Chiral Lagrangian for Karsten-Wilczek Minimally Doubled Fermions

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The Karsten-Wilczek Action

$$S = \frac{1}{2}\sum_{x,\mu}\left[\overline{\psi}_x\gamma_\mu\psi_{x+\mu} - \overline{\psi}_{x+\mu}\gamma_\mu\psi_x\right] - i\frac{\lambda}{2}\sum_{x,j}\left[\overline{\psi}_x\gamma_4\psi_{x+j} + \overline{\psi}_{x+j}\gamma_4\psi_x - 2\overline{\psi}_x\gamma_4\psi_x\right]$$

…. for massless quarks

Symmetries:

1. Parity
2. Charge Conjugation × Time Reversal
3. Site-Reflection
4. ~~Lorentz Group~~ {Spatial Rotations}
5. Full Chiral Group

$U(1)_L \times SU(2)_L \times U(1)_R \times SU(2)_R$

Transformations of the Single-Taste Wavefunction

Transformation	$\psi_x \to$
Parity	$\gamma_4 \psi_{-\mathbf{x},x_4}$
Charge Conjugation X Time Reversal	$\gamma_2 \gamma_5 \overline{\psi}^T_{\mathbf{x},-x_4}$
Site-Reflection	$\gamma_4 \gamma_5 \overline{\psi}^T_{\mathbf{1}-\mathbf{x},x_4}$

Two Approaches

1. *Single-Taste Method*

★ Assume there is *no fermion doubling* → Single quark wavefunction

2. *Double-Taste Method*

Single quark wavefunction → (Point splitting) → Two separated wavefunctions

…later

Building a Lattice ChiPT

The Workflow

Lattice Action

↓

Symanzik Effective Action

↓ Spurion analysis

Chiral Lagrangian

[Bär, 2010]

Symanzik Effective Theory

"A lattice field theory can be described by an effective continuum theory in which the lattice spacing dependence is made explicit."

$$S_{Sym} = S_0 + aS_1 + a^2 S_2 + \ldots$$

$$S_k = \sum_i \int d^4x \bar{c}_i^{(k+4)} O_i^{(k+4)}$$

unknown LECs

dim $(k+4)$
local operators
obeying *all* symmetries

[Symanzik, 1983] [Bär, 2010]

Symanzik Effective Theory

Example: Dimension-3 term: $i\bar{\psi}_x\gamma_4\psi_x$

$$i\bar{\psi}_x\gamma_4\psi_x \xrightarrow{P} i\bar{\psi}_x\gamma_4\psi_x$$

$$i\bar{\psi}_x\gamma_4\psi_x \xrightarrow{CT} i\bar{\psi}_x\gamma_4\psi_x$$

$$i\bar{\psi}_x\gamma_4\psi_x \xrightarrow{SR} i\bar{\psi}_x\gamma_4\psi_x$$

$i\bar{\psi}_x\gamma_4\psi_x$ is *chirally invariant*

$i\bar{\psi}_x\gamma_4\psi_x$ is *invariant under spatial rotations*

Single-Taste Symanzik Action

Dimension-3 Terms:

$i\overline{\psi}_x \gamma_4 \psi_x$

Dimension-4 Terms:

$\overline{\psi}_x \gamma_k \partial_k \psi_x$

$\overline{\psi}_x \gamma_4 \partial_4 \psi_x$

$i\overline{\psi}_x m \psi_x$

Dimension-5 Terms:

$i\overline{\psi}_x m^2 \gamma_4 \psi_x$

$\overline{\psi}_x m \gamma_k \partial_k \psi_x$

$\overline{\psi}_x m \gamma_4 \partial_4 \psi_x$

$i\overline{\psi}_x m^2 \psi_x$

$i\overline{\psi}_x \gamma_4 \partial_4 \partial_4 \psi_x$

$i\overline{\psi}_x \gamma_4 \partial_k \partial_k \psi_x$

$i\overline{\psi}_x \gamma_k \partial_k \partial_4 \psi_x$

Single-Taste Symanzik Action

$$S_{Sym} = \int d^4x \Big[a^{-1}\Big\{ c^{(1)} i\overline{\psi}_x \gamma_4 \psi_x \Big\} + a^0 \Big\{ b^{(1)} \overline{\psi}_x \gamma_k \partial_k \psi_x + b^{(2)} \overline{\psi}_x \gamma_4 \partial_4 \psi_x + b^{(3)} i\overline{\psi}_x m \psi_x \Big\}$$

$$+ a^1 \Big\{ d^{(0)} i\overline{\psi} m^2 \psi_x + d^{(1)} i\overline{\psi}_x m^2 \gamma_4 \psi_x + d^{(3)} \overline{\psi}_x m \gamma_4 \partial_4 \psi_x + d^{(4)} i\overline{\psi}_x \gamma_4 \partial_4 \partial_4 \psi_x$$

$$+ d^{(5)} i\overline{\psi}_x \gamma_4 \partial_k \partial_k \psi_x + d^{(6)} i\overline{\psi}_x \gamma_k \partial_k \partial_4 \psi_x \Big\} \Big]$$

Spurions

LECs $\xrightarrow[a\text{-dependence}]{\textit{Spurious transformations}}$ Spurions

Spurious transformations

a-dependence

Chiral Transformations

$$\psi_L \stackrel{G}{\to} g_L \psi_L$$

$$\psi_R \stackrel{G}{\to} g_R \psi_R$$

…..$G \equiv$ chiral group

…..for the next slide

Spurions

Example: Wilson Fermions

Dirac term in the Symanzik Effective Action: $S_1 = a\bar{c}_{SW}\int d^4x\overline{\psi}(x)\sigma_{\mu\nu}G_{\mu\nu}(x)\psi(x)$

breaks chiral symmetry

We promote $a\bar{c}_{SW}$ to a spurion A such that

$$A \xrightarrow{G} g_L A g_R^\dagger$$
$$A \xrightarrow{P} A^\dagger$$
$$A \xrightarrow{C} A^T$$

With these transformations, Dirac term becomes invariant under all symmetries.

Transformation of Spurions

$$m \xrightarrow{G} g_L m g_R^\dagger$$

$$c^{(1)} \xrightarrow{C/T} -c^{(1)}$$

$$c^{(1)} \xrightarrow{\Lambda} \Lambda^{-1}_{\mu 4} c^{(1)}_4$$

$$b^{(1)} \xrightarrow{C/T} b^{(1)}$$

$$b^{(1)} \xrightarrow{\Lambda} \Lambda^{-1}_{\mu k} \Lambda_{\alpha m} b^{(1)}_{km}$$

$$b^{(2)} \xrightarrow{C/T} b^{(2)}$$

$$b^{(1)} \xrightarrow{\Lambda} \Lambda^{-1}_{\mu 4} \Lambda_{\alpha 4} b^{(1)}_{44}$$

$$d^{(1)} \xrightarrow{C/T} -d^{(1)}$$

$$d^{(1)} \xrightarrow{\Lambda} \Lambda^{-1}_{\mu 4} d^{(1)}_4$$

$$d^{(2)} \xrightarrow{C/T} d^{(2)}$$

$$d^{(2)} \xrightarrow{\Lambda} \Lambda^{-1}_{\mu k} \Lambda_{\alpha m} d^{(2)}_{km}$$

$$d^{(3)} \xrightarrow{C/T} d^{(3)}$$

$$d^{(3)} \xrightarrow{\Lambda} \Lambda^{-1}_{\mu 4} \Lambda_{\alpha 4} d^{(3)}_{44}$$

$$d^{(4)} \xrightarrow{C/T} -d^{(4)}$$

$$d^{(4)} \xrightarrow{\Lambda} \Lambda^{-1}_{\mu 4} \Lambda_{\alpha 4} \Lambda^{-1}_{\rho 4} d^{(4)}_{444}$$

$$d^{(5)} \xrightarrow{C/T} -d^{(5)}$$

$$d^{(5)} \xrightarrow{\Lambda} \Lambda^{-1}_{\mu 4} \Lambda_{\alpha k} \Lambda^{-1}_{\rho m} d^{(5)}_{4km}$$

$$d^{(6)} \xrightarrow{C/T} -d^{(6)}$$

$$d^{(6)} \xrightarrow{\Lambda} \Lambda^{-1}_{\mu k} \Lambda_{\alpha m} \Lambda^{-1}_{\rho 4} d^{(6)}_{km4}$$

The Pion Field

Exponential Parametrization:

$$\Sigma(x) = \exp(i\pi^a(x)T^a)$$

.... T^a are broken generators

Transformations:

$$\Sigma(x) \xrightarrow{P} \Sigma^\dagger(x)$$

$$\Sigma(x) \xrightarrow{C} \Sigma^T(x)$$

$$\Sigma(x) \xrightarrow{T} \Sigma(x)$$

$$\Sigma(x) \xrightarrow{G} g_L \Sigma(x) g_R^\dagger$$

Constructing the Chiral Lagrangian

Invariant Objects $\left\{ \text{Spurions} + \text{Pion Field} + \text{Derivatives} \right\} \Rightarrow \mathscr{L}_\chi$

$c^{(1)2}$

$< \partial_4 \Sigma \partial_4 \Sigma >$

….. example

$c^{(1)2} < \partial_4 \Sigma \partial_4 \Sigma >$ enters the Chiral Lagrangian.

The Chiral Lagrangian

Terms with two derivatives

$$\mathscr{L}_{p^2} = a^0 \left\{ A_1 < \partial_k \Sigma \partial_k \Sigma^\dagger > + A_2 < \partial_4 \Sigma \partial_4 \Sigma^\dagger > \right\}$$

$$\mathscr{L}_{p^2/a^2} = \frac{1}{a^2} B_1 < \partial_4 \Sigma \partial_4 \Sigma^\dagger >$$

$$\mathscr{L}_{p^2 a} = a \left\{ C_1 < \partial_k \Sigma \partial_k \Sigma^\dagger > + C_2 < \partial_4 \Sigma \partial_4 \Sigma^\dagger > \right\}$$

$$\mathscr{L}_{p^2 a^2} = a^2 D_1 < \partial_4 \Sigma \partial_4 \Sigma^\dagger >$$

The Chiral Lagrangian

Terms with the matrix M

$$\mathscr{L}_M = E_1 < M\Sigma + M\Sigma^\dagger >$$

$$\mathscr{L}_{M^2} = F_1 < M\Sigma + M\Sigma^\dagger >^2 + F_2 < M\Sigma - M\Sigma^\dagger >^2 + F_3 < \Sigma M \Sigma M + M\Sigma^\dagger M \Sigma^\dagger >$$

The Chiral Lagrangian

Terms with M and two derivatives

$$\mathscr{L}_{p^2M} = H_1 < \partial_k\Sigma\partial_k\Sigma^\dagger > < M\Sigma^\dagger + \Sigma M > + H_2 < \partial_4\Sigma\partial_4\Sigma^\dagger > < M\Sigma^\dagger + \Sigma M >$$

$$+H_3 < [\partial_k\Sigma\partial_k\Sigma^\dagger][M\Sigma^\dagger + \Sigma M] > + H_4 < [\partial_4\Sigma\partial_4\Sigma^\dagger][M\Sigma^\dagger + \Sigma M] >$$

$$\mathscr{L}_{p^2M/a^2} = \frac{1}{a^2}\left\{ G_1 < \partial_4\Sigma\partial_4\Sigma^\dagger > < M\Sigma^\dagger + \Sigma M > + G_2 < [\partial_4\Sigma\partial_4\Sigma^\dagger][M\Sigma^\dagger + \Sigma M] > \right\}$$

Conclusions

- Simple Model to Study
- Can study the Lorentz symmetry breaking patterns
- Tells us what to expect in the 2-taste Chiral Lagrangian

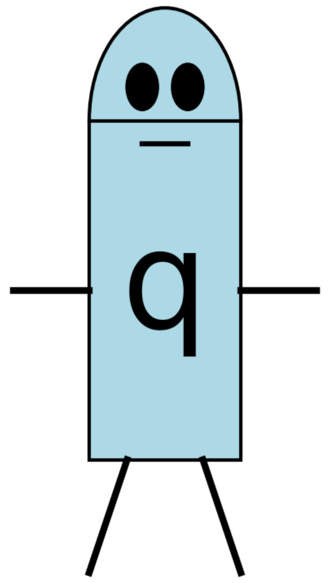
Two Approaches

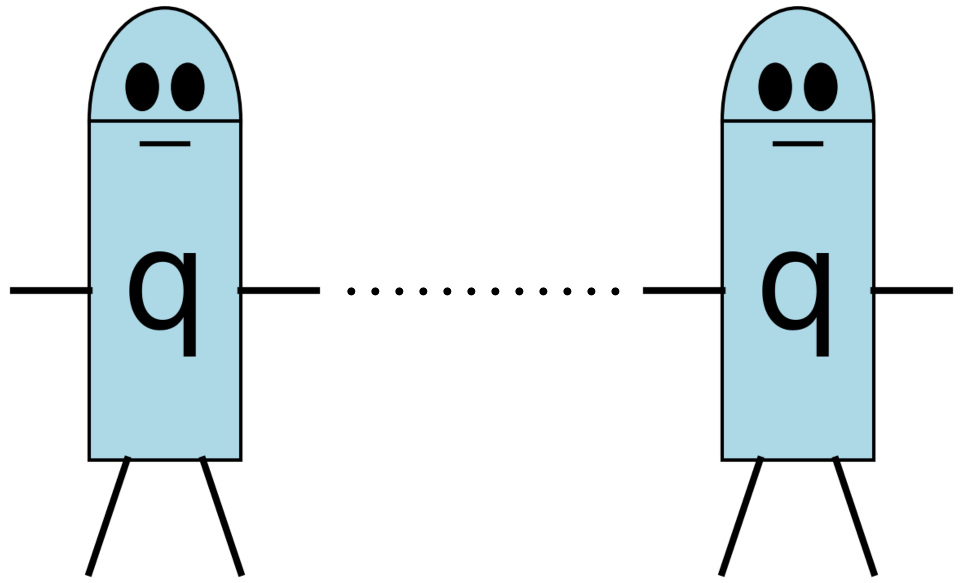
1. *Single-Taste Method*

Assume there is <u>no fermion doubling</u> → Single quark wavefunction

2. *Double-Taste Method*

★ Single quark wavefunction —Point splitting→ Two separated wavefunctions





Karsten-Wilczek Propagator

$$S(p) = \frac{-i\sum_{\mu=1}^{4}\gamma_\mu \sin p_\mu - 2i\gamma_4 \sum_{k=1}^{3}\sin^2\frac{p_k}{2}}{\sum_{\mu=1}^{4}\sin^2 p_\mu + 4\sin p_4 \sum_{k=1}^{3}\sin^2\frac{p_k}{2} + 4[\sum_{k=1}^{3}\sin^2\frac{p_k}{2}][\sum_{l=1}^{3}\sin^2\frac{p_l}{2}]}$$

Poles at $\{0,0,0,0\}$ and $\{0,0,0,\pi\}$

"Natural" pole — $\{0,0,0,0\}$

"Doubler" pole — $\{0,0,0,\pi\}$

[Capitani et al, 2010]

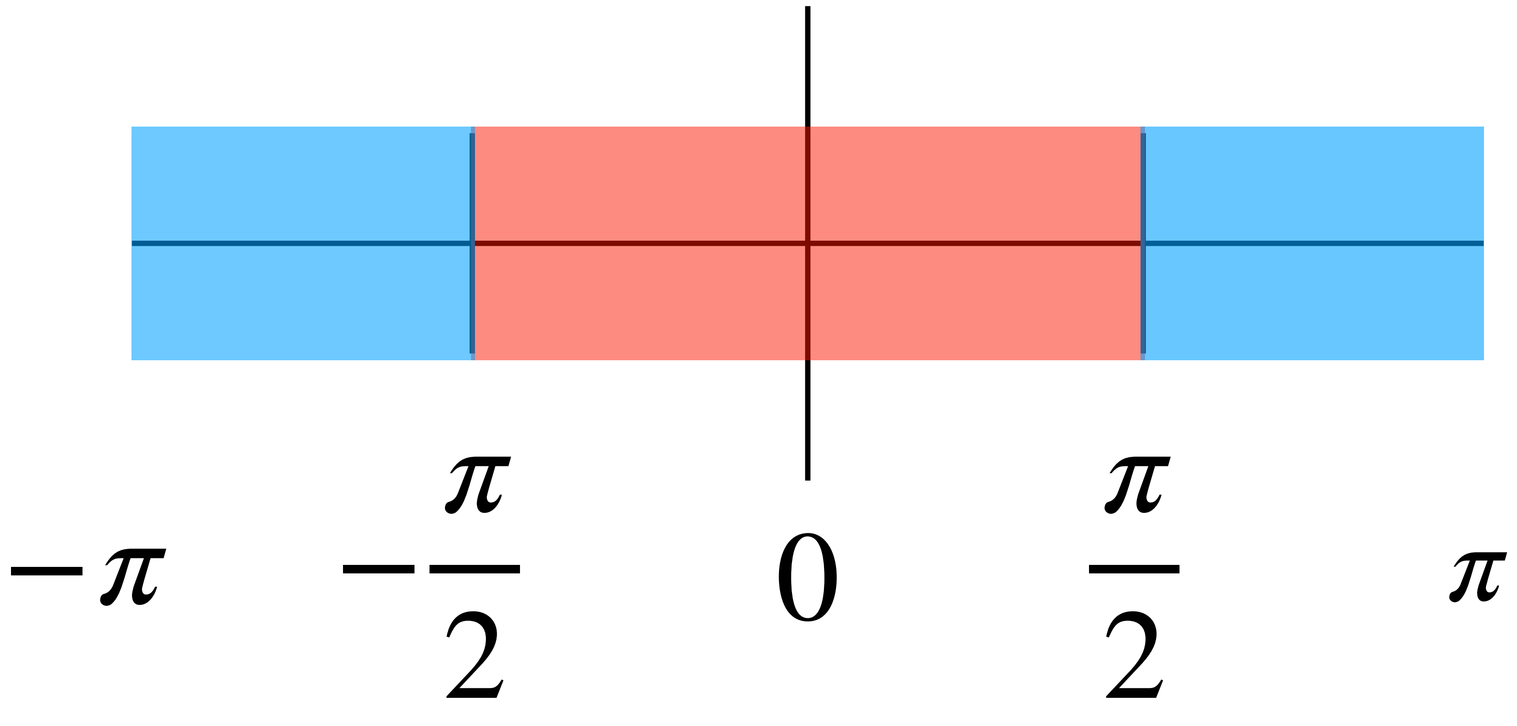
Karsten-Wilczek Propagator

Perturbing the propagator around $\{0,0,0,\pi\}$ changes its form.

We apply a unitary transformation to the *half BZ* around $\{0,0,0,\pi\}$.

[Capitani et al, 2010]

Point-Splitting



$$\mathscr{B} \equiv \{k_\mu : |k_4| < \frac{\pi}{2}\} \equiv$$

$$\overline{\mathscr{B}} \equiv \{k_\mu : |k_4| \not< \frac{\pi}{2}\} \equiv$$

$$\psi(k)\Big|_{k_\mu \in \mathscr{B}} = \psi^{(1)}(k)$$

$$\psi(k)\Big|_{k_\mu \notin \mathscr{B}} = \gamma_4\gamma_5\psi^{(2)}(T_{\pi 4}k)$$

"Point-Splitting Relations"

$$\ldots\ldots T_{\pi 4}k_\mu = [\vec{k}, (k_4 + \pi) \mod 2\pi]$$

[Tiburzi, 2010]

Point-Split Action

Create an *isospinor*:

$$\Psi(k) = \begin{pmatrix} \psi^{(1)}(k) \\ \psi^{(2)}(k) \end{pmatrix}$$

Momentum-Space Action:

$$S = \int_{\mathscr{B}} \frac{d^4k}{(2\pi)^4} \overline{\Psi}(k) \left[\sum_{\mu} i(\gamma_\mu \otimes 1) sin k_\mu - i\lambda(\gamma_4 \otimes \sigma_3) \sum_j (\cos k_j - 1) \right] \Psi(k)$$

[Tiburzi, 2010]

Time-Smeared Wavefunction

Let's come to the coordinate space now.

Under symmetry transformations, ψ_x is *not* the fundamental object.

Instead

$$\delta_{\mathscr{B}}\psi_x = \int_{\mathscr{B}} \frac{dk_4}{2\pi} \sum_{y_4} e^{ik_4(x_4-y_4)} \psi_{\vec{x},y_4}$$

$$\delta_{\overline{\mathscr{B}}}\psi_x = \int_{\overline{\mathscr{B}}} \frac{dk_4}{2\pi} \sum_{y_4} e^{ik_4(x_4-y_4)} \psi_{\vec{x},y_4}$$

are the actual fundamental objects.

Transformations of the Time-Smeared Wavefunction

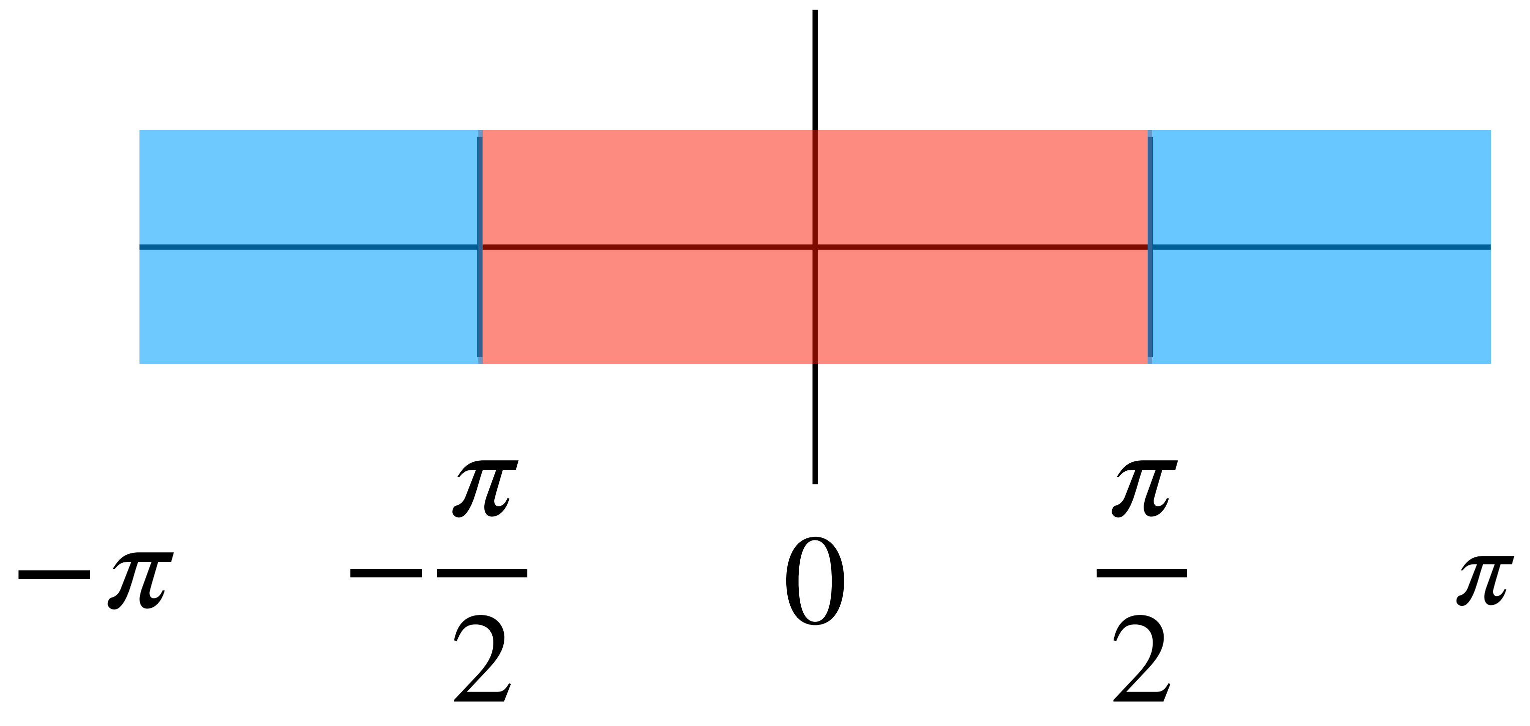
Transformation	$\delta_{\mathscr{B}}\psi_x$	$\delta_{\overline{\mathscr{B}}}\psi_x$
Site-Reflection	$\gamma_4\gamma_5\delta_{\mathscr{B}}\overline{\psi}^T_{\mathbf{1-x},x_4}$	$\gamma_4\gamma_5\delta_{\overline{\mathscr{B}}}\overline{\psi}^T_{\mathbf{1-x},x_4}$
Parity	$\gamma_4\delta_{\mathscr{B}}\psi_{-\mathbf{x},x_4}$	$-\gamma_4\delta_{\overline{\mathscr{B}}}\psi_{-\mathbf{x},x_4}$
C X T	$\gamma_2\gamma_5\delta_{\mathscr{B}}\overline{\psi}^T_{\mathbf{x},-x_4}$	$-\gamma_2\gamma_5\delta_{\overline{\mathscr{B}}}\overline{\psi}^T_{\mathbf{x},-x_4}$
T X Taste Rotation	$ie^{i\pi x_4}e^{i\theta}\delta_{\overline{\mathscr{B}}}\psi_{\mathbf{x},-x_4}$	$-ie^{i\pi x_4}e^{-i\theta}\delta_{\mathscr{B}}\psi_{\mathbf{x},-x_4}$
U(1) Isosinglet Vector	$e^{i\theta}\delta_{\mathscr{B}}\psi_x$	$e^{i\theta}\delta_{\overline{\mathscr{B}}}\psi_x$
U(1) Isovector Axial	$e^{i\theta\gamma_5}\delta_{\mathscr{B}}\psi_x$	$e^{i\theta\gamma_5}\delta_{\overline{\mathscr{B}}}\psi_x$
U(1) Isovector Vector	$e^{i\theta}\delta_{\mathscr{B}}\psi_x$	$e^{-i\theta}\delta_{\overline{\mathscr{B}}}\psi_x$
U(1) Isosinglet Axial	$e^{i\theta\gamma_5}\delta_{\mathscr{B}}\psi_x$	$e^{-i\theta\gamma_5}\delta_{\overline{\mathscr{B}}}\psi_x$

Two-Taste Symanzik Action

Symanzik Action breaks into two seperate parts

$$B.Z. = \mathscr{B} + \overline{\mathscr{B}}$$

$$S_{Sym} = S^{\mathscr{B}}_{Sym} + S^{\overline{\mathscr{B}}}_{Sym}$$



Separate Symanzik actions for both the "*species*"

Two-Taste Symanzik Action

$$S^{\mathscr{B}}_{Sym} = \int d^4x \Big[a^{-1} \Big\{ c^{(1)} i\delta_{\mathscr{B}}\overline{\psi}\gamma_4\delta_{\mathscr{B}}\psi \Big\} + a^0 \Big\{ b^{(1)}\delta_{\mathscr{B}}\overline{\psi}\gamma_k\partial_k\delta_{\mathscr{B}}\psi + b^{(2)}\delta_{\mathscr{B}}\overline{\psi}\gamma_4\partial_4\delta_{\mathscr{B}}\psi$$

$$+ b^{(3)} i\delta_{\mathscr{B}}\overline{\psi} m\delta_{\mathscr{B}}\psi \Big\} + a^1 \Big\{ d^{(0)} i\delta_{\mathscr{B}}\overline{\psi} m^2\delta_{\mathscr{B}}\psi_x + d^{(1)} i\delta_{\mathscr{B}}\overline{\psi}_x m^2\gamma_4\delta_{\mathscr{B}}\psi_x$$

$$+ d^{(3)}\delta_{\mathscr{B}}\overline{\psi}_x m\gamma_4\partial_4\delta_{\mathscr{B}}\psi_x + d^{(4)} i\delta_{\mathscr{B}}\overline{\psi}_x\gamma_4\partial_4\partial_4\delta_{\mathscr{B}}\psi_x + d^{(5)} i\delta_{\mathscr{B}}\overline{\psi}_x\gamma_4\partial_k\partial_k\delta_{\mathscr{B}}\psi_x$$

$$+ d^{(6)} i\overline{\psi}_x\gamma_k\partial_k\partial_4\psi_x \Big\} \Big]$$

$S^{\overline{\mathscr{B}}}_{Sym}$ has a similar action with *identical coefficients*.

Because of $T \times$ Taste Rotation

Chiral Group

Chiral Group G for two tastes:

$$G = U(1)_V^{\mathscr{B}} \times U(1)_V^{\overline{\mathscr{B}}} \times U(1)_A^{\mathscr{B}} \times U(1)_A^{\overline{\mathscr{B}}}$$

G is spontaneously broken to H.

$$H = U(1)_V^{\mathscr{B}} \times U(1)_V^{\overline{\mathscr{B}}}$$

Two abelian generators (axial-vectors) are spontaneously broken.

Goldstone's Theorem

Two axial vector generators are spontaneously broken.

↓ Goldstone's Theorem

2 Massless Goldstone Bosons (Pions)

$U(1)_A^{\mathscr{B}} \equiv$ Pion from $\{0,0,0,0\}$

$U(1)_A^{\overline{\mathscr{B}}} \equiv$ Pion from $\{0,0,0,\pi\}$

The Pion Field

Broken Group:

$$G/H \equiv U(1)_A^{\mathscr{B}} \oplus U(1)_A^{\overline{\mathscr{B}}}$$

Pion Field:

$$\pi(x) = \pi^a(x)T^a$$

$$= \pi(x)\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \tilde{\pi}(x)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \pi(x) & 0 \\ 0 & \tilde{\pi}(x) \end{pmatrix}$$

The Pion Field

Exponential Parametrization:

$$\Sigma(x) = \exp i\pi(x) = \begin{pmatrix} e^{i\pi(x)} & 0 \\ 0 & e^{i\tilde{\pi}(x)} \end{pmatrix}$$

Transformations:

$$\Sigma(x) \xrightarrow{P} \Sigma^\dagger(x)$$

$$\Sigma(x) \xrightarrow{C} \Sigma^T(x)$$

$$\Sigma(x) \xrightarrow{T} \Sigma(x)$$

$$\Sigma(x) \xrightarrow{G} g_L \Sigma(x) g_R^\dagger$$

$$\Sigma(x) \xrightarrow{T \times taste} \sigma_1 \Sigma(x) \sigma_1$$

New symmetry

The Pion Field

where,

$$g_L = \begin{pmatrix} e^{i\theta_L^{\mathscr{B}}} & 0 \\ 0 & e^{i\theta_L^{\overline{\mathscr{B}}}} \end{pmatrix} \qquad g_R = \begin{pmatrix} e^{i\theta_R^{\mathscr{B}}} & 0 \\ 0 & e^{i\theta_R^{\overline{\mathscr{B}}}} \end{pmatrix}$$

and,

$$\begin{pmatrix} e^{i\pi(x)} & 0 \\ 0 & e^{i\tilde{\pi}(x)} \end{pmatrix} \xrightarrow{T\times taste} \begin{pmatrix} e^{i\tilde{\pi}(x)} & 0 \\ 0 & e^{i\pi(x)} \end{pmatrix}$$

We now perform the same spurion analysis we did in the Single-Taste approach.

The Chiral Lagrangian

Terms with two derivatives

$$\mathcal{L}_{p^2} = A_1 < \partial_k \Sigma \partial_k \Sigma^\dagger > + A_2 < \partial_4 \Sigma \partial_4 \Sigma^\dagger >$$

$$\mathcal{L}_{p^2/a^2} = \frac{1}{a^2} B_1 < \partial_4 \Sigma \partial_4 \Sigma^\dagger >$$

$$\mathcal{L}_{p^2 a} = a\{ C_1 < \partial_k \Sigma \partial_k \Sigma^\dagger > + C_2 < \partial_4 \Sigma \partial_4 \Sigma^\dagger > \}$$

$$\mathcal{L}_{p^2 a^2} = a^2 D_1 < \partial_4 \Sigma \partial_4 \Sigma^\dagger >$$

The Chiral Lagrangian

Terms with the matrix M

$$\mathscr{L}_M = E_1 < M\Sigma + M\Sigma^\dagger >$$

$$\mathscr{L}_{M^2} = F_1 < M\Sigma + M\Sigma^\dagger >^2 + F_2 < M\Sigma - M\Sigma^\dagger >^2 + F_3 < \Sigma M \Sigma M + M\Sigma^\dagger M \Sigma^\dagger >$$

The Chiral Lagrangian

Terms with M and two derivatives

$$\mathscr{L}_{p^2M} = H_1 < \partial_k\Sigma\partial_k\Sigma^\dagger > < M\Sigma^\dagger + \Sigma M > + H_2 < \partial_4\Sigma\partial_4\Sigma^\dagger > < M\Sigma^\dagger + \Sigma M >$$

$$+H_3 < [\partial_k\Sigma\partial_k\Sigma^\dagger][M\Sigma^\dagger + \Sigma M] > + H_4 < [\partial_4\Sigma\partial_4\Sigma^\dagger][M\Sigma^\dagger + \Sigma M] >$$

$$\mathscr{L}_{p^2M/a^2} = \frac{1}{a^2}\{G_1 < \partial_4\Sigma\partial_4\Sigma^\dagger > < M\Sigma^\dagger + \Sigma M > + G_2 < [\partial_4\Sigma\partial_4\Sigma^\dagger][M\Sigma^\dagger + \Sigma M] > \}$$

The Chiral Lagrangian

Single-Taste Chiral Lagrangian and Two-Taste Chiral Lagrangians *are identical*.

Except for the structure of $\Sigma(x)$ which brings subtle changes.

Conclusions

- Considering point-splitting adds structure to the pion field.

- Comparison with Lattice data on Minimally Doubled fermions will be interesting.

Thanks!