Lattice QCD calculation of hadron mass splitting

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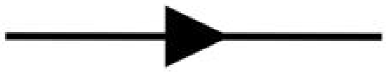
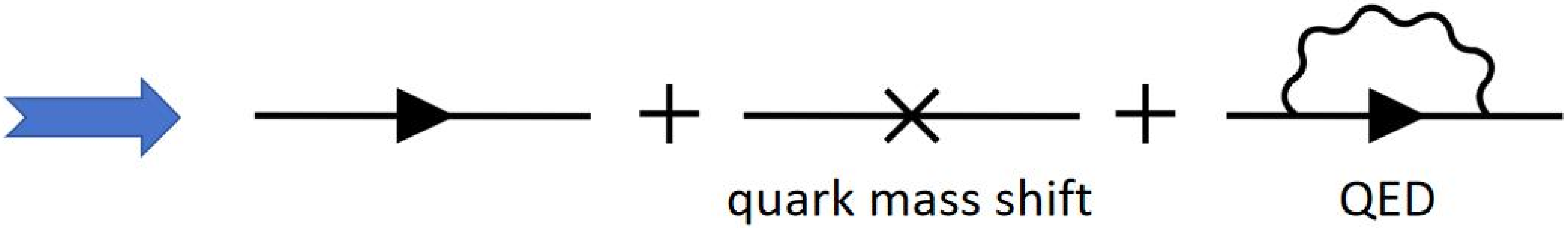
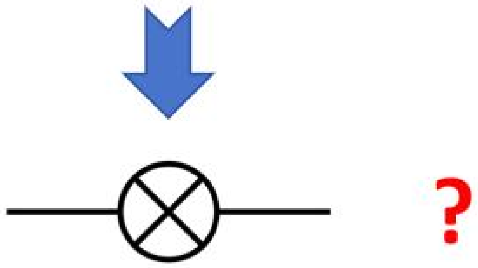
Isospin Breaking by Perturbation

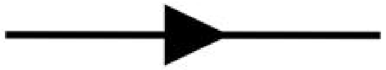
bare quark propagator: 

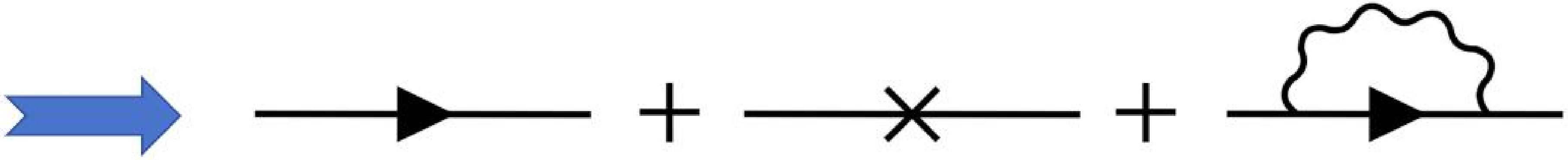


QED self-energy counter term needs renormalization. 

Isospin Breaking by Perturbation

bare quark propagator:





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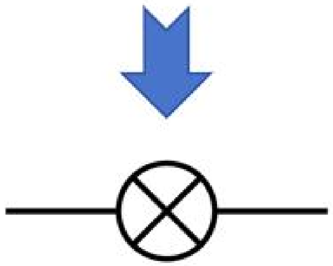
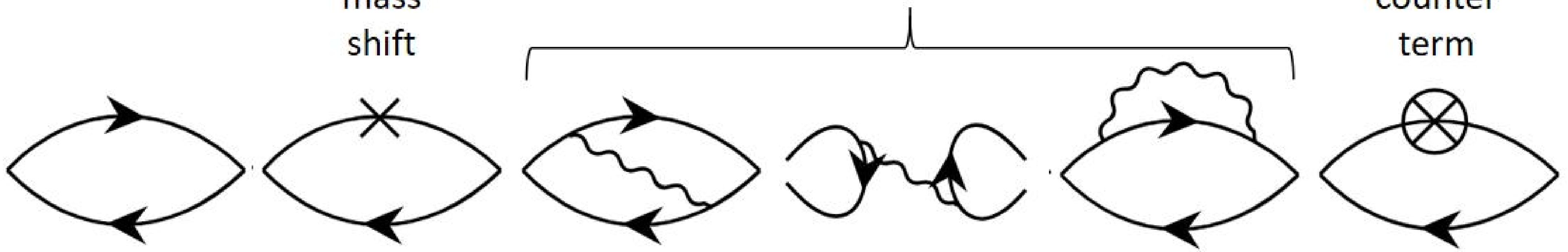


?

However, such counter term does not appear in some cases of hadron mass splittings.

Example: π

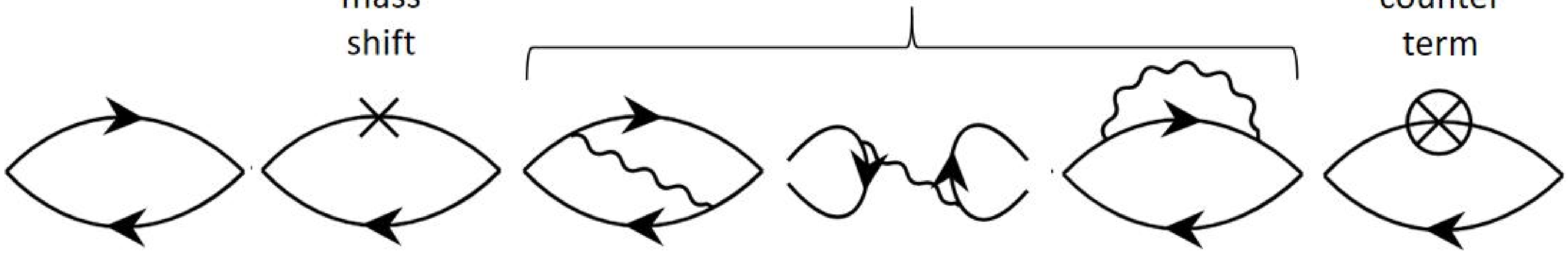


$m_{\pi^\pm}$	1	$F_s(0)\cdot\Delta m_q$	$2e^2/9$	0	$-5e^2/9$	$-5e^2/9\cdot CT$
m_{π^0}	1	$F_s(0)\cdot\Delta m_q$	$-5e^2/18$	$e^2/2$	$-5e^2/9$	$-5e^2/9\cdot CT$
$m_{\pi^\pm}-m_{\pi^0}$	0	0	$e^2/2$	$-\,e^2/2$	0	0

No ultra-violet divergence nor counter term in $m_{\pi^\pm}-m_{\pi^0}$.

For baryons made of uds, the same thing happens in $m_{\Sigma^+}+m_{\Sigma^-}-2m_{\Sigma^0}$.

Example: π



coefficients of diagrams		quark mass shift	QED			counter term
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How to make it general?

Generalize to Any Hadron

- Within pure QCD

used on lattice

$$m_H = m_H^0 + \langle H|\bar{q}q|H\rangle(m_q^{phy} - m_q) + O\left((m_q^{phy} - m_q)^2\right)$$

u-d mass splitting and fine tuning.

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- We square it for mesons

$$m_\pi^2 = (m_\pi^0)^2 + \underline{2m_\pi^0\langle\pi|\bar{q}q|\pi\rangle}(m_q^{phy} - m_q) + O\left((m_q^{phy} - m_q)^2\right)$$

relates mildly to pion mass.

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relates mildly to pion mass.

cancel $m_q^{phy} - m_q$

lattice calculation

prediction

$$m_H = m_H^0 + \frac{m_\Omega^{phy}\langle H|\bar{q}q|H\rangle}{2m_\pi^0\langle \pi|\bar{q}q|\pi\rangle}\frac{m_\pi^2 - (m_\pi^0)^2}{m_\Omega^{phy}} + O\left((m_q^{phy} - m_q)^2\right)$$

Z_s cancels   input

Introduce QED Effect

UV divergence is cut off by lattice spacing

$$m_H = m_H^0 + \langle H|\bar{q}q|H\rangle\left(m_q^{phy} - m_q + CT\right) + \Delta m_H^{QED} + O\left(\left(m_q^{phy} - m_q\right)^2\right)$$

QED self-energy counter term of quark

$$m_\pi^2 = (m_\pi^0)^2 + 2m_\pi^0\langle \pi|\bar{q}q|\pi\rangle\left(m_q^{phy} - m_q + CT\right) + 2m_\pi^0\Delta m_\pi^{QED} + O\left(\left(m_q^{phy} - m_q\right)^2\right)$$

counter term & unphysical part cancel

lattice calculation

prediction

$$m_H = m_H^0 + \frac{m_\Omega^{phy}\langle H|\bar{q}q|H\rangle}{2m_\pi^0\langle \pi|\bar{q}q|\pi\rangle}\,\frac{m_\pi^2 - (m_\pi^0)^2 - 2m_\pi^0\Delta m_\pi^{QED}}{m_\Omega^{phy}} + \Delta m_H^{QED} + O\left(\left(m_q^{phy} - m_q\right)^2\right)$$

$= \Delta\bar{m}_\pi$

input

Lattice Calculation

- For lattice with 3 flavors of quarks, we need 3 kinds of mesons

$$m_H = m_H^0 + \sum_q \langle H|\overline{q}q|H\rangle\left(m_q^{phy} - m_q + CT\right) + \Delta m_H^{QED} + O\left(\left(m_q^{phy} - m_q\right)^2\right)$$
$$= m_H^0 + \alpha\Delta\overline{m}_{\pi^\pm} + \beta\Delta\overline{m}_{K^\pm} + \gamma\Delta\overline{m}_{K^0} + \Delta m_H^{QED} + O\left(\left(m_q^{phy} - m_q\right)^2\right)$$

α, β, γ: combinations of ratios of $F_s(0)$ ➡ need to compute properties of π & K.

Lattice Calculation

- For lattice with 3 flavors of quarks, we need 3 kinds of mesons

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α, β, γ: combinations of ratios of $F_s(0)$ ➡ need to compute properties of π & K.

- Both input and output need lattice spacing to convert between lattice and physical unit

$$a^{QCD} m_H^{phy} = m_H^0$$

The lattice spacing changes while introducing QED. We need another hadron (eg.Ω)

$$a^{full} m_\Omega^{phy} = m_\Omega = m_\Omega^0 + \alpha\Delta\bar{m}_{\pi^\pm} + \beta\Delta\bar{m}_{K^\pm} + \gamma\Delta\bar{m}_{K^0} + \cdots$$

as input to solve the new spacing a^{full}.

include a^{full}

Numerical Analysis

We compute:

- m^0, by 2pt
- $Z_S F_S(0)$, by 3pt
- Δm^{QED}, by 4pt

of several particles:

- π
- K
- Ω — lattice spacing
- N
- Σ
- Ξ
- $\vdots$

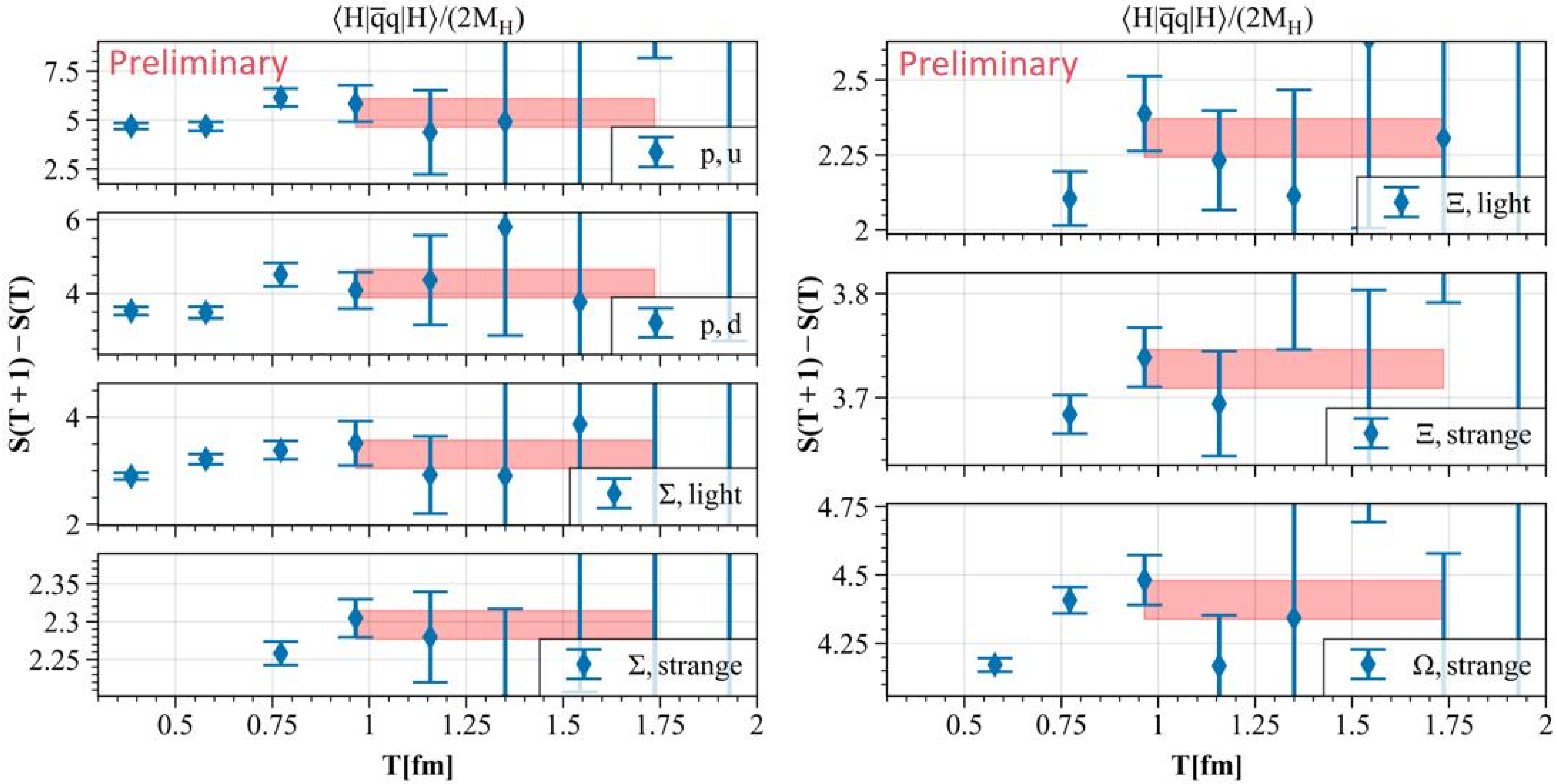
(N, Σ, Ξ, $\vdots$ — prediction)

Name	N	L	T	a	m_π
24D	94	24	64	0.19fm	143MeV

Generated by RBC/UKQCD, 2+1 flavor

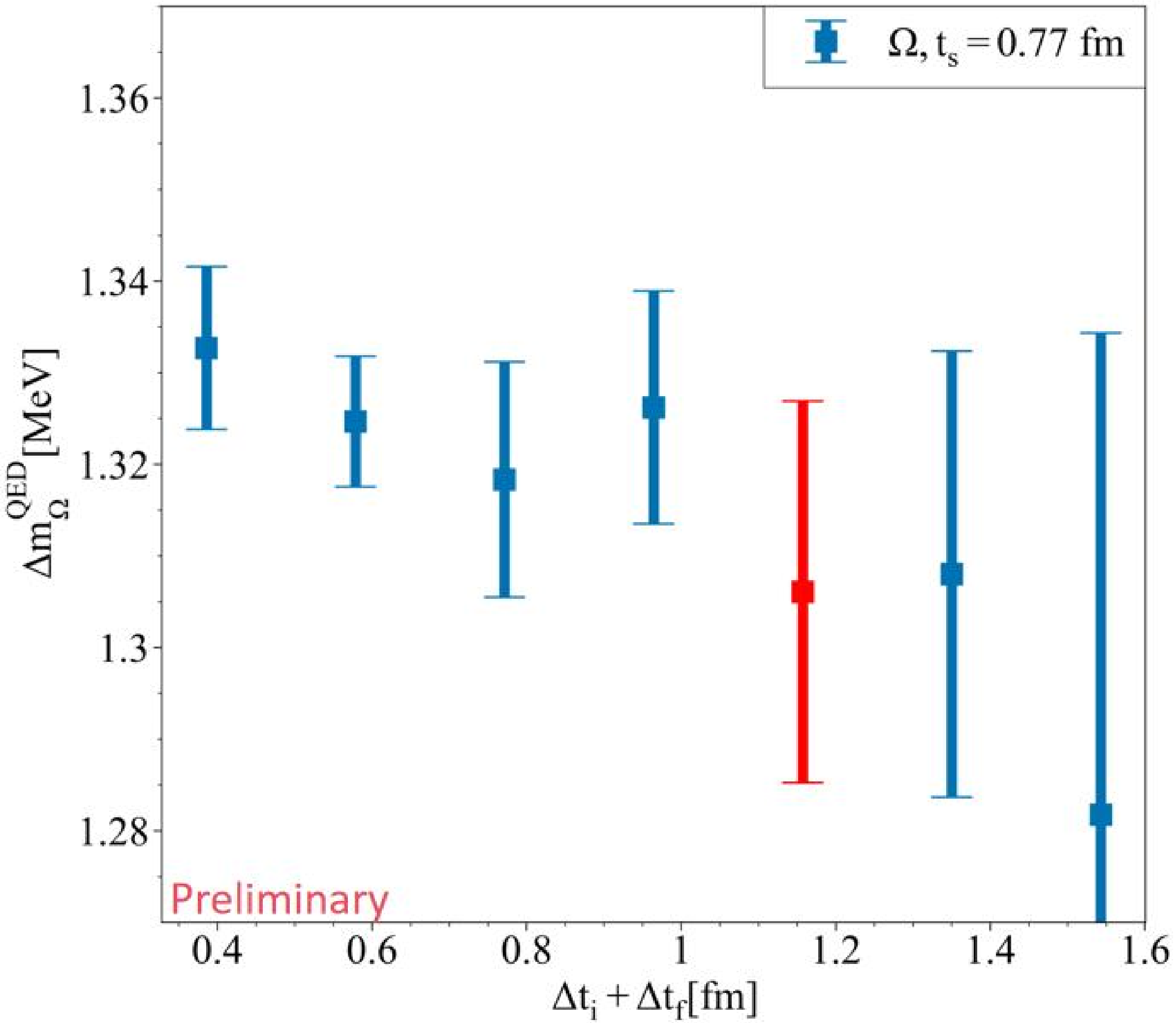
$$\begin{cases} m^0, & \text{by 2pt} \\ Z_S F_S(0), & \text{by 3pt} \\ \Delta m^{QED}, & \text{by 4pt} \end{cases}$$

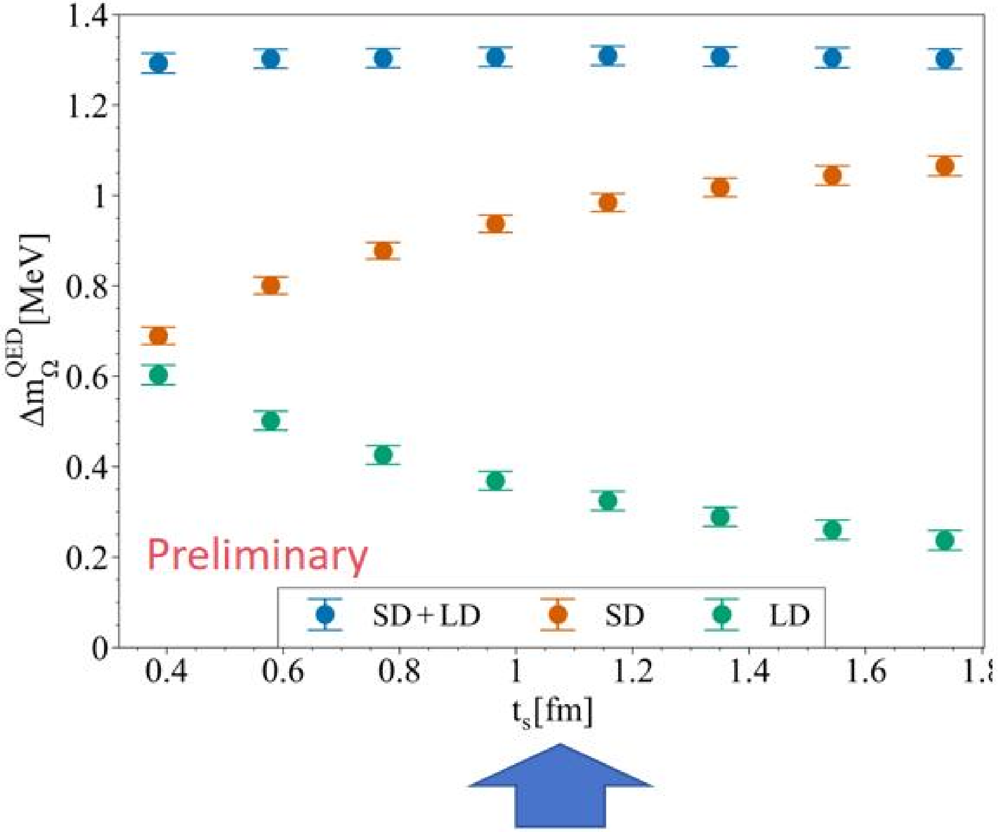
$$S(T) = \sum_{\tau=0}^{T} \langle O_H(T) J_S(\tau) O_H^{+}(0) \rangle = C_0 + \langle H|\overline{q}q|H\rangle \cdot T + (excited\ states)$$



$$\begin{cases} m^0, & \text{by 2pt} \\ Z_s F_s(0), & \text{by 3pt} \\ \Delta m^{QED}, & \text{by 4pt} \end{cases}$$

$$\Delta m^{QED} = \frac{e^2}{2} \int d^4x \langle H | J^\mu(x) J^\nu(0) | H \rangle S^\gamma_{\mu\nu}(x)$$





IVR[1]

Exponentially suppressed finite volume effects

[1]X.Feng, L.Jin, PRD.**100**.094509

Preliminary Results

$$a^{full} m_H^{phy} = m_H = m_H^0 + \alpha \Delta \bar{m}_{\pi^\pm} + \beta \Delta \bar{m}_{K^\pm} + \gamma \Delta \bar{m}_{K^\pm} + \Delta m_H^{QED} + O\left(\left(m_q^{phy} - m_q\right)^2\right)$$

- For Ω

$$\alpha = -1.198(23), \qquad \frac{a^{full\ theory}}{a^{pure\ QCD}} = 1.0195(50)$$

$$\beta = \gamma = 1.264(20)$$

- To do next...
    - $N, \Sigma, \Xi, \Delta \cdots$, for example, $\alpha_N, \beta_N, \gamma_N, \Delta m_N^{QED}, a^{full} \rightarrow \Delta m_N$
    - Disconnected diagrams
    - Add finer lattice to do continuum extrapolation

Conclusion

- We develop a new method to compute isospin breaking effect on hadron mass splitting on lattice.
- Using this method we can compute hadron mass splitting without ultraviolet divergence nor renormalization.
- We apply it on a real lattice to compute several hadrons.

Thank you for your listening.