Fractionally charged particles of the Standard Model

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Introduction

We know that the Standard Model is based on the gauge group:

$$G = SU(3)_c \times SU(2)_L \times U(1)_Y \,, \tag{1}$$

which describes all fundamental forces in nature apart from gravity. Its elementary matter fields can be grouped in the table below.

	q_L	u_R	d_R	ℓ_L	e_R	H
$U(1)_Y$	$1/6$	$2/3$	$-1/3$	$-1/2$	-1	$1/2$
$SU(2)_L$	2	1	1	2	1	2
$SU(3)_c$	3	3	3	1	1	1

Table: Table with SM representations and our convention for hypercharge. When conveying the same information not in table format we will have that e.g. q_L is in $(3, 2)_{1/6}$.

The Standard Model Group

The SM group can be moded by a discrete symmetry Z_p. [2404.03438]
Taking the quotient by each of the possible subgroups yields

$$G_p \equiv SU(3)_c \times SU(2)_L \times U(1)_Y / Z_p \,, \tag{2}$$

$$Z_p = \begin{cases} Z_1 & \xi_{(6)} \\ Z_2 & \left\{1\,,\, \xi_{(3)}\right\} \\ Z_3 & \left\{1\,,\, \xi_{(2)}\,,\, \xi_{(2)}^2\right\} \\ Z_6 & \left\{1\,,\, \xi\,,\, \xi^2\,,\, \xi^3\,,\, \xi^4\,,\, \xi^5\right\} \end{cases} \tag{3}$$

where the generating elements are defined as

$$\xi = e^{2\pi(1+6k)Q_Y i}\, e^{2\pi i n_c/3}\, e^{i\pi n_L} \tag{4}$$

$$\xi_{(2)} = e^{4\pi(1+3k)Q_Y i}\, e^{4\pi i n_c/3} \tag{5}$$

$$\xi_{(3)} = e^{6\pi(1+2k)Q_Y i}\, e^{i\pi n_L} \tag{6}$$

$$\xi_{(6)} = e^{12\pi(1+k)Q_Y i} \tag{7}$$

and n_c, n_L are the n-alities under $SU(3)_c$, $SU(2)_L$.

Hypercharge quantisation

The invariance of all SM representations under the action of Z_p leads to quantisation conditions for hyperchage.

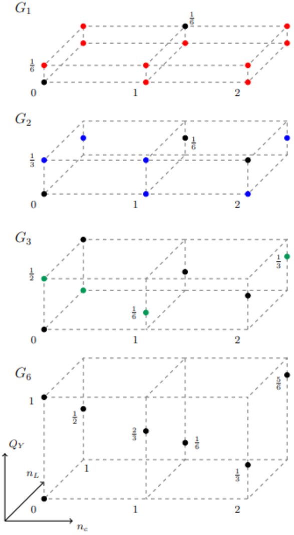
$$G_6: \quad \xi R = R, \quad \frac{n_c}{3} + \frac{n_L}{2} + Q_Y = \mathbb{Z}\,, \tag{8}$$

$$G_3: \quad \xi_{(2)} R = R, \quad \frac{2n_c}{3} + 2Q_Y = \mathbb{Z}\,, \tag{9}$$

$$G_2: \quad \xi_{(3)} R = R, \quad \frac{n_L}{2} + 3Q_Y = \mathbb{Z}\,, \tag{10}$$

$$G_1: \quad \xi_{(6)} R = R, \quad 6Q_Y = \mathbb{Z}\,. \tag{11}$$

Constituent blocks of allowed hypercharge



Electromagnetic charge spectrum

$$Q_{em} = T_{3L} + Q_Y \tag{12}$$

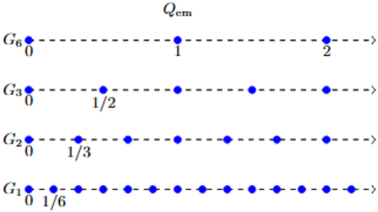
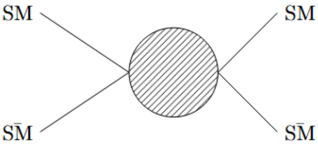


Figure: Electric charge spectrum for hadrons and leptons in the case $k = 0$.

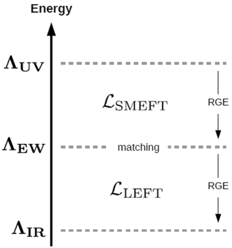
How do these fractional states couple to SM?

$$
\begin{aligned}
G_6 :& \quad \bar{e}_R \quad \bar{\ell}_L \quad d_R \\
& (1,1)_1\,,\ (1,2)_{1/2}\,,\ (3,1)_{-1/3} && (13)\\
G_3 :& \quad \Xi \quad \Lambda \quad \Omega && (14)\\
& (1,1)_{1/2}\,,\ (1,2)_0\,,\ (3,1)_{1/6}\,, && (15)\\
G_2 :& \quad \Sigma \quad \Delta \quad \Theta \\
& (1,1)_{1/3}\,,\ (1,2)_{1/6}\,,\ (3,1)_0\,, && (16)\\
G_1 :& \quad \Phi \quad \Lambda \quad \Theta \\
& (1,1)_{1/6}\,,\ (1,2)_0\,,\ (3,1)_0\,. && (17)
\end{aligned}
$$



Effective Field Theory approach

The idea with EFTs is to approximate such interactions at low energies to be point-like considering a heavy mediator.



$$\int \mathcal{D}\mathrm{Fr}\,\mathcal{D}W\,\mathcal{D}u\,\mathcal{D}d\ldots e^{iS} = \int \mathcal{D}W\,\mathcal{D}u\,\mathcal{D}d\ldots e^{iS_{\mathrm{eff},\,\mathrm{W}}} = \int \mathcal{D}u\,\mathcal{D}d\ldots e^{iS_{\mathrm{eff},\,\mathrm{MeV}}}$$

Matching from a UV theory to the EFT can be done by evaluating the path integral to integrate out heavy fields.

SMEFT

SMEFT is an effective field theory that describes SM interactions with higher dimension operators. We can build every possible operator at each order, in mass dimension, from the existing Standard Model fields:

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{d=6}} + \dots \tag{18}$$

$$\mathcal{L}_{d=6} \supset -\frac{2\delta}{v^2}\left(a_c J_c^b J_c^b + a_L J_L^I J_L^I + a_Y J_Y J_Y\right) \tag{19}$$

with b the colour index, I the isospin, the coefficients a_c, a_Y

$$\delta = \frac{a_s \mathrm{d}_L \mathrm{d}_c v^2}{(4\pi)^2 240 M^2}\,, \qquad a_c = \frac{\mathrm{I}_c g_c^4}{\mathrm{d}_c}\,, \tag{20}$$

$$a_L = \frac{\mathrm{I}_L g^4}{\mathrm{d}_L}\,, \qquad a_Y = Q_Y^2 (g_Y)^4\,. \tag{21}$$

and the $SU(2)_W, U(1)_Y$ Higgs currents

$$\begin{aligned} J_{L,\mu,a} &= i(H^\dagger T_{L,a} D_\mu H - (D_\mu H)^\dagger T_{L,a} H) \\ J_{Y,\mu} &= i Q_Y (H^\dagger D_\mu H - (D_\mu H)^\dagger H). \end{aligned} \tag{22}$$

BSM electroweak precision observables

$$\mathcal{L} \supset -\frac{g}{\sqrt{2}} W_+^\mu J_{L,\mu}^- (1 - \delta a_L) + h.c. \tag{23}$$

$$- \frac{g}{c_w} Z[(c_w^2 - a_L \delta) J_{L,3} - (s_w^2 - a_Y \delta) J_Y], \tag{24}$$

where $J_{L,-} = (J_{L,1} + i J_{L,2})$, and

$$M_W^2 = \frac{g^2 v^2}{4} (1 - a_L \delta) \ , \tag{25}$$

$$M_Z^2 = \frac{g^2 v^2}{4 c_w^2} (1 - a_L \delta - a_Y \delta) \tag{26}$$

$$G_F = \frac{1}{\sqrt{2} v^2} \ . \tag{27}$$

Conclusion

- Detection of fractionally charged particles would showcase the actual Standard Model Group.
- More fundamentally, they may help us understand charge quantisation.

Thank you!

Supplementary material

Next we substitute g, s_w, v for $M_W, \bar{s}_w, G_F$ in our theory predictions, the observables we choose to display the correlation with are

$$\rho_{\Gamma 3} \equiv \frac{1}{6}\frac{M_Z^3 \Gamma_W}{M_W^3 \Gamma_Z^{\rm inv}} = (1 + \delta a_Y)\,, \quad (28)$$

$$\rho_{\Gamma 5} \equiv \frac{1}{6}\frac{(1-\bar{s}_w^2) M_Z^5 \Gamma_W}{M_W^5 \Gamma_Z^{\rm inv}} = (1 - \bar{t}_w^2 \delta a_L)\,, \quad (29)$$