February 28, 2024

QCD@50:
Key ideas and issues going forward

Keith Ellis
IPPP, Durham

With an attempt to provide links to references

QCD and Asymptotic Freedom

Current Algebra: Quarks and What Else?

HARALD FRITZSCH*†

and

MURRAY GELL-MANN**†

CERN, Geneva, Switzerland

Talk presented at the XVI International Conference on High Energy Physics,

Chicago, September, 1972

Volume 47B, number 4 PHYSICS LETTERS 26 November 1973

ADVANTAGES OF THE COLOR OCTET GLUON PICTURE*

H. FRITZSCH*, M. GELL-MANN and H. LEUTWYLER**

California Institute of Technology, Pasadena, Calif. 91109, USA

Received 1 October 1973

It is pointed out that there are several advantages in abstracting properties of hadrons and their currents from a Yang–Mills gauge model based on colored quarks and color octet gluons.

…ET JOURNAL OF NUCLEAR PHYSICS VOLUME 10, NUMBER 2 FEBRUARY, 1970

…EN'S FUNCTIONS IN THEORIES WITH A NON-ABELIAN GAUGE GROUP

KHRIPLOVICH

Institute for Nuclear Physics, Siberian Section USSR Academy of Sciences

Submitted December 21, 1968

Yad. Fiz. 10, 409–424 (August, 1969)

The Yang-Mills field is considered in the radi… …unction of the independent canonical variables is … …tant. Therefore, when the diagram tech… …counterpart in quantum electrodynamics. … …ion Hamiltonian in the form of an N-product … becomes inadmissible in scalar electro… …ral structure of the Green's functions of … …unction is then computed to second order …

Symanzik's reply was one of interest but skepticism. It looked too good to be true. If true, this result would be very important, he told me, and he advised me to publish it soon. I ignored this sensible advice however, because I found it necessary to first write down elaborately my methods [13] which deviated from what was then conventional. I did mention my result of eq. (5) at the discussion session after Symanzik's talk at the conference.

THE BIRTH OF ASYMPTOTIC FREEDOM

Gerard 't HOOFT

*Institut voor Theoretische Fysica, Rijksuniversiteit, Utrecht, The Netherlands**

G 't Hooft, unpublished Proc Colloquium on Renormalization of Yang-Mills Fields and applications to Particle Physics, 19 Jun 1972, Marseilles, France

Nucl. Phys. B254 (1985), 31

A Field Theory with Computable Large-Momenta Behaviour.

K. Symanzik

Deutsches Elektronen-Synchrotr…

(ricevuto il 12 Dicembre 1972)

LETTERE AL NUOVO CIMENTO VOL. 7, N. 2 12 Maggio 1973

Deep Inelastic Scattering in a Field Theory with Computable Large-Momenta Behaviour.

G. Parisi

Laboratori Nazionali di Frascati del CNEN - Frascati

(ricevuto il 7 Febbraio 1972)

Ultraviolet Behavior of Non-Abelian Gauge Theories*

David J. Gross† and Frank Wilczek

Joseph Henry Laboratories, Princeton University, Princeton, New Jers…

(Received 27 April 1973)

It is shown that a wide class of non… rithmic corrections … scaling …

Reliable Perturbative Results for Strong Interactions?*

H. David Politzer

Jefferson Physical Laboratories, Harvard University, Cambridge, Massachusetts 02138

(Received 3 May 1973)

An explicit calculation shows perturbation theory to be arbitrarily good for the deep Euclidean Green's functions of any Yang-Mills theory and of many Yang-Mills theories with fermions. Under the hypothesis that spontaneous symmetry breakdown is of dynamical origin, these symmetric Green's functions are the asymptotic forms of the physically significant spontaneously broken solution, whose coupling could be strong.

References

Aug-1968: Khriplovich Yad Fiz 10 (1968) 409-424

12-Dec-1972: Symanzik

7-Feb-1972: Parisi

19-Jun-1972: 't Hooft, Marseille Conference

6-13 Sep-1972: Fritzsch & Gell-Mann, ICHEP 16 (Batavia)

27-Apr-1973: Gross-Wilczek

3-May-1973: Politzer,

1-Oct-1973: Fritzsch, Gell-Mann and Leutwyler

Now the interesting question has been raised lately whether we should regard the gluons as well as the quarks as being non-singlets with respect to color.[5] For example, they could form a color octet of neutral vector fields

5. J. Wess (Private communication to B. Zumino).

Birth of QCD

Watergate Break-in 17th June 1972

95th Year .... No. 197 © 1972, The Washington Post Co. MOND

GOP Security Aide Among 5 Arrested In Bugging Affair

By Bob Woodward and Carl Bernstein
Washington Post Staff Writers

One of the five men arrested early Saturday in the attempt to bug the Democratic National Committee headquarters here is the salaried security coordinator for President Nixon's reelection committee.

The suspect, former CIA employee James W. McCord Jr., 53, also holds a separate contract to provide security services to the Republican National Committee, GOP national chairman Bob Dole said yesterday.

Former Attorney General John N. Mitchell, head of the Committee for the Re-Election of the President, said yesterday McCord was employed to help install that committee's own se-

Dole issued a similar statement, adding that "we deplore action of this kind in or out of politics." An aide to Dole said he was unsure at this time exactly what security services McCord was hired to perform by the National Committee.

Police sources said last night that they were seeking a sixth man in connection with the attempted bugging. The sources would give no other details.

Other sources close to the investigation said yesterday that there still was no explanation as to why the five suspects might have attempted to bug Democratic headquarters in the Watergate at 2600 Virginia Ave. NW,

JAMES W. McCORD
...retired CIA employee

ugliest questions about the integrity of the political process that I have encountered in a quarter century.

"No mere statement of innocence by Mr. Nixon's campaign manager will dispel

Sovie Hope On T

Podgor Moscow Help Er

CALCUTTA, (AP)—Soviet Pr olai Podgorny, day mission to pleted, predicted that the stalled talks will resum indicated that t would work to e success.

"The Soviet U everything possib calation of the V a smiling Podgor men on a 30-min

Birth of QCD

9th August, 1974

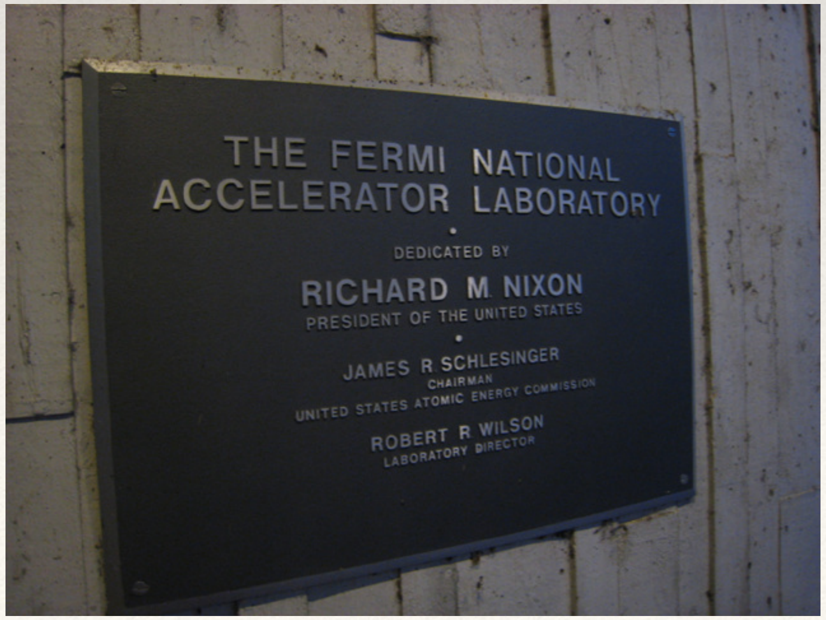
The Washington Post

FINAL

Nixon Resigns

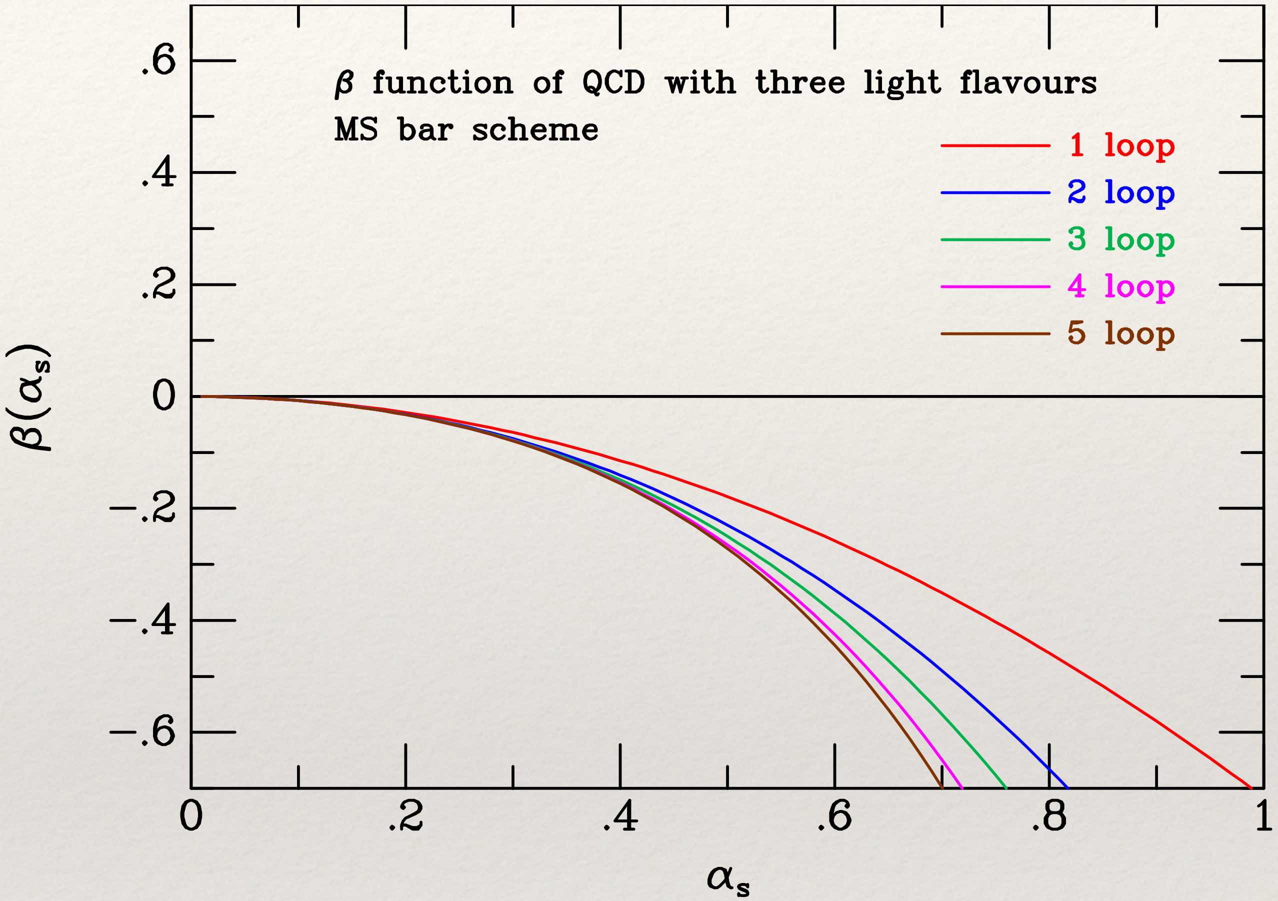
Ford Assumes Presidency Today

May 11, 1974: Fermilab Dedication



Beta function

- Running of the QCD coupling α_s is determined by the β function, which has the expansion $\beta(\alpha_s) = -b\alpha_s^2(1 + b'\alpha_s) + O(\alpha_s^4)$
- $b = \dfrac{(33 - 2n_f)}{12\pi}$ (April/May 1973)
- The first two coefficients b, b' are invariant under scheme change.



1loop: Politzer, Gross-Wilczek
2loop: Caswell, Jones
3loop: Tarasov et al, Larin et al
4loop: Ritbergen et al, Czakon
5loop: Baikov et al, Herzog et al

Acceptance of QCD

- Asymptotic freedom gave one the ability to immediately calculate a limited number of quantities in strong interactions, based on operator product expansion.
    - Approximate scaling in Deep-Inelastic scattering, Gross-Wilczek, Georgi-Politzer
    - e^+e^- total cross section, Appelquist-Georgi, Zee
    - $\Delta I = \frac{1}{2}$ rule, Gaillard-Lee, Altarelli-Maiani
- However acceptance of the new theory was not immediate.

PHYSICAL REVIEW D VOLUME 15, NUMBER 9 1 MAY 1977

Quark elastic scattering as a source of high-transverse-momentum mesons*

R. D. Field and R. P. Feynman
California Institute of Technology, Pasadena, California 91125
(Received 20 October 1976)

We disregard the theoretical argument that this elastic cross section [which we write as $d\hat{\sigma}/d\hat{t}(\hat{s},\hat{t})$, where $\hat{s}$ and $\hat{t}$ are the s, t invariants for the quark collision] must vary as $\hat{s}^{-2}f(\hat{t}/\hat{s})$ and, instead, leave it as an unknown function to be determined empirically by the data. It will vary more like $\hat{s}^{-N}f(\hat{t}/\hat{s})$ with N about 4.

LEPTOPRODUCTION AND DRELL-YAN PROCESSES BEYOND THE LEADING APPROXIMATION IN CHROMODYNAMICS *

G. ALTARELLI
Istituto di Fisica dell' Università, Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Rome 00185, Italy

R.K. ELLIS
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

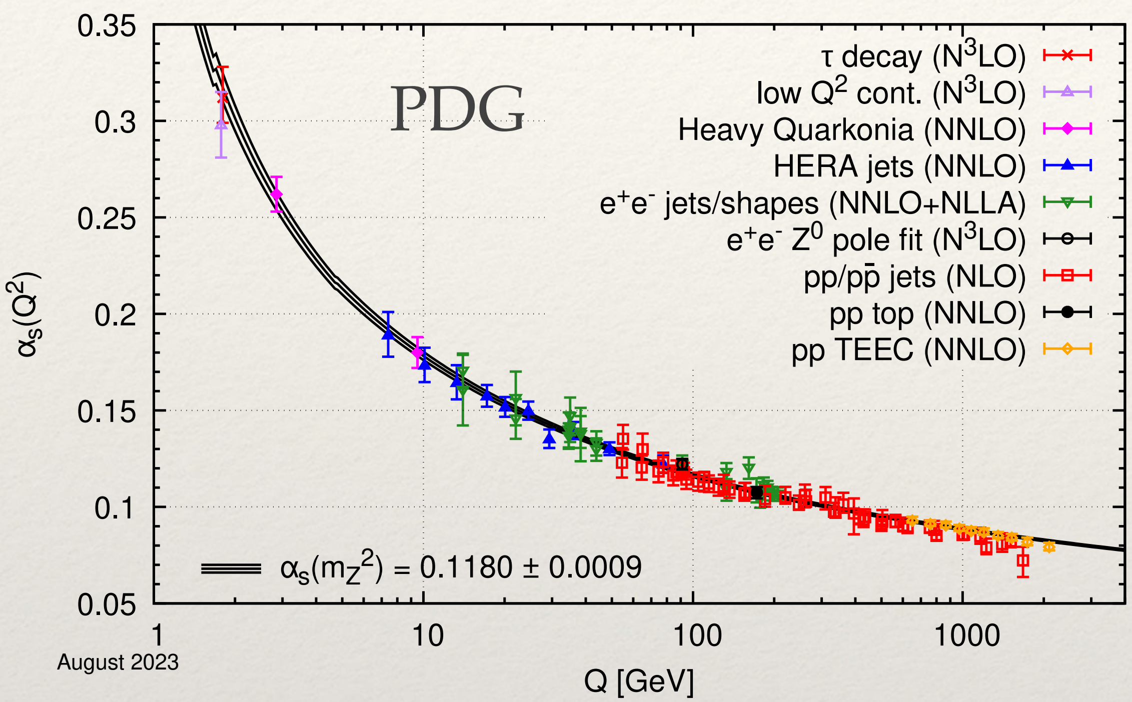
G. MARTINELLI
Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Frascati, Frascati 00044, Italy

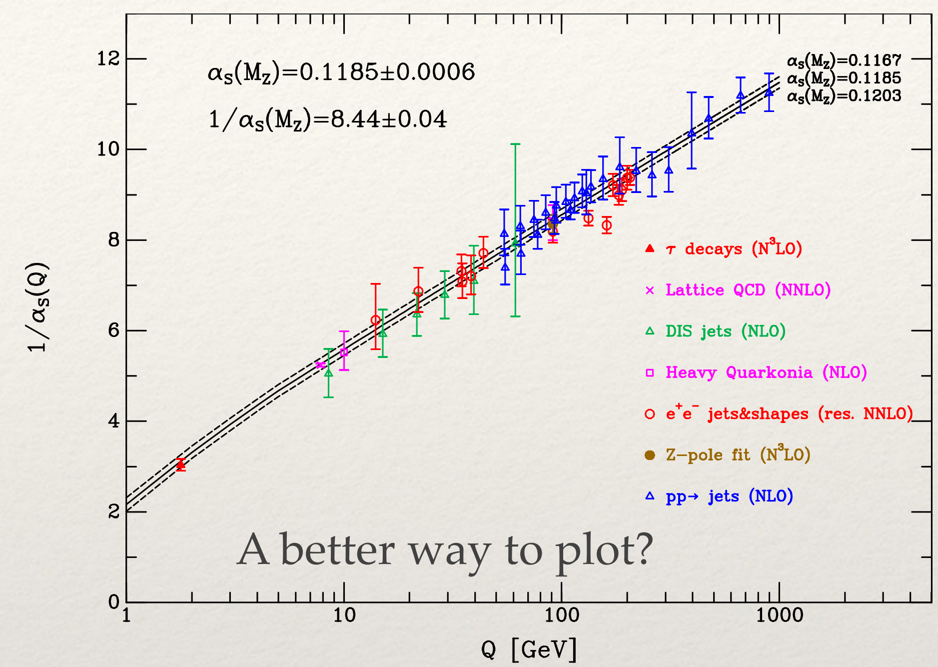
Received 3 July 1978

1. Introduction

The gauge theory of colored quarks and gluons (QCD) ** is at present the best candidate for a fundamental theory of the strong interactions. The asymptotic free-

Plotting α_s





Heavy quark potential from lattice gauge theory, fitted to linear +coulomb potential

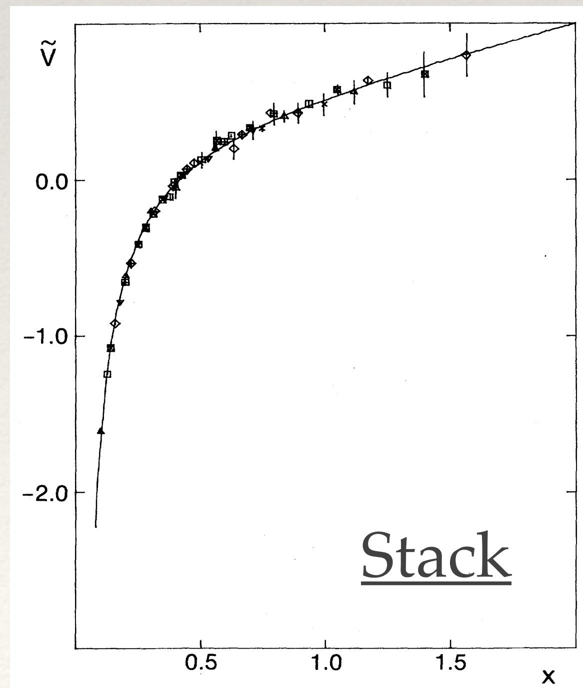


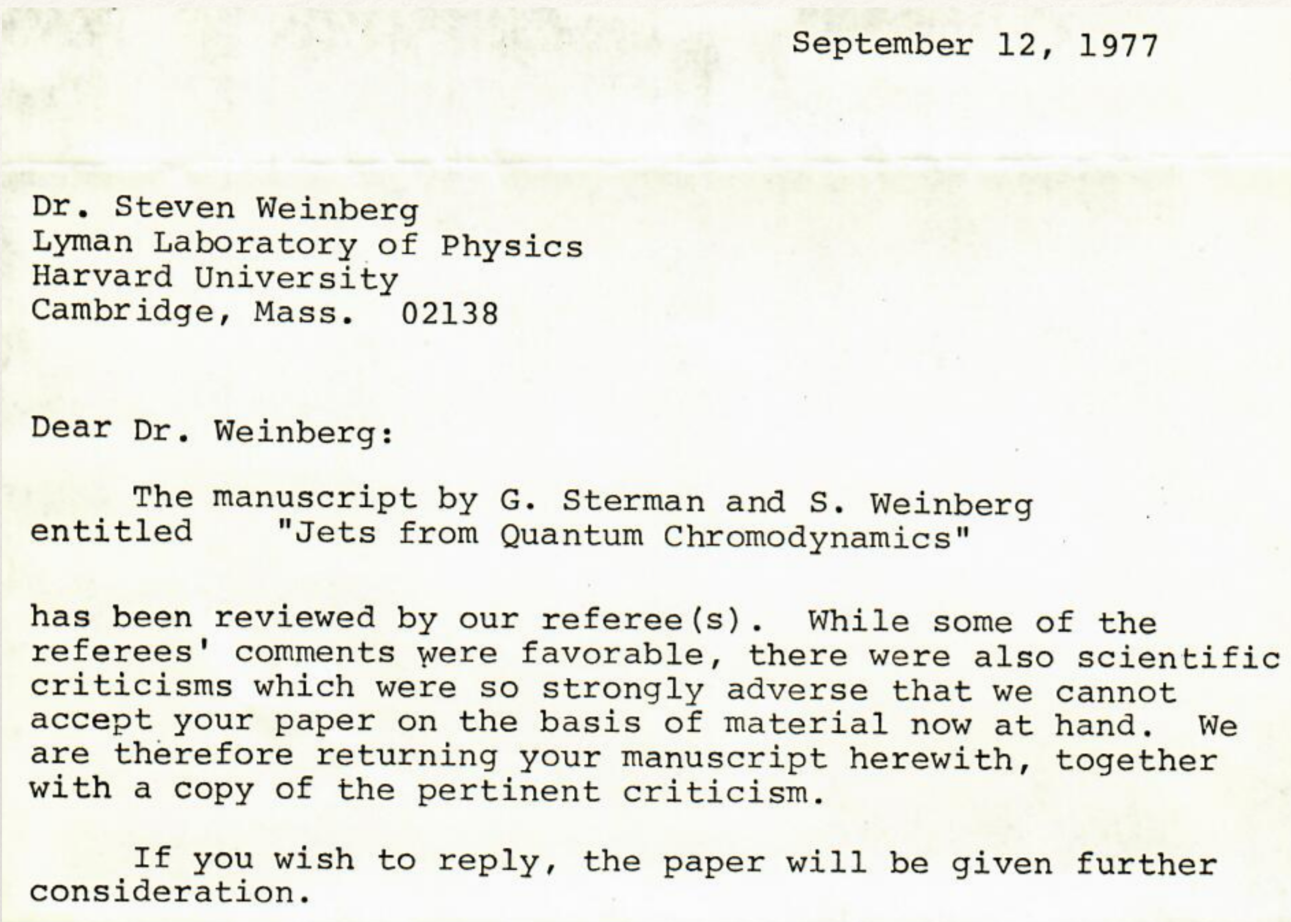
FIG. 3. Linear-plus-Coulomb fit to $\tilde{V}$.

Extension beyond processes governed by operator product expansion

Infrared safety

- In the 1977 paper of Sterman and Weinberg a final state is classified as two-jet like if all but a fraction ϵ of the energy is contained in a pair of cones of half-angle δ.
- $$f_2 = 1 - 8C_F\frac{\alpha_s}{2\pi}\left\{\ln\frac{1}{\delta}\left[\ln\left(\frac{1}{2\epsilon}-1\right)-\frac{3}{4}+3\epsilon\right]+\frac{\pi^2}{12}-\frac{7}{16}-\epsilon+\frac{3}{2}\epsilon^2+O(\delta^2\ln\epsilon)\right\}$$
- The jet measure proposed by Sterman and Weinberg was not important for itself, but because it established the zero-mass limit as a diagnostic for perturbative calculability.
- An observable is infrared and collinear safe if,
  in the limit of a collinear splitting, or the emission of an infinitely soft particle, the observable remains unchanged.

IR safety


September 12, 1977

Dr. Steven Weinberg
Lyman Laboratory of Physics
Harvard University
Cambridge, Mass. 02138

Dear Dr. Weinberg:

The manuscript by G. Sterman and S. Weinberg entitled "Jets from Quantum Chromodynamics"

has been reviewed by our referee(s). While some of the referees' comments were favorable, there were also scientific criticisms which were so strongly adverse that we cannot accept your paper on the basis of material now at hand. We are therefore returning your manuscript herewith, together with a copy of the pertinent criticism.

If you wish to reply, the paper will be given further consideration.

George Sterman
50 years of Quantum Chromodynamics

- PRL reconsidered after the acceptance of papers exploiting infrared safety by Farhi (thrust) and by Georgi & Machacek (spherocity), both listed as received on Sept. 26, 1977.

Jet structure: IR safe sequential recombination algorithms

- Calculate the distances between particles:

$$d_{ij} = min(k_{Ti}^{2p}, k_{Tj}^{2p})\frac{\Delta y^2 + \Delta\phi^2}{R^2}$$

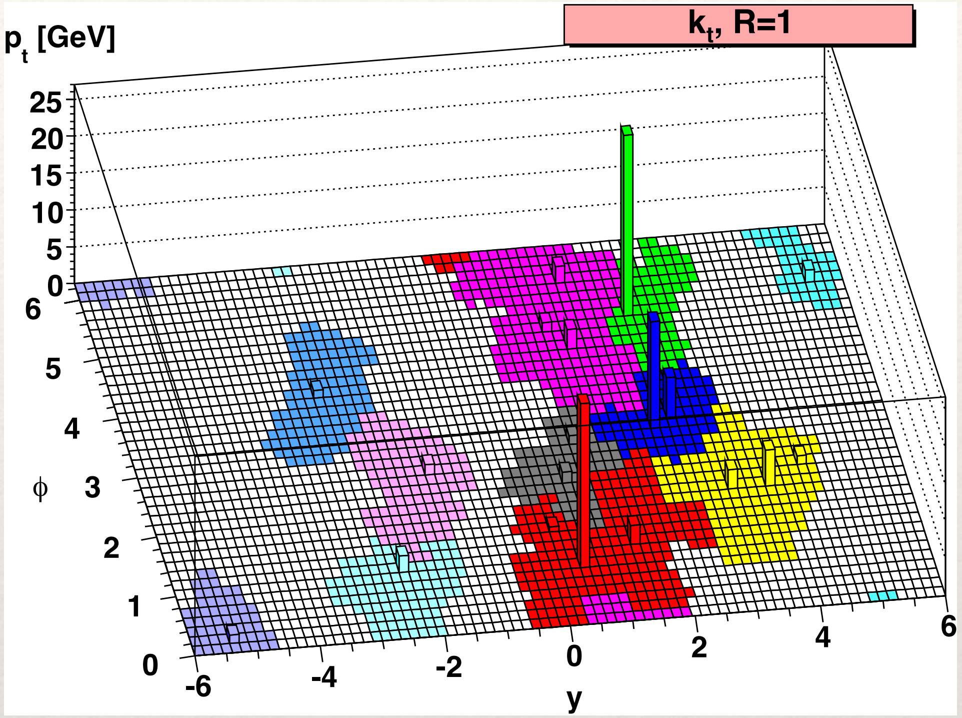
- Calculate the beam distances: $d_{iB} = k_{Ti}^{2p}$;
- Combine particles with smallest distance or, if d_{iB} is smallest, call it a jet;
- Find again smallest distance and repeat procedure until no pseudo-particles are left.

Sequential recombination algorithms

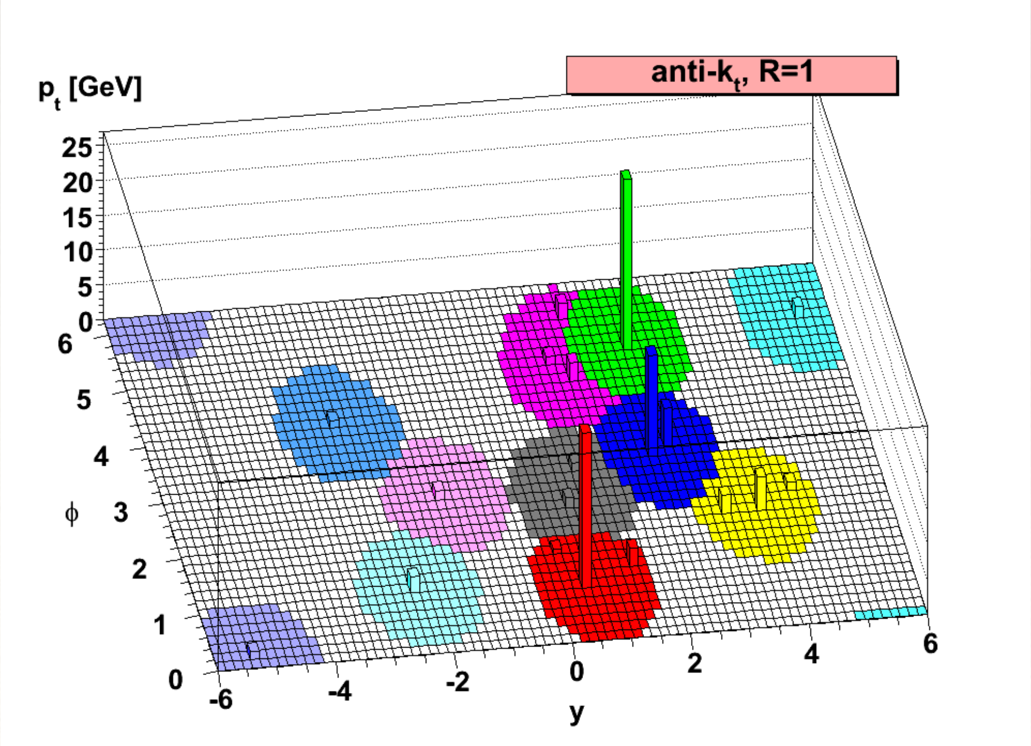
- $d_{ij} = min(k_{Ti}^{2p}, k_{Tj}^{2p})\frac{\Delta y^2 + \Delta\phi^2}{R^2}$
- p=1 (inclusive k_T algorithm)
  - Soft particle ($k_T \to 0$) means that $d \to 0 \Rightarrow$ clustered first, no effect on jets
    Collinear particle ($\Delta y^2 + \Delta\phi^2 \to 0$) means that $d \to 0 \Rightarrow$ clustered first, no effect on jets
- p=0
  - Soft particle ($k_T \to 0$) can be new jet of zero momentum $\Rightarrow$ no effect on hard jets
    Collinear particle ($\Delta y^2 + \Delta\phi^2 \to 0$) means that $d \to 0 \Rightarrow$ clustered first, no effect on jets
- p<0 (anti-k_T algorithm)
  - Soft particle ($k_T \to 0$) means d $\to\infty \Rightarrow$ clustered last or new zero-jet, no effect on hard jets
    Collinear particle ($\Delta y^2 + \Delta\phi^2 \to 0$) means that $d \to 0 \Rightarrow$ clustered first, no effect on jets

k_T and Anti-k_T

- k_T algorithm: motivated by QCD branching structure



- Anti-k_T: hard particles cluster first; if no other hard particles are close by, the algorithm will give perfect cones
- Some what ironic that anti-k_T algorithm leads to conical jets.



Salam, Towards Jetography

Factorization

Asymptotic freedom expands it scope

- The publication of the DGLAP equation Altarelli-Parisi 1977, Dokshitser (Sov. Phys. JETP, 46,641) with its physical picture of parton evolution, raised the issue of whether the Drell-Yan model could be extended to QCD.
- Politzer (1977) deserves credit for outlining the factorization idea.
- Unlike in the parton model, the transverse momentum is now unbounded.
- Transverse momentum in Drell-Yan processes (APP) and AEM (1979) followed Politzer's lead regulating collinear/soft singularities by continuing off-shell, (which turned out to be a tricky procedure).

cf, Sachrajda, 2/1978 - Lepton pair production and the Drell-Yan formula in QCD

Collinear factorization

$$\sigma(P_1,P_2) = \sum_{i,j} \int dx_1 \int dx_2\ f_i(x_1,\mu^2)\, f_j(x_2,\mu^2)\ \hat{\sigma}_{ij}(p_1,p_2,\alpha_s(\mu^2),Q^2/\mu^2) + O(1/Q^2)$$

Universal parton distributions

Hard scattering cross section

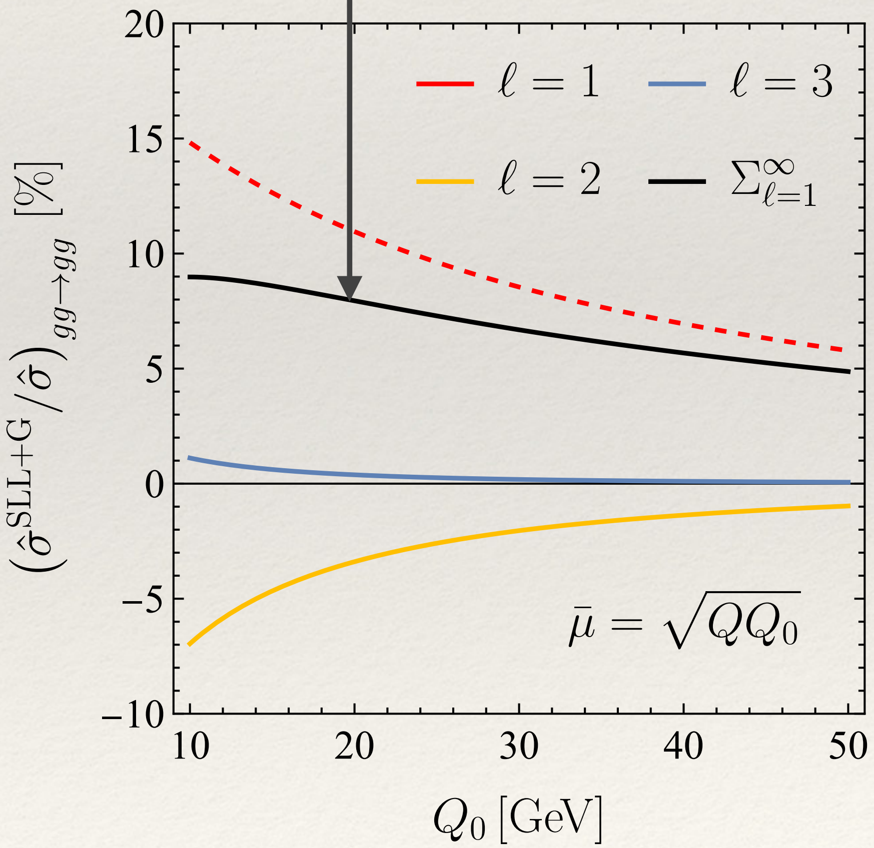
- QCD factorization should hold for inclusive quantities Collins,Soper,Sterman
  - DIS: $e + A \to e' + X$
  - Semi-inclusive e^+ e^- annihilation: $e^+ + e^- \to A + X$
  - Drell-Yan processes: $A + B \to (\mu^+ + \mu^-, W, Z) + X$
  - Inclusive jet production: $A + B \to \text{jet} + X$
  - Heavy quark production: $A + B \to \text{heavy quark}(\mathrm{m_Q} \gg \Lambda) + X$

Non-global effects

- Real physical measurements are not inclusive, because of limited detector acceptance and vetoes imposed to identify jet signatures.
- In proton-proton collisions Glauber (Coulomb) phases can spoil the cancellation of collinear singularities in soft observables giving rise to super-leading logarithms, $L = \ln(Q/Q_0)$ where Q_0 is jet-veto scale; Forshaw, Kyrieleis, Seymour
- Thus for gap-between-jets cross-sections we get super-leading logs
- $$\sigma^{\text{SLL}} \sim \frac{\alpha_s L}{\pi N_c} \left(\frac{N_c \alpha_s}{\pi} \pi^2 \right) \sum_{n=0}^{\infty} c_{1,n} \left(\frac{N_c \alpha_s}{\pi} L^2 \right)^{n+1} \equiv \frac{\alpha_s L}{\pi N_c} w_\pi \sum_{n=0}^{\infty} c_{1,n} w^{n+1},$$

  $$w = \frac{N_c \alpha_s}{\pi} L^2, \; w_\pi = \frac{N_c \alpha_s}{\pi} \pi^2$$
- Summing the effects of Glauber gluons (alternating series) we have, $\sigma^{\text{SLL+G}} \sim \frac{\alpha_s L}{\pi N_c} \sum_{\ell=1}^{\infty} \sum_{n=0}^{\infty} c_{\ell,n} w_\pi^\ell w^{n+\ell}$,

$O(7\%)$ effects for $O(20 \text{ GeV})$ jet veto cuts in small angle gluon-gluon scattering



Boer, Hager, Neubert, Stillger, Xu (2023)

Time to start treating violations of factorization as a feature, rather than a bug

Amplitudes

Spinor techniques

Weyl representation $\gamma^\mu = \begin{pmatrix} \mathbf{0} & \sigma^\mu \\ \bar{\sigma}^\mu & \mathbf{0} \end{pmatrix}$, $(\mu = 0,3)$ $\quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{+1} \end{pmatrix}$

$$\sigma^\mu = (\mathbf{1}, \sigma^i), \bar{\sigma}^\mu = (\mathbf{1}, -\sigma^i)$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Weyl spinors
- Explicitly we find in terms of the components of $p^\mu = (p^0, p^1, p^2, p^3)$,

$$p_\mu \gamma^\mu = \begin{pmatrix} \mathbf{0} & p^{\dot{\alpha}\beta} \\ p_{\alpha\dot{\beta}} & \mathbf{0} \end{pmatrix},$$

$$p^{\dot{\alpha}\beta} = \begin{pmatrix} p^- & -\bar{p}_\perp \\ -p_\perp & p^+ \end{pmatrix}, \ p_{\alpha\dot{\beta}} = \begin{pmatrix} p^+ & \bar{p}_\perp \\ p_\perp & p^- \end{pmatrix}, \ \text{where } p^\pm = p^0 \pm p^3, p_\perp = p^1 + ip^2, \bar{p}_\perp = p^1 - ip^2 .$$

Lance Dixon, Calculating Scattering Amplitudes Efficiently, Elvang and Huang, Scattering amplitudes

Spinor products

Calkul, Xu, Zhang and Chang

- Take all particles to be outgoing.
- So we need outgoing particles $\bar{u}_\pm(p)$ and outgoing antiparticles $v_\pm(p)$
- Spinor products, $\langle ij\rangle[ji] = 2p_i \cdot p_j$
- Gluon polarizations require an auxiliary light-like vector b

$$\bar{u}_\pm(p) p_\mu \gamma^\mu = 0, \qquad \bar{u}_-(p) = \left(\mathbf{0}, \langle p|^\beta\right)$$

$$\bar{u}_+(p) = \left([p|_{\dot\beta}, \mathbf{0}\right)$$

$$p_\mu \gamma^\mu \, v_\pm(p) = 0 \qquad v_-(p) = \begin{pmatrix} \mathbf{0} \\ |p\rangle_\alpha \end{pmatrix}, \quad v_+(p) = \begin{pmatrix} |p]^{\dot\alpha} \\ \mathbf{0} \end{pmatrix}$$

$$\bar{u}_+(p_i) v_+(p_j) = [i|_{\dot\beta} \, |j]^{\dot\beta} = [ij], \quad \bar{u}_-(p_i) v_-(p_j) = \langle i|^\alpha \, |j\rangle_\alpha = \langle ij\rangle,$$

$$\varepsilon_+^\mu(k,b) = \frac{[k|\gamma^\mu|b\rangle}{\sqrt{2}\langle bk\rangle}, \quad \varepsilon_-^\mu(k,b) = \frac{\langle k|\gamma^\mu|b]}{\sqrt{2}[kb]}$$

$\langle ij\rangle \sim \sqrt{2p_i \cdot p_j}$

Dotted (undotted) indices come together with an south-west, north-east, (north-west, south-east) summation convention, which is neatly handled by the angle and square bracket notation.

Maximal Helicity Violating amplitudes

- $(m-1)!$ colour sub-amplitudes defined in terms of traces of fundamental SU(3) matrices

$$\mathscr{A}_m^{\text{tree}}(1,2,3,\ldots m) = g^{m-2} \sum_{\mathscr{P}(2,3,\ldots m)} \text{Tr}[t^{A_1} t^{A_2} t^{A_3} \cdots t^{A_m}]\, A_m^{tree}(1,2,3,\ldots m)$$

- $A_m^{tree}(1^+, 2^+, 3^+, \ldots m^+)$ and $A_m^{tree}(1^-, 2^+, 3^+, \ldots m^+) = 0$ $(\varepsilon_i \cdot \varepsilon_j = 0 \text{ for all } i, j)$

- Maximal helicity violating amplitude has a simple form for all m

$$A_m^{tree}(1^-, 2^-, 3^+, \ldots m^+) = i \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle m1 \rangle}$$

Parke-Taylor

Berends-Giele

- Simple expression for m=3 (in complex kinematics) and m=4

$$A_3^{tree}(1^-, 2^-, 3^+) = i \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \quad A_3^{tree}(1^+, 2^+, 3^-) = i \frac{[12]^4}{[12][23][31]}$$

$$A_4^{tree}(1^-, 2^-, 3^+, 4^+) = i \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle}$$

- The "Little group" is the group of transformations that leave the momentum of an on-shell particle invariant, i.e for a massless particle rotation in xy plane= SO(2)=U(1) $|p\rangle \to t|p\rangle,\ |p] = t^{-1}|p]$

Angle and square spinors for Weyl spinor scale as t^{-2h} for $h = \pm \frac{1}{2}$

Tree-level gluon amplitudes

- Colour decomposition

$$\mathcal{A}_m^{\text{tree}}(1,2,3,\ldots m) = g^{m-2} \sum_{\mathcal{P}(2,3,\ldots m)} \text{Tr}[t^{A_1}t^{A_2}t^{A_3}\cdots t^{A_m}]\, A_m^{tree}(1,2,3,\ldots m)$$

$(m-1)!$

- However these amplitudes are not all independent, Kleiss-Kuijf, DDM

$$\mathcal{A}_m^{\text{tree}} = g^{m-2} \sum_{\sigma \in S_{m-2}} (F^{A_{\sigma_2}} \cdots F^{A_{\sigma_3}})_{1m}\, A_m^{\text{tree}}(1,\sigma_2,\ldots \sigma_{m-1}, m)$$

$(m-2)!$

- Further reduction in independent amplitudes, because of BCJ relations. c_j are colour factors subject to Jacobi identity, n_j kinematic factors, satisfying the same algebra as color factors, d_{i_j}, ordinary Feynman propagators.

$$\mathcal{A}_m^{\text{tree}} = g^{m-2} \sum_j \frac{c_j\, n_j}{\prod_{i_j} d_{i_j}}$$

$(m-3)!$

Sum over j runs over distinct m-point graphs with only three point vertices

Review of BCJ and double copy results

Explicit forms for MHV 4-gluon amplitude

Parke-Taylor

$$A_{st} = A_4^{\text{tree}}(1^-,2^-,3^+,4^+) = \frac{n_s}{s} - \frac{n_t}{t}$$

$$= -i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} = i\frac{\langle 12\rangle^2[34]^2}{st}$$

$$A_{tu} = A_4^{\text{tree}}(1^-,3^+,2^-,4^+) = \frac{n_t}{t} - \frac{n_u}{u}$$

$$= -i\frac{\langle 12\rangle^4}{\langle 13\rangle\langle 32\rangle\langle 24\rangle\langle 41\rangle} = i\frac{\langle 12\rangle^2[34]^2}{tu}$$

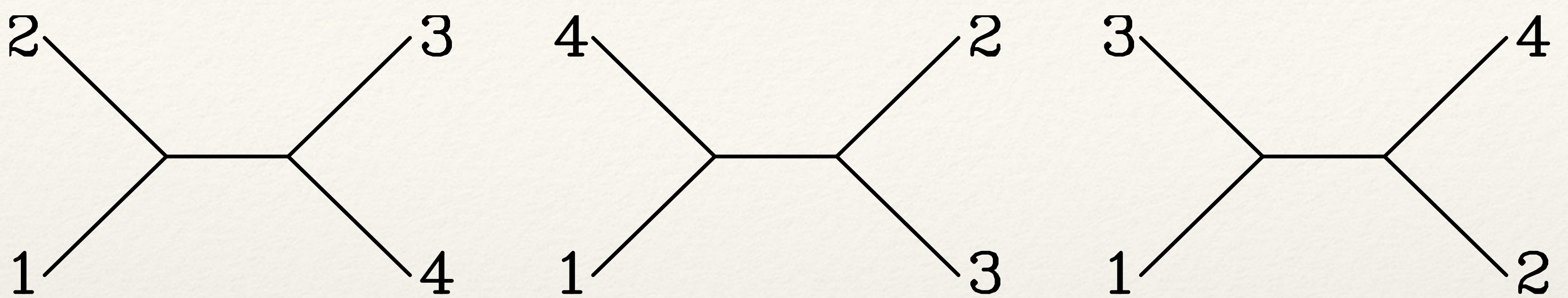
$$A_{us} = A_4^{\text{tree}}(1^-,2^-,4^+,3^+) = \frac{n_u}{u} - \frac{n_s}{s}$$

$$= -i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 24\rangle\langle 43\rangle\langle 31\rangle} = i\frac{\langle 12\rangle^2[34]^2}{su}$$

BCJ relation: $st\, A_{st} = ut\, A_{tu} = su\, A_{us}$

Additional simplifications: BCJ for 4 point diagrams

gluon-gluon scattering



$c_s = f^{A_1 A_2 B} f^{B A_3 A_4}$ $\quad c_t = f^{A_1 A_4 B} f^{B A_2 A_3}$ $\quad c_u = f^{A_1 A_3 B} f^{B A_4 A_2}$

Jacobi Identity: $c_s + c_t + c_u = 0$

Full amplitude can be written in a form where kinematic part obeys the same algebra as the color part

$$\mathscr{A}_4 = g^2 \left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u} \right)$$

$$c_s + c_t + c_u = 0 \quad \Longrightarrow n_s + n_t + n_u = 0$$

$$\mathscr{A}_4 = g^2 \left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u} \right) = g^2 [c_s A_{st} - c_u A_{tu}]$$

Bern, Carrasco, Johansson

MHV - Graviton Scattering

- Double copy result $\mathcal{M}(1^-,2^-,3^+,4^+) = \left(\frac{\kappa}{2}\right)^2 \left[\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}\right]$

$$= -\left(\frac{\kappa}{2}\right)^2 \left[s\, A_{st}\, A_{us}\right]$$

- In agreement with BGK (1988) result

$$\mathcal{M}(1^-,2^-,3^+,4^+) = -\left(\frac{\kappa}{2}\right)^2 \frac{\langle 12\rangle^7 s}{N(4)\,\langle 34\rangle}$$

- $N(4) = \langle 12\rangle\langle 13\rangle\langle 14\rangle\langle 23\rangle\langle 24\rangle\langle 34\rangle$ and $\kappa = \sqrt{32\pi G}$

Kawai, Lewellen, Tye

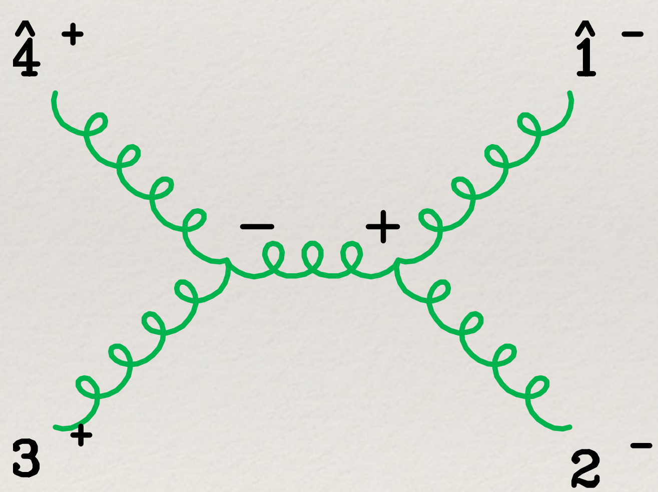
BCFW-combining on-shell amplitudes

- Continue to complex momenta, such that $\hat{p}_1^2 = \hat{p}_4^2 = 0$, $\hat{p}_1 + \hat{p}_4 = p_1 + p_4$

$$|\hat{1}] = |1] - z|4]$$
$$|\hat{4}\rangle = |4\rangle + z|1\rangle$$

$$\hat{p}_1 = \frac{1}{2}\langle 1|\gamma^\mu|\hat{1}] = \frac{1}{2}\langle 1|\gamma^\mu|1] - \frac{z}{2}\langle 1|\gamma^\mu|4]$$
$$\hat{p}_4 = \frac{1}{2}[4|\gamma^\mu|\hat{4}\rangle = \frac{1}{2}[4|\gamma^\mu|\hat{4}\rangle + \frac{z}{2}[4|\gamma^\mu|1\rangle$$

$$A(z) = A_3(-\hat{P}^-, p_3^+, \hat{p}_4^+)\,\frac{i}{\hat{P}^2(z)}\,A_3(\hat{P}^+, \hat{p}_1^-, p_2^-) \text{ where } P = p_3 + \hat{p}_4$$



$$A_3(-\hat{P}^-, p_3^+, \hat{p}_4^+) = -i\frac{[34]^3}{[4|\hat{P}][\hat{P}|3]} \qquad A_3(\hat{P}^+, \hat{p}_1^-, p_2^-) = -i\frac{\langle 12\rangle^3}{\langle 2\hat{P}\rangle\langle \hat{P}1\rangle}$$

$$A(z) = -i\frac{[34]^3\langle 12\rangle^3}{\langle 1|\hat{P}|3]\,\langle 2|\hat{P}|4]}\,\frac{1}{\hat{P}^2(z)} \text{ has a simple pole at } z_0$$

$$\langle 1|\hat{P}|3] = \langle 14\rangle[43],\ \ \langle 2|\hat{P}|4] = \langle 23\rangle[34],\ \hat{P}^2(z) = [43]\langle 31\rangle\,(z+z_0),\ \ z_0 = \langle 34\rangle/\langle 31\rangle$$

$$\text{by Cauchy } A(z) = \frac{c_i}{(z-z_0)} \Longrightarrow A(0) = -\frac{c_i}{z_0} \qquad A(0) = -i\frac{\langle 12\rangle^3}{\langle 23\rangle\,\langle 34\rangle\,\langle 41\rangle}$$

BCFW

Spinor techniques for massive particles

- If amplitudes purport to be a complete description, it is necessary to be able to handle massive particles.
- For massive particles we can go to the rest-frame, where the little group is $O(3) \equiv SU(2)$.
- In a general frame $p^\mu = (E, P\sin\theta\cos\phi, P\sin\theta\sin\phi, P\cos\theta), \quad P^\pm = E \pm P$
- Arkani-Hamed et al define spin-spinors with an SU2(2) index I,
- $$\lambda^I_\alpha = |\mathbf{p}^I\rangle_\alpha = \sqrt{P^-}\begin{pmatrix} -s^* \\ c \end{pmatrix}, \quad \tilde{\lambda}_{I\,\dot\alpha} = [\mathbf{p}_I|_{\dot\alpha} = \sqrt{P^-}\begin{pmatrix} -s \\ c \end{pmatrix} \quad \text{for } I = 1$$
$$\lambda^I_\alpha = |\mathbf{p}^I\rangle_\alpha = \sqrt{P^+}\begin{pmatrix} c \\ s \end{pmatrix}, \qquad \tilde{\lambda}_{\dot\alpha\,I} = [\mathbf{p}_I|_{\dot\alpha} = \sqrt{P^+}\begin{pmatrix} c \\ s^* \end{pmatrix} \quad \text{for } I = 2$$

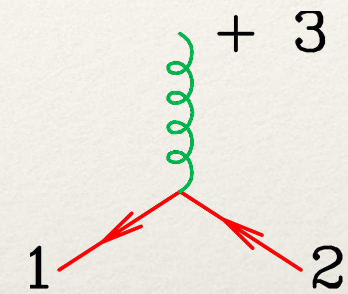
$$c = \cos(\frac{\theta}{2}),\ s = \sin(\frac{\theta}{2})\exp(i\phi),\ s^* = \cos(\frac{\theta}{2})\exp(-i\phi)$$

Arkani-Hamed et al

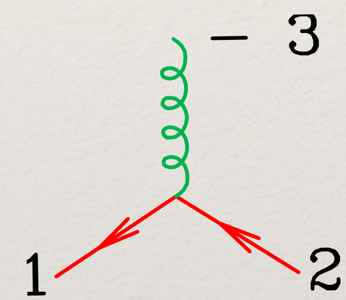
Examples for Top production

- Simple results for massive amplitudes

$$-iA_3(\mathbf{1}_Q, 3^+, \mathbf{2}_{\bar{Q}}) = -\frac{\langle \mathbf{12} \rangle \langle q | \mathbf{1} | 3]}{m \langle q3 \rangle}$$



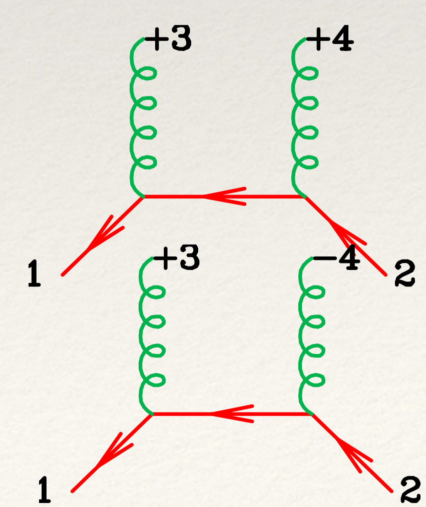
$$-iA_3(\mathbf{1}_Q, 3^-, \mathbf{2}_{\bar{Q}}) = -\frac{\langle \mathbf{12} \rangle \langle q | \mathbf{1} | 3]}{m \langle q3 \rangle}$$



- the two primitive leading-colour amplitudes are given by,

$$\text{-}iA_4(\mathbf{1}_Q, 3_g^+, 4_g^+, \mathbf{2}_{\bar{Q}}) = m \frac{[34]}{\langle 34 \rangle} \frac{\langle \mathbf{12} \rangle}{(s_{13} - m^2)}$$

$$-iA_4(\mathbf{1}_Q, 3_g^+, 4_g^-, \mathbf{2}_{\bar{Q}}) = \frac{\langle 4 | \mathbf{1} | 3] \big([\mathbf{1}3] \langle 4\mathbf{2} \rangle + \langle \mathbf{1}4 \rangle [3\mathbf{2}] \big)}{(s_{13} - m^2) s_{34}}$$



Helicity amplitudes for QCD with massive quarks, Ochirov   see also Ellis & Campbell

NLO (in hadron-hadron reactions)

The beginning…

VOLUME 25, NUMBER 21 PHYSICAL REVIEW LETTERS 23 NOVEMBER 1970

Observation of Massive Muon Pairs in Hadron Collisions*

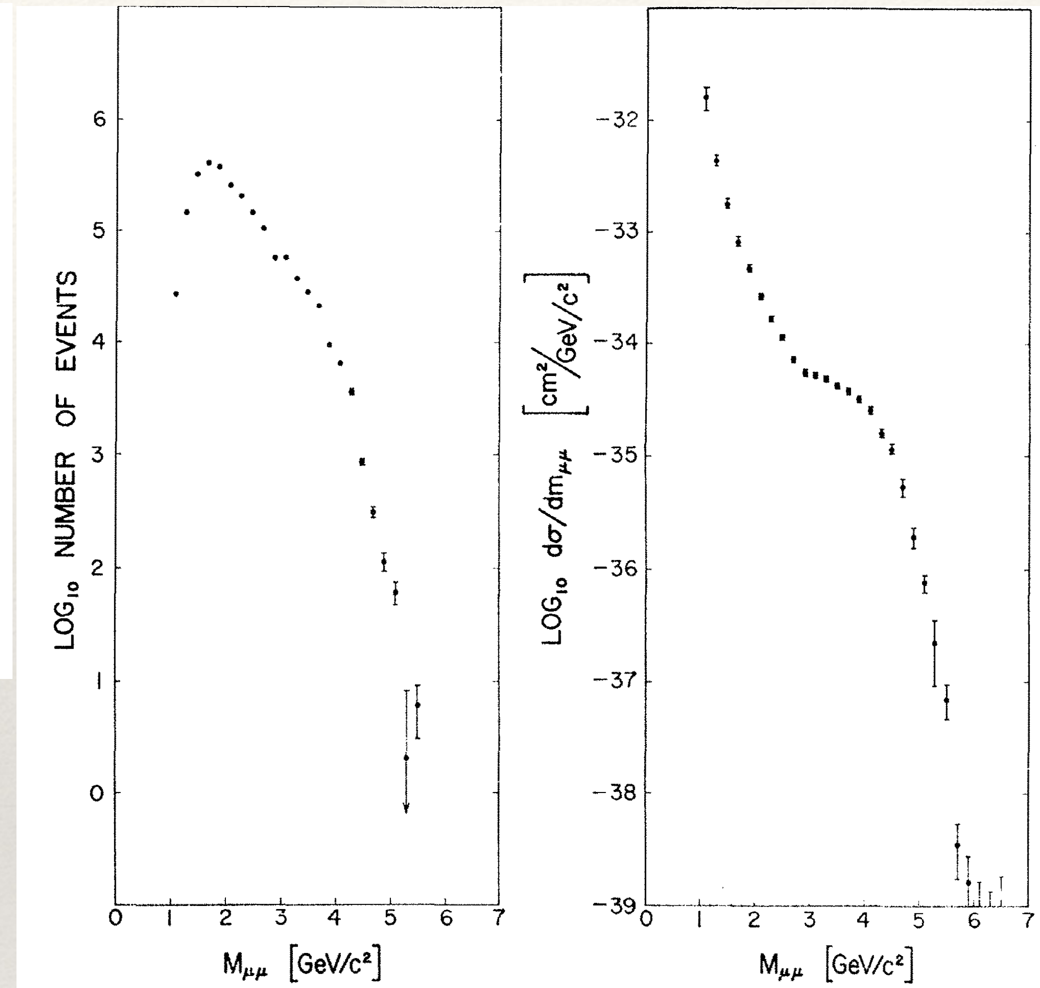
J. H. Christenson, G. S. Hicks, L. M. Lederman, P. J. Limon, and B. G. Pope
Columbia University, New York, New York 10027, and Brookhaven National Laboratory, Upton, New York 11973

and

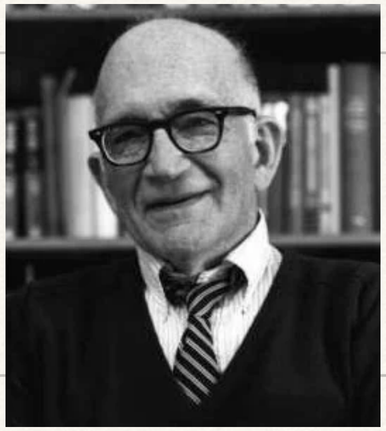
E. Zavattini
CERN Laboratory, Geneva, Switzerland
(Received 8 September 1970)

8th September 1970

Muon pairs in the mass range $1 < m_{\mu\mu} < 6.7$ GeV/c^2 have been observed in collisions of high-energy protons with uranium nuclei. At an incident energy of 29 GeV, the cross section varies smoothly as $d\sigma/dm_{\mu\mu} \approx 10^{-32}/m_{\mu\mu}{}^5$ cm^2 (GeV/c)$^{-2}$ and exhibits no resonant structure. The total cross section increases by a factor of 5 as the proton energy rises from 22 to 29.5 GeV.



- Lederman credits Yamaguchi and Okun for suggesting lepton pair processes.
- “As seen both in the mass spectrum and the resultant cross section there is no forcing evidence of any resonant structure.”
- “Indeed, in the mass region near 3.5 GeV/c², the observed spectrum may be reproduced by a composite of a resonance and a steeper continuum.”
- Drell and Yan had seen the Christenson et al data at the spring APS meeting



Drell-Yan

- Drell and Yan (1970) showed that the parton model could be derived if the impulse approximation was valid.
- To accomplish this, they had to impose a transverse momentum cut-off for the particles that appeared in the quantum field theory.

Assumed anti-parton distributions= parton distributions!

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha}{3Q^2}\frac{1}{Q^2}\mathscr{F}(\tau) = \frac{4\pi\alpha}{3Q^2}\frac{1}{Q^2}\int_0^1 dx_1 \int_0^1 dx_2\, \delta(x_1 x_2 - \tau) \sum_a \lambda_a^{-2} F_{2a}(x_1) F'_{2\bar{a}}(x_2)$$

No colour factor!

Unknown! parton charges

- Rapid fall-off of the cross section, despite the fact that the partons were point-like particles (in contrast to DIS).

cf, Altarelli, Brandt & Preparata, PRL (1970)

The first Drell Yan prediction

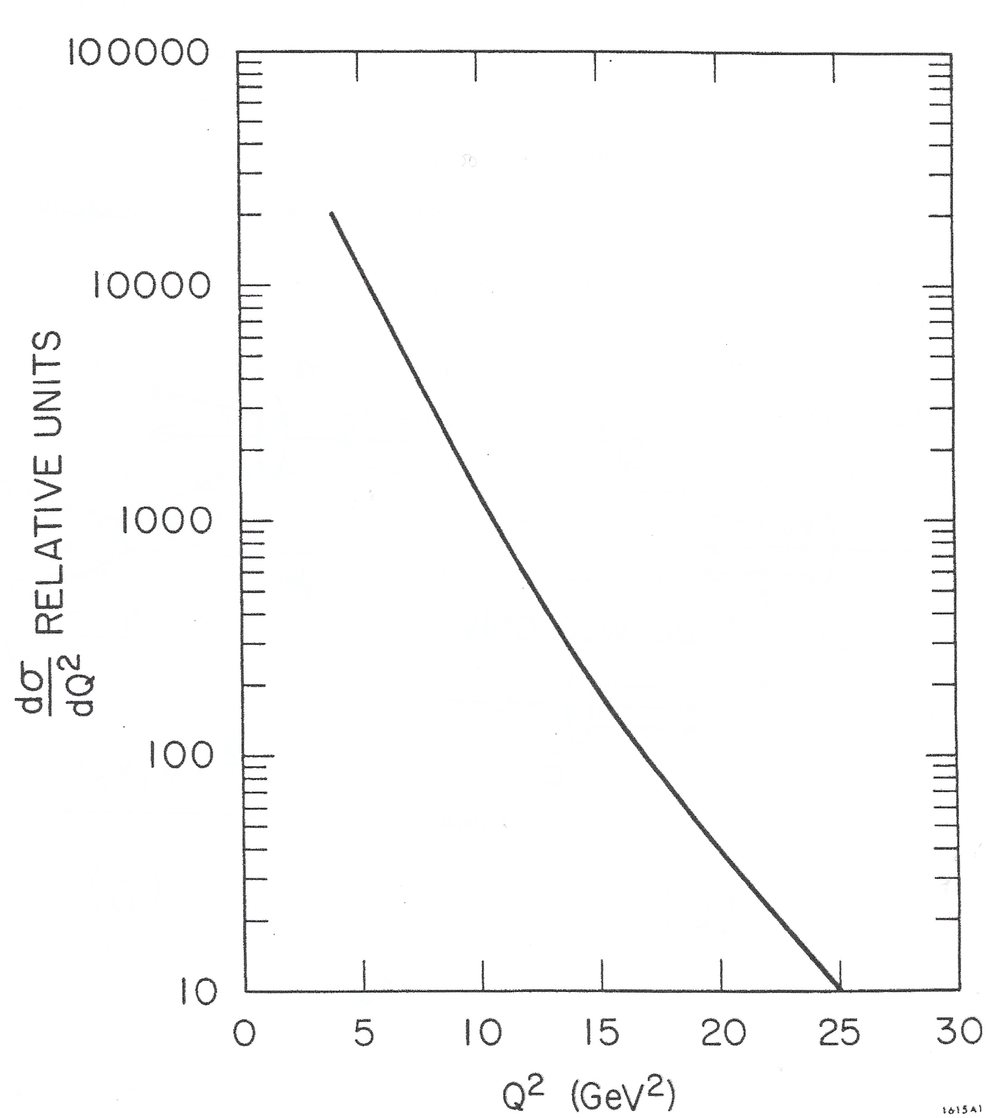
MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

May1970!

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron–hadron inelastic collisions in the limiting region, $s \to \infty$, Q^2/s finite, Q^2 and s being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and inelastic electron scattering are discussed. In particular, a rapid section as $Q^2/s \to 1$ is predicted as a consequence of the observed elastic scattering structure function νW_2 near threshold.

- Predictions are
  - approximate scaling $\dfrac{Q^3\, d\sigma}{dQ} = F(\tau),\ \tau = Q^2/s,$
  - angular dependence, $(1+\cos^2\theta)$
  - A^1 dependence on nucleon number.



Radiative corrections to Drell-Yan

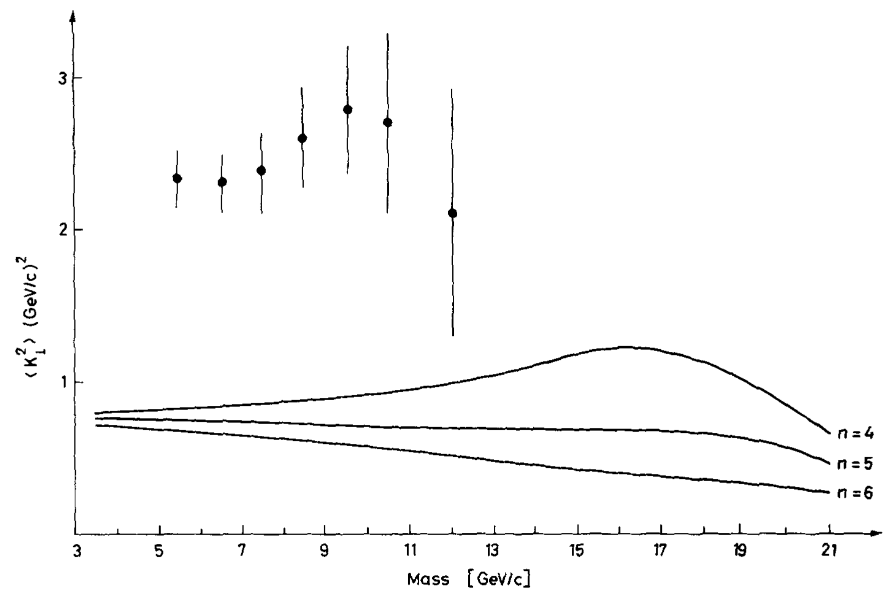


Fig. 3. The hard component of the ⟨$k_\perp^2$⟩ of the muon pair as a function of their invariant mass is compared with the experimental points taken from ref. [9] for three different powers n = 4, 5, 6 of the gluon distribution, following the procedure described in the text.

Altarelli, RKE, Martinelli had written a previous paper mainly on radiative corrections to DIS, including corrections to DY as a (erroneous) postscript

LARGE PERTURBATIVE CORRECTIONS TO THE DRELL-YAN PROCESS IN QCD *

G. ALTARELLI
Istituto di Fisica dell' Università,
Istituto Nazionale di Fisica Nucleare, Sezione di Roma,
Rome 00185, Italy

R.K. ELLIS
Center for Theoretical Physics,
Laboratory for Nuclear Science and Department of Physics,
Massachusetts Institute of Technology,
Cambridge, Massachusetts 02139, USA

G. MARTINELLI
Istituto Nazionale di Fisica Nucleare,
Laboratori Nazionali di Frascati,
Frascati 00044, Italy

Received 17 April 1979

AEM

- QCD predicts an approximate linear rise of $\langle k_T^2 \rangle$ with s or Q^2, but only at fixed τ.
- Intrinsic k_T needed.

Transverse momentum in DY processes, Altarelli, Parisi and Petronzio (1977)

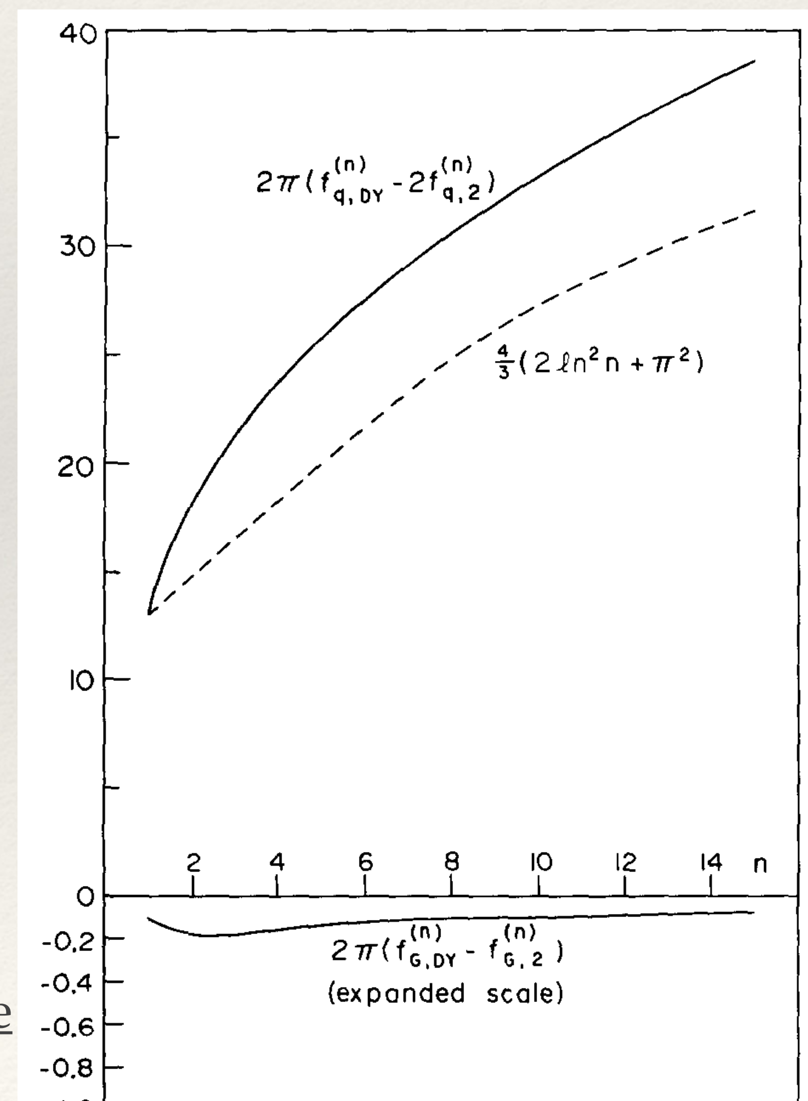
Marciano(1975) - Dimensional Regularization and Mass singularities

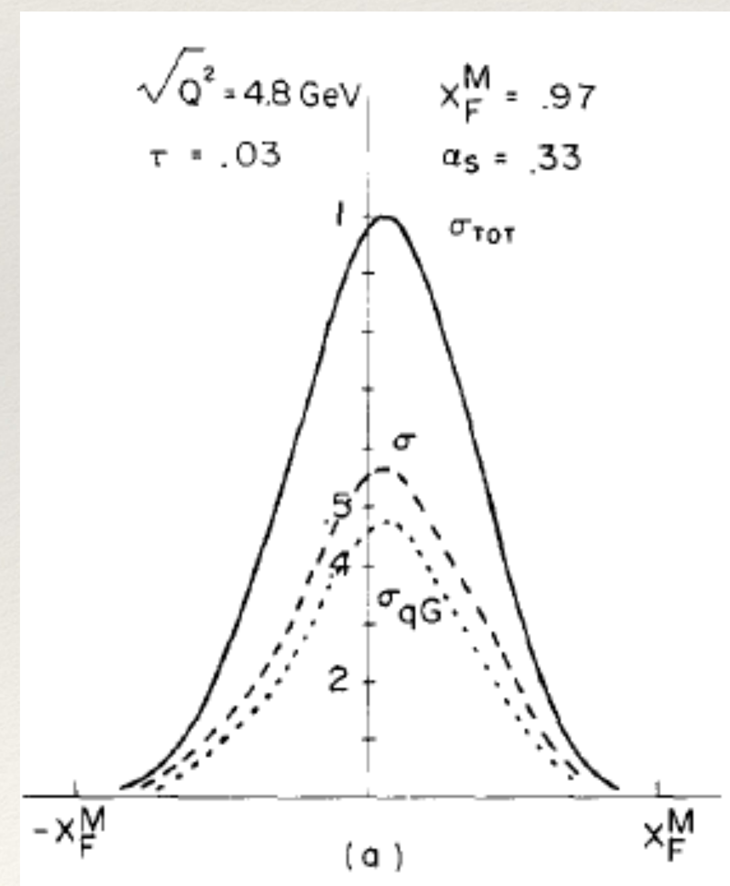
QCD corrections for hadron-hadron interactions

$$\alpha_s f_q(z) = C_F \frac{\alpha_s}{2\pi}\Big[\Big(1+\frac{4\pi^2}{3}\Big)\delta(1-z) + 2(1+z^2)\Big(\frac{\ln(1-z)}{1-z}\Big)_+ + \frac{3}{(1-z)_+} - 6 - 4z\Big]$$

$$\alpha_S f_G(z) = \frac{1}{2}\frac{\alpha_s}{2\pi}\Big[(z^2+(1-z)^2)\,\ln(1-z) + \frac{9}{2}z^2 - 5z + \frac{3}{2}\Big]$$

- Correction relative to DIS
- $\dfrac{\alpha_S}{2\pi} \approx \dfrac{1}{20}$
- Simple origin for the large size of the corrections;
- Phenomenology, x_F distribution;



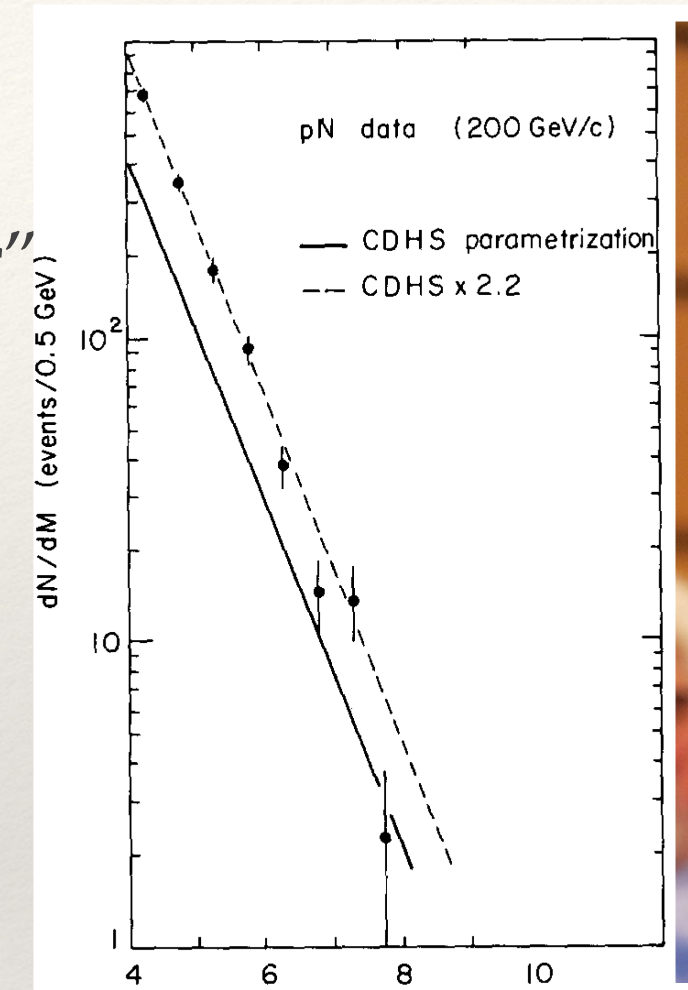


Altarelli, Ellis, Martinelli, see also Kubar-Andre and Paige, and Abad and Humpert

Drell-Yan data and K-factor

- Data lay above the naive DY prediction, leading to the introduction of a "K-factor"
- $$\left(\frac{d\sigma}{dQ^2}\right)_{EXP} = K \left(\frac{d\sigma}{dQ^2}\right)_{NAIVE\ D.Y.}$$

  From ~4 experiments $K \geq 2$
- Telegdi question (N_c or not?)

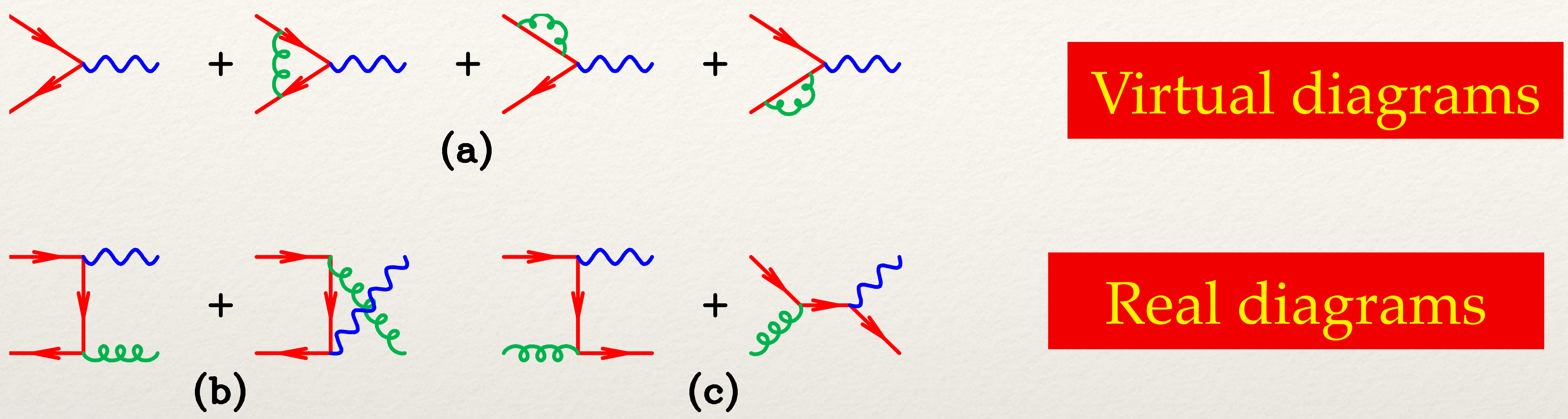


$K = (d^2\sigma/dx_1dx_2)_{exp}/(d^2\sigma/dx_1dx_2)_{DY\ model}$.

Reaction	pN	$\bar{p}$N	π^-N	π^+N	π^-H_2	$(\pi^- - \pi^+)$N
K	2.2 ± 0.4	2.4 ± 0.5	2.2 ± 0.3	2.4 ± 0.4	2.4 ± 0.4	2.2 ± 0.4
Events	960	44	5607	2073	138	–

NA3, Badier et al,

Subtraction/slicing method at NLO



- $\sigma_{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$
- Jet definition can be arbitrarily complicated, but IR/collinear safe
- $d\sigma^R = PS_{m+1} |\mathcal{M}_{m+1}|^2 F^J_{m+1}(p_1, \ldots p_{m+1})$
- We need to combine without knowledge of F^J
- Divergences regularized in $d = 4 - 2\epsilon$ dimensions.
- Two solutions: slicing and subtraction.

One-dimensional example

- The full cross section in d dimensions is

$$\sigma = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathscr{M}(x) F_1^J(x) + \frac{1}{\epsilon} \nu F^J(0)$$

- x is the energy of the emitted gluon
  - KLN cancellation theorem, $\mathscr{M}(0) = \nu$
  - Infrared safety: $F_1^J(0) = F_0^J$

$$\sigma = \int_0^1 \frac{dx}{x^{1+\epsilon}} \Big[\mathscr{M}(x) F_1^J(x) - \mathscr{M}(0) F_1^J(0) \Big] + \int_0^1 \frac{dx}{x^{1+\epsilon}} \nu F_0^J + \frac{1}{\epsilon} \nu F_0^J$$

$$= \int_0^1 \frac{dx}{x} \Big[\mathscr{M}(x) F_1^J(x) - \mathscr{M}(0) F_1^J(0) \Big] + O(1) \nu F_0^J$$

Subtraction

$$\sigma = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathscr{M}(x) F_1^J(x) + \frac{1}{\epsilon} \nu F_0^J \approx \int_0^\delta \frac{dx}{x^{1+\epsilon}} \nu F_0^J + \frac{1}{\epsilon} \nu F_0^J + \int_\delta^1 \frac{dx}{x} \mathscr{M}(x)\, F_I^J(x)$$

$$\approx \ln \delta\, \nu\, F_0^J + \int_\delta^1 \frac{dx}{x} \mathscr{M}(x)\, F_I^J(x)$$

Slicing

NLO QCD solved!

- NLO order is a solved problem numerically, (with the exception of processes first occurring at one-loop level, and processes with a large number of external partons). NLO electroweak corrections also often included. In some cases matched with parton shower.
- MadGraph5_aMC@NLO, Recola, Openloops 2, Gosam, POWHEG(Box)
- Ingredients required -
    - Tree-level and one-loop diagram generation, (or equivalent for processes beginning at one-loop order);
    - Reduction to known integrals (Generalized Unitarity, OPP, Tensor reduction to scalar integrals, Passarino&Veltman Collier, On the fly reduction);
    - Complete basis set of one-loop scalar integrals ('tHooft & Veltman, Denner Nierste & Scharf, RKE & Zanderighi).
    - Subtraction procedure to cancel soft and collinear divergences between real and virtual (ERT, Catani-Seymour, FKS);

RepresentativeNLO results

	Process	μ	n_{lf}	Cross section (pb) LO	NLO
a.1	$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
a.2	$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
a.3	$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
a.4	$pp \to t\bar{b}j$	$m_{top}/4$	4	31.37 ± 0.03	32.86 ± 0.04
a.5	$pp \to t\bar{b}jj$	$m_{top}/4$	4	11.91 ± 0.006	7.299 ± 0.05
b.1	$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
b.2	$pp \to (W^+ \to)e^+\nu_e\, j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
b.3	$pp \to (W^+ \to)e^+\nu_e\, jj$	m_W	5	298.8 ± 0.4	289.7 ± 0.3
b.4	$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
b.5	$pp \to (\gamma^*/Z \to)e^+e^-\, j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
b.6	$pp \to (\gamma^*/Z \to)e^+e^-\, jj$	m_Z	5	54.24 ± 0.02	54.1 ± 0.6
c.1	$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
c.2	$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
c.3	$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
c.4	$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
c.5	$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
d.1	$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
d.2	$pp \to W^+W^-\, j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
d.3	$pp \to W^+W^+\, jj$	$2m_W$	4	0.07048 ± 0.00004	0.08241 ± 0.0004
e.1	$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
e.2	$pp \to HW^+\, j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
e.3	$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
e.4	$pp \to HZ\, j$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
e.5	$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
e.6	$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
e.7	$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.333 ± 0.002

Table 2: Results for total rates, possibly within cuts, at the 7 TeV LHC, obtained with MadFKS and MadLoop. The errors are due to the statistical uncertainty of Monte Carlo integration. See the text for details.

Madloop+MadFKS,Hirschi et al

MadGraph5_aMC@NLO. Alwall et al,

NNLO

NNLO results

- In a recent paper (2202.07738) I tried to document all the processes calculated at NNLO.
- About 50% are available in MCFM.
- We use both q_T slicing and jettiness slicing.

Process	MCFM	Process	MCFM
$H + 0$ jet [8–14]	✓ [15]	$W^{\pm} + 0$ jet [16–18]	✓ [15]
$Z/\gamma^* + 0$ jet [11, 17–19]	✓ [15]	ZH [20]	✓ [21]
$W^{\pm}\gamma$ [18, 22, 23]	✓ [24]	$Z\gamma$ [18, 25]	✓ [25]
$\gamma\gamma$ [18, 26–28]	✓ [29]	single top [30]	✓ [31]
$W^{\pm}H$ [32, 33]	✓ [21]	WZ [34, 35]	✓
ZZ [1, 18, 36–40]	✓	W^+W^- [18, 41–44]	✓
$W^{\pm} + 1$ jet [45, 46]	[3]	$Z + 1$ jet [47, 48]	[4]
$\gamma + 1$ jet [49]	[5]	$H + 1$ jet [50–55]	[6]
$t\bar{t}$ [56–61]		$Z + b$ [62]	
$W^{\pm}H$+jet [63]		ZH+jet [64]	
Higgs WBF [65, 66]		$H \to b\bar{b}$ [67–69]	
top decay [31, 70, 71]		dijets [72–74]	
$\gamma\gamma$+jet [75]		$W^{\pm}c$ [76]	
$b\bar{b}$ [77]		$\gamma\gamma\gamma$ [78]	
HH [79]		HHH [80]	

Most apart from heavy quark and jet production are generalizations of Drell-Yan

Examples of NNLO results from MCFM

Process		target		MCFM		
	$\sigma_{NLO^\star}$	σ_{NNLO}	δ_{NNLO}	σ_{NNLO}	δ_{NNLO}	
$pp \to H$	29.78(0)	39.93(3)	10.15(3)	39.91(5)	10.13(5)	nb
$pp \to Z$	56.41(0)	55.99(3)	−0.42(3)	56.03(3)	−0.38(3)	nb
$pp \to W^-$	79.09(0)	78.33(8)	−0.76(8)	78.41(6)	−0.68(6)	nb
$pp \to W^+$	106.2(0)	105.8(1)	−0.4(1)	105.8(1)	−0.4(1)	nb
$pp \to \gamma\gamma$	25.61(0)	40.28(30)	14.67(30)	40.19(20)	14.58(20)	pb
$pp \to e^-e^+\gamma$	2194(0)	2316(5)	122(5)	2315(5)	121(5)	pb
$pp \to e^-\bar{\nu}_e\gamma$	1902(0)	2256(15)	354(15)	2251(2)	349(2)	pb
$pp \to e^+\nu_e\gamma$	2242(0)	2671(35)	429(35)	2675(2)	433(2)	pb
$pp \to e^-\mu^-e^+\mu^+$	17.29(0)	20.30(1)	3.01(1)	20.30(2)	3.01(2)	fb
$pp \to e^-\mu^+\nu_\mu\bar{\nu}_e$	243.7(1)	264.6(2)	20.9(3)	264.9(9)	21.2(8)	fb
$pp \to e^-\mu^-e^+\bar{\nu}_\mu$	23.94(1)	26.17(2)	2.23(3)	26.18(3)	2.24(2)	fb
$pp \to e^-e^+\mu^+\nu_\mu$	34.62(1)	37.74(4)	3.12(5)	37.78(4)	3.16(3)	fb
$pp \to ZH$	780.0(4)	846.7(5)	66.7(6)	847.3(7)	67.3(6)	fb
$pp \to W^\pm H$	1446.5(7)	1476.1(7)	29.6(10)	1476.7(8)	30.2(4)	fb

Table 4. NLO results, computed using MCFM with NNLO PDFs (denoted $\sigma_{NLO^\star}$), total NNLO cross sections from `vh@nnlo` ($W^\pm H$ and ZH only) and MATRIX (remaining processes, using the extrapolated result from Table 6 of Ref. [24]) and the target NNLO coefficients (δ_{NNLO}, with $\delta_{NNLO} = \sigma_{NNLO} - \sigma_{NLO^\star}$). The result of the MCFM calculation (0-jettiness, fit result b_0 from Eq. (3.9)) is shown in the final column.

NNLO by slicing

$$\sigma_{NNLO} = \int d\Phi_N |\mathcal{M}_N|^2 + \int d\Phi_{N+1} |\mathcal{M}_{N+1}|^2 \theta_N^< + \int d\Phi_{N+2} |\mathcal{M}_{N+2}|^2 \theta_N^<$$

Unresolved

$$+ \int d\Phi_{N+1} |\mathcal{M}_{N+1}|^2 \theta_N^> + \int d\Phi_{N+2} |\mathcal{M}_{N+2}|^2 \theta_N^>$$

Resolved

$$\equiv \sigma_{NNLO}(\tau < \tau_{cut}) + \sigma_{NNLO}(\tau > \tau_{cut}) \,.$$

$$\theta_N^< = \theta(\tau_{cut} - \tau) \text{ and } \theta_N^> = \theta(\tau - \tau_{cut})$$

- Unresolved is subject to a factorization formula and power corrections.
- Resolved radiation contribution obtained from NLO calculation with one additional jet, available by subtraction in MCFM.
- As the cut on the resolved radiation becomes smaller, neglected power corrections are also smaller, but cancellation between resolved and unresolved is bigger.

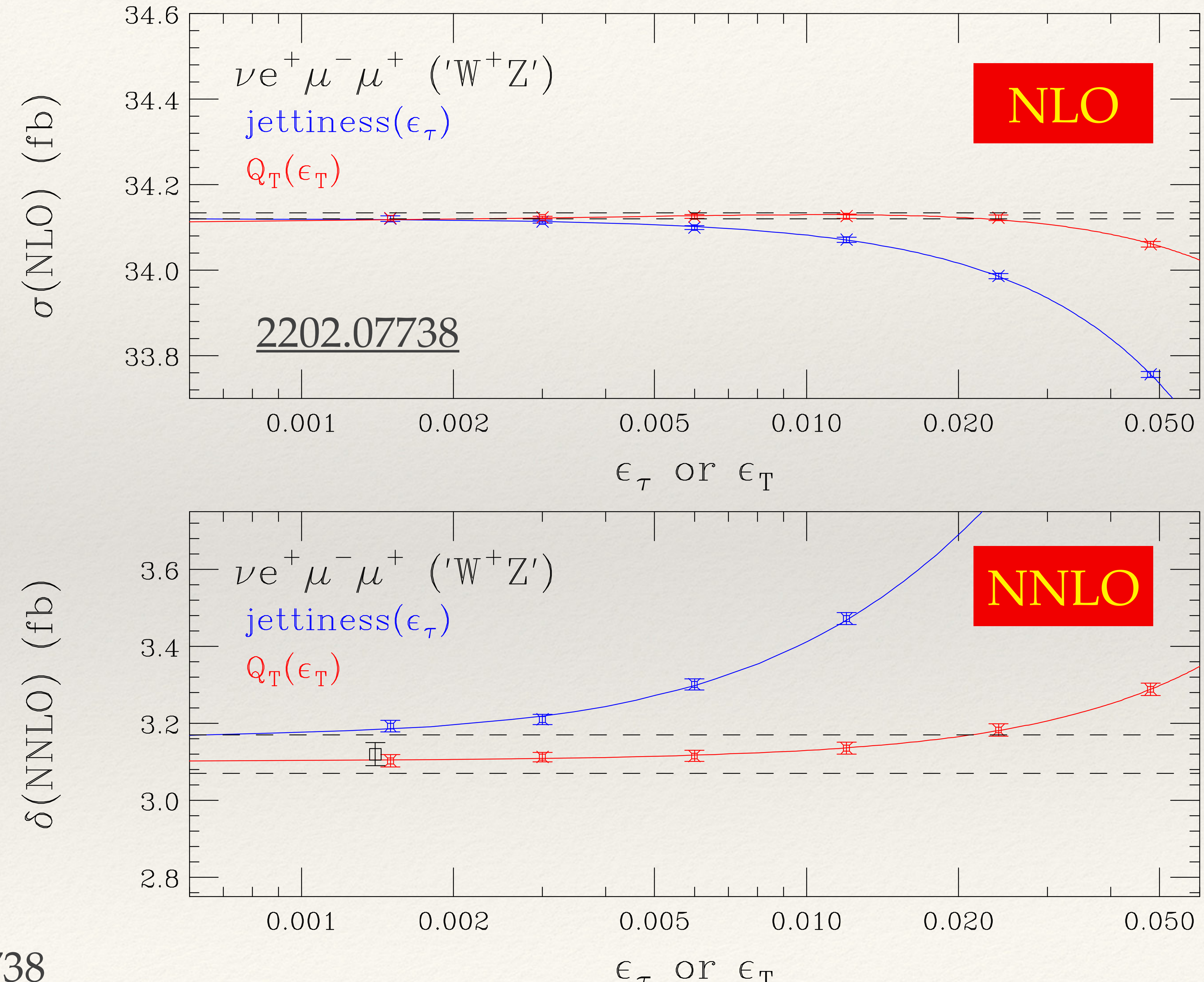
$$\sigma(\tau < \tau_{cut}) = \int H \otimes B \otimes B \otimes S \otimes \left[\prod_n^N J_n\right] + \cdots \,.$$

Slicing parameters

- For color singlet production, "q_T" of produced color singlet object, (Catani et al hep-ph/0703012v2)
- "N-jettiness" (Boughezal et al) 1505.03893 $\mathcal{T}_N = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i} \right\}$
    - The p_i are light-like reference vectors for each of the initial beams and final-state jets in the problem
    - q_k denote the four-momenta of any final-state radiation.
    - $Q_i = 2E_i$ is twice the lab-frame energy of each jet
    - Can handle coloured final states, e.g. H+jet
- Recent new parameter "Jet veto" (Gavardi et al), 2308.11577

NNLO results: dependence on slicing procedure

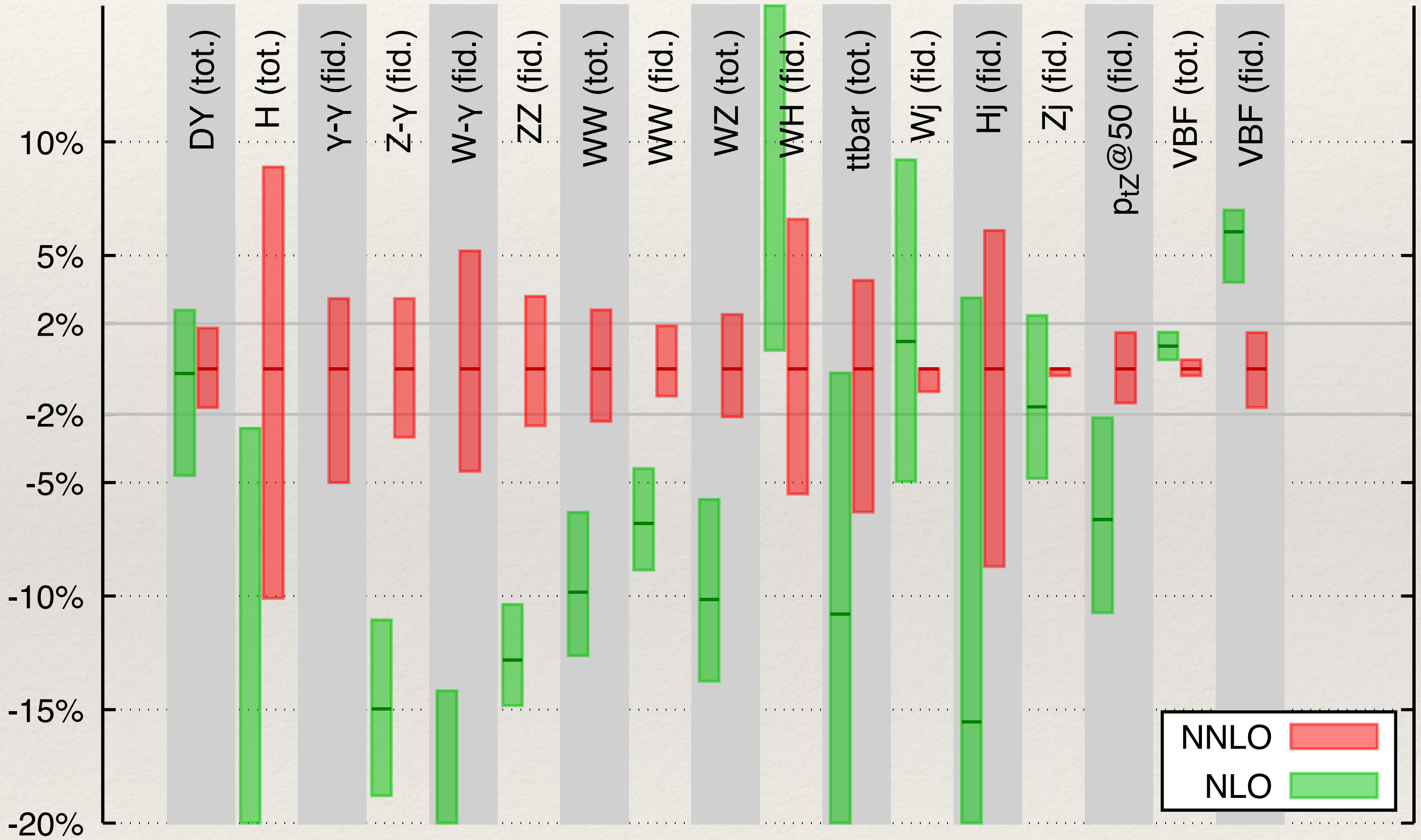
- For most (but not all) processes the power corrections are smaller for Q_T slicing than for jettiness.
- Factor of two in the exponent difference between the leading form factors for q_T and jettiness
- removed by defining $\epsilon_T = q_T^{\text{cut}}/Q$ and $\epsilon_\tau = (\tau^{\text{cut}}/Q)^{\frac{1}{\sqrt{2}}}$



Campbell et al, 2202.07738

Precision QCD

- We compute higher orders in QCD to increase the precision of our predictions i.e. to reduce the theoretical error.
- As we accumulate higher order terms we can ask how our error estimates in lower order perform.
- The NNLO central value lies within the NLO error band in only 4 out of the 17 cases shown.

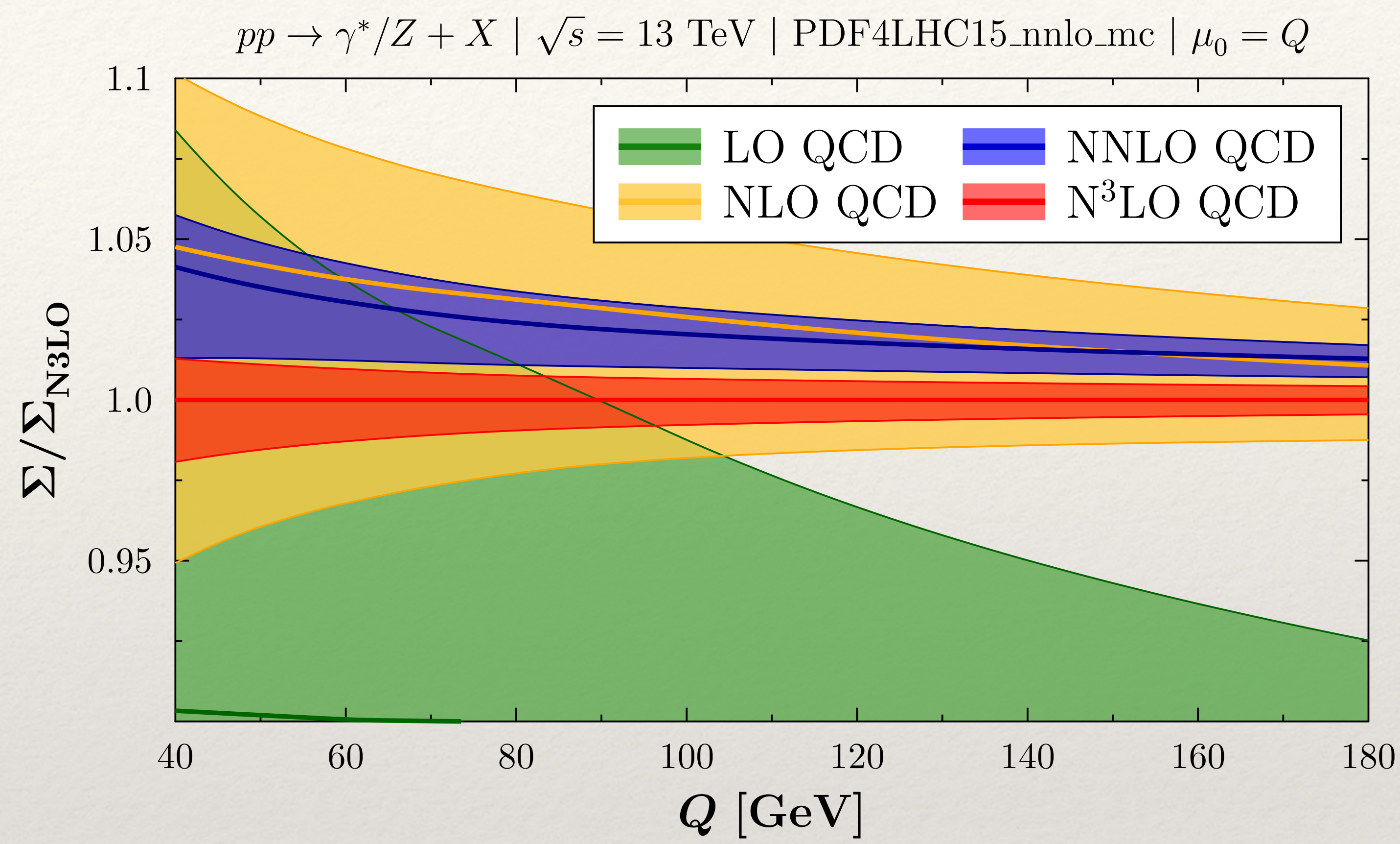


Gavin Salam, (LHCP2016)

N^3LO

N^3LO results for inclusive Z/γ^* etc

- Results for $Z, W^{\pm}, H, WH, ZH$ normalized to N^3LO.
- Both μ_R and μ_F are varied by a factor 2 about their central values respecting the constraint $\frac{1}{2} < \frac{\mu_R}{\mu_F} < 2$, "7-point scale variation"
- In most of the analyzed cases the seven point scale variation at NNLO does not capture the N3LO central value.



Baglio et al, 2209.06138,
c.f. Mistlberger

Conclusions

- The future for (perturbative) QCD is bright.
- Only ~10% of the final LHC luminosity of $3ab^{-1}$ has been collected.
- Paucity of BSM signatures, emphasizes the importance of precision QCD for LHC (and ultimately for planned successor machines, FCC).
- Electron Ion Collider, expected to perform 3-d tomography of the proton, is expected in the early 2030's