

# Axial-vector and scalar bottom-charm tetraquarks from Lattice QCD

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In collaboration with

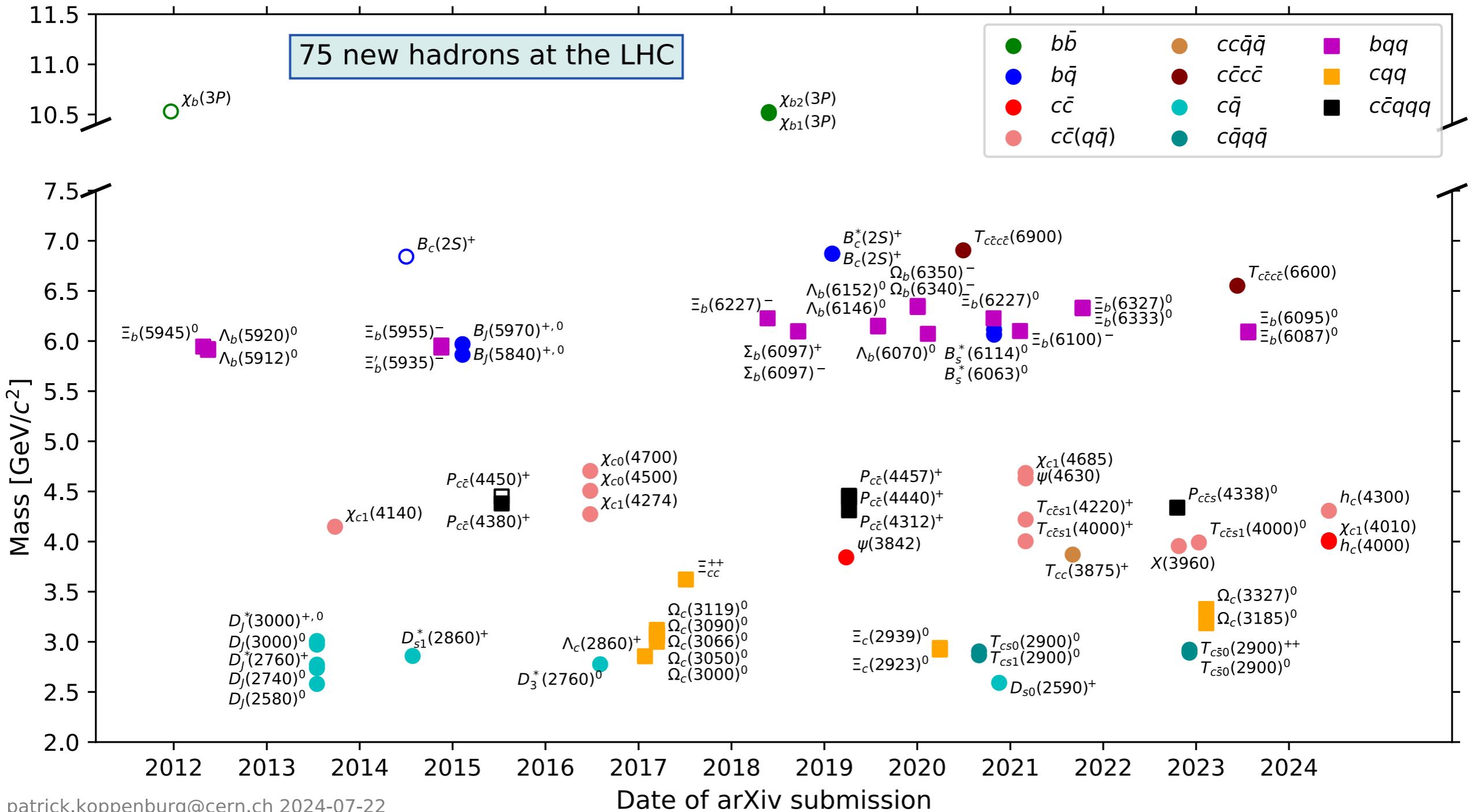


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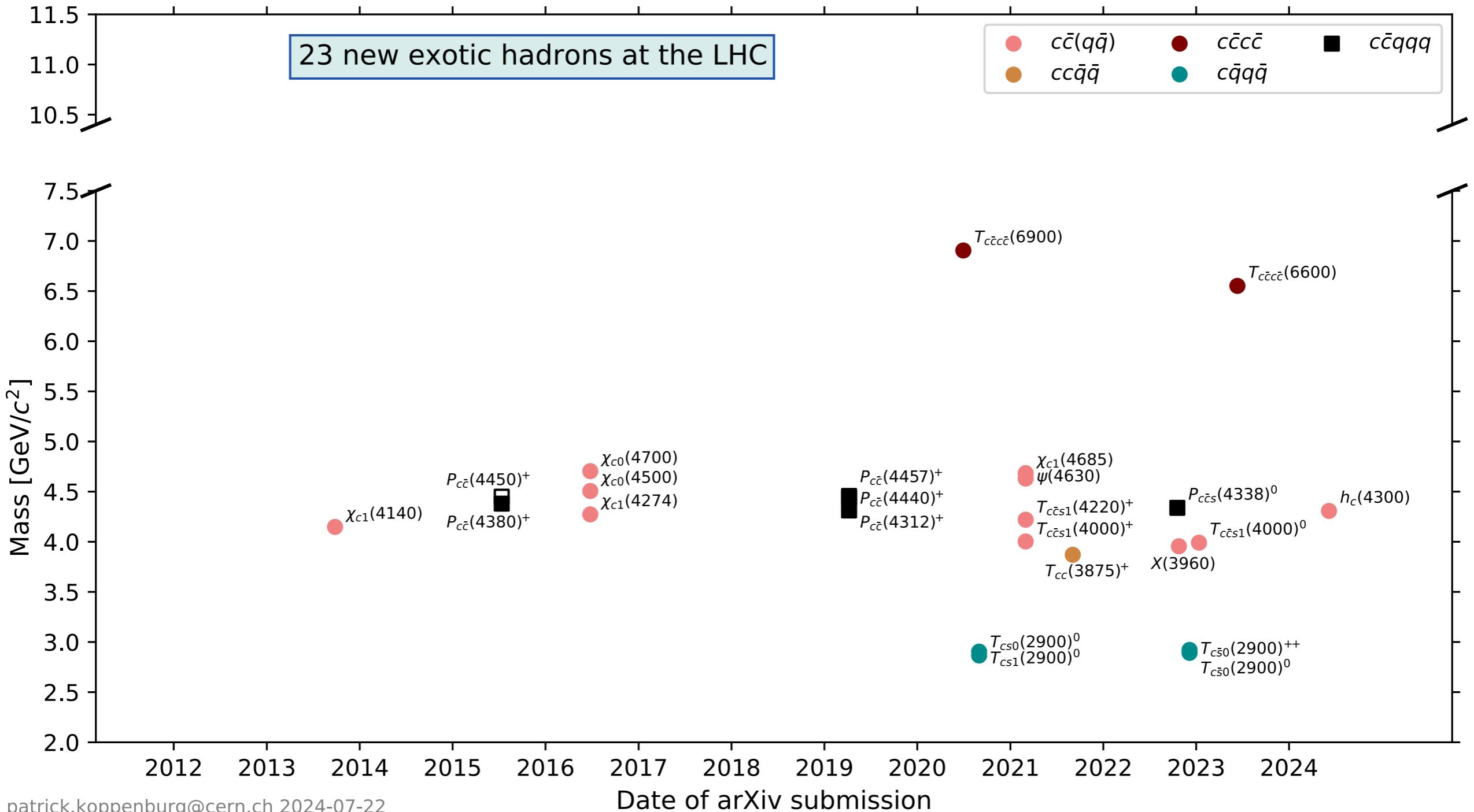


# Experimental Status



patrick.koppenburg@cern.ch 2024-07-22

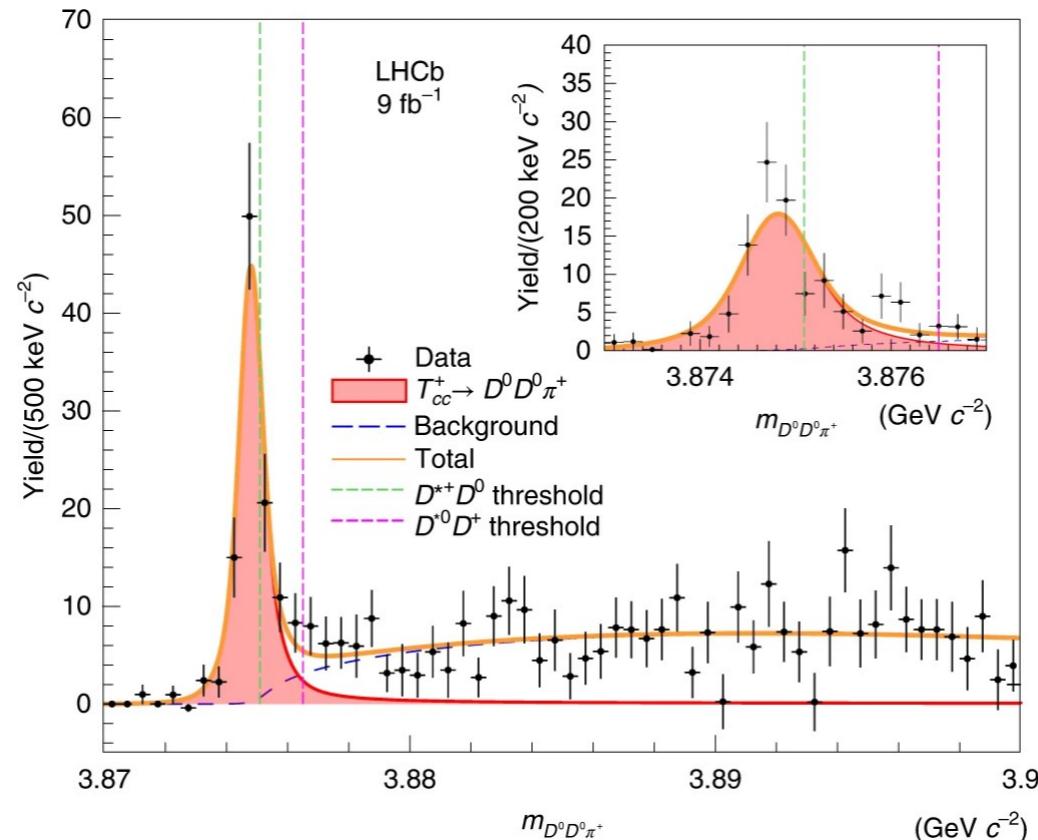
# Experimental Status



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# Discovery of $T_{cc}^+$ and the prospects for other $QQ'qq'$ states

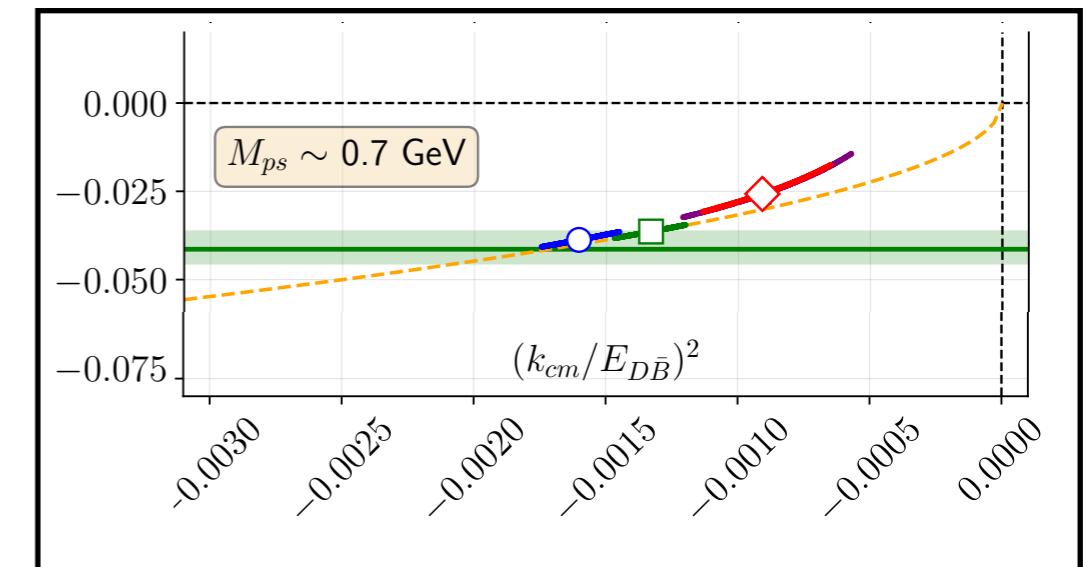
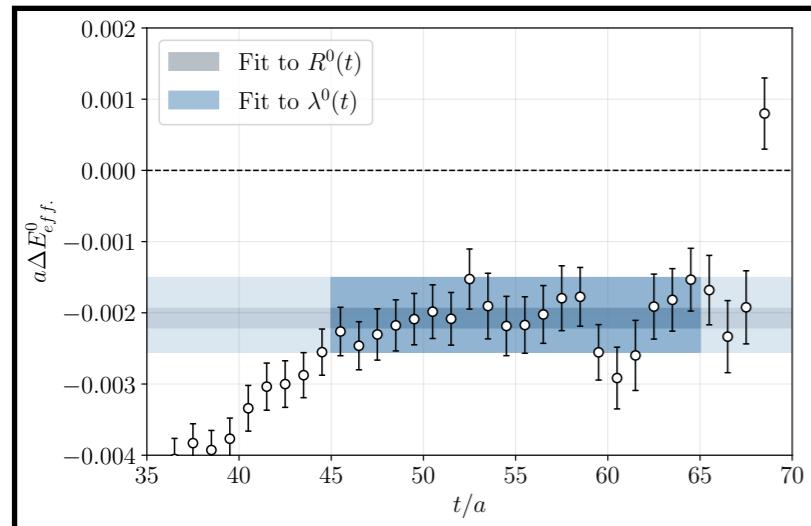
Nat. Phys. **18**, 751–754 (2022)



- Doubly-heavy channels are promising places to look for long-lived exotic tetraquarks
- $cc\bar{d}\bar{u}$  observed in 2021 by LHCb
- Where should experiments look to find more such states?
- What does theory tell us?

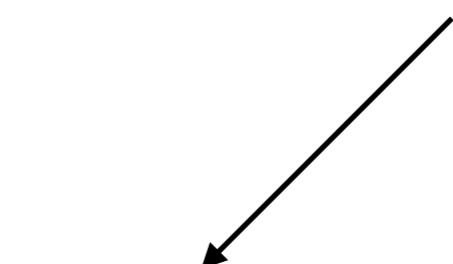
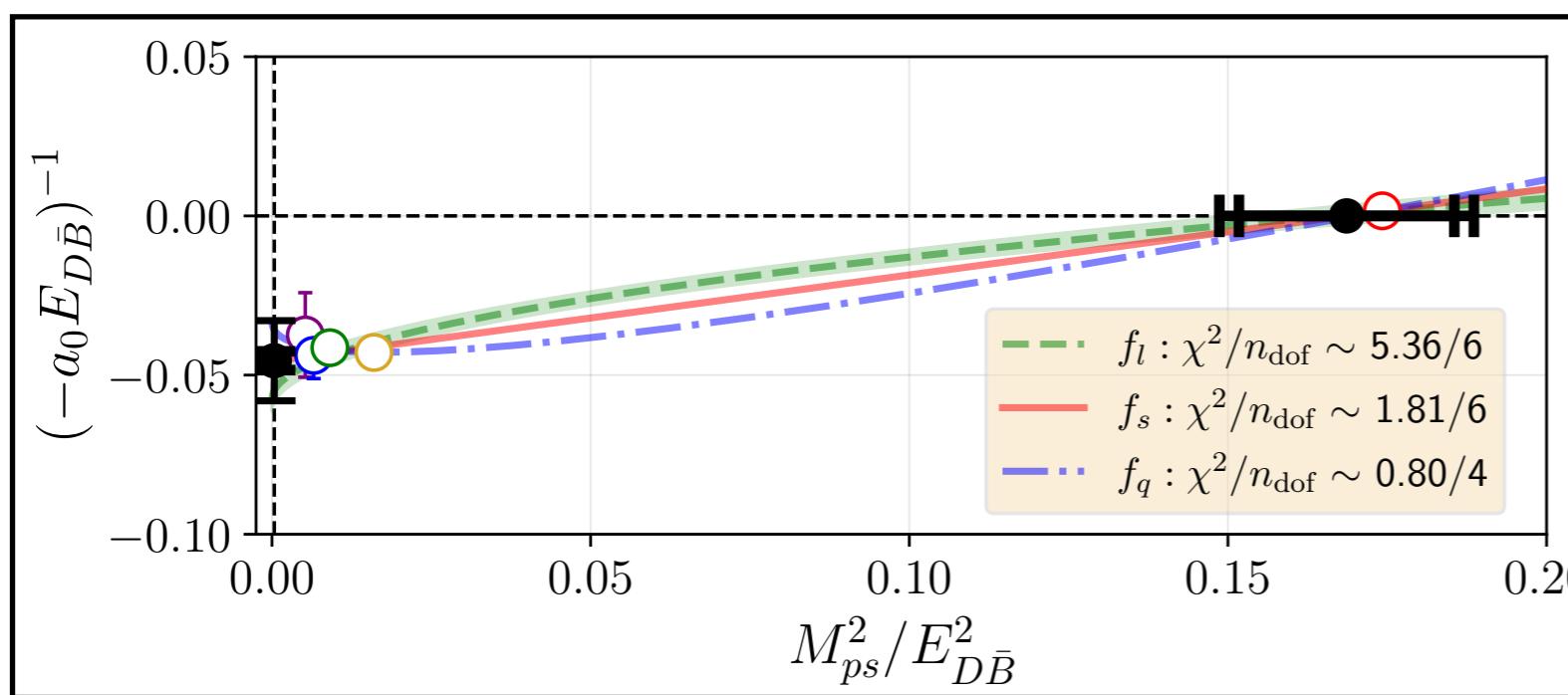
Other possible  $QQ'qq'$  states:  $bb\bar{u}\bar{d}$  and  $bc\bar{u}\bar{d}$

# Lattice Methodology



The time dependence of two point correlation functions gives discrete energy levels that depend on the volume of the lattice

Map the discrete energy levels to the scattering amplitudes

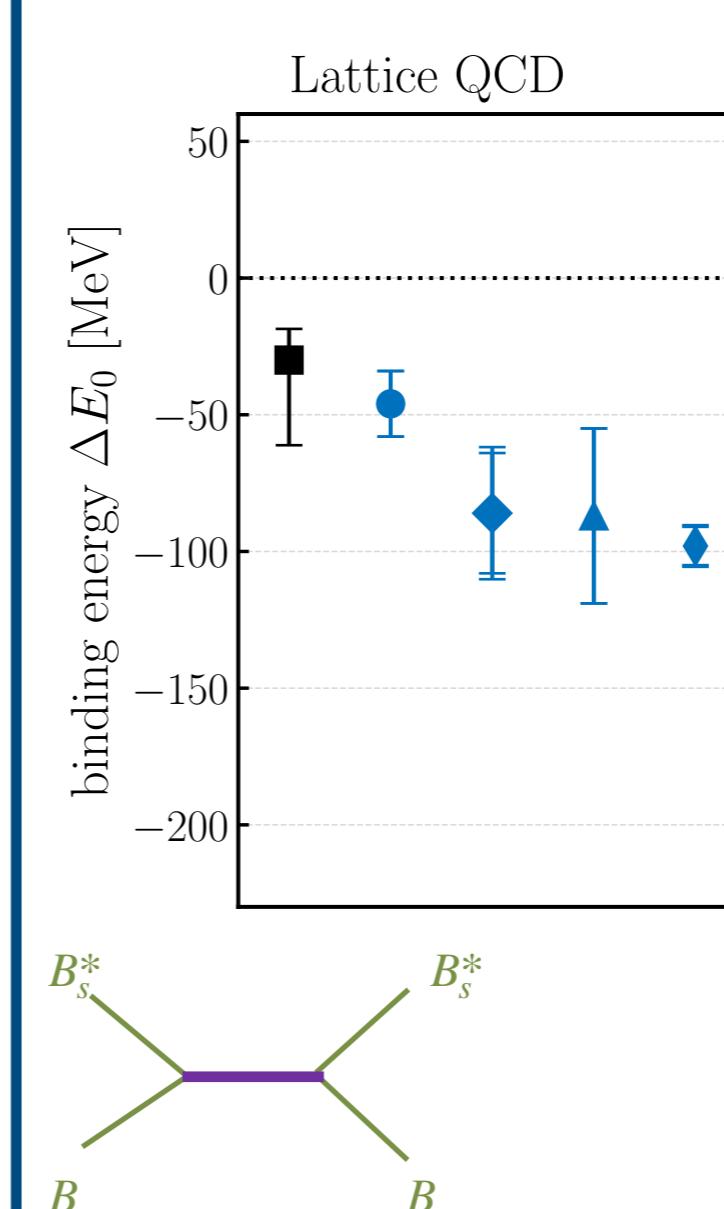
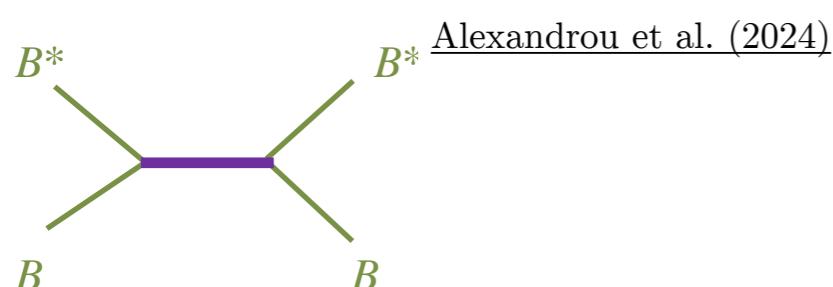
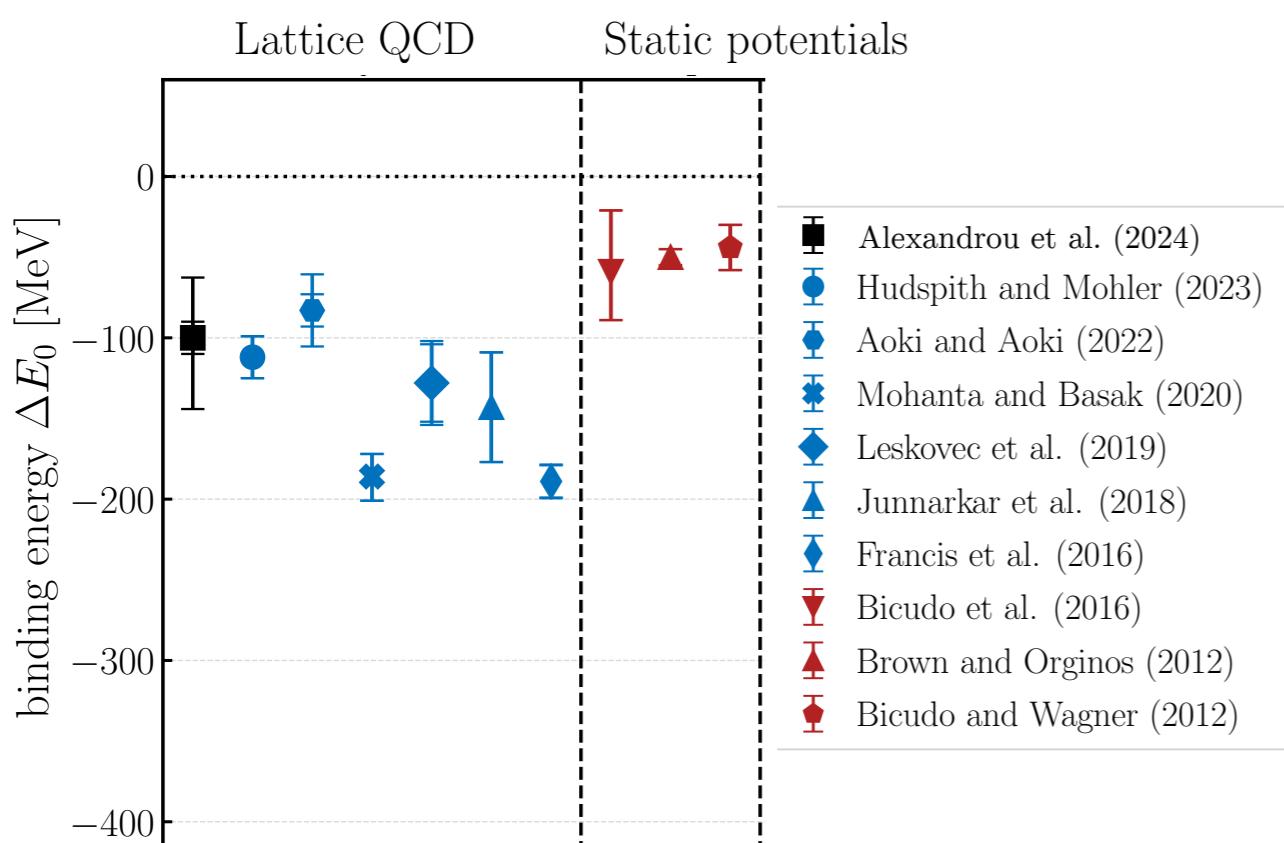


Continuum and chiral extrapolation to get the binding energies

# Motivation from $T_{bb}$ and $T_{cc}$

States well below threshold

$bb\bar{u}\bar{d}$  0(1<sup>+</sup>)



$bb\bar{u}\bar{s}$  0(1<sup>+</sup>)

- Alexandrou et al. (2024)
- Hudspith and Mohler (2023)
- ◆ Meinel et al. (2022)
- ▲ Junnarkar et al. (2018)
- ◆ Francis et al. (2016)

Alexandrou et al. (2024)

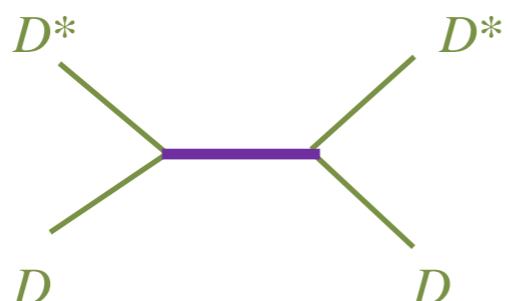
Surely bound and numerous calculations have been performed using LQCD - assuming not substantial finite volume effects - **a reliable claim for deeply bound states with heavy quarks**

These states may be discovered in future, but the energies might be too high for current experiments to reliably explore

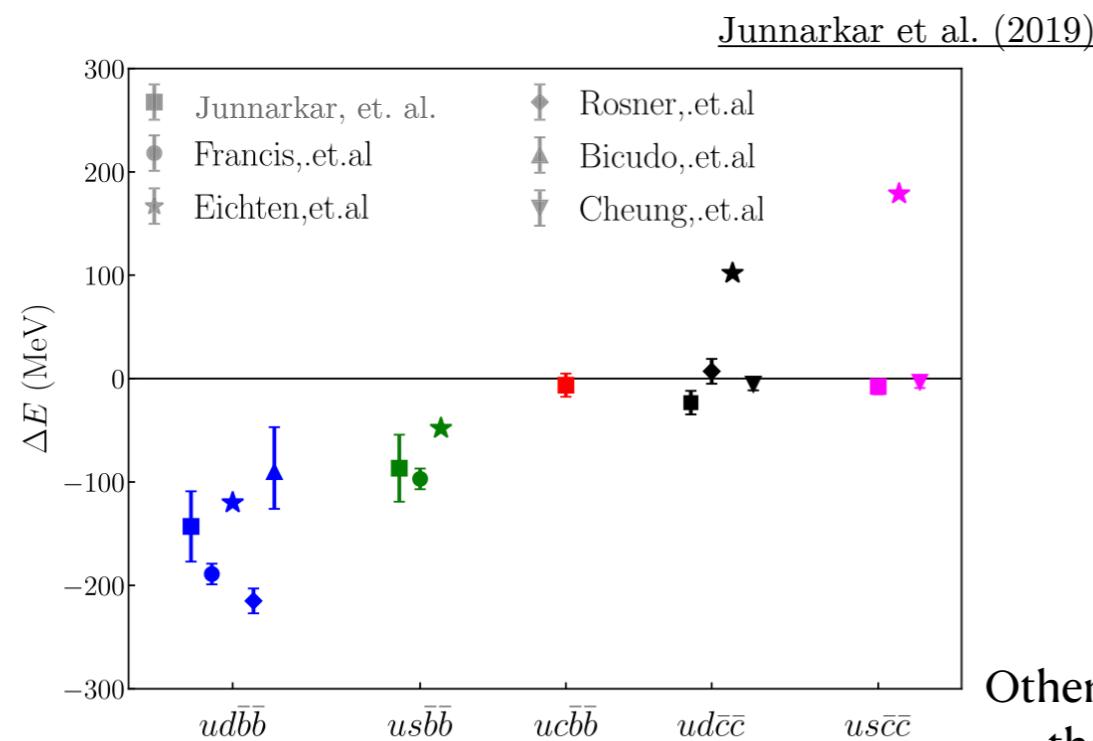
# Motivation from $T_{bb}$ and $T_{cc}$

## States near threshold

$cc\bar{u}\bar{d}$   $0(1^+)$



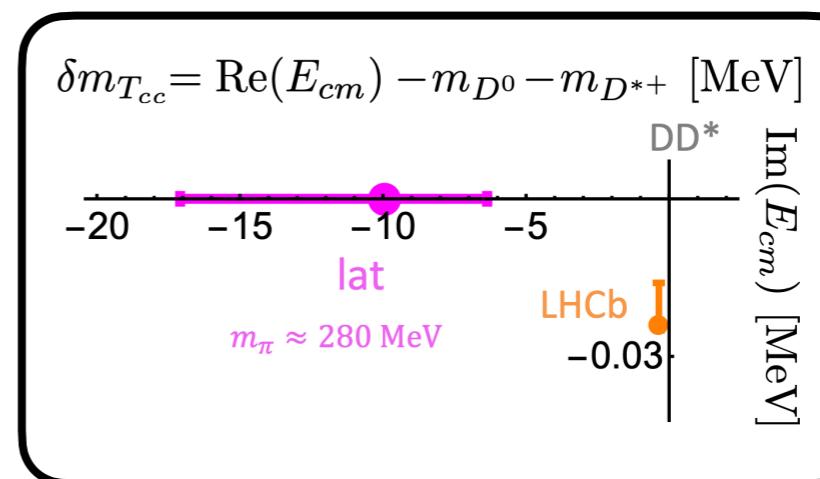
Lattice had seen indications of a bound state  
- later discovered by LHCb!



Other recent lattice studies confirm  
the presence the bound state

Paper	Results
Padmanath et al. [PRL 129,032002 (2022)]	$-9.9^{+3.6}_{-7.1}$ MeV
Chen et al. [PLB 833,137391 (2022)]	$I = 0, 1$ (attractive,repulsive) no info about the pole
Lyu et al. [PRL 131,161901 (2023)]	$-45^{(+41)}_{(-78)}$ keV
Collins et al. [arXiv:2402.14715 (2024)]	quark mass dependence checked
Whyte et al. [arXiv:2405.15741 (2024)]	$-62 \pm 31$ MeV

Need to find the poles in the scattering amplitude to extract  
(virtual) bound poles:

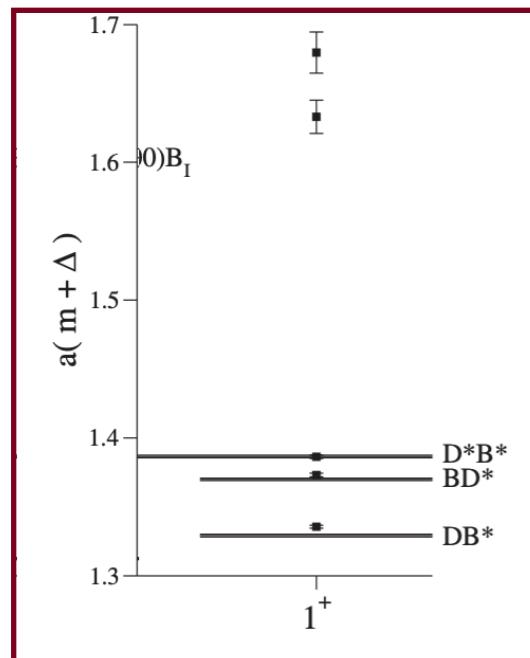


Exploring tetraquarks with  
bottom and  
charm may be accessible to  
experiments  
Motivates the study of  $bc\bar{u}\bar{d}$   
but significant volume effects  
possible - close to threshold

# What does LQCD tell us?

States near threshold

$bc\bar{u}\bar{d}$   $0(1^+)$



Hudspith et al. Phys. Rev. D 102, 114506

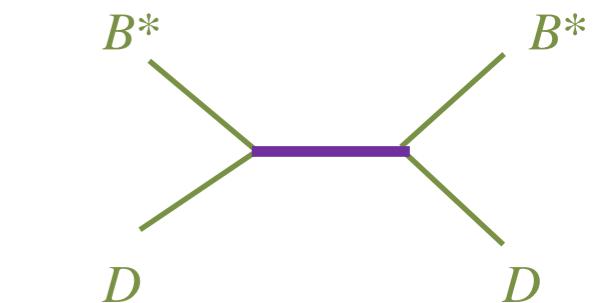
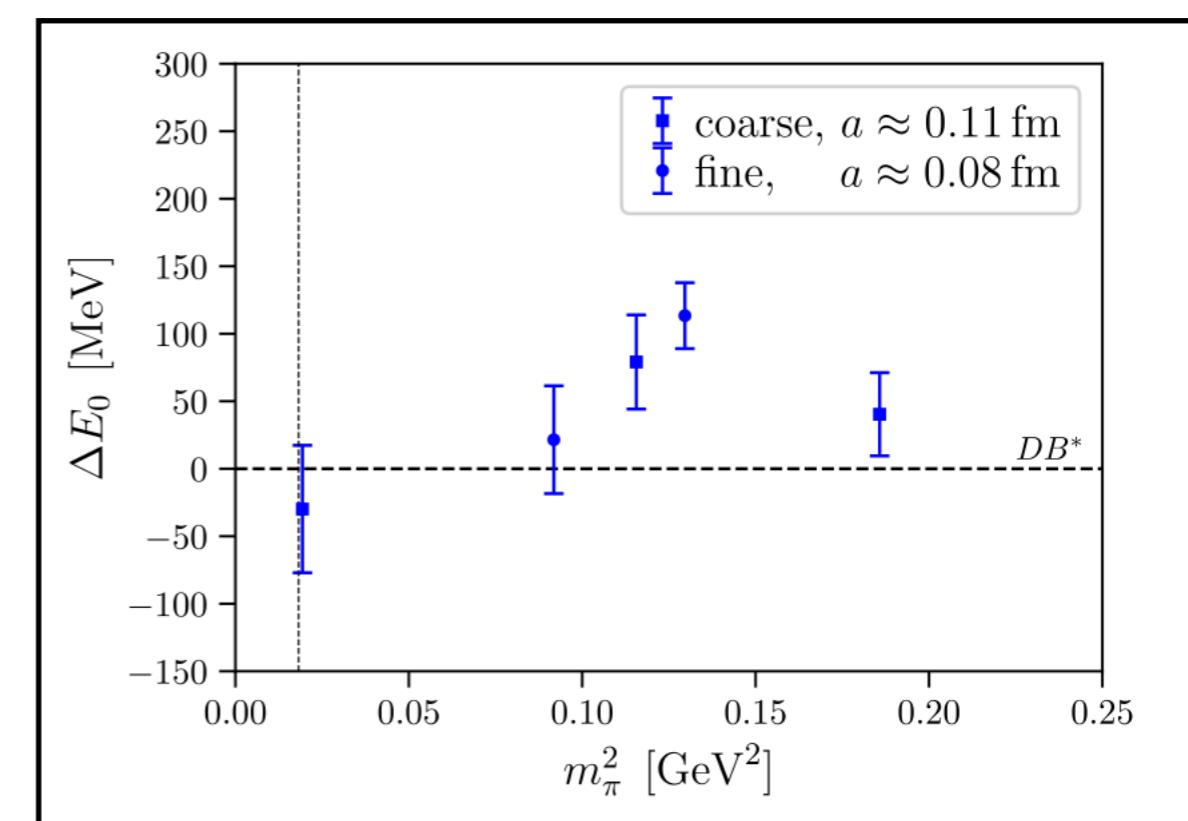
Need more such studies  
of  $bc\bar{u}\bar{d}$

Infinite volume scattering  
amplitude was not  
extracted

Tetraquarks with charm and  
bottom can be currently  
accessible to experiments -  
theoretical predictions can  
help in such searches!

Meinel et al. Phys. Rev. D 106, 034507

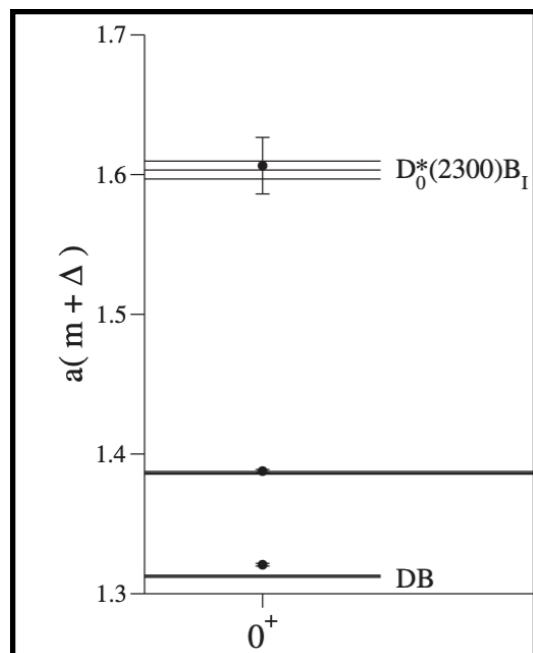
No conclusive  
evidence of bound  
states



# What does LQCD tell us?

States near threshold

$bc\bar{u}\bar{d}$   $0(0^+)$



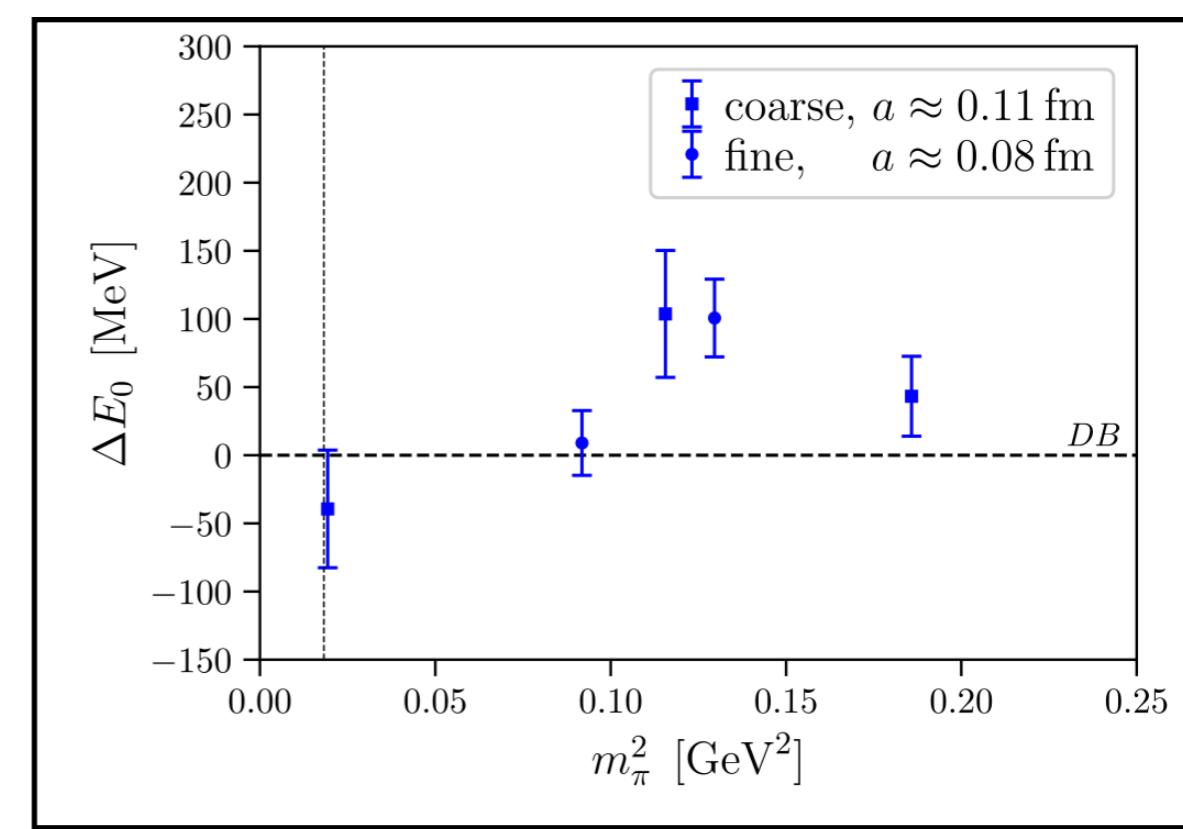
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Need more such studies of  $bc\bar{u}\bar{d}$

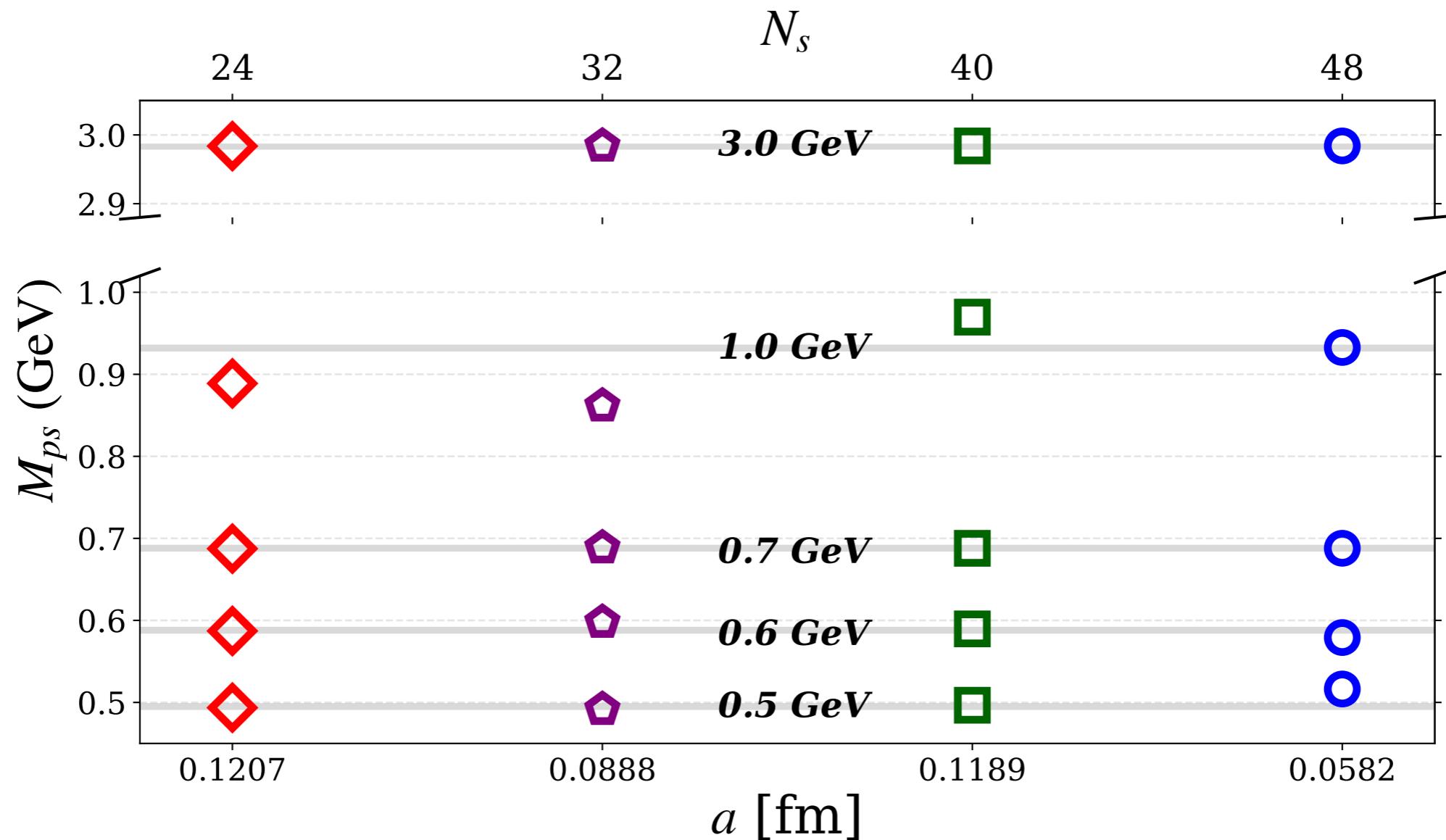
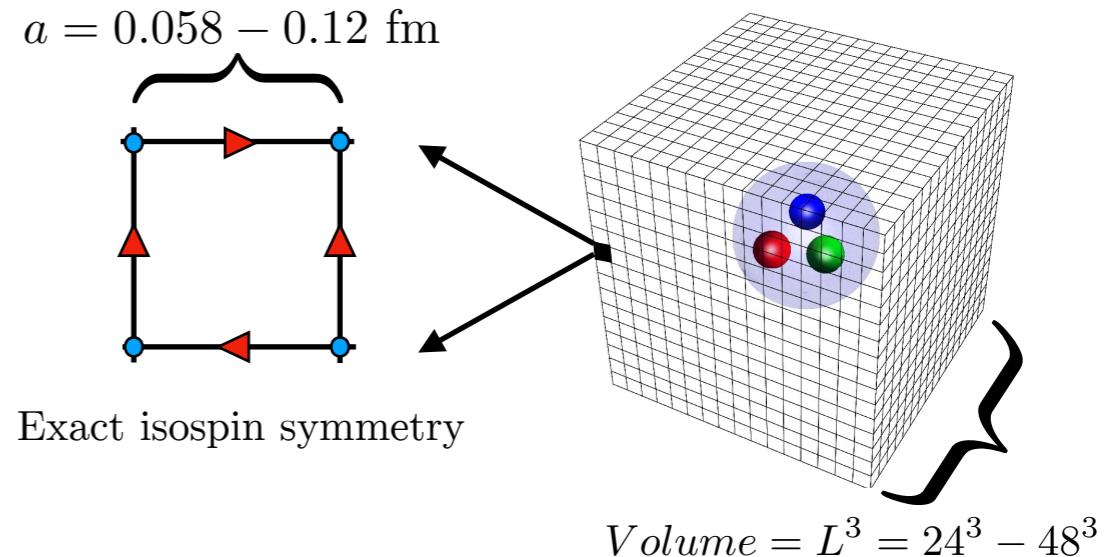
Tetraquarks with charm and bottom can be currently accessible to experiments - theoretical predictions can help in such searches!

Meinel et al. Phys. Rev. D 106, 034507



# Lattice details

- MILC ensembles NF=2+1+1 HISQ action.
- valence quarks were implemented using an overlap action ( $u, d, s, c$ ).
- the evolution of the bottom quark was studied within a non-relativistic QCD framework.
- $b, c, s$  at the physical point and light quark mass is varied.



# Extraction of the energy spectrum

The time dependence of Euclidean two point correlation function gives the energy,

$$C(t) = \langle 0 | \Omega_f(t) \Omega_i^\dagger(0) | 0 \rangle$$

Adding a complete set of states,

$$C(t) = \sum_n e^{-E_n t} Z_n^f Z_n^i$$

↑  
time-evolution      ↑  
                          overlaps

at large times they are dominated by  
the ground state

Local 2 two-meson-like interpolators and one diquark-antidiquark-like interpolator

$$\begin{aligned}\mathcal{O}_1(x) &= [\bar{u} \gamma_i b][\bar{d} \gamma_5 c](x) - [\bar{d} \gamma_i b][\bar{u} \gamma_5 c](x), \\ \mathcal{O}_2(x) &= [\bar{u} \gamma_5 b][\bar{d} \gamma_i c](x) - [\bar{d} \gamma_5 b][\bar{u} \gamma_i c](x), \\ \mathcal{O}_3(x) &= [(\bar{u}^T \Gamma_5 \bar{d} - \bar{d}^T \Gamma_5 \bar{u})(b \Gamma_i c)](x).\end{aligned}$$

$$\begin{bmatrix} C(t)_{00} & C(t)_{01} & \dots \\ \vdots & \ddots & \\ C(t)_{N0} & C(t)_{NN} \end{bmatrix} v_a = \lambda_a(t, t_0) \begin{bmatrix} C(t_0)_{00} & C(t_0)_{01} & \dots \\ \vdots & \ddots & \\ C(t_0)_{N0} & C(t_0)_{NN} \end{bmatrix} v_a$$

best linear combination of the basis to interpolate a specific state

$\rightarrow \Omega_{K\pi}^\alpha = \sum_i v_\alpha^i \mathcal{O}_i$

**Optimized Operators**

# Extraction of the energy spectrum

The time dependence of Euclidean two point correlation function gives the energy,

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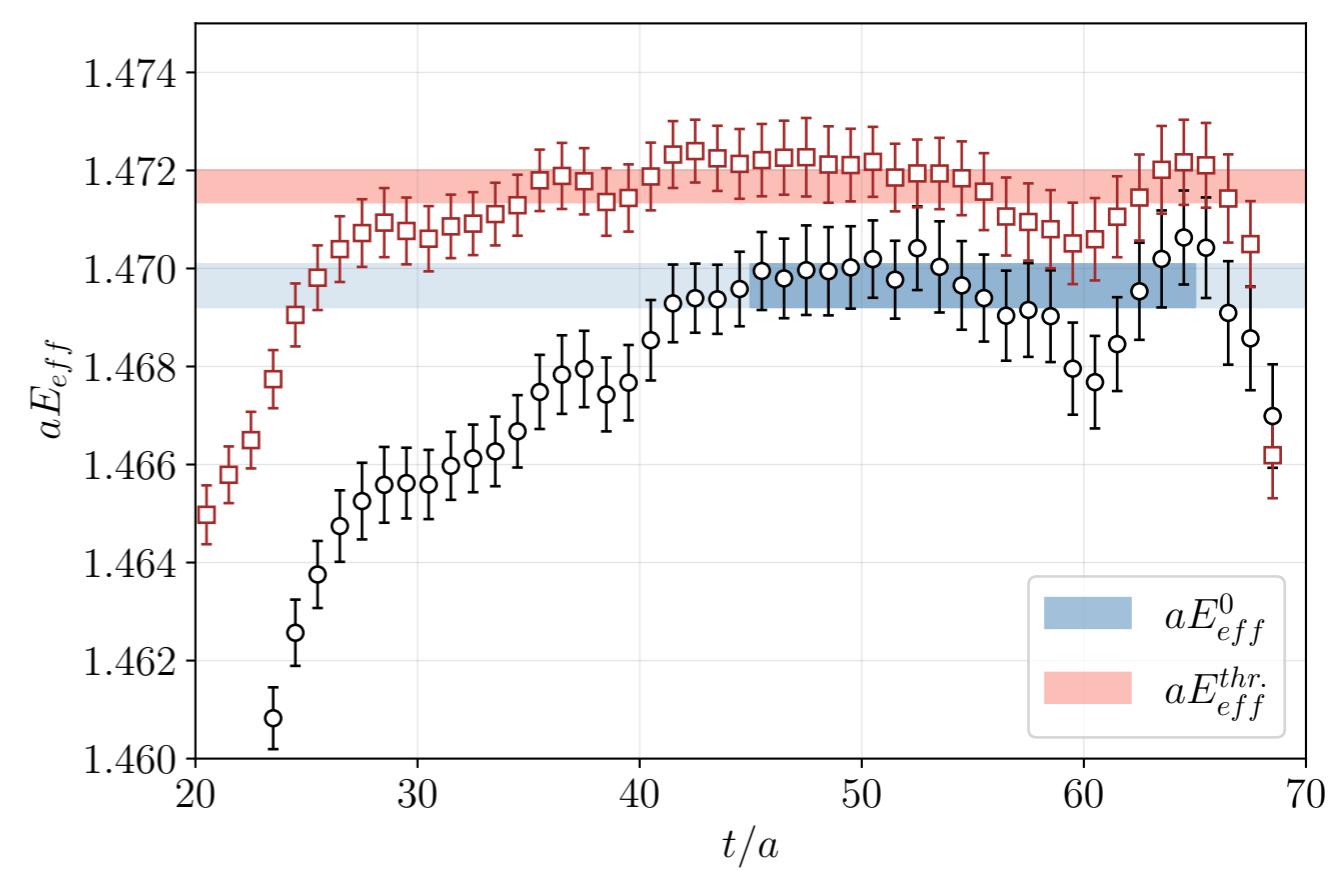
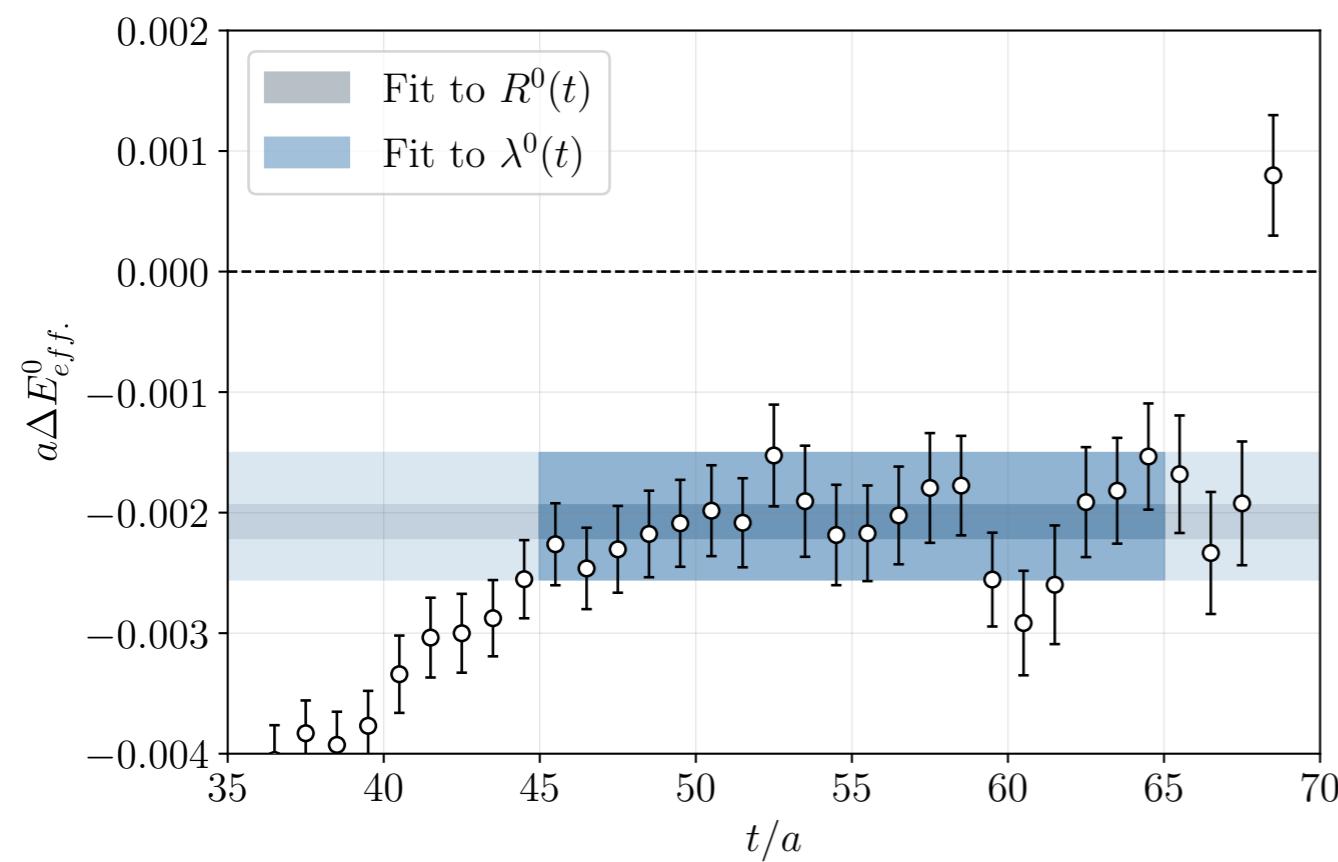
Adding a complete set of states,

$$C(t) = \sum_n e^{-E_n t} Z_n^f Z_n^i$$

↑  
time-evolution      ↑  
                          overlaps

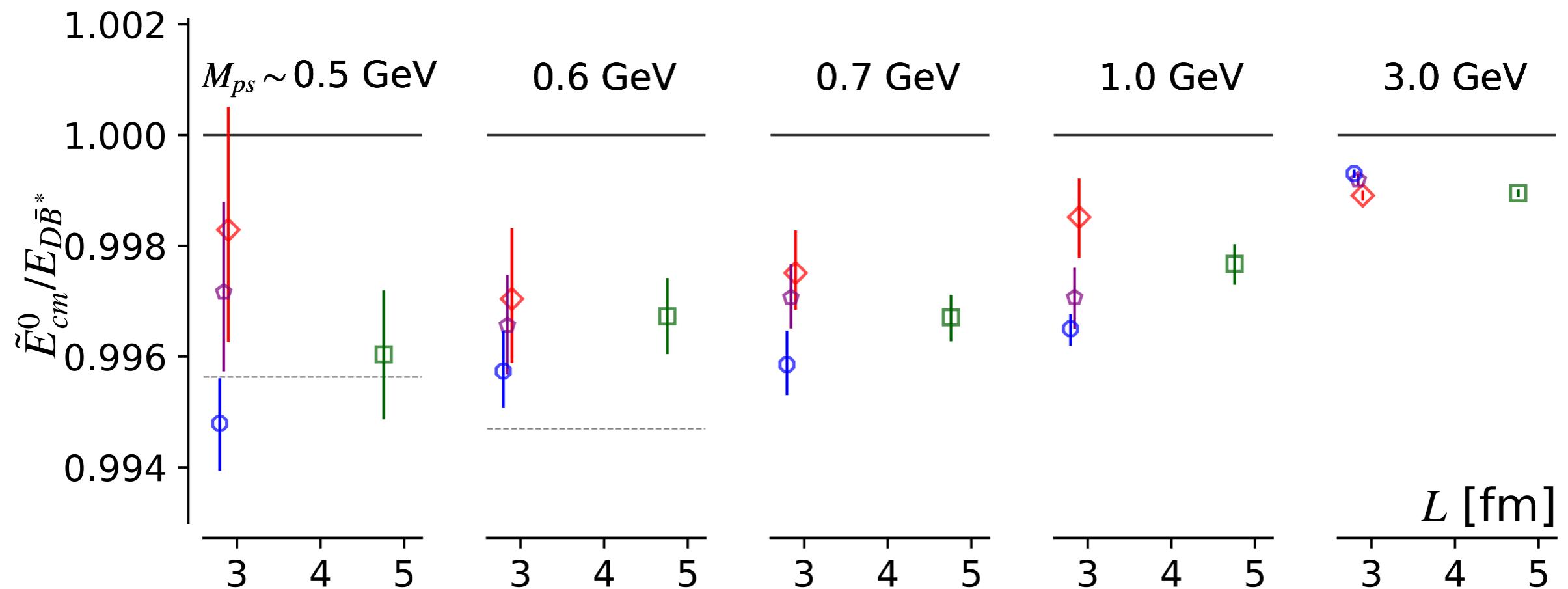
at large times they are dominated by  
the ground state

$$m_{eff} = \ln \frac{C(t)}{C(t+1)}$$



# Finite Volume Spectra - $\bar{b}\bar{c}ud$ $0(1^+)$

M. Padmanath, AR, Nilmani Mathur [PRL.132.201902]



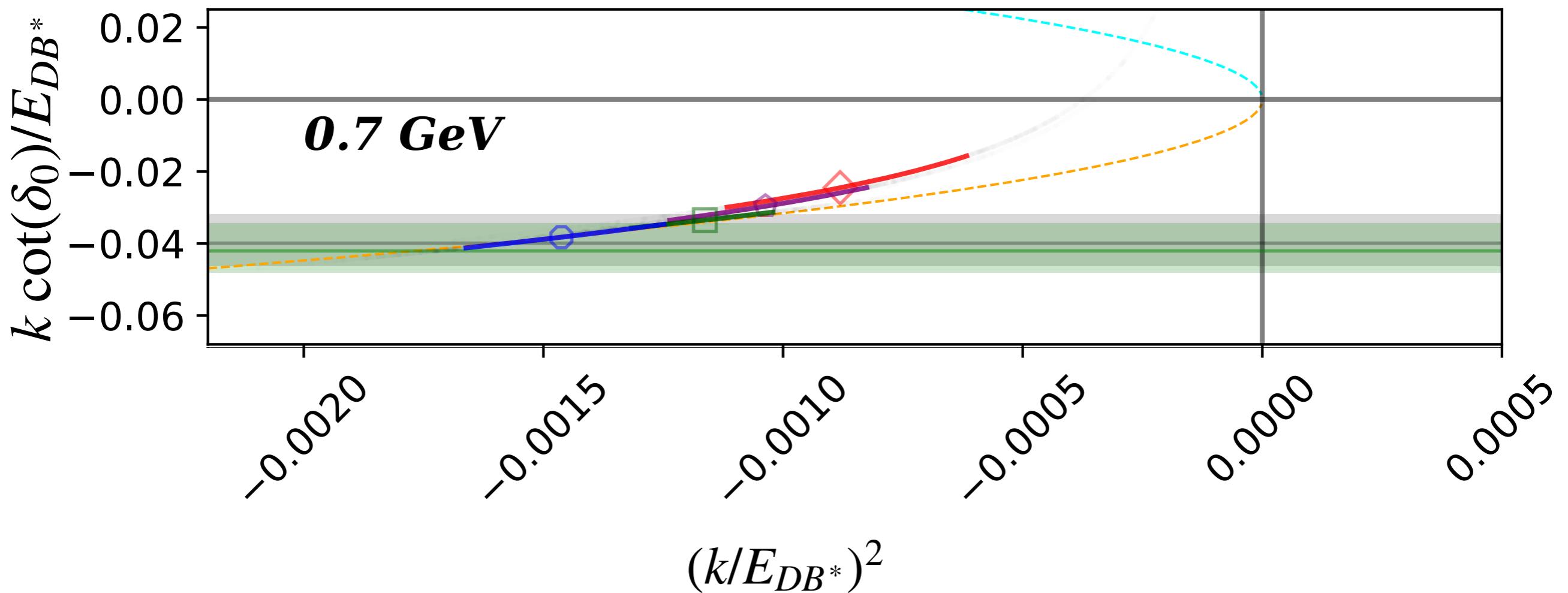
A state **below threshold** is seen  
consistently for all volumes in the five  
different pion masses — indicates an  
attractive interaction

Extract the scattering amplitudes

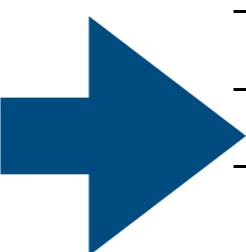
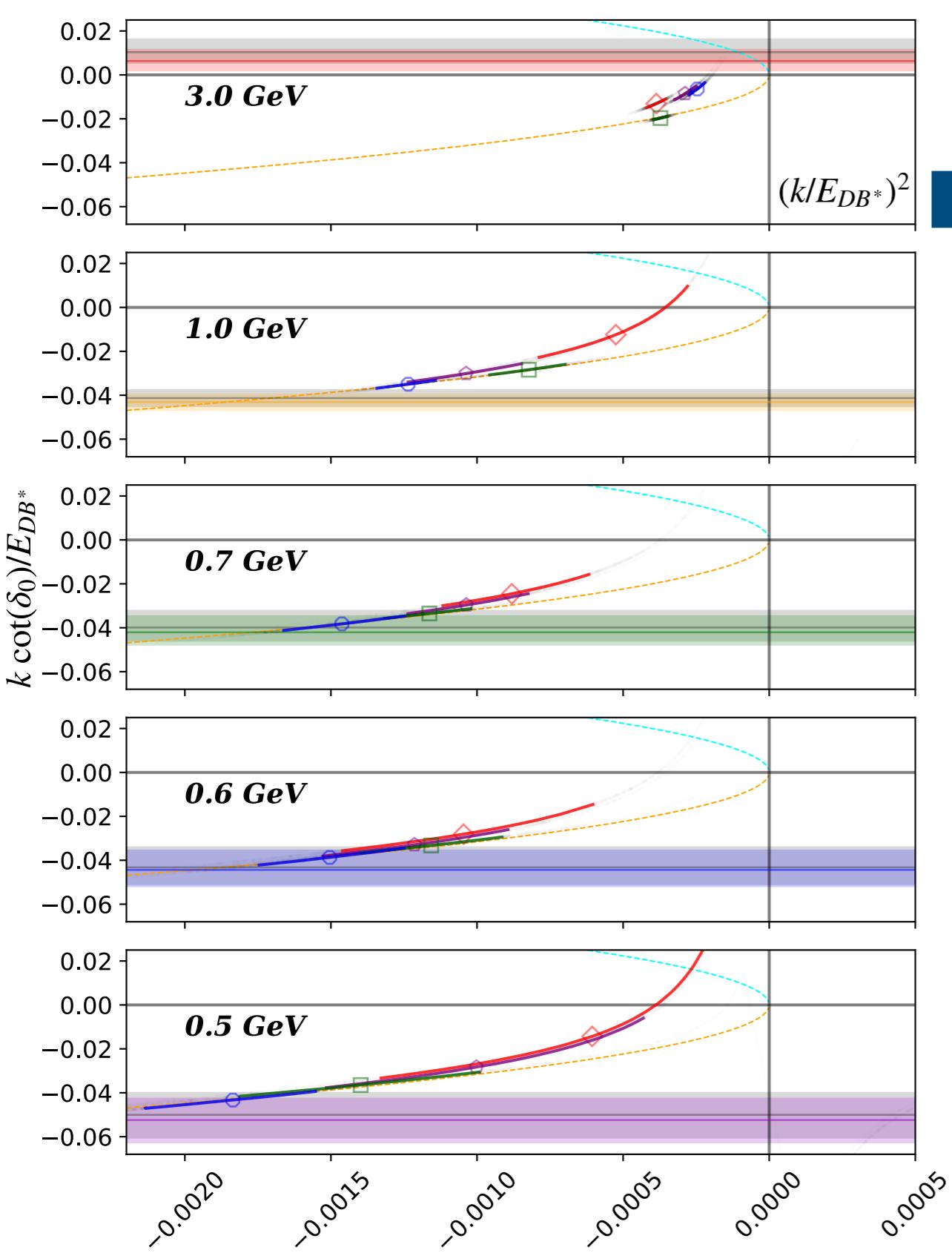
# Extracted infinite volume amplitude with Lüscher- $\bar{b}\bar{c}ud$ 0(1<sup>+</sup>)

M. Padmanath, AR, Nilmani Mathur [PRL.132.201902]

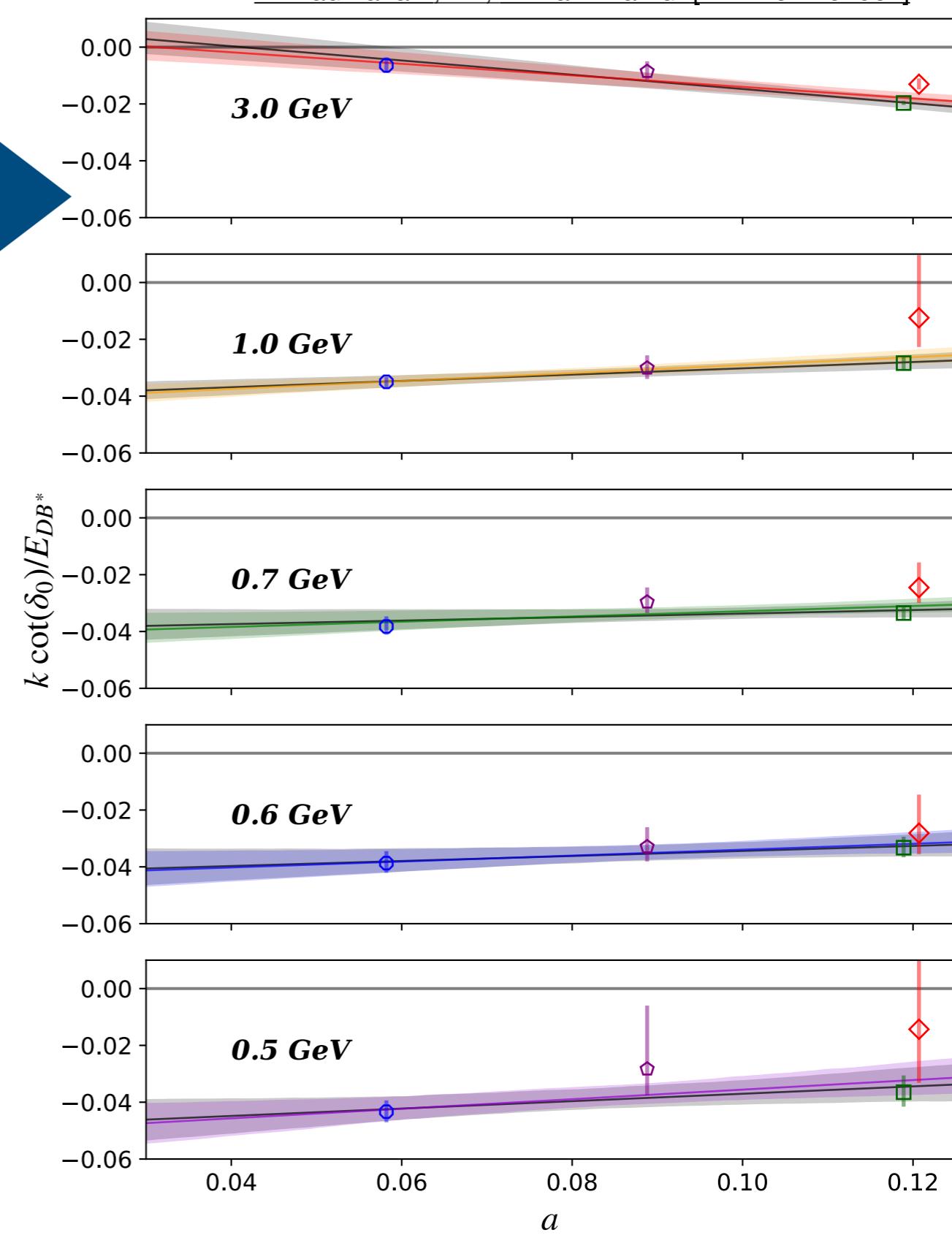
Amplitude:  $k \cot \delta_0 \sim -1/a_0$

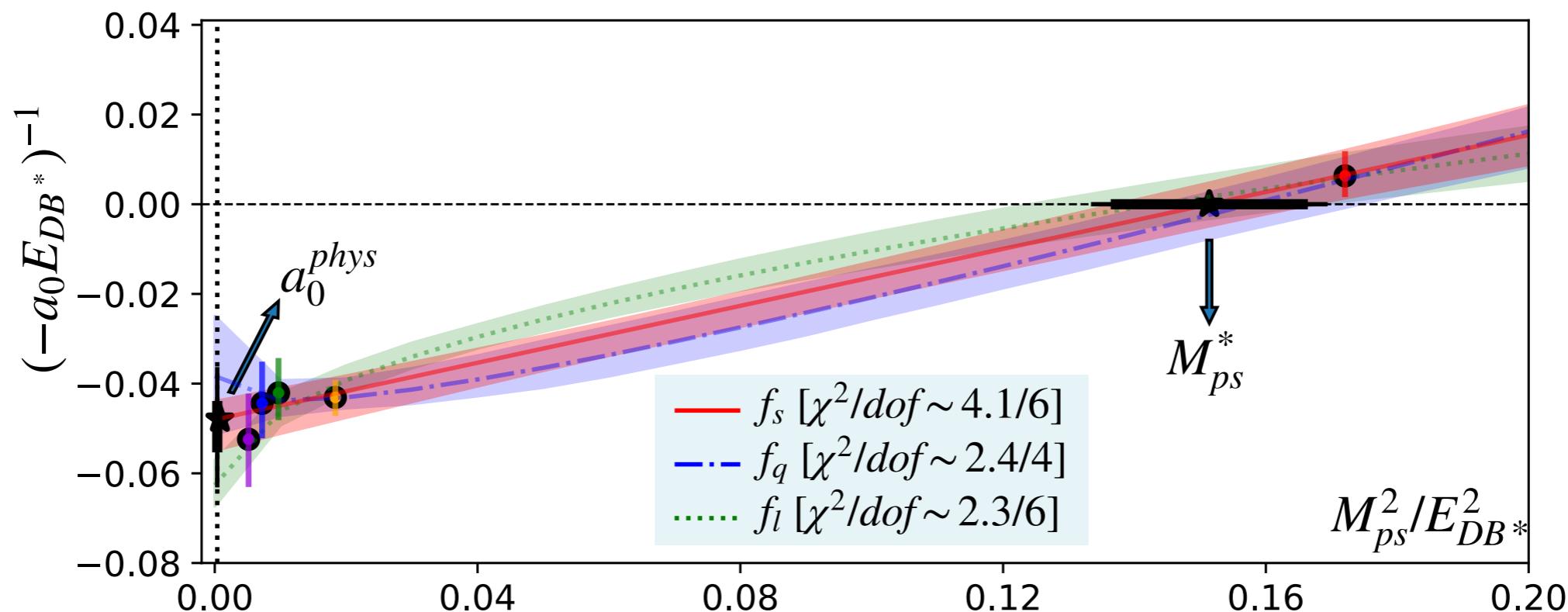


- Non-trivial quark mass dependence and lattice spacing dependence is seen.
- Perform a continuum extrapolation and chiral extrapolation to get the physical amplitudes - then find poles in the amplitude to look for bound states or resonances.



M. Padmanath, AR, Nilmani Mathur [PRL.132.201902]





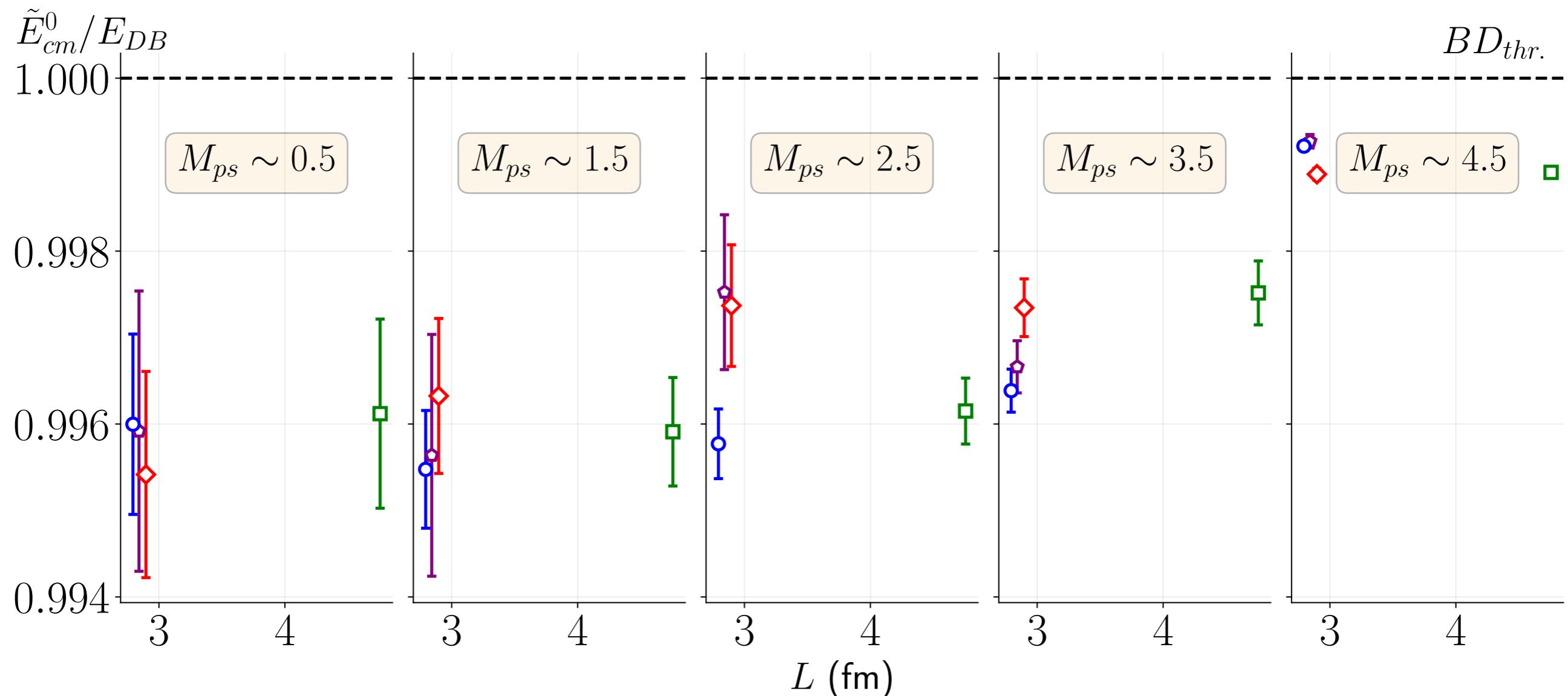
- The binding energy decreases with the increasing light quark mass.
- Indicates the **presence of a real bound state** at physical quark mass.

$$a_o = 57^{(+4)}_{(-5)}(17) \text{ fm}$$

$$\delta m_{T_{bc}} = -43^{(+6)}_{(-7)}(^{+14}_{-24}) \text{ MeV}$$

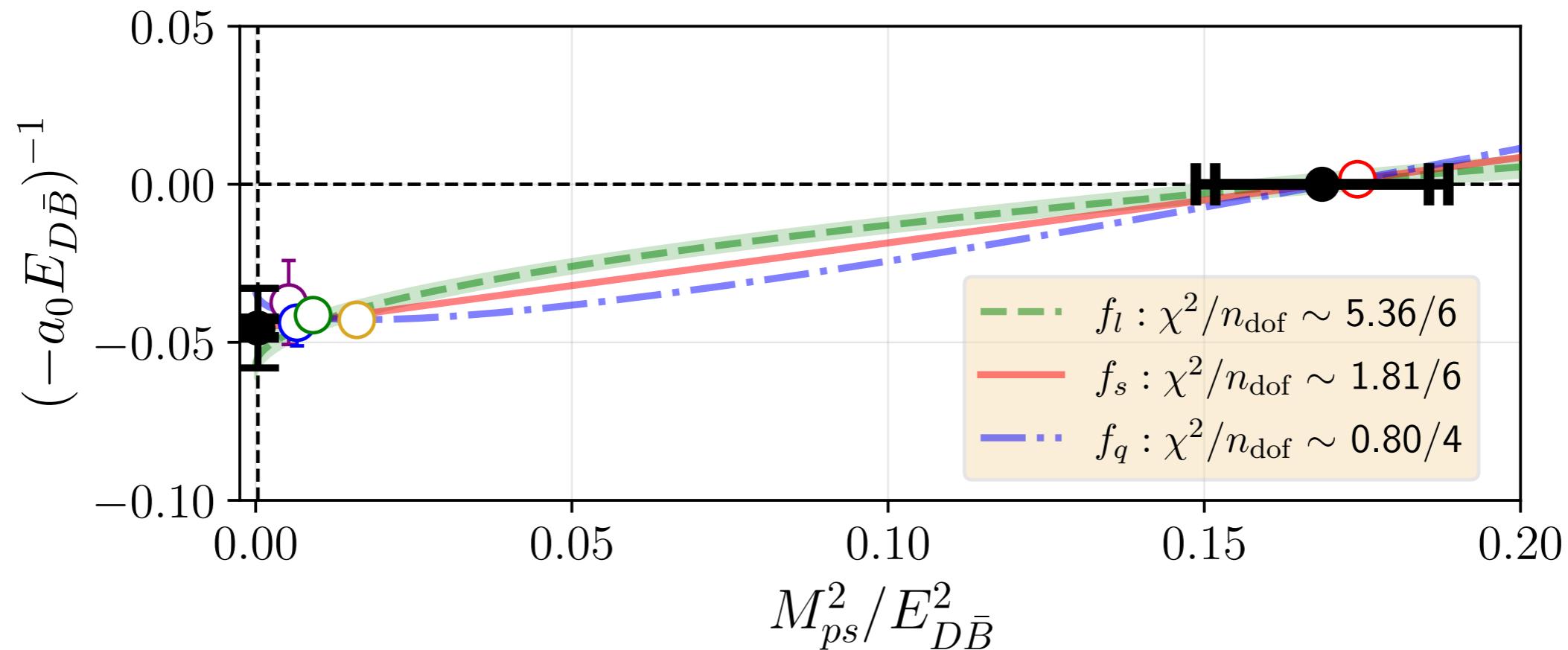
# Finite Volume Spectra - $\bar{b}\bar{c}ud\ 0(0^+)$

AR, M. Padmanath, Nilmani Mathur [arXiv:2307.14128]



A state **below threshold** is seen  
consistently for all volumes in the five  
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Extract the scattering amplitudes



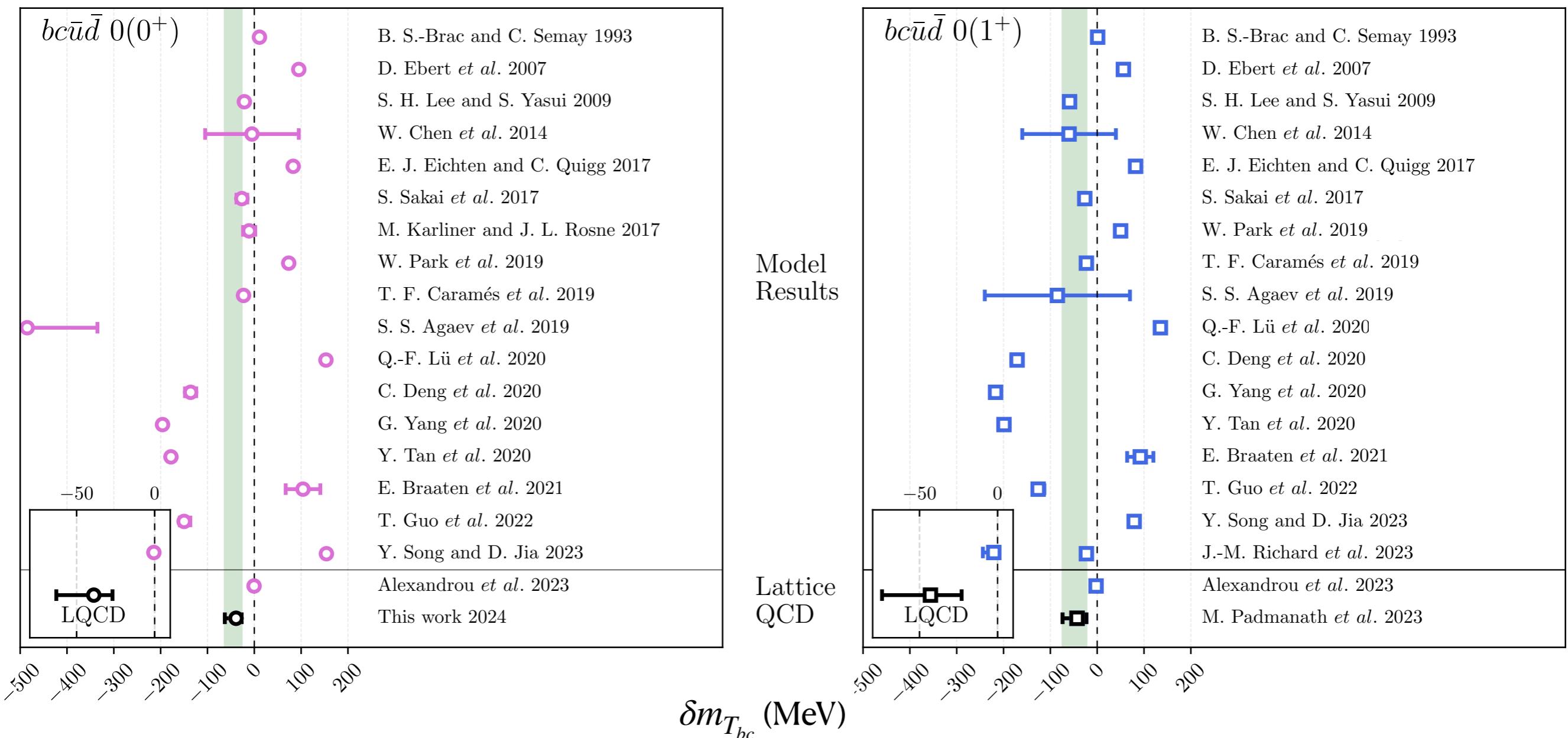
- The binding energy decreases with the increasing light quark mass.
- Indicates the **presence of a real bound state** at physics quark mass.

$$a_0 = 0.58^{(+3)}_{(-4)}(18) \text{ fm}$$

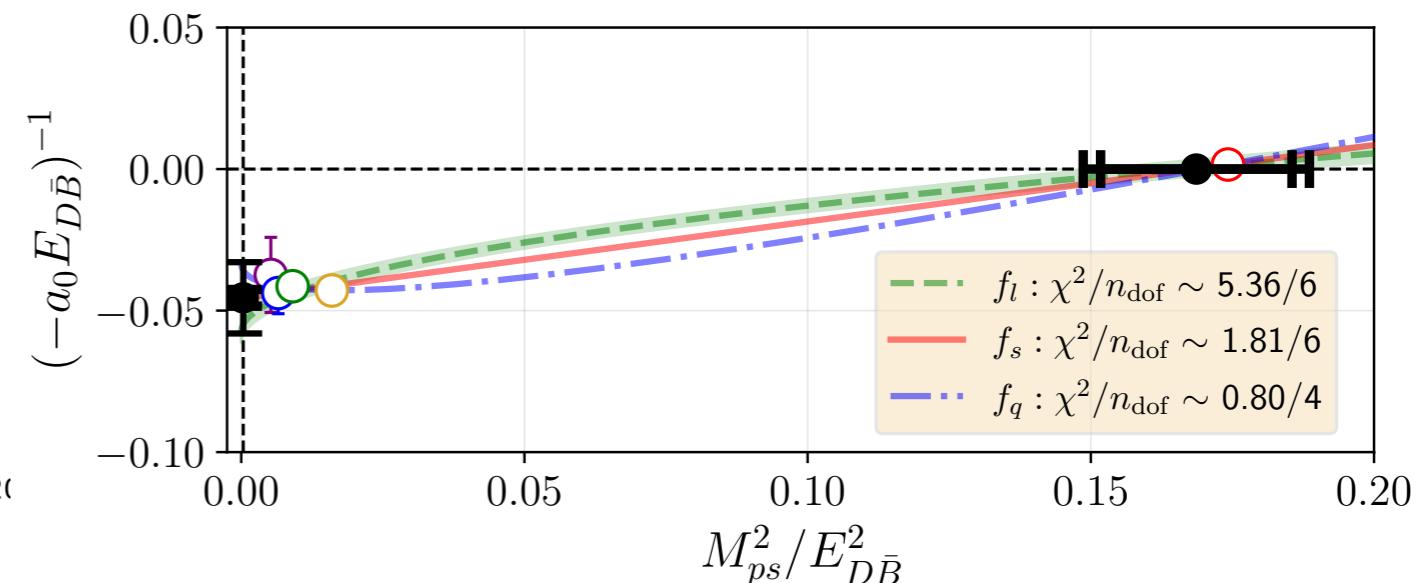
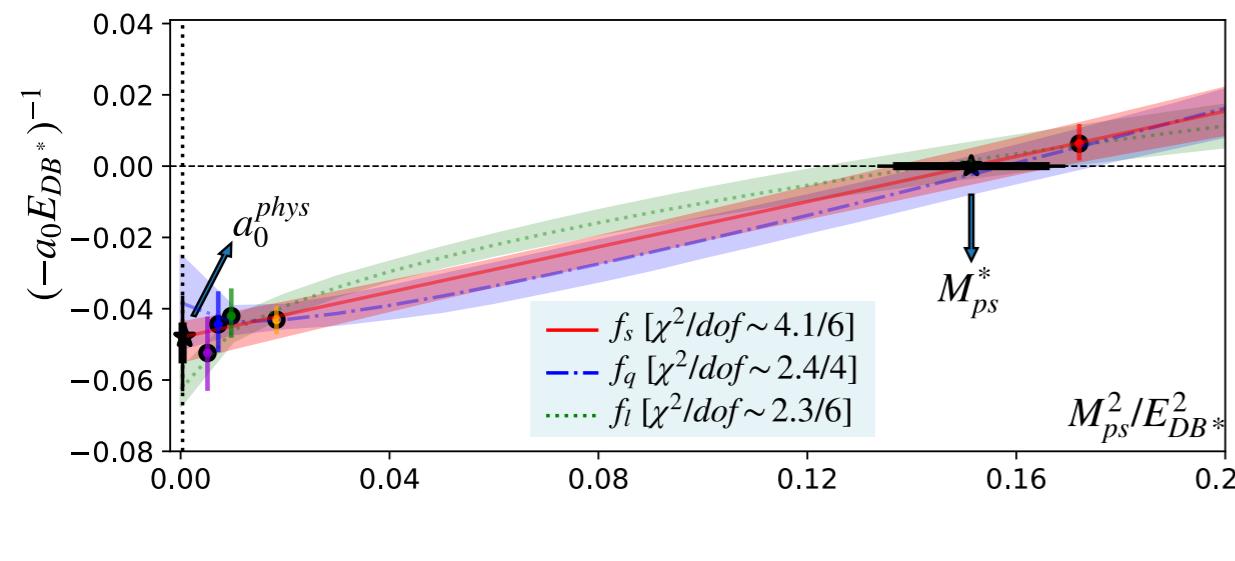
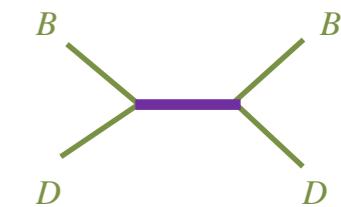
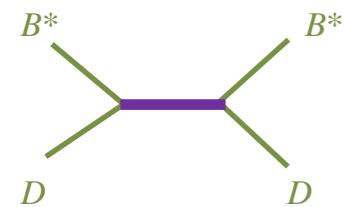
$$\delta m_{T_{bc}} = -42^{(+4)}_{(-6)}(^{+24}_{-12}) \text{ MeV}$$

# Comparison of $T_{bc}$ binding energy results

AR, M. Padmanath, Nilmani Mathur [arXiv:2307.14128]



# Summary



- Lattice QCD can play an important role in understanding these states from QCD and guide such efforts.
- Our preliminary findings suggest the possibility of an attractive interaction between a  $B^*$  and  $D$  meson (and  $B$  and  $D$  meson).
- Hope this will motivate experimental searches and complementary LQCD efforts.

## BACKUP

# Checking the “robustness of the ground state”

$1^+$

$0^+$

