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HMC and gradient flow with machine-learned classically perfect fixed point actions

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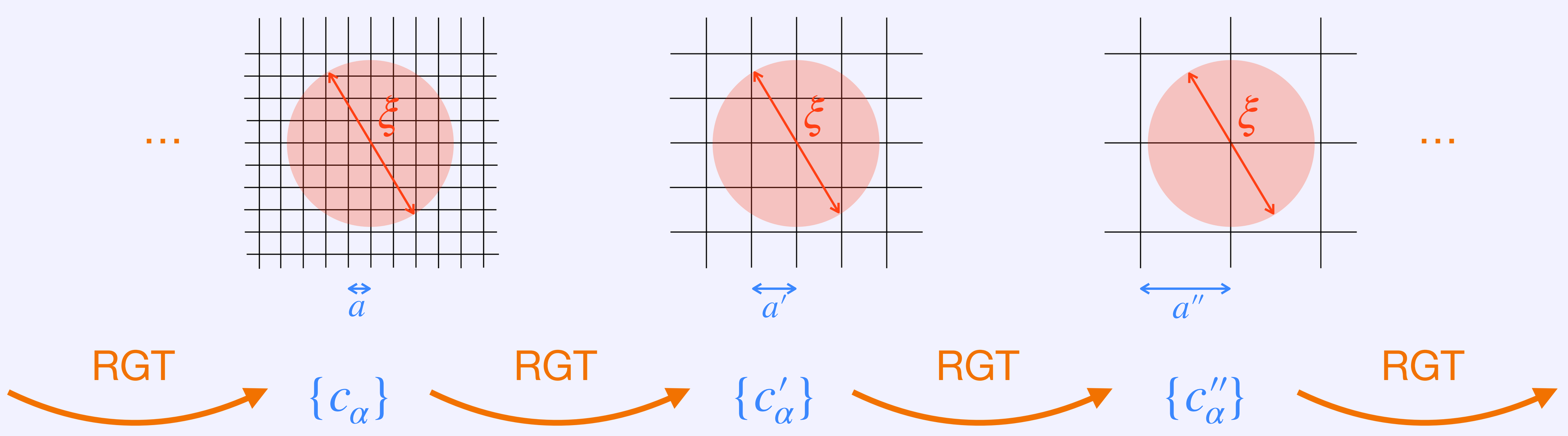
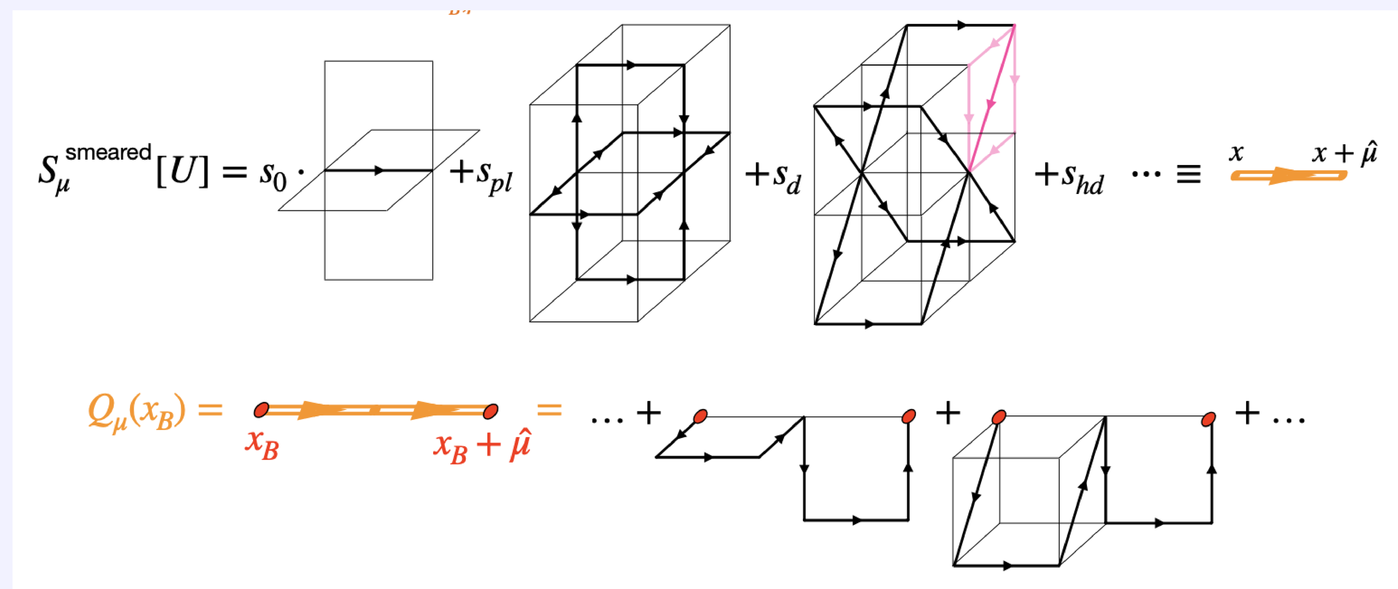
Renormalization group transformation



- Coarse lattices can lead to large lattice artifacts
- Fine lattices can lead to critical slowing down & topological freezing
- The aim of a **renormalization group transformation (RGT)** is to describe the physics of the fine lattice on a coarse lattice

Renormalization group blocking



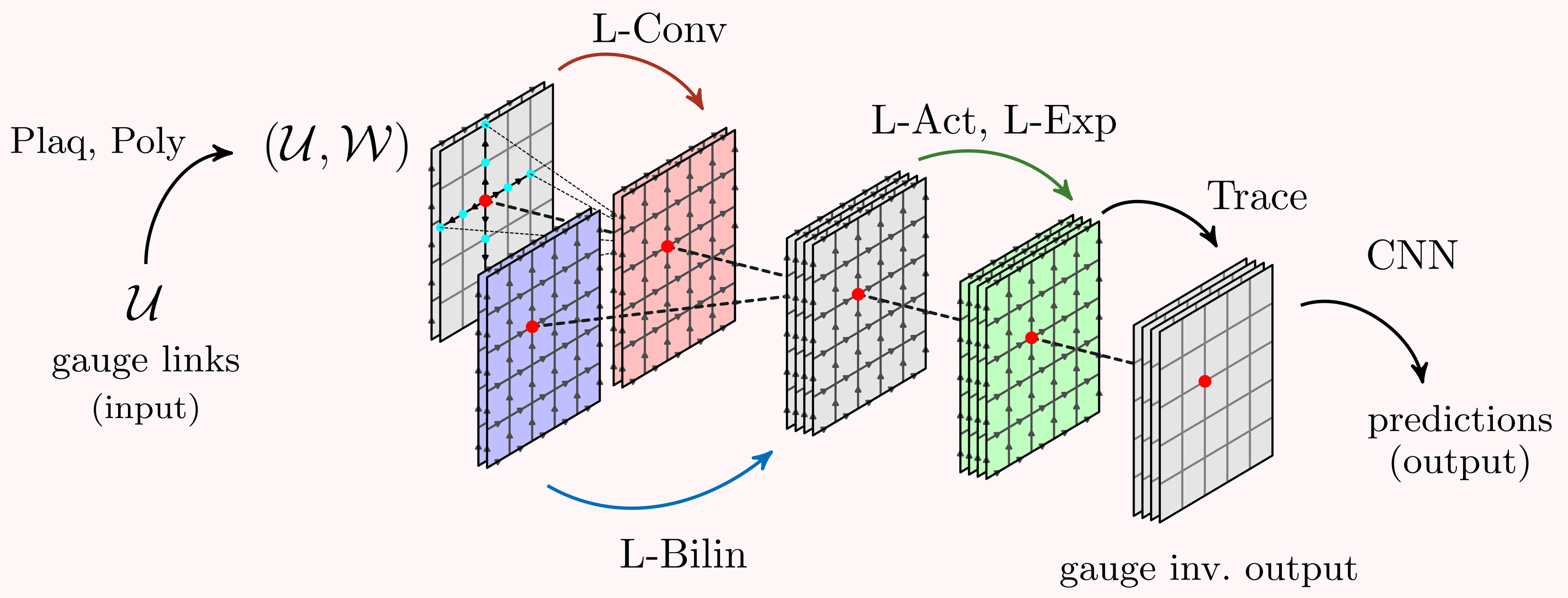
- **Fixed point action** defined through RG blocking
- Many gauge links over **hypercube** contribute to blocked link

Preserving gauge symmetry in neural networks

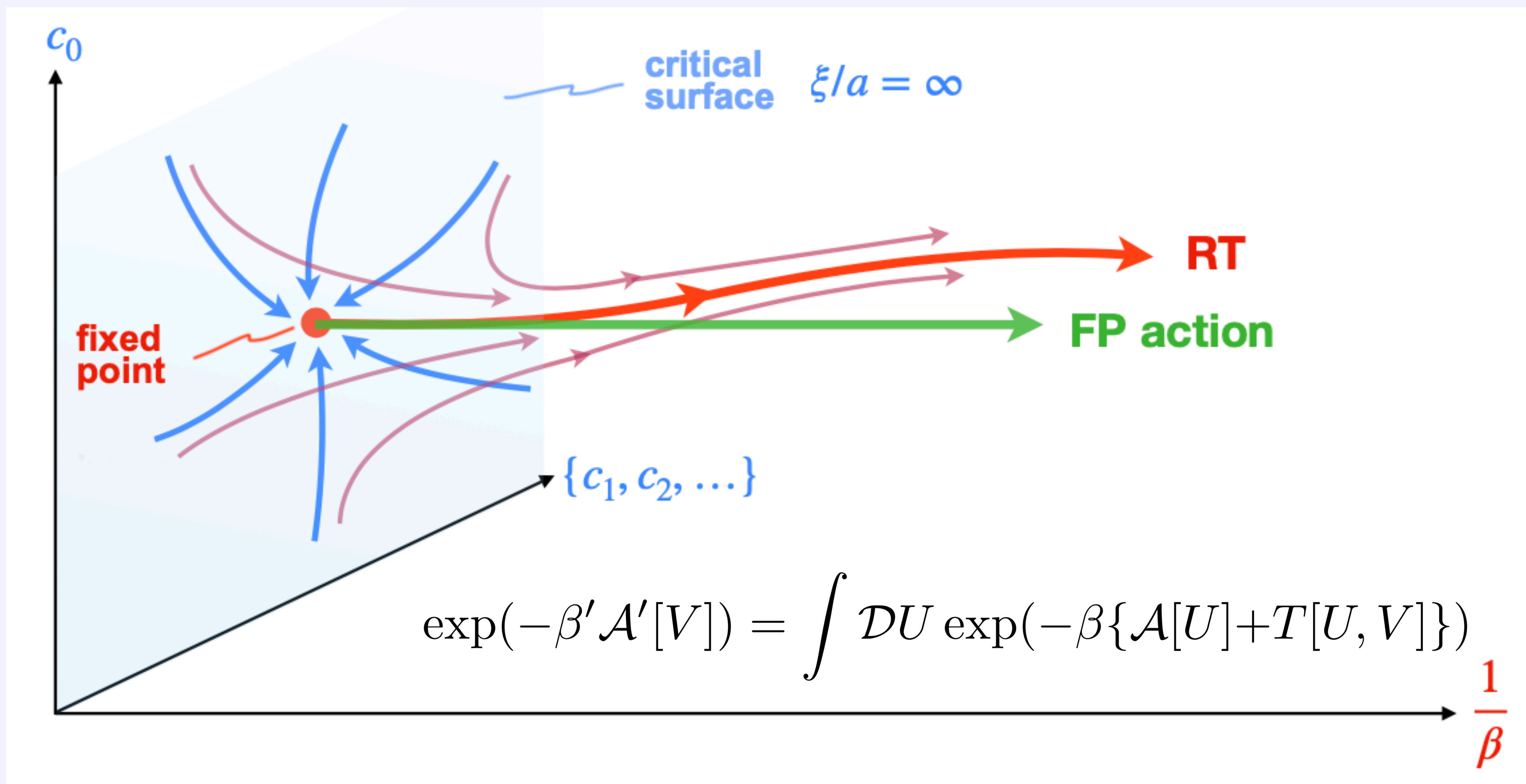
- **4D lattice gauge theory**: input $W(\vec{x}) \in \mathbb{C}^{N_c \times N_c}$, $U_\mu(\vec{x}) \in \mathbb{C}^{N_c \times N_c}$
  - **Gauge transformations**:

$$W(\vec{x}) \to \Omega(\vec{x}) W(\vec{x}) \Omega^\dagger(\vec{x}), \qquad \Omega(\vec{x}) \in \mathbb{C}^{N_c \times N_c}$$
$$U_\mu(\vec{x}) \to \Omega(\vec{x}) U_\mu(\vec{x}) \Omega^\dagger(\vec{x} + \vec{\mu})$$

- **New architecture:** lattice gauge equivariant CNNs (L-CNNs) [1]
- Convolutional layers with **parallel transport** retain gauge symmetry



Fixed point action

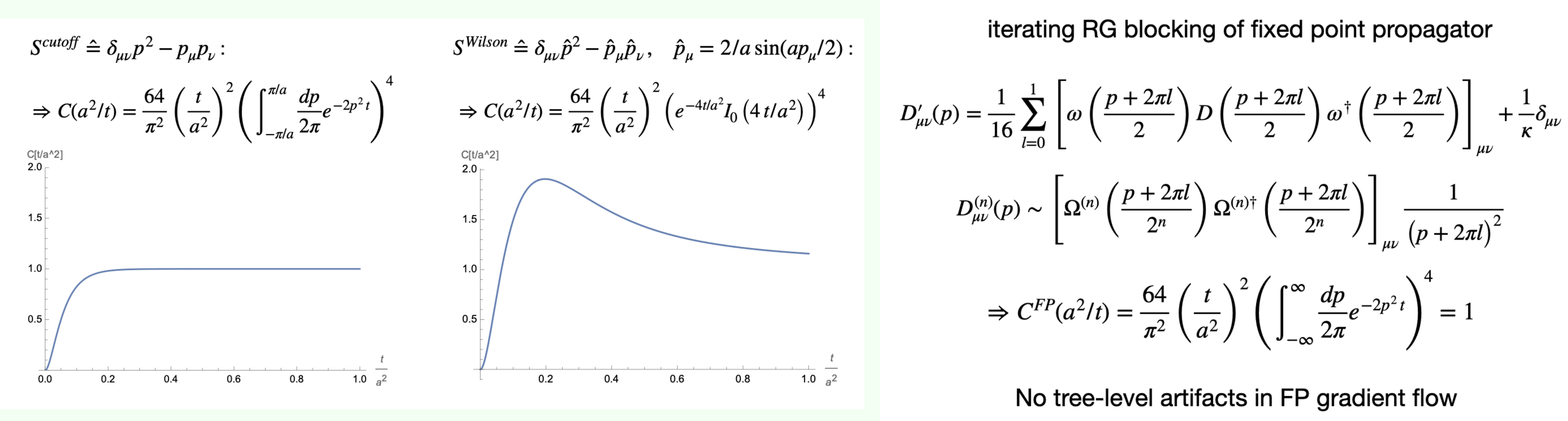


- Define a RGT for lattice action $\mathcal{A}$ from fine lattice U to coarse lattice V with blocking kernel $T[U,V]$ and gauge coupling β
- **Renormalized trajectory (RT)** at weak coupling expected to stay close to fixed point (FP) action with fixed couplings c_i
- **Fixed point**: $\beta \to \infty$ $\quad \mathcal{A}^{\text{FP}}[V] = \min_U \left(\mathcal{A}^{\text{FP}}[U] + T[U,V]\right)$
- **Parametrization**: $\mathcal{A}^{\text{FP}}$ form unknown, FP eq gives data for each V

Gradient flow cutoff artifacts

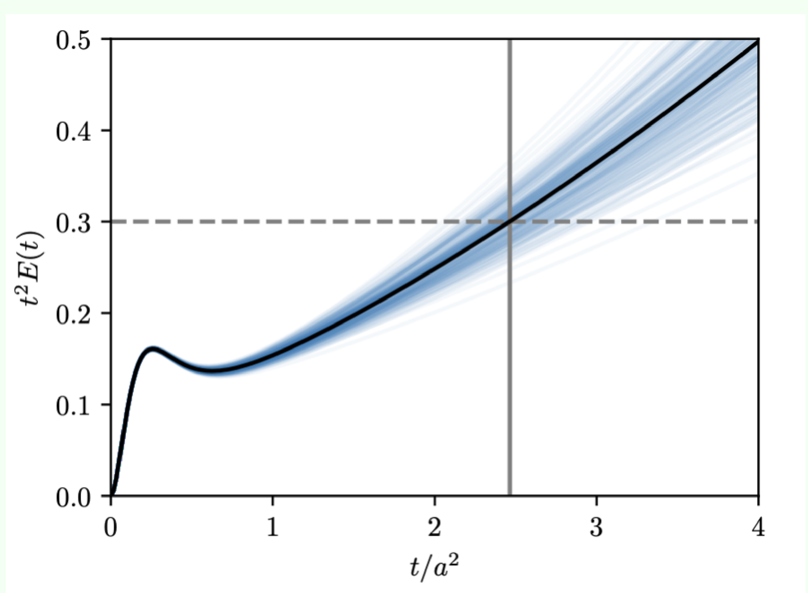
- Cutoff artifacts in gradient flow at **tree level**
  $\mathcal{A}^f$: flow action $\quad \mathcal{A}^g$: MC ensemble $\quad \mathcal{A}^e$: observable

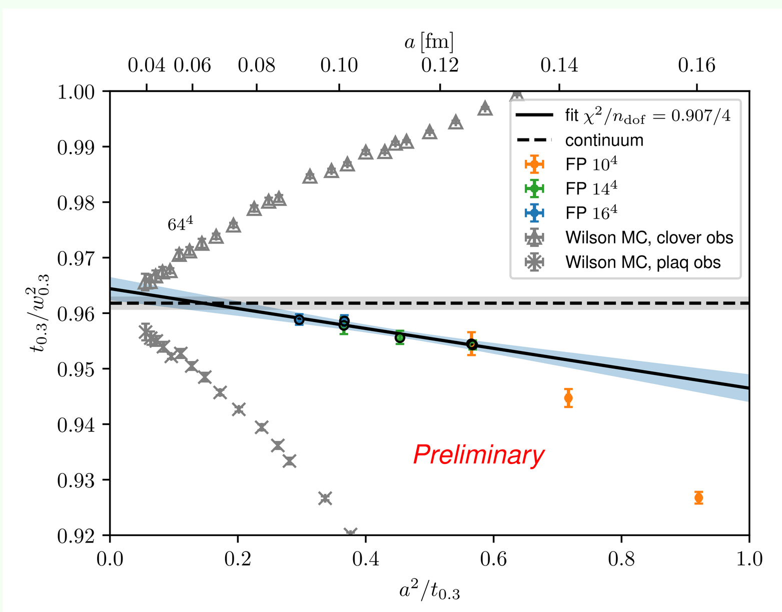
$$\langle t^2 E(t) \rangle = \frac{(N^2-1) g_0^2 t^2}{2} \int_{-\pi/a}^{\pi/a} \frac{d^4 p}{(2\pi)^4} \operatorname{Tr}\left[e^{-t(\mathcal{A}^f + \mathcal{G})} (\mathcal{A}^g + \mathcal{G})^{-1} e^{-t(\mathcal{A}^f + \mathcal{G})} \mathcal{A}^e \right]$$
$$\langle t^2 E(t) \rangle = \frac{3(N^2-1) g_0^2}{128\pi^2} \left[C(a^2/t) + \mathcal{O}(g_0^2) \right] \qquad \text{tree-level artifacts: } C(a^2/t)$$



- **FP action**: $C(a^2/t) = 1$ no tree-level artifacts, classically perfect

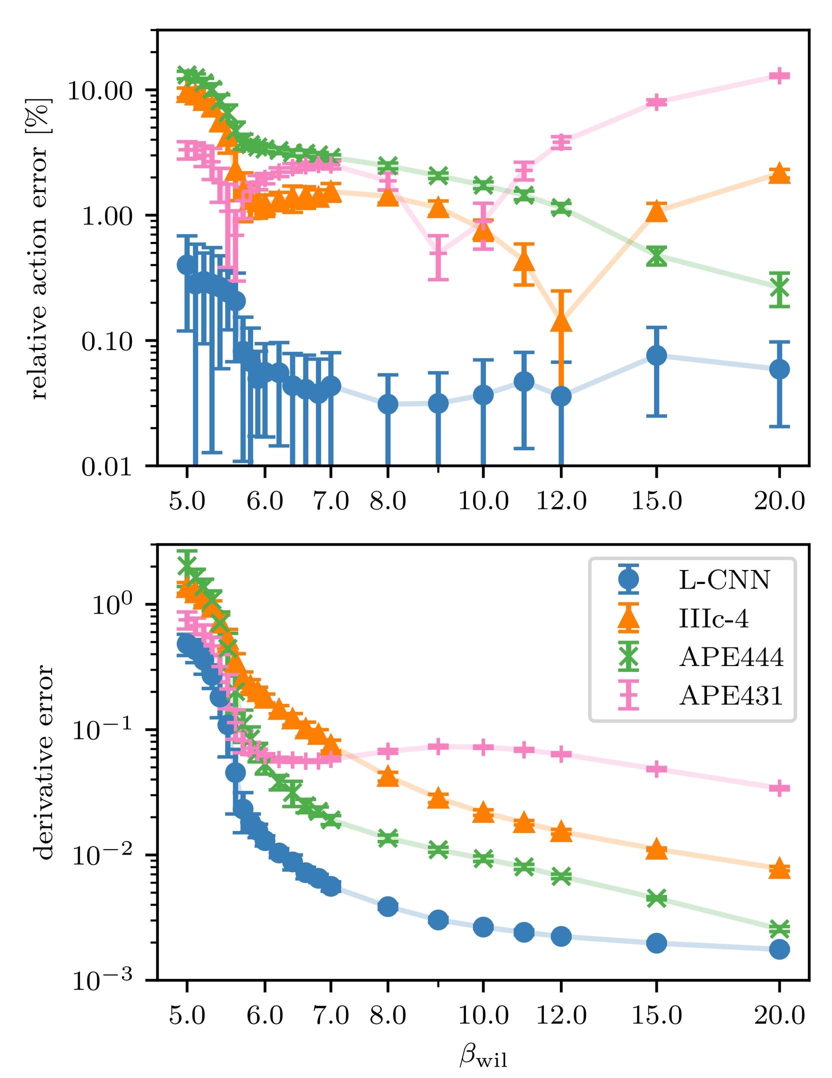
Gradient flow scales



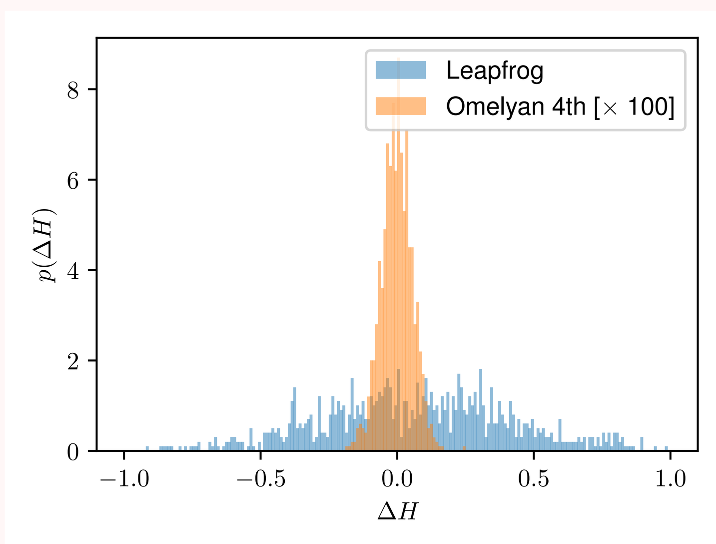


- Gradient flow with FP action $\quad dV_\mu/dt = -\delta\mathcal{A}^{\text{FP}}/\delta V_\mu$
- FP action also used for action density observable $\langle t^2 E(t) \rangle$
- possible physical scales: $t^2 \langle E \rangle|_{t=t_0} = 0.3, \quad t\frac{d}{dt}\left(t^2 \langle E \rangle\right)|_{t=w_0^2} = 0.3$
- Flow equation integrated with 3rd order Runge-Kutta scheme
- MC ensembles also generated with FP action

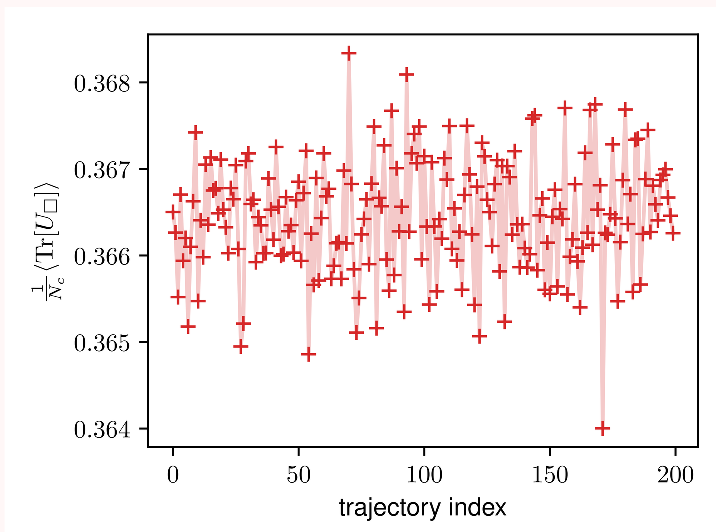
Parametrization and simulation with fixed point action



- **Best model**: L-CNN with 3 layers with 12, 24, 24 channels and kernel size 2, 2, 1
- **L-CNN superior** to older FP parametrizations [2]



- **HMC algorithm** Omelyan integrator, correct Hamiltonian distribution



- **MC history** plaquette

- **Algorithmic**: use backpropagation through L-CNN for $\delta\mathcal{A}^{\text{FP}}/\delta V$ in parametrization of FP action, HMC updates and gradient flow

References

[1] M. Favoni, A. Ipp, D. I. Müller and D. Schuh, PRL **128** (2022) [arXiv:2012.12901].

[2] K. Holland, A. Ipp, D. I. Müller and U. Wenger, *arXiv:2401.06481*.