

Neutrino Physics

History, electroweak interactions and neutrino scattering

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Resources

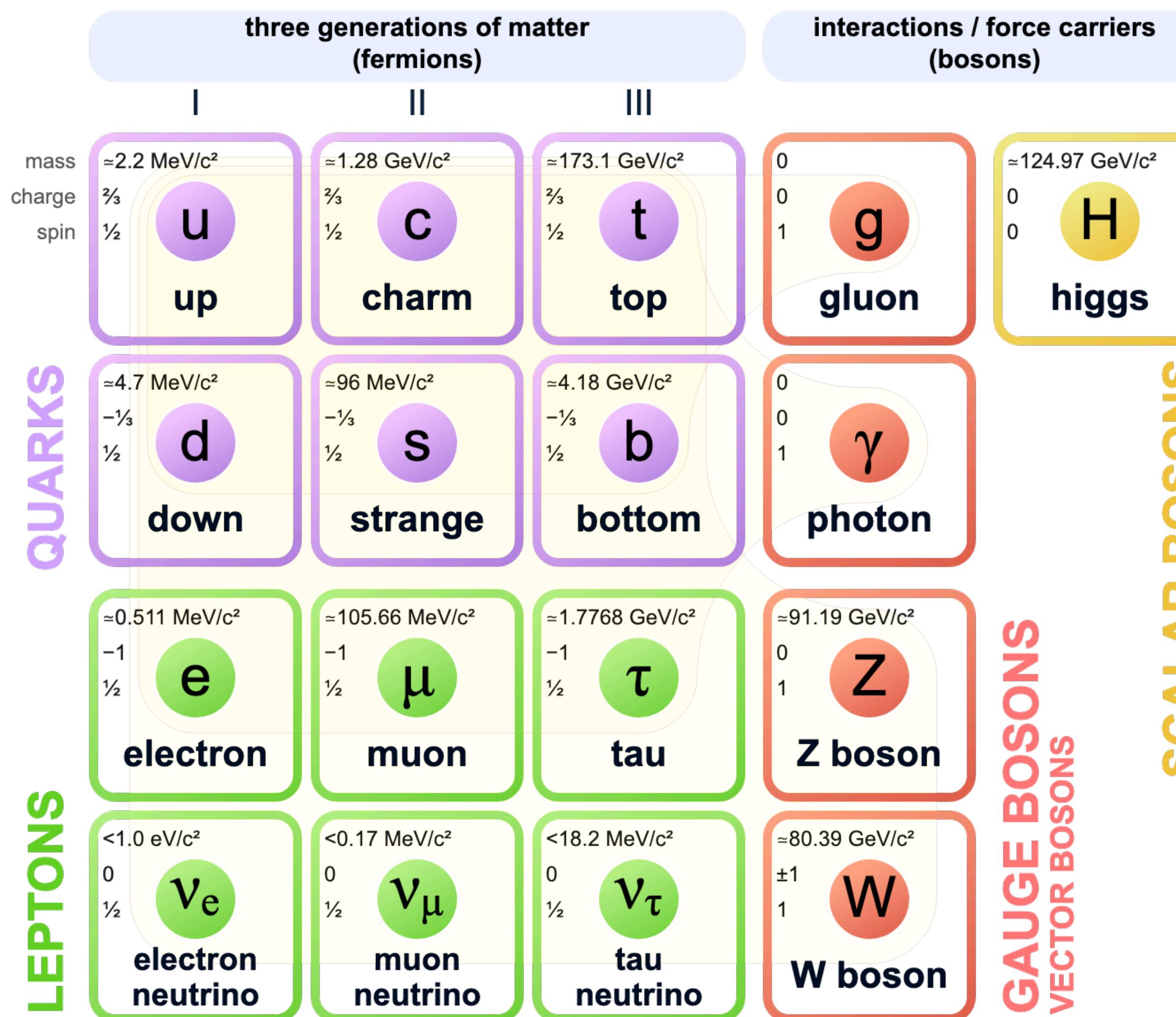
- Athar & Singh: “The Physics of Neutrino Interactions”
- de Gouvea: “TASI lectures on Neutrino ” available [hep-ph/0411274.pdf](#)
- Giunti & Kim: “Fundamentals of Neutrino Physics and Astrophysics”
- Hernandez: “Neutrino Physics ” available [hep-ph/1708.01046.pdf](#)
- Pascoli: “Neutrino Physics” available [Neutrino Physics](#)
- Thompson: Chapter 13 “Modern Particle Physics”

Outline

- Lecture 1: Historical Overview, Neutrinos in the SM, Neutrino Interactions
- Lecture 2: Neutrino Oscillations and Phenomenology
- Lecture 3: Neutrino Oscillations in Matter
- Lecture 4: Neutrino Masses, Models and Consequences
- Lecture 5: Neutrinos in the Early Universe

Neutrinos in the Standard Model

Standard Model of Elementary Particles



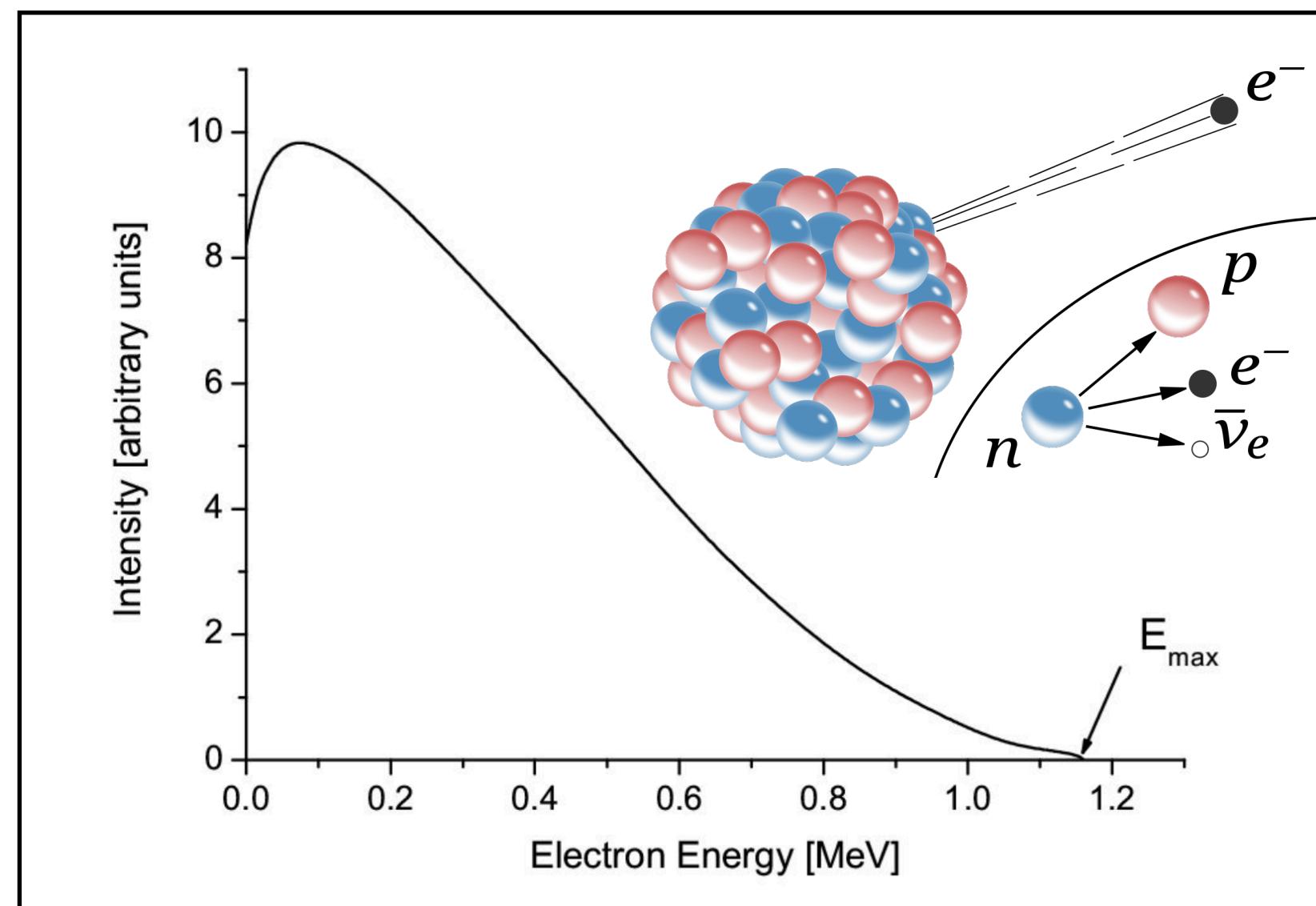
- ν electrically neutral fermions which are part of $SU(2)_L$ doublet
- ν undergo weak interactions with force carriers, W and Z boson ($m(W^\pm) \sim 80 \text{ GeV}, m(Z) \sim 91 \text{ GeV}$)
- $\sum_i m_{\nu_i} \lesssim 1 \text{ eV}$
- Neutrino in SM are described by Weyl spinors with left chirality

$$P_L = \frac{1}{2} (1 - \gamma^5)$$

Discoveries of the Neutrino

- 1800s was an extraordinary time for radioactivity discovery: α , β , γ discovered
- α - Helium nucleus, discovered by Rutherford 1899
- γ - electromagnetic radiation (photon) arising from the radioactive decay of atomic nuclei 1900
- β - electron emitted by radioactive nuclei, discovered by Rutherford 1899

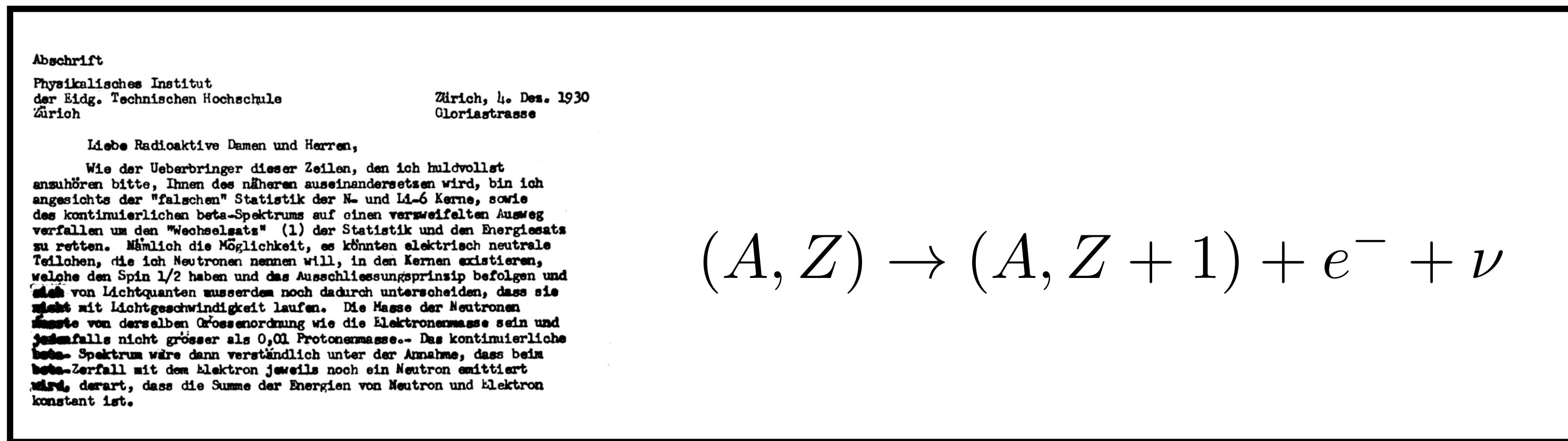
$$(A, Z) \rightarrow (A, Z + 1) + e^- \implies E_e = M(A, Z + 1) - M(A, Z)$$



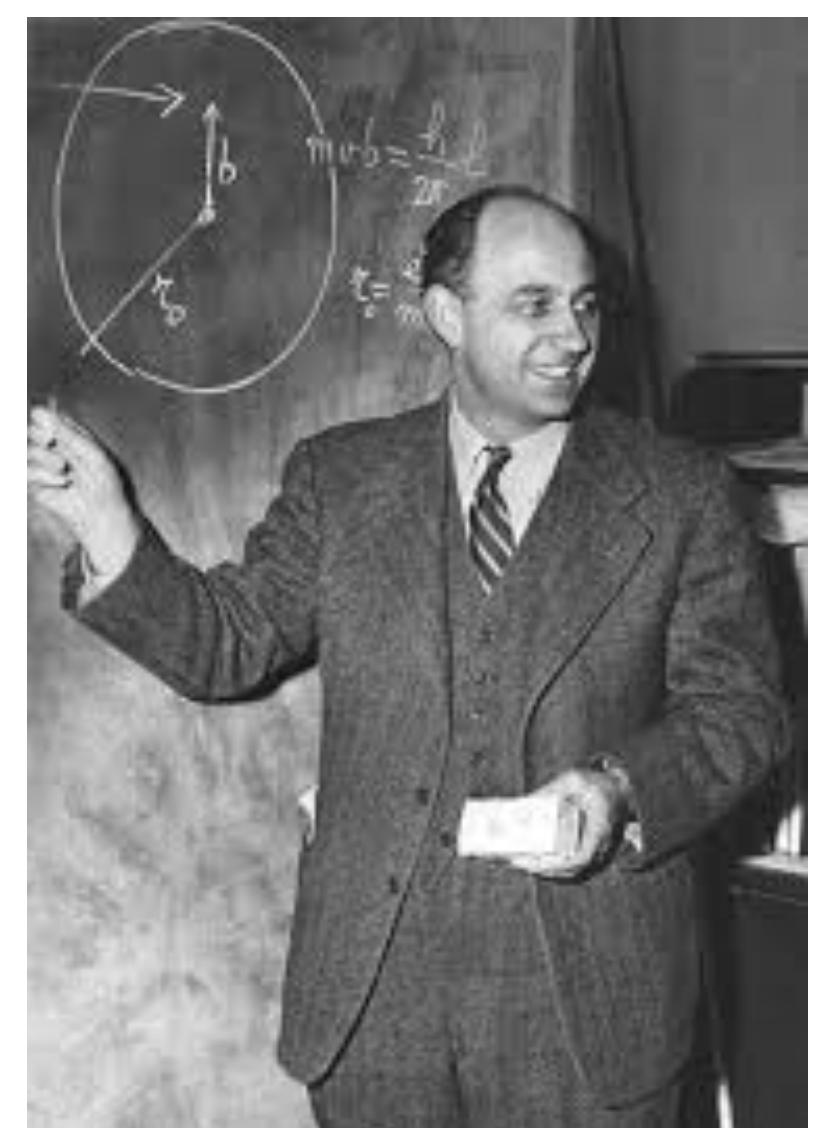
From energy conservation, the electron should have had a fixed energy, not a spectrum which was what was observed.

Discoveries of the Neutrino

- Some scientist thought that energy conservation principle must be violated.
- In his famous letter to “Radioactive Ladies and Gentlemen” Pauli (1930) proposed the existence of a new and yet undiscovered electrically neutral particle which would explain the continuous spectrum observed in beta decay.



$$(A, Z) \rightarrow (A, Z + 1) + e^- + \nu$$



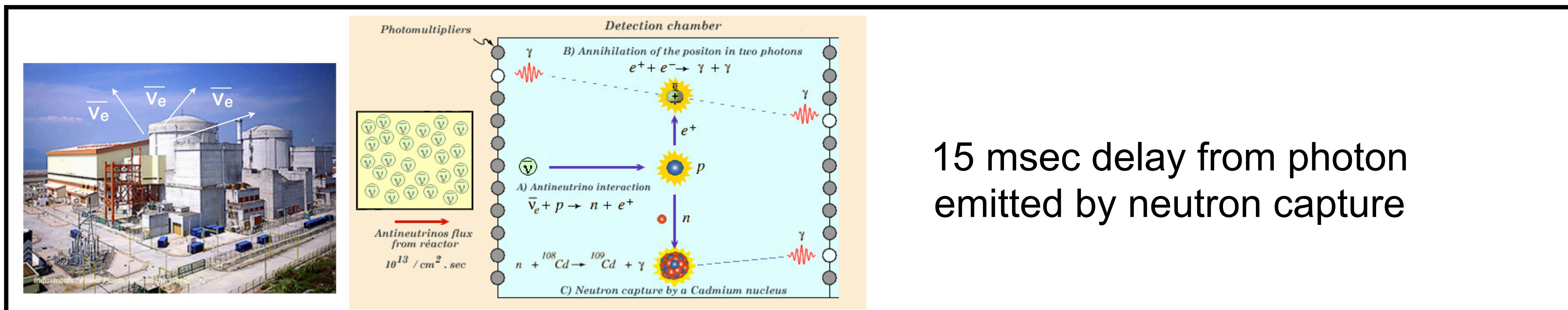
- Fermi proposed the new particle should be **light, spin 1/2 and electrically neutral**. He dubbed this new particle the “**neutrino**”.

Discoveries of the Neutrino

- 1934: Bethe and Peierls showed the interaction cross section of the neutrino should be extremely small \Rightarrow a neutrino can traverse the Earth without deviation

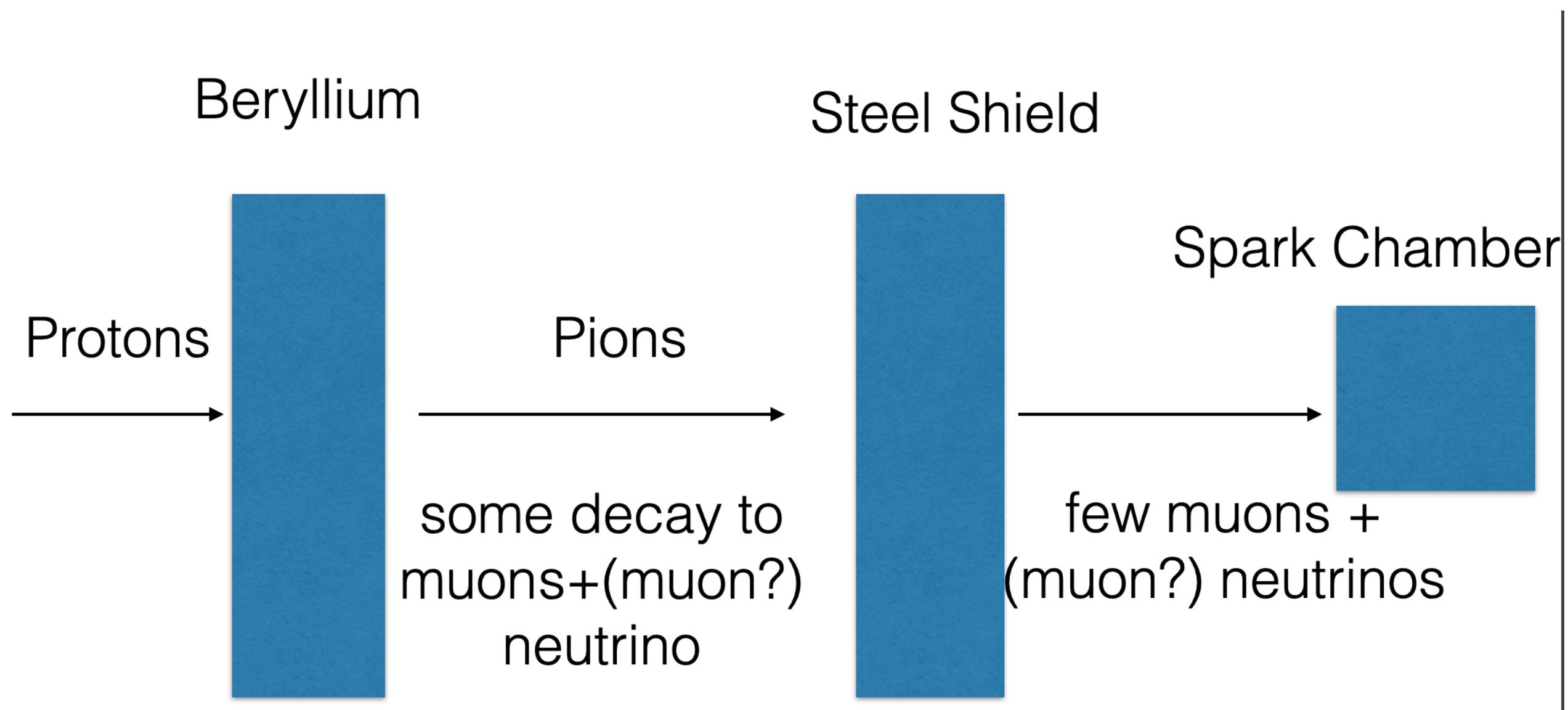
“Today I have done something which no theoretical physicist should ever do in his life: I have predicted something which shall never be detected experimentally!”

- Walter Baade, had great faith in his experiment colleagues and bet Pauli the neutrino would be discovered. The bet was a crate of champagne.
- 1956: Reines and Cowan placed neutrino detector (vat of water combined with cadmium chloride) in front a fission reactor.



Discoveries of the Neutrino

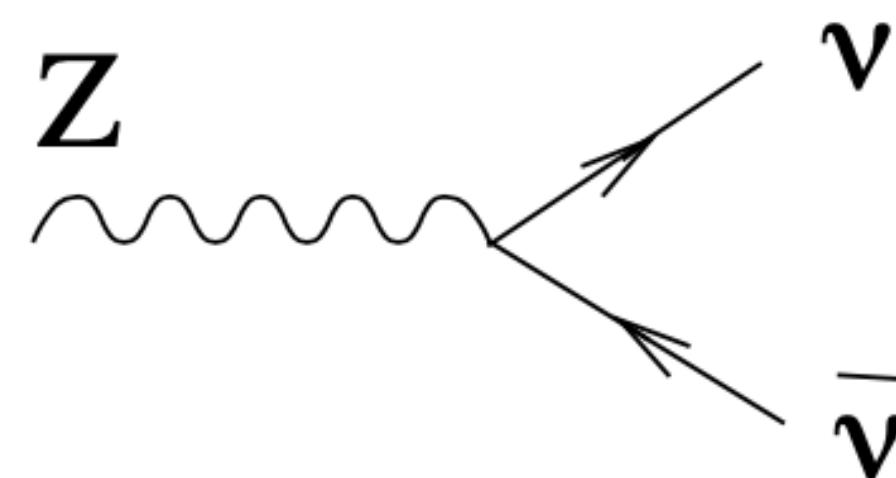
- The electron antineutrino had been discovered because the neutrinos coming from fission reactors is emitted together with electrons.
- Muon discovered (cosmic ray showers) in 1936. Was there an associated neutrino?
- 1959 Gaillard, Lederman, Schwartz and Steinberger built a spark chamber (chambers of a stack of metal plates placed in a sealed box filled with a gas such as helium, neon) to detect muon neutrino coming from pion decays



$$\nu_e \equiv \nu_\mu \implies N(\mu^-) = N(e^-)$$
$$36 \mu^-, 4 e^-$$
$$36 \mu^-, 4 e^- \implies \nu_\mu \neq \nu_e$$

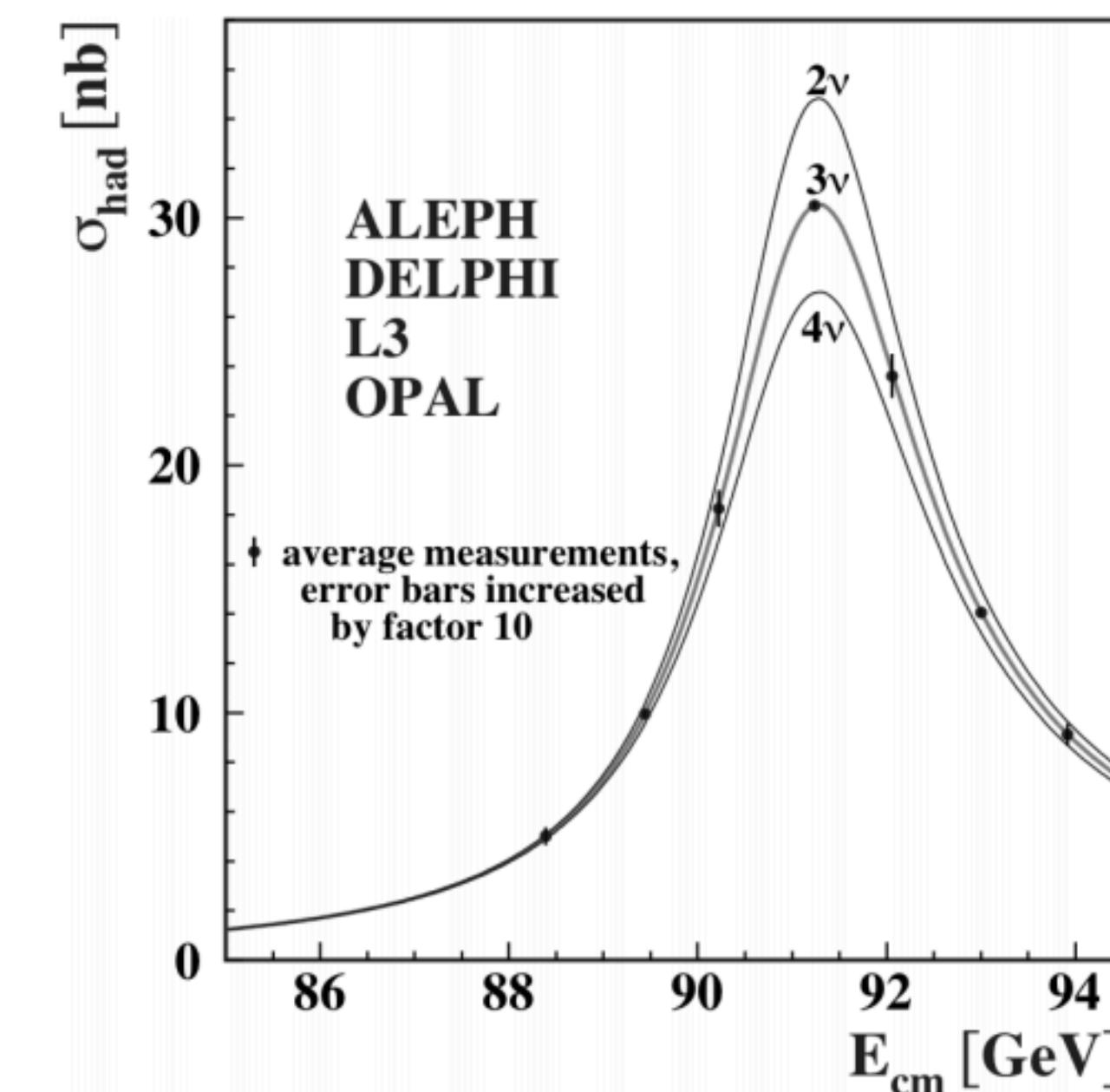
Discoveries of the Neutrino

- neutrinos couple to weak gauge bosons and will modify their decay width
Z boson width measurement by LEP in 1989 confirmed 3 generations of neutrinos



$$N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_\nu} = \frac{\Gamma_{\text{total}} - \Gamma_{\text{visible}}}{\Gamma_\nu} = \frac{\Gamma_{\text{total}} - \Gamma_{\text{had}} - 3\Gamma_{\text{lep}}}{\Gamma_\nu}$$

Requires $N_\nu = 2.98 \pm 0.082$

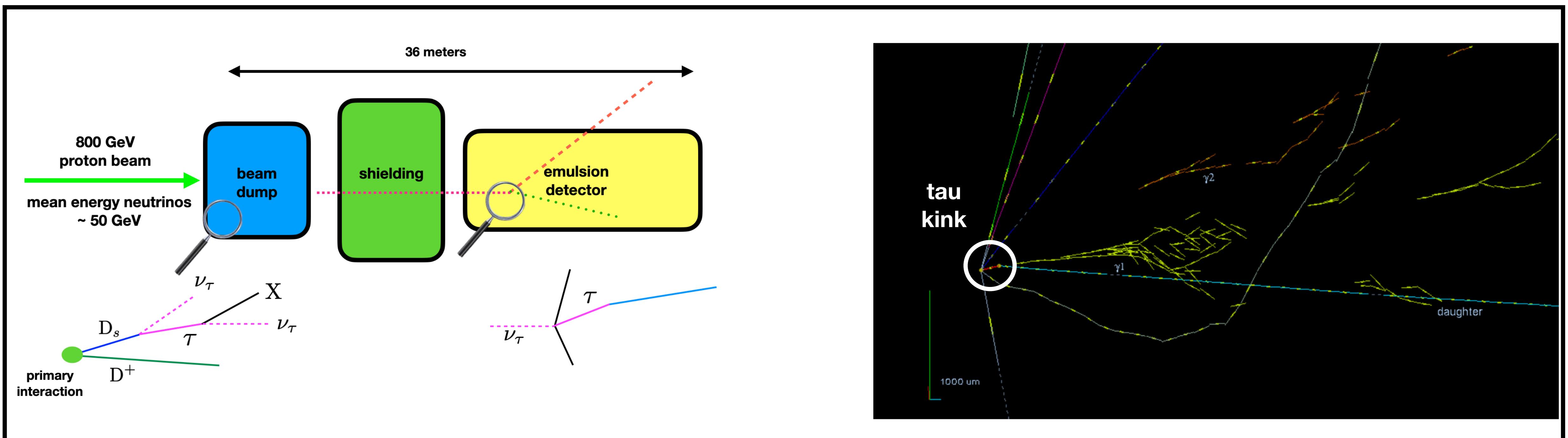


Discoveries of the Neutrino

- Tau lepton was discovered in 1975 by Perls and colleague

$$e^+ + e^- \rightarrow \tau^+ + \tau^- \rightarrow e^\pm + \mu^\pm + 4\nu$$

- Was there a neutrino associated to this new heavy lepton?
- Confirmed in 1997 by the DONUT experiment: they observed 4 tau neutrinos



- OPERA experiment observed about 10 tau neutrinos. These are the least well measured Standard Model particles!!

Neutrinos in the SM

- The Standard Model (SM) gauge group based on the following symmetry:

$$SU(3)_C \times \textcolor{red}{SU(2)_L} \times \textcolor{blue}{U(1)_Y} \rightarrow SU(3)_C \times U(1)_{EM}$$

- We have 3 generations of fermions: generations with identical gauge quantum number but different masses

$$Q_{EM} = T_{L_3} + Y$$

$(1, 2, -\frac{1}{2})$	$(3, 2, \frac{1}{6})$	$(1, 1, -1)$	$(3, 1, \frac{2}{3})$	$(3, 1, -\frac{1}{3})$
L_L	Q_L^i	E_R	U_R^i	D_R^i
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u_R^i	d_R^i
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c_R^i	s_R^i
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t_R^i	b_R^i

T_{L3} weak component of isospin

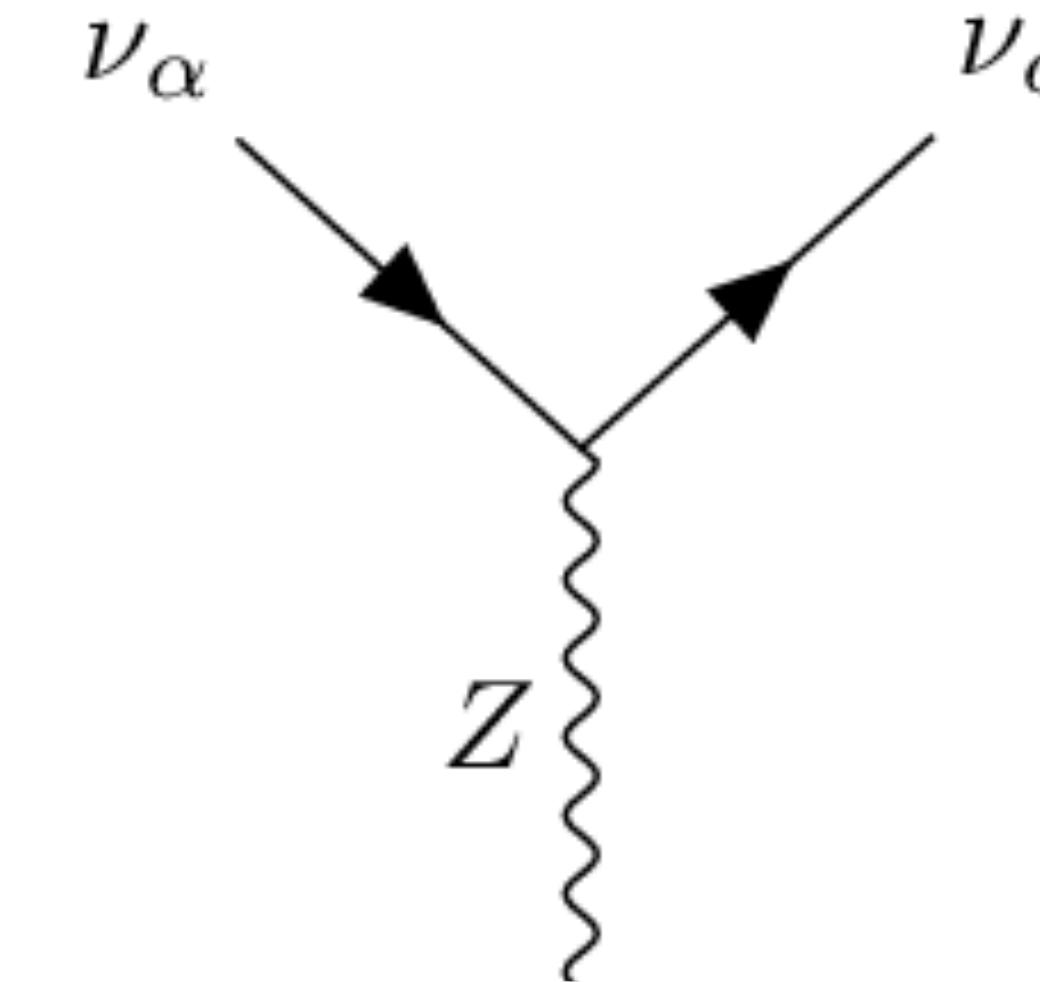
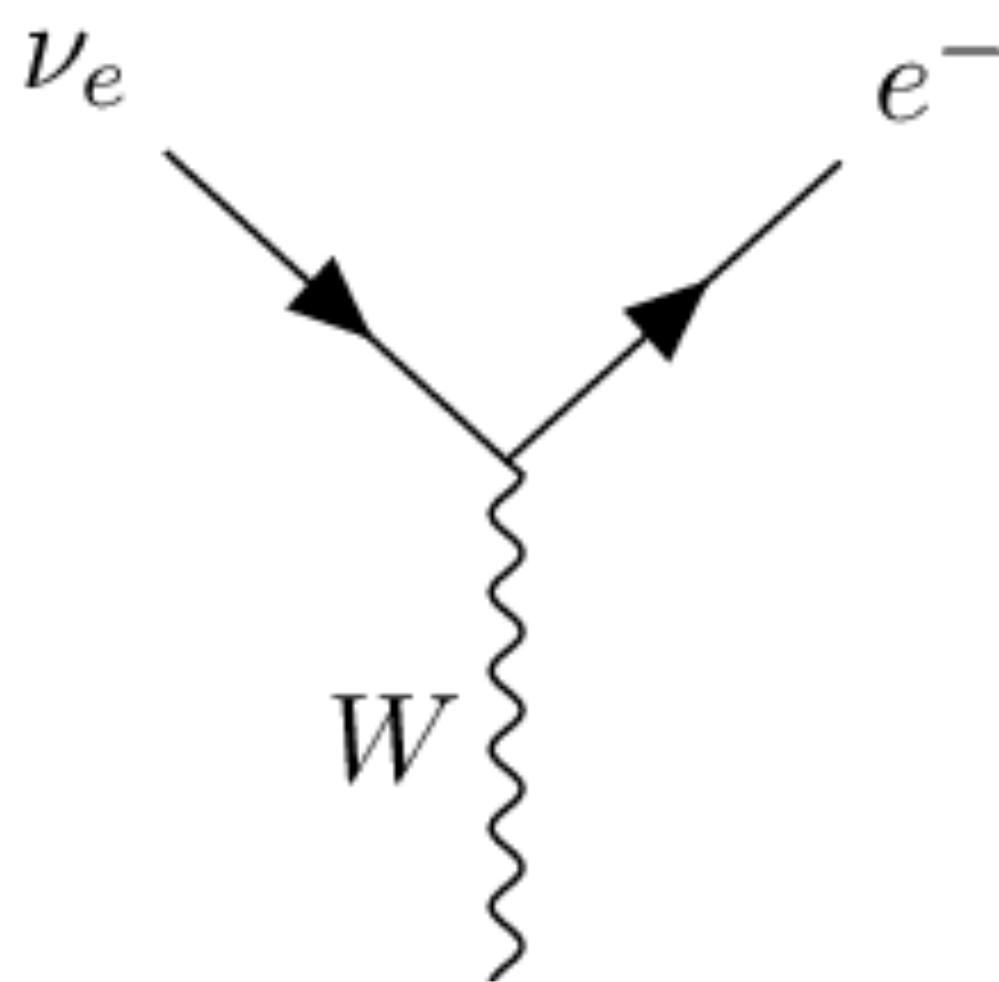
Neutrinos $T_{L3} = 1/2$

Neutrinos have no strong or EM interactions

No right handed neutrinos (ν_R) in the SM

SM content: ν_L & $\bar{\nu}_R$

Electroweak Theory for neutrinos



GSW Theory of Leptons

Electroweak Theory for neutrinos

Leptonic $SU(2)_L$ **doublet** interact via their **left-handed** components:

$$\psi_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \Psi_L = P_L \Psi \quad P_L = \frac{1}{2} (1 - \gamma^5)$$

Right-handed components are $SU(2)_L$ **singlets**: e_R, ν_R

Build weak interaction Lagrangian for single flavour. **Kinetic Term for free leptons**:

$$\mathcal{L} = i\bar{\psi}_L \not{\partial} \psi_L + i\bar{e}_R \not{\partial} e_R + i\bar{\nu}_R \not{\partial} \nu_R$$

Add gauge interactions, promote derivative to **covariant derivative**: $\partial^\mu \rightarrow D^\mu$

$$D^\mu = \partial^\mu + i\frac{g}{2} \vec{\tau} \cdot \vec{W}^\mu + i\frac{g'}{2} Y B^\mu$$

Electroweak Theory for neutrinos

$$\mathcal{L} = i\bar{\psi}_L \not{D} \psi_L + i\bar{e}_R \not{D} e_R + i\bar{\nu}_R \not{D} \nu_R \quad (1)$$

Lagrangian is invariant under $SU(2)_L \times U(1)_Y$

$$D^\mu \psi_L = \left(\partial^\mu + \frac{ig}{2} \vec{\tau} \cdot \vec{W}^\mu - \frac{ig'}{2} B^\mu \right) \psi_L \quad (2)$$

$$D^\mu e_R = (\partial^\mu - ig' B^\mu) e_R$$

$$D^\mu \nu_R = \partial^\mu \nu_R$$

Pauli matrices are generators of $SU(2)_L$

$$\vec{\tau} \cdot \vec{W}^\mu = \sum_{i=1}^3 \tau^i W^{i\mu} = \begin{pmatrix} W^{3\mu} & W^{1\mu} - iW^{2\mu} \\ W^{1\mu} + iW^{2\mu} & -W^{3\mu} \end{pmatrix} \quad (3)$$

Electroweak Theory for neutrinos

Exercise: Substitute (2) into (3) and then into (1) to show **interaction terms** are:

$$\mathcal{L}_{EW} = -\frac{g}{2} (\bar{\nu}_e \bar{e}_L) \gamma_\mu \begin{pmatrix} W^{3\mu} & W^{1\mu} - iW^{2\mu} \\ W^{1\mu} + iW^{2\mu} & -W^{3\mu} \end{pmatrix} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \frac{g'}{2} (\bar{\nu}_e \bar{e}_L) \gamma_\mu B^\mu \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \frac{g'}{2} \bar{e}_R \gamma_\mu B^\mu e_R$$

Electroweak Theory for neutrinos

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$$W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}$$

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$$\begin{aligned}
 \mathcal{L}_{EW} &= -\frac{g}{2} (\bar{\nu}_e \bar{e}_L) \gamma_\mu \begin{pmatrix} W^{3\mu} & W^{1\mu} - iW^{2\mu} \\ W^{1\mu} + iW^{2\mu} & -W^{3\mu} \end{pmatrix} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \frac{g'}{2} (\bar{\nu}_e \bar{e}_L) \gamma_\mu B^\mu \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \frac{g'}{2} \bar{e}_R \gamma_\mu B^\mu e_R \\
 &= -\frac{g}{2} (\bar{\nu}_e \bar{e}_L) \gamma_\mu \begin{pmatrix} W^{3\mu} & W^{+\mu} \\ W^{-\mu} & -W^{3\mu} \end{pmatrix} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \frac{g'}{2} (\bar{\nu}_e \bar{e}_L) \gamma_\mu B^\mu \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \frac{g'}{2} \bar{e}_R \gamma_\mu B^\mu e_R \\
 &= -\frac{g}{\sqrt{2}} (\bar{\nu}_e \gamma_\mu W^{\mu+} e_L + \bar{e}_L \gamma_\mu W^{\mu-} \nu_L) - \frac{1}{2} \bar{\nu}_e \gamma_\mu (gW^{3\mu} - g'B^\mu) \nu_L + \frac{1}{2} \bar{e}_L \gamma_\mu (gW^{3\mu} + g'B^\mu) e_L + \frac{g'}{2} \bar{e}_R \gamma_\mu B^\mu e_R \\
 &\quad \sqrt{g^2 + g'^2} Z^\mu \quad \frac{(g^2 - g'^2) Z^\mu + 2gg' A^\mu}{\sqrt{g^2 + g'^2}}
 \end{aligned}$$

Basis Transformation

$$\boxed{Z_\mu = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}} \quad A_\mu = \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2 + g'^2}} \quad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \frac{1}{\sqrt{g^2 + g'^2}} \begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B_\mu \end{pmatrix}}$$

Electroweak Theory for neutrinos

Basis Transformation

$$Z_\mu = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}} \quad A_\mu = \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2 + g'^2}} \quad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \frac{1}{\sqrt{g^2 + g'^2}} \begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B_\mu \end{pmatrix}$$

A_μ remains massless and is the electromagnetic field of the photon

Z_μ is the neutral weak gauge boson which has a mass

$$\begin{aligned} \mathcal{L}_{EW} = & -\frac{g}{2\sqrt{2}} (\bar{\nu}_e \gamma^\mu (1 - \gamma_5) e W_\mu^+ + \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e W_\mu^-) - \frac{\sqrt{g^2 + g'^2}}{2} \bar{\nu}_{eL} \gamma^\mu \nu_{eL} Z_\mu \\ & + \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e} \gamma^\mu e A_\mu + \frac{Z_\mu}{\sqrt{g^2 + g'^2}} \left[-g'^2 \bar{e}_R \gamma^\mu e_R + \frac{g^2 - g'^2}{2} \bar{e}_L \gamma^\mu e_L \right] \end{aligned}$$

Charged Current Interactions

$$\begin{aligned}\mathcal{L}_{EW} = & -\frac{g}{2\sqrt{2}} (\bar{\nu}_e \gamma^\mu (1 - \gamma_5) e W_\mu^+ + \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e W_\mu^-) - \frac{\sqrt{g^2 + g'^2}}{2} \bar{\nu}_{eL} \gamma^\mu \nu_{eL} Z_\mu \\ & + \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e} \gamma^\mu e A_\mu + \frac{Z_\mu}{\sqrt{g^2 + g'^2}} \left[-g'^2 \bar{e}_R \gamma^\mu e_R + \frac{g^2 - g'^2}{2} \bar{e}_L \gamma^\mu e_L \right]\end{aligned}$$

$$\mathcal{L}_{EW}^{\text{CC}} = -\frac{g}{2\sqrt{2}} \bar{\nu}_e \gamma^\mu (1 - \gamma_5) e W_\mu^+ + \text{h.c.}$$

$$\left(\frac{g}{2\sqrt{2}} \right)^2 \frac{1}{M_W^2} = \frac{G_F}{\sqrt{2}} \quad G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$$

Neutral Current Interactions

$$\mathcal{L}_{EW} = -\frac{g}{2\sqrt{2}} (\bar{\nu}_e \gamma^\mu (1 - \gamma_5) e W_\mu^+ + \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e W_\mu^-) - \frac{\sqrt{g^2 + g'^2}}{2} \bar{\nu}_{eL} \gamma^\mu \nu_{eL} Z_\mu$$
$$+ \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e} \gamma^\mu e A_\mu + \frac{Z_\mu}{\sqrt{g^2 + g'^2}} \left[-g'^2 \bar{e}_R \gamma^\mu e_R + \frac{g^2 - g'^2}{2} \bar{e}_L \gamma^\mu e_L \right]$$

$$\mathcal{L}_{EW}^{NC} = -\frac{\sqrt{g^2 + g'^2}}{2} \bar{\nu}_{eL} \gamma^\mu \nu_{eL} Z_\mu$$

$$\mathcal{L}^{EM} = \underbrace{\frac{gg'}{\sqrt{g^2 + g'^2}}}_{e} \bar{e} \gamma^\mu e A_\mu$$

Z and photon are different linear combinations of W^3 and B weighted by different factors. We can conveniently parametrise this in terms of the “**weak mixing angle**” θ_W

$$\begin{aligned} Z_\mu &= \cos \theta_W W_{3\mu} - \sin \theta_W B_\mu \\ A_\mu &= \sin \theta_W W_{3\mu} + \cos \theta_W B_\mu \end{aligned}$$

$$\cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \quad \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \quad M_W = M_Z \cos \theta_W$$

Exercise: rewrite \mathcal{L}_{EW} in terms of weak mixing angle:

$$\mathcal{L}^{\text{em}} = e \bar{e} \gamma^\mu e A_\mu$$

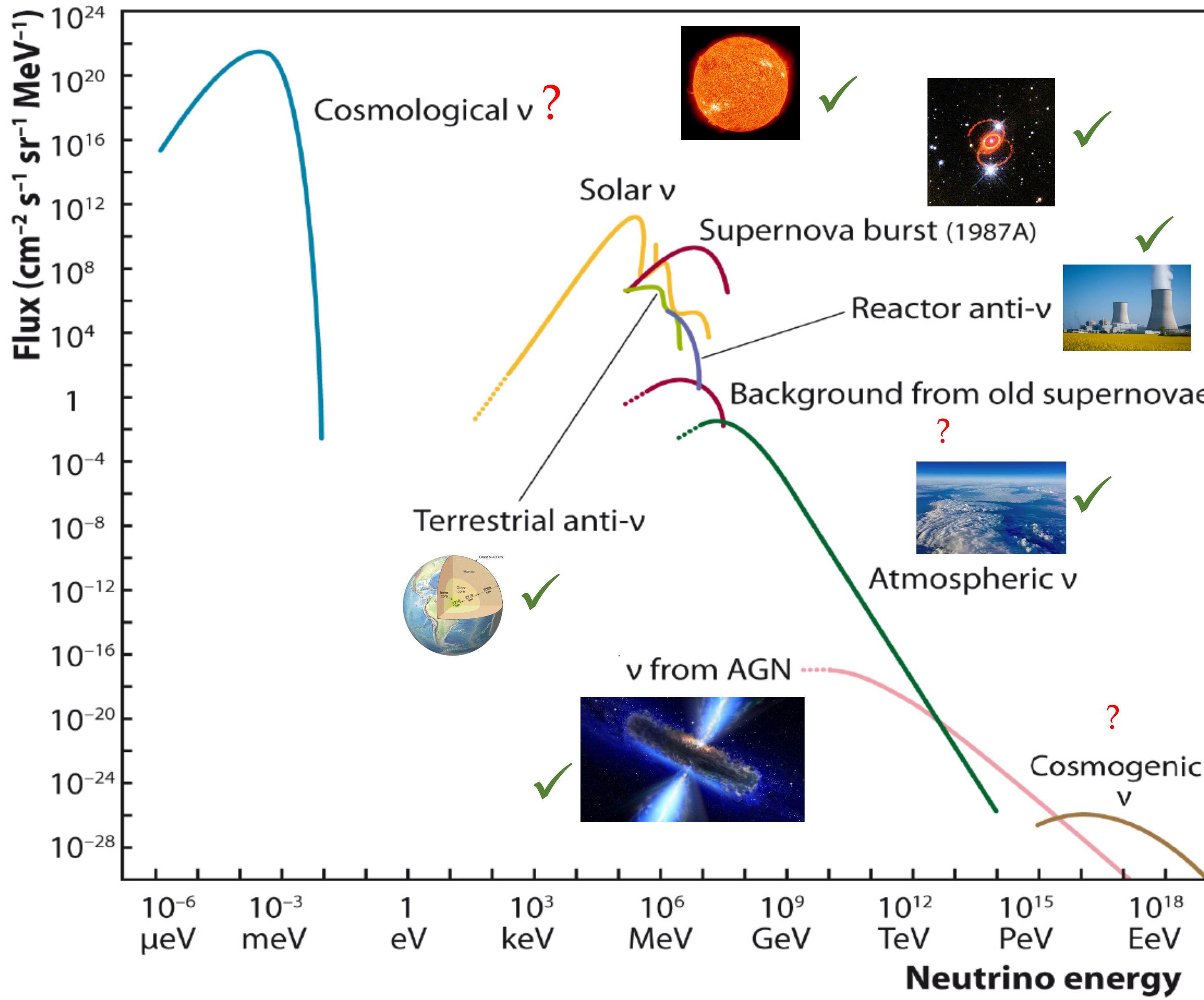
$$\mathcal{L}^{\text{CC}} = -\frac{g}{2\sqrt{2}} [\bar{\nu}_e \gamma^\mu (1 - \gamma_5) e W_\mu^+ + \text{h.c.}]$$

$$\mathcal{L}^{\text{NC}} = -\frac{g}{4 \cos \theta_W} [\bar{\nu}_e \gamma^\mu (1 - \gamma_5) \nu_e + \bar{e} \gamma^\mu (g_V^e - g_A^e \gamma_5) e] Z_\mu$$

$$g_V^e = 4 \sin^2 \theta_W, \quad g_A^e = -1, \quad \frac{g}{2\sqrt{2}} = \left(\frac{G_F M_W^2}{\sqrt{2}} \right)^{\frac{1}{2}}, \quad \frac{g}{4 \cos \theta_W} = \frac{1}{\sqrt{2}} \left(\frac{G_F M_Z^2}{\sqrt{2}} \right)^{\frac{1}{2}}$$

Board

Neutrino Sources



Neutrino Sources

- By 1990s we knew there were three neutrinos: extremely light, electrically neutral and very weakly interacting → least well understood particle of the SM

BB: $E_\nu \sim 10^{-4}$ eV $\rho_\nu \sim 330/\text{cm}^3$

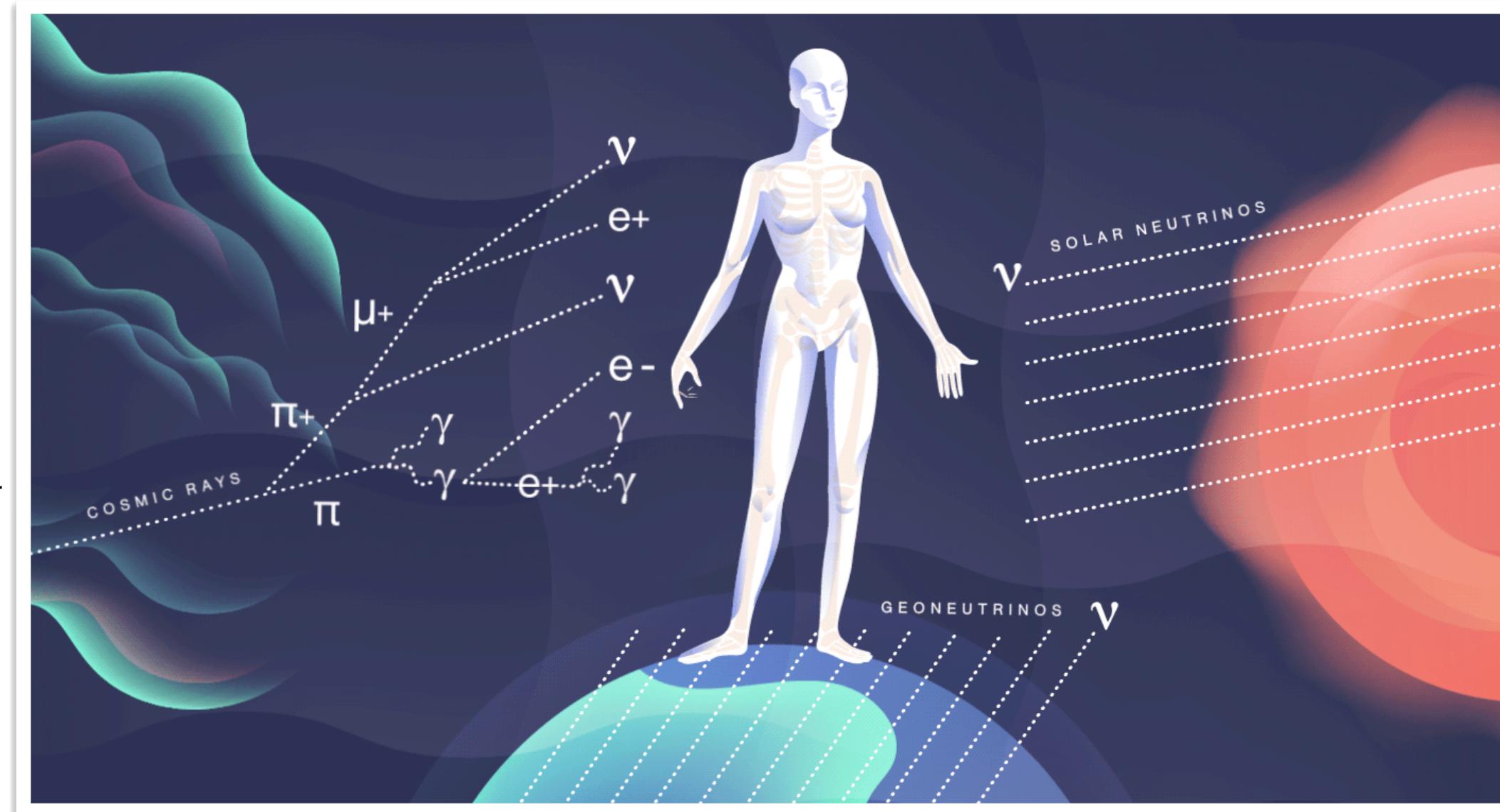


atmospheric neutrino:
 $E_\nu \sim \text{MeV} - \text{PeV}$
 $\Phi_\nu \sim 1 \nu/\text{cm}^2\text{s}$



$E_\nu \sim \text{MeV}$

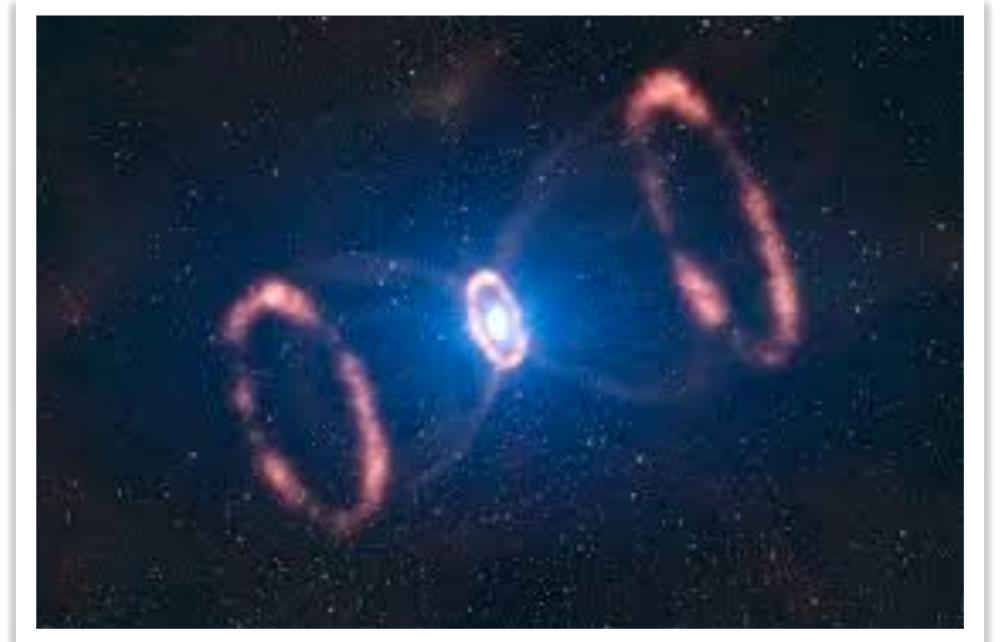
Human:
 $\Phi_\nu \sim 360 \times 10^6 \nu/\text{day}$



Earth: $E_\nu \sim \text{MeV}$
 $\Phi_\nu \sim 6 \times 10^6 \nu/\text{cm}^2\text{s}$

$E_\nu \sim \text{GeV}$

SN1986: $E_\nu \sim \text{MeV}$



Consider atmospheric muon neutrinos with mean energy of a GeV. The interaction cross-section with a proton is

$$\sigma_{\nu p} \sim 10^{-38} \text{ cm}^2 \frac{E_\nu}{\text{GeV}}$$

How many atmospheric muon neutrinos would interact with you in your lifetime?

$$\Phi = \frac{1 \nu}{\text{cm}^2 \text{sec}} \quad \langle E_\nu \rangle \sim \text{GeV}$$

$$N_{\text{events}} = \sigma \times \Phi \times N_{\text{target}} \times \text{Time}$$

$$1 \text{ kg water} \implies 3.3 \times 10^{25} \text{ H}_2\text{O} \implies 1 \text{ kg water} \implies 2.6 \times 10^{26} \text{ protons}$$

Say you're 60 kg and live to a ripe old age

$$N_{\text{target}} \sim 1.5 \times 10^{28} \text{ protons} \quad \text{Time} = 80 \text{ years} = 2 \times 10^9 \text{ secs}$$

$$N_{\text{events}} \sim 1.5 \times 10^{28} \times 2 \times 10^9 \times 10^{-38} \sim 1!$$

It's likely you will have 1 CC interaction from atmospheric neutrinos your whole life! Not sure why cinema paints neutrinos in such a menacing light (see Alien Covenant and The Core) **Physics point: need huge detectors with long exposures!**