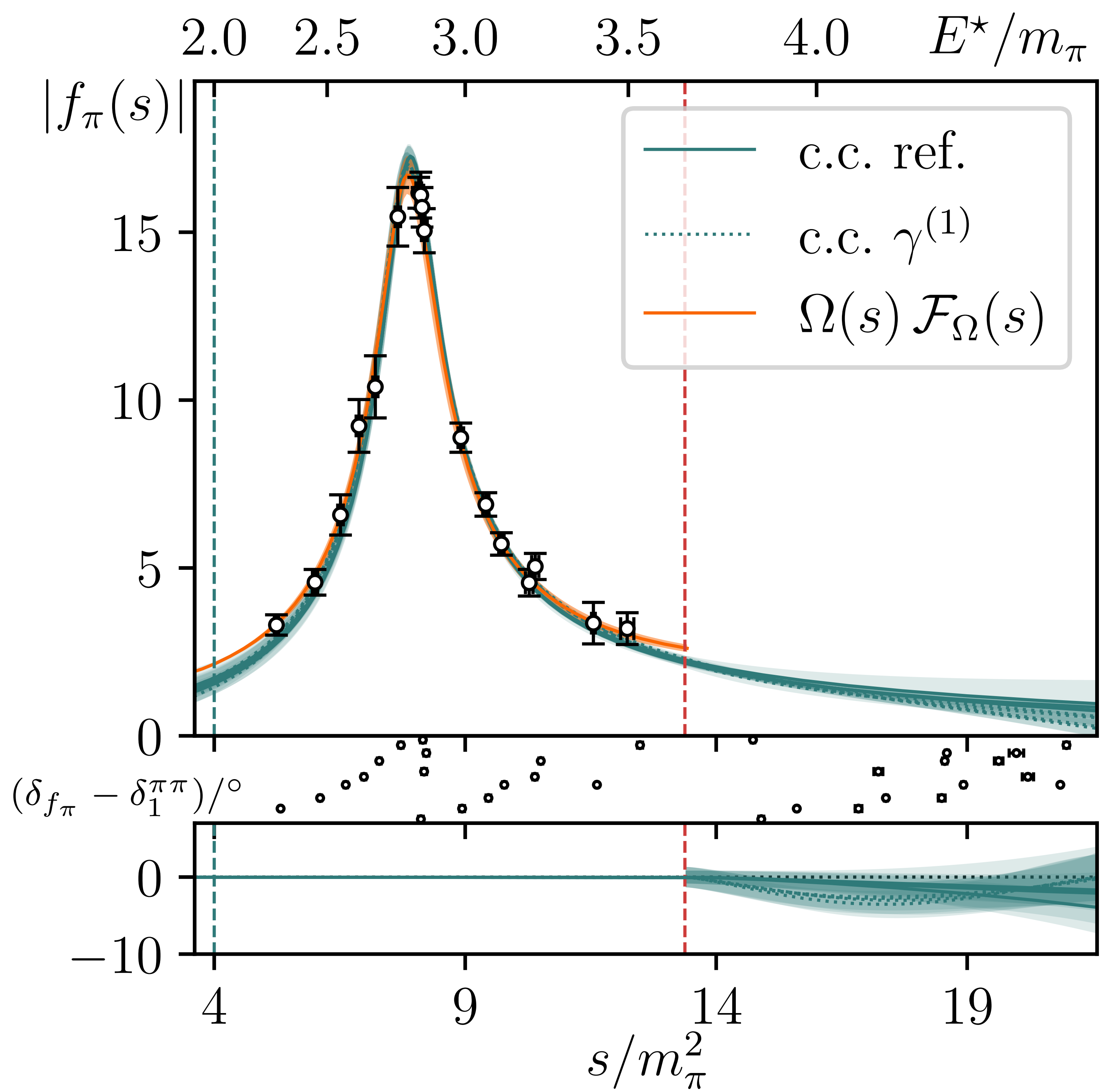
$$\mathcal{F}_a(s) = \sum_{n=0}^{N_a} h_{n,a}\, s^n$$







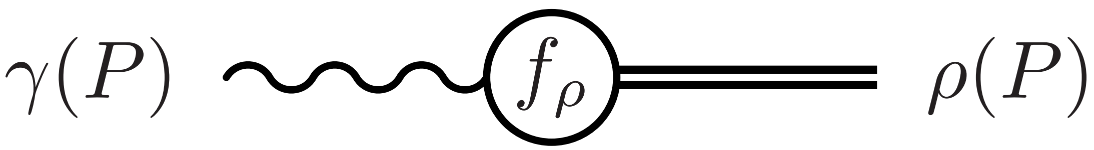
Comparison dispersive and coupled channel

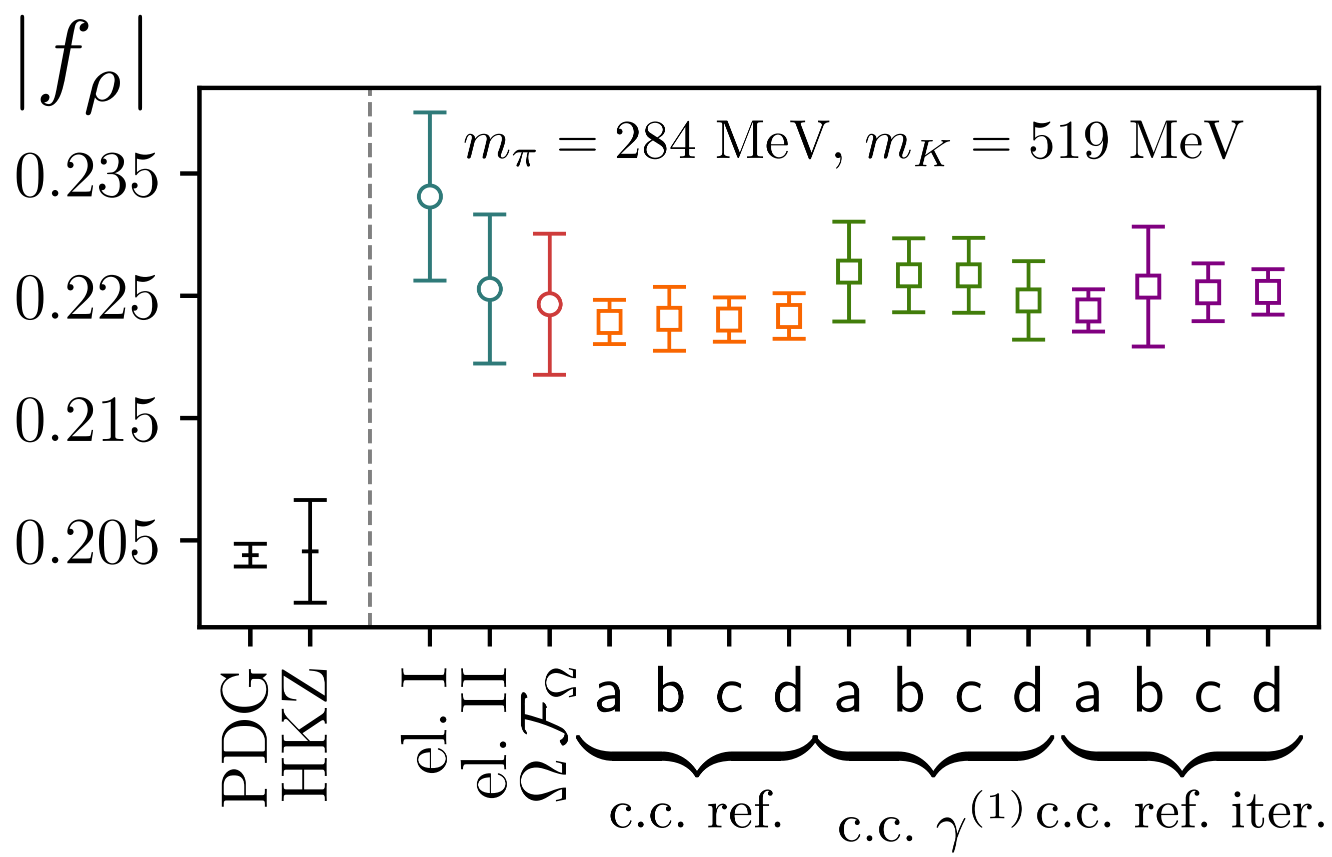


ρ meson decay constant

$$f_a(s \sim s_R) \propto \sqrt{16\pi}\,\hat{c}_a \frac{1}{s - s_R} f_\rho$$

$$\langle \rho(\vec{P}, m) | \mathcal{J}^\mu | 0 \rangle = \epsilon^{\mu *}(P, m)\, f_\rho\, m_R^2$$



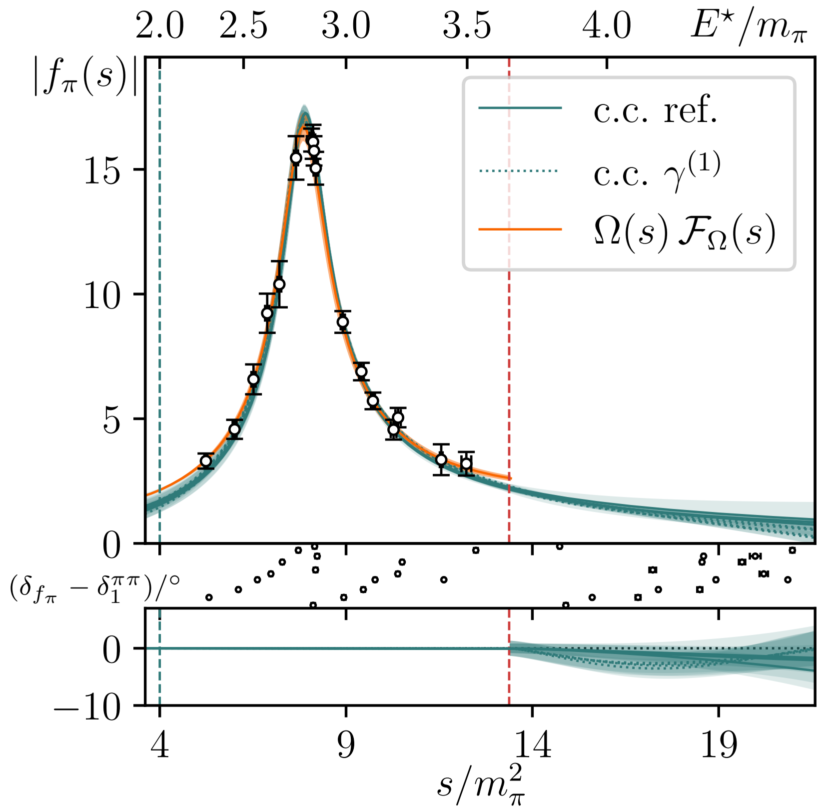


$$\mathcal{F}_a(s) = \sum_{n=0}^{N_a} h_{n,a}\, s^n$$

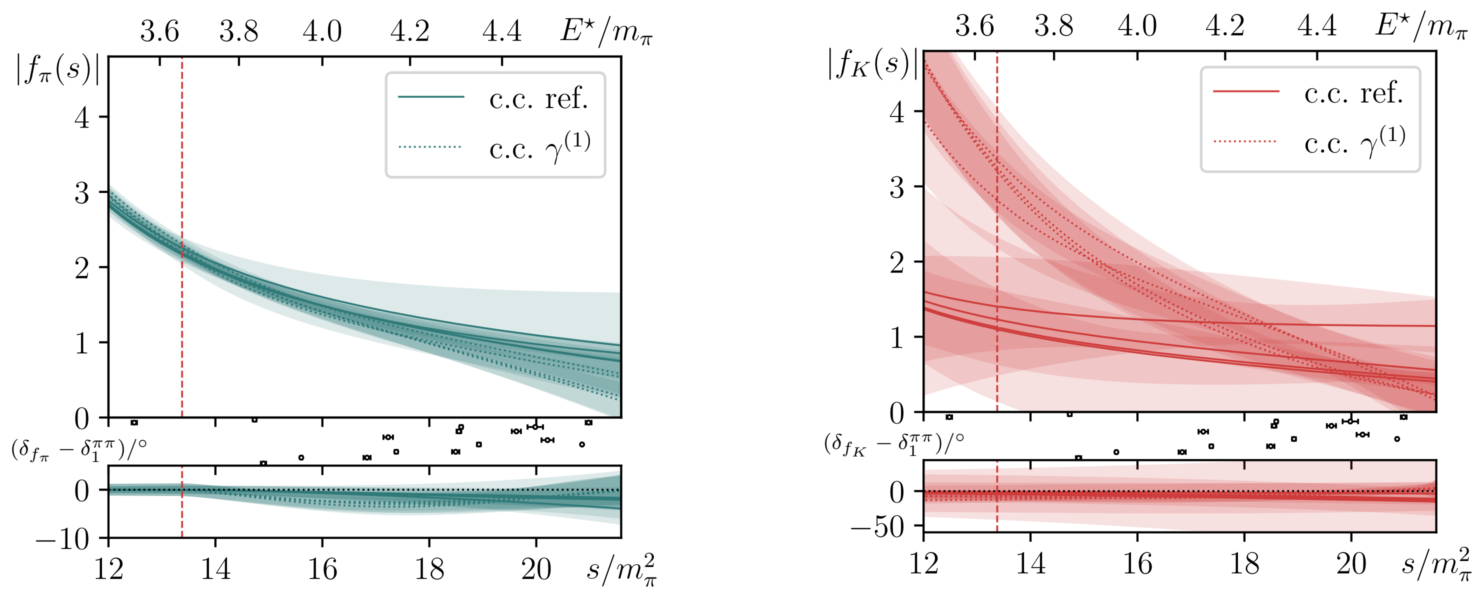
Model	$N_{\pi\pi}$	$N_{K\overline{K}}$
a	1	0
b	0	1
c	1	1
d	2	0

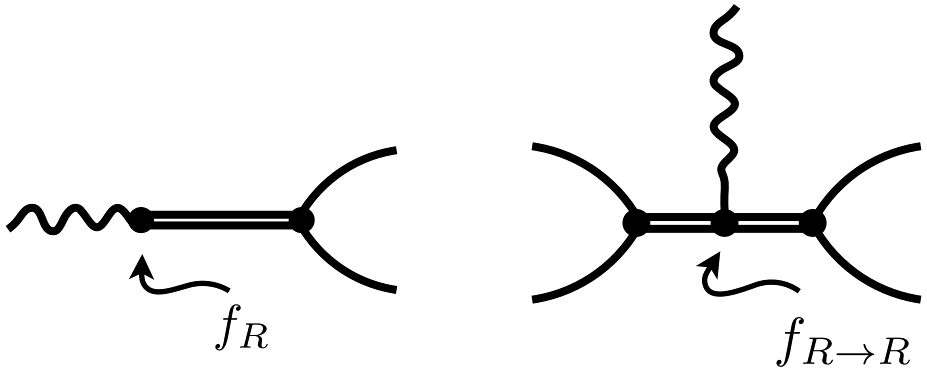
Summary and outlook

- ☑ Coupled channel $\pi\pi/KK$ scattering.
- Pseudoscalar vector form factor
  - ☑ Coupled channel region.
- ☐ Form factors of exotic resonances.
- ☐ Rare decays: e.g. D to $\pi\pi/KK$.
- Learn more about the internal structure of resonances.



$m_\pi = 284$ MeV, $m_K = 519$ MeV





$|f_\rho| = 0.224(6)$ $\langle r_\rho^2 \rangle = ?$

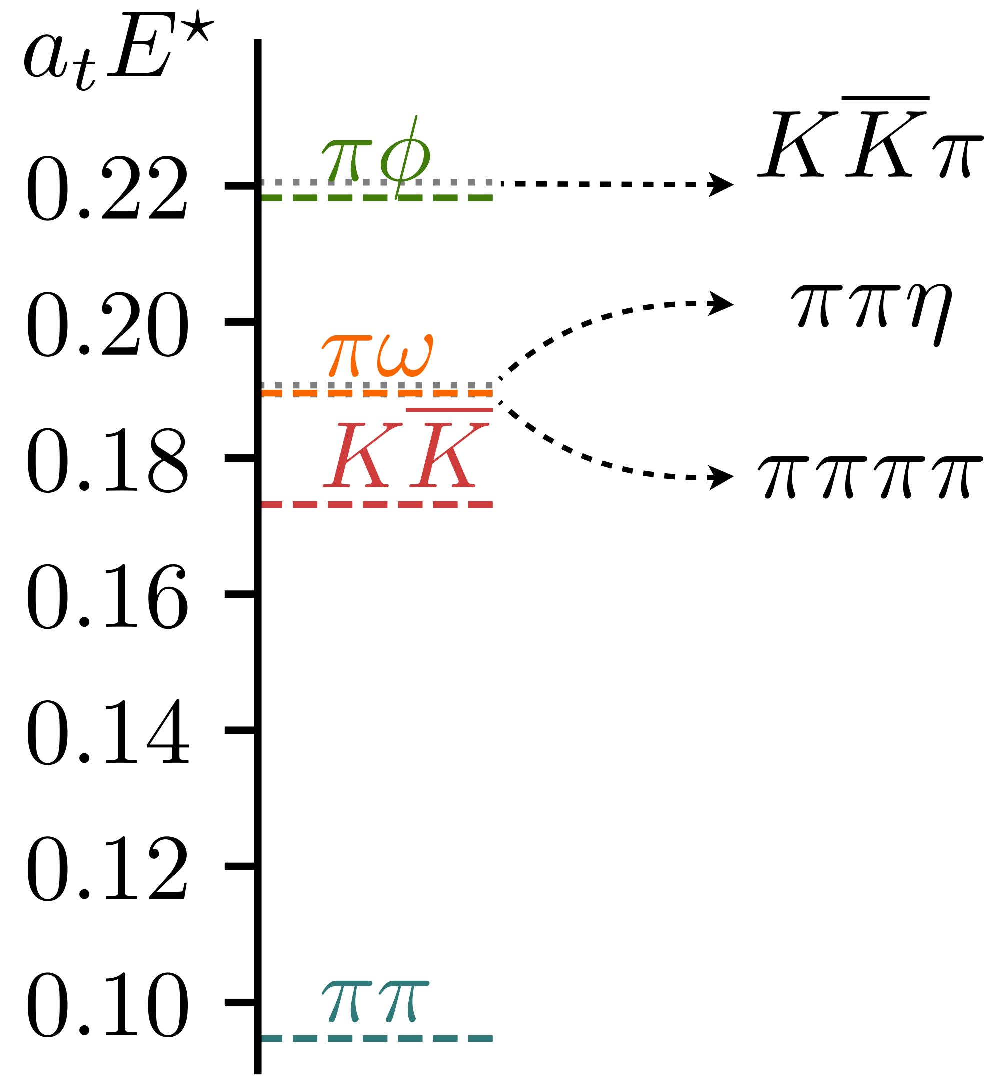
Back up

Multimeson tresholds

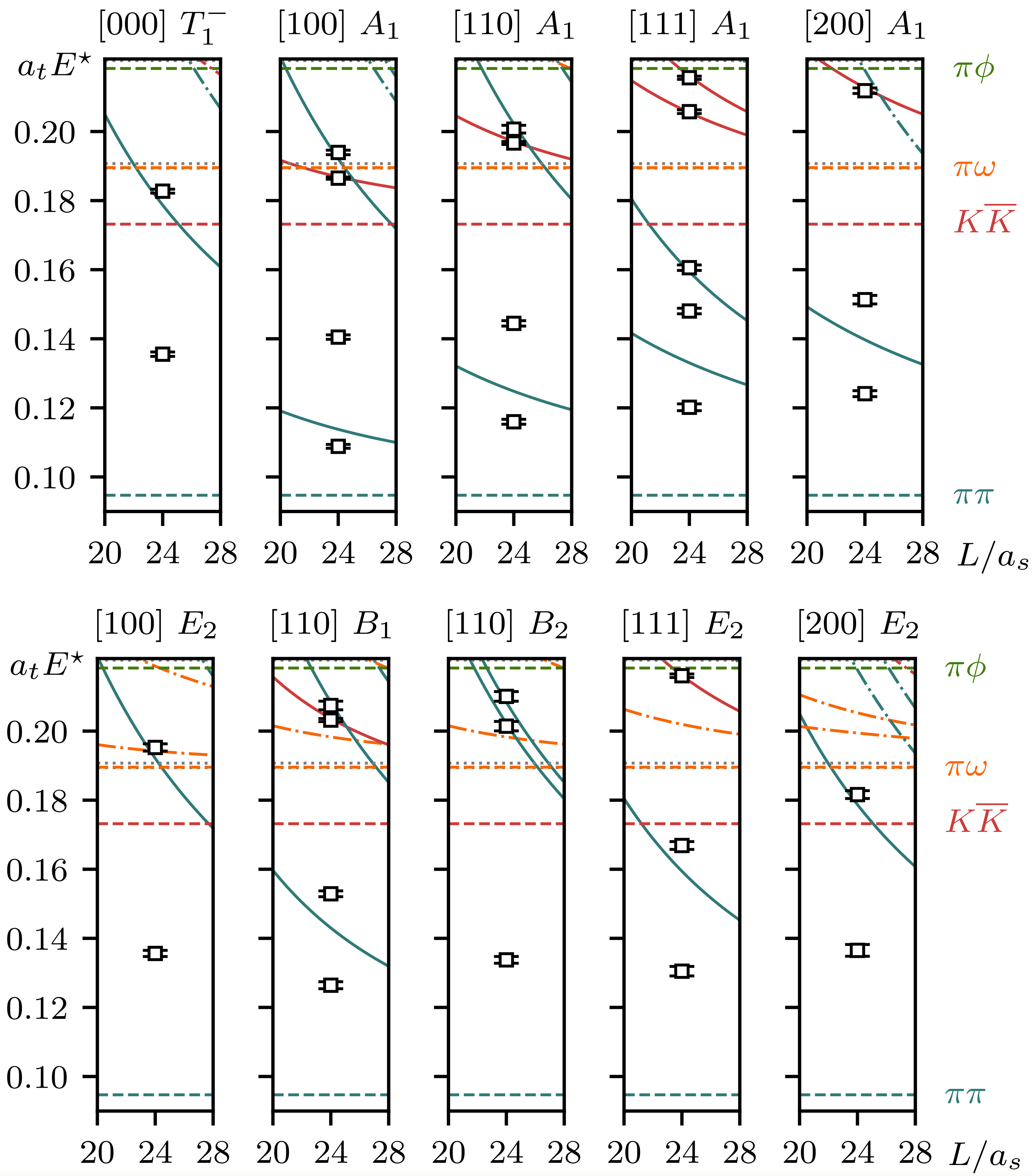
$J^P(I^G)=1^-(1^+)$

meson	$a_t m$
π	0.0474
K	0.0866
η	0.0960
ω	0.1422
ϕ	0.1709

threshold	$a_t E_{\mathrm{th}}$
$\pi\pi$	0.0947
$K\overline{K}$	0.1732
$\pi\pi\pi\pi$	0.1894
$\pi\omega$	0.1896
$\pi\pi\eta$	0.1908
$\pi\phi$	0.2183
$\pi K\overline{K}$	0.2206

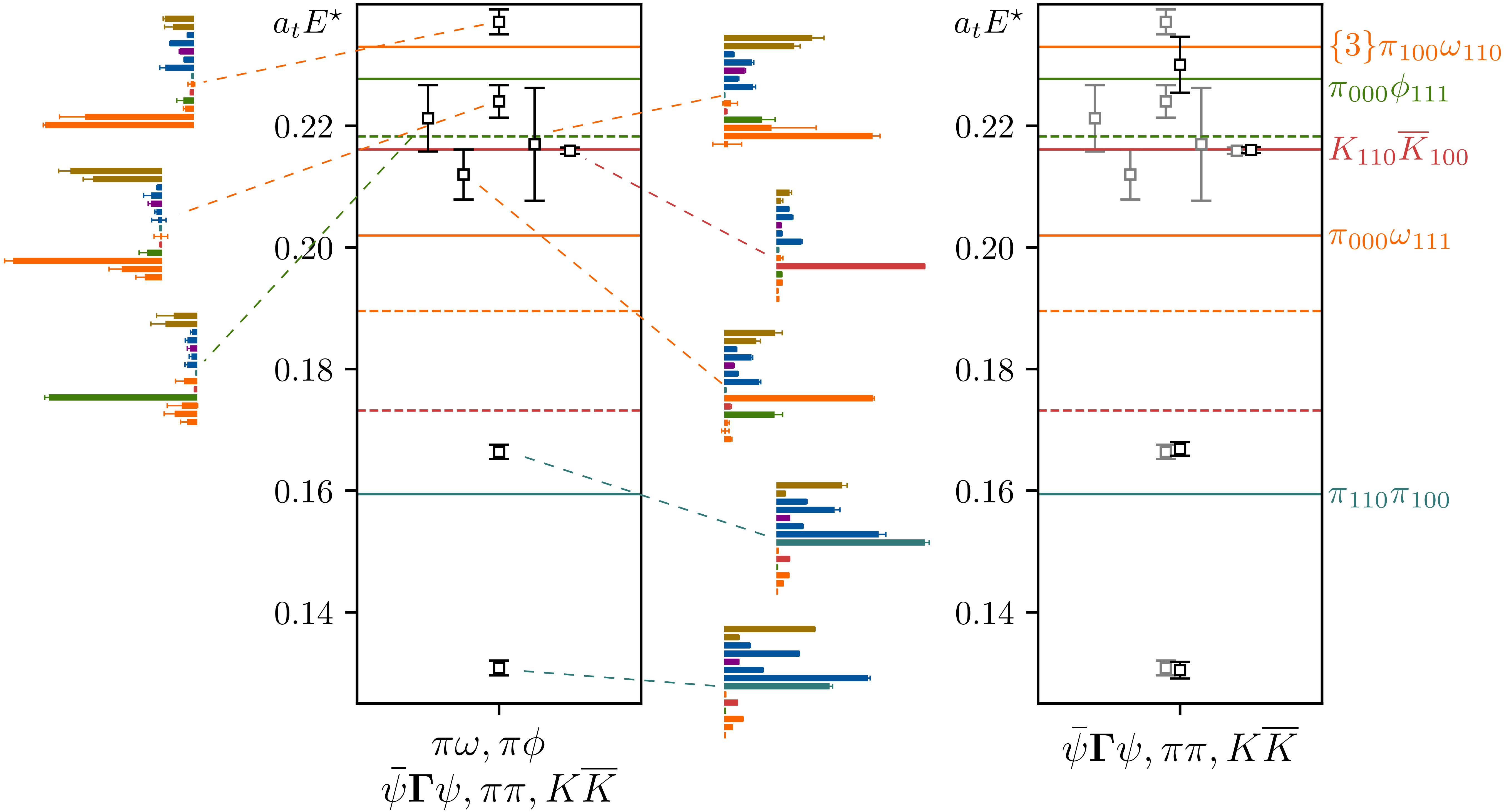


Finite volume spectrum



- 17 elastic levels
- 15 above KK threshold

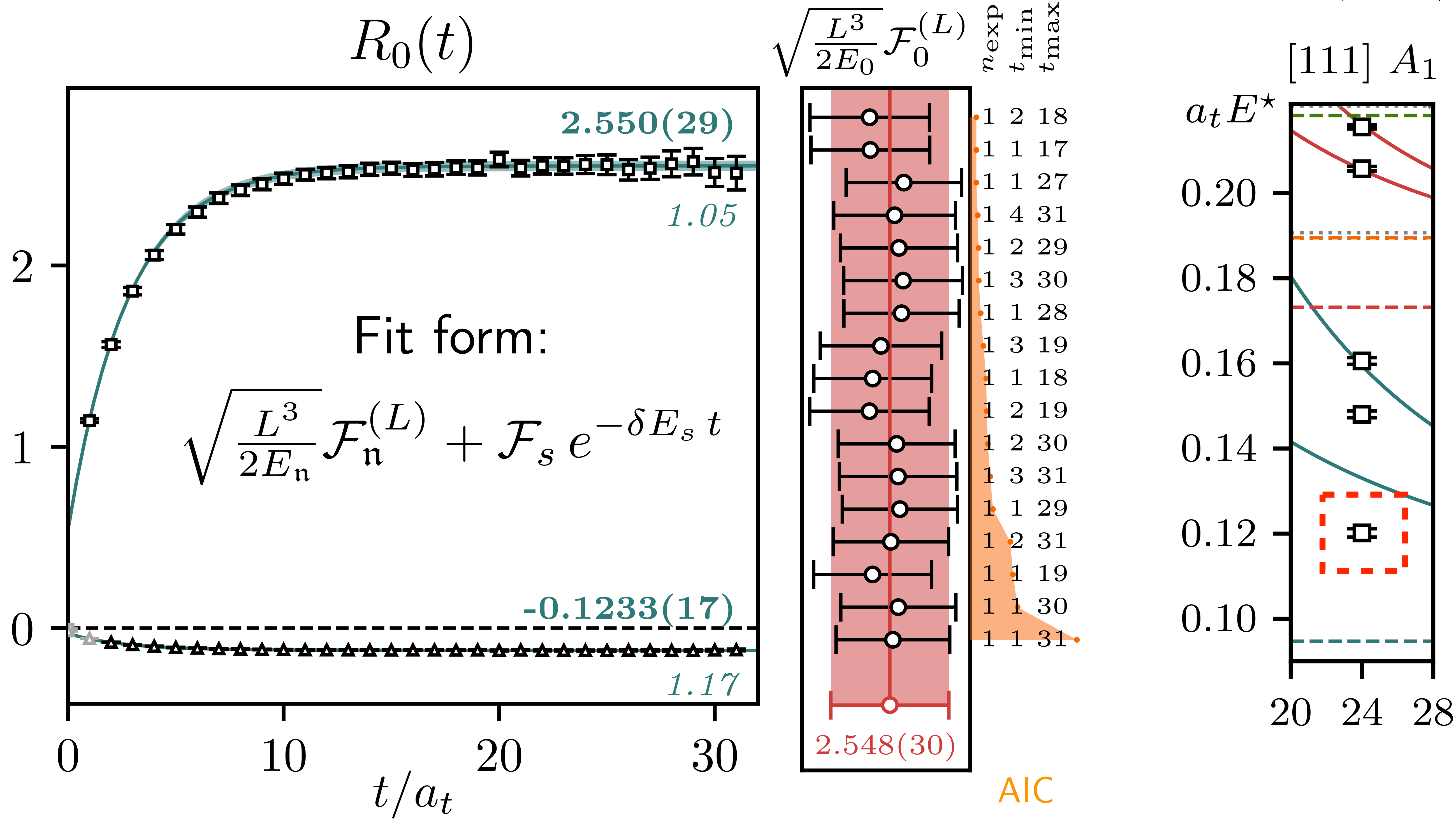
Effects of vector-pseudoscalar channels



$[111]\, E_2$ $q\bar{q}$ *like* operators ρ ρ_2 b_1

AIC fits to matrix elements

$R_n(t) = \frac{\sqrt{2E_n}}{K} \frac{\langle \mathcal{J}(t)\Omega_n^\dagger(0)\rangle}{\langle \Omega_n(t)\Omega_n^\dagger(0)\rangle}$



$$\langle 0|\mathcal{J}^i_{\mathrm{impro}}|\mathfrak{n}\rangle = \langle 0|\mathcal{J}^i|\mathfrak{n}\rangle - \frac{1}{4}(1-\xi)a_t E_\mathfrak{n}\epsilon_{ijk}\langle 0|\bar{\psi}\gamma^j\gamma^k\psi|\mathfrak{n}\rangle$$

Lellouch-Lüscher factor

$$\widetilde{\mathcal{R}}(E_{\mathfrak{n}}, L) = 2E_{\mathfrak{n}} \lim_{E\to E_{\mathfrak{n}}} \frac{E - E_{\mathfrak{n}}}{\mathcal{M}(s) + F^{-1}(E, L)} = \frac{2E_{\mathfrak{n}}}{\lambda_0'} \mathbf{v}_0 \, \mathbf{v}_0^{\mathsf{T}}$$

$$r_{\mathfrak{n},a}(L) = \sqrt{\frac{2E_{\mathfrak{n}}^{\star}}{\lambda_0'^{\star}}} \frac{(\mathbf{v}_0)_a}{k_a^{\star}}$$

$$-\mathcal{M}\widetilde{\mathcal{R}}\mathcal{M} = 2E_{\mathfrak{n}} \cdot \lim_{E\to E_{\mathfrak{n}}} \frac{E - E_{\mathfrak{n}}}{\mathcal{M}^{-1}(s) + F(E, L)} = \frac{2E_{\mathfrak{n}}^{\star}}{\mu_0'^{\star}} \mathbf{w}_0 \, \mathbf{w}_0^{\mathsf{T}}$$

$$\tilde{r}_{\mathfrak{n},a}(L) = \sqrt{\frac{2E_{\mathfrak{n}}^{\star}}{-\mu_0'^{\star}}} \frac{(\mathbf{w}_0)_a}{k_a^{\star}}$$