Analytical approximations for curved primordial power spectra

Based on work from:

[Thavanesan et al. (arXiv: 2009.05573)]

Presented by

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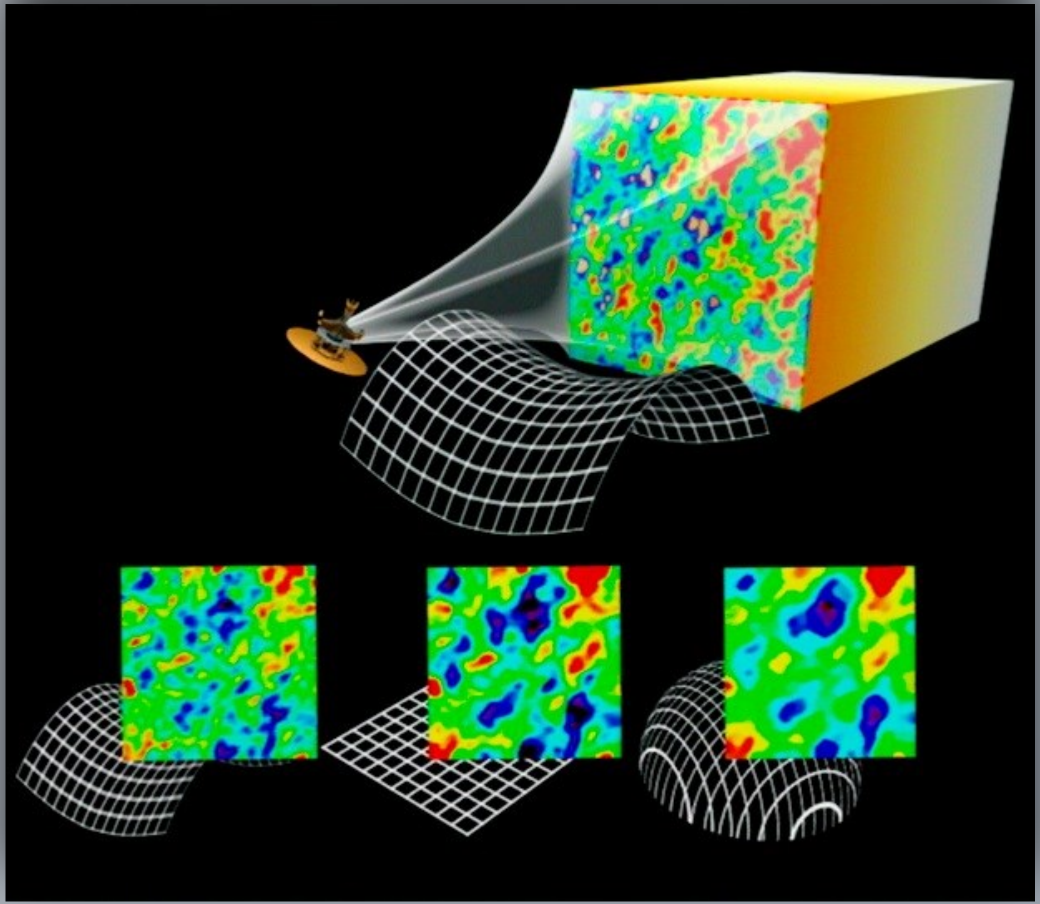

Figure from:

NASA/GSFC

Image sources:
https://www.kicc.cam.ac.uk/directory/wh260
https://plus.maths.org/content/women-mathematics-anne-christine-davis
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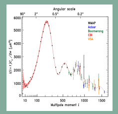
Curvature in Cosmology

Kinetic Dominance

Solving curved inflationary dynamics



Curved Primordial Power Spectra

ΛCDM model

$$H(a) = \frac{\dot{a}}{a} = H_0\sqrt{\left(\Omega_{c,0} + \Omega_{b,0}\right)a^{-3} + \Omega_{r,0}a^{-4} + \Omega_{k,0}a^{-2} + \Omega_\Lambda}$$

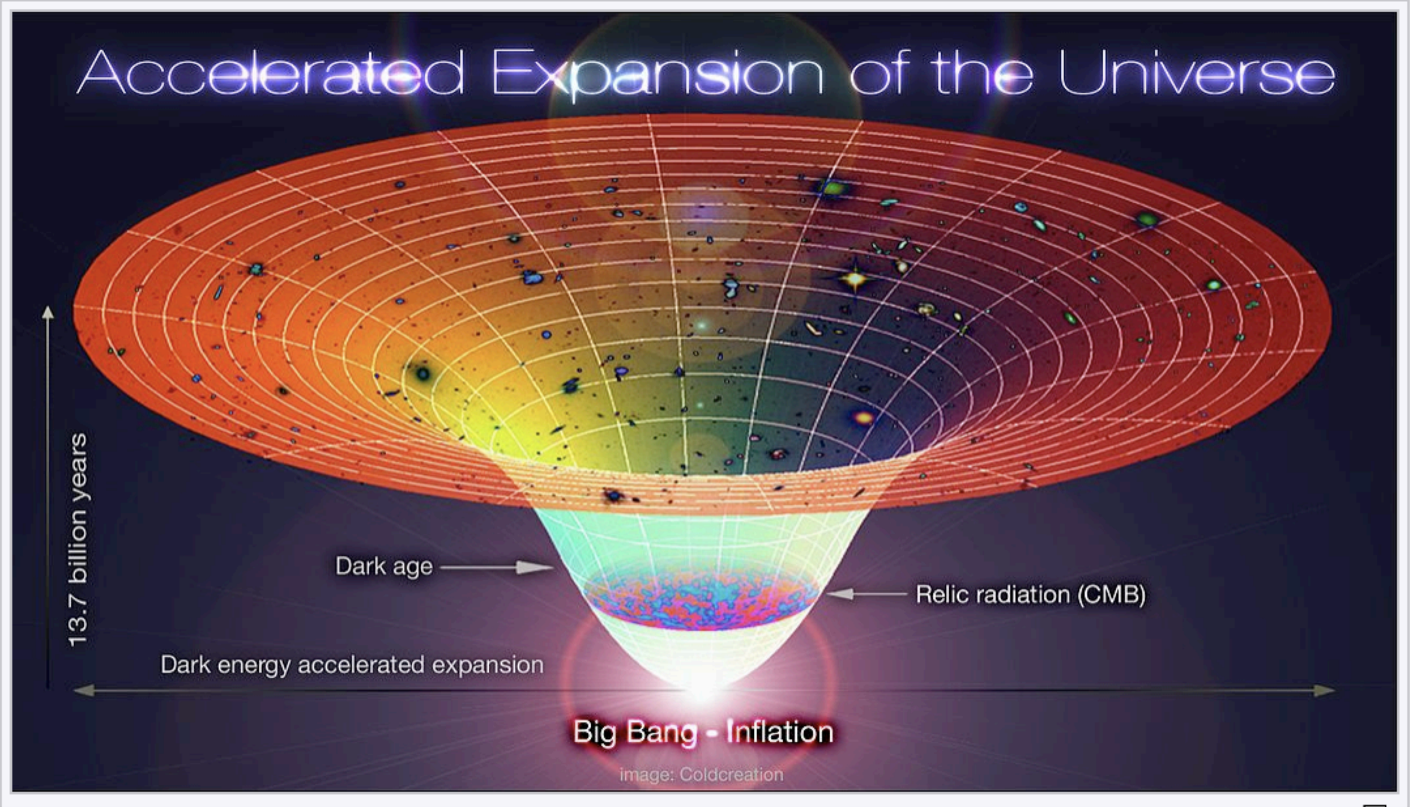


Figure from:

https://en.wikipedia.org/wiki/Lambda-CDM_model#/media/File:Lambda-Cold_Dark_Matter,_Accelerated_Expansion_of_the_Universe,_Big_Bang-Inflation.jpg

Standard model of Cosmology (ΛCDM) consists of:

- Ω_m – matter
- Ω_Λ – dark energy
- Ω_r – radiation
- and $\Omega_k = 0$
- *Flat* universe → *K=0*
- *Open* universe → *K=-1*
- *Closed* universe → *K=+1*

Why curved inflation?

Eternal inflation

Just-enough inflation

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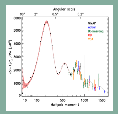

Curved Primordial Power Spectra

Slow-roll (SR)

$$w_\phi = \frac{P_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi} - V(\phi)}{\frac{1}{2}\dot{\phi} + V(\phi)}, \quad \Longrightarrow \quad \dot{\phi}^2 \ll V(\phi)$$

Kinetic-Dominance (KD)

$$\dot{\phi}^2 \gg V(\phi)$$

$$\Downarrow$$

$$w_\phi = \frac{P_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi} - V(\phi)}{\frac{1}{2}\dot{\phi} + V(\phi)}, \longrightarrow \dot{\phi}^2 \ll V(\phi)$$

Curvature tension in Cosmology

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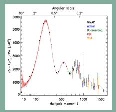
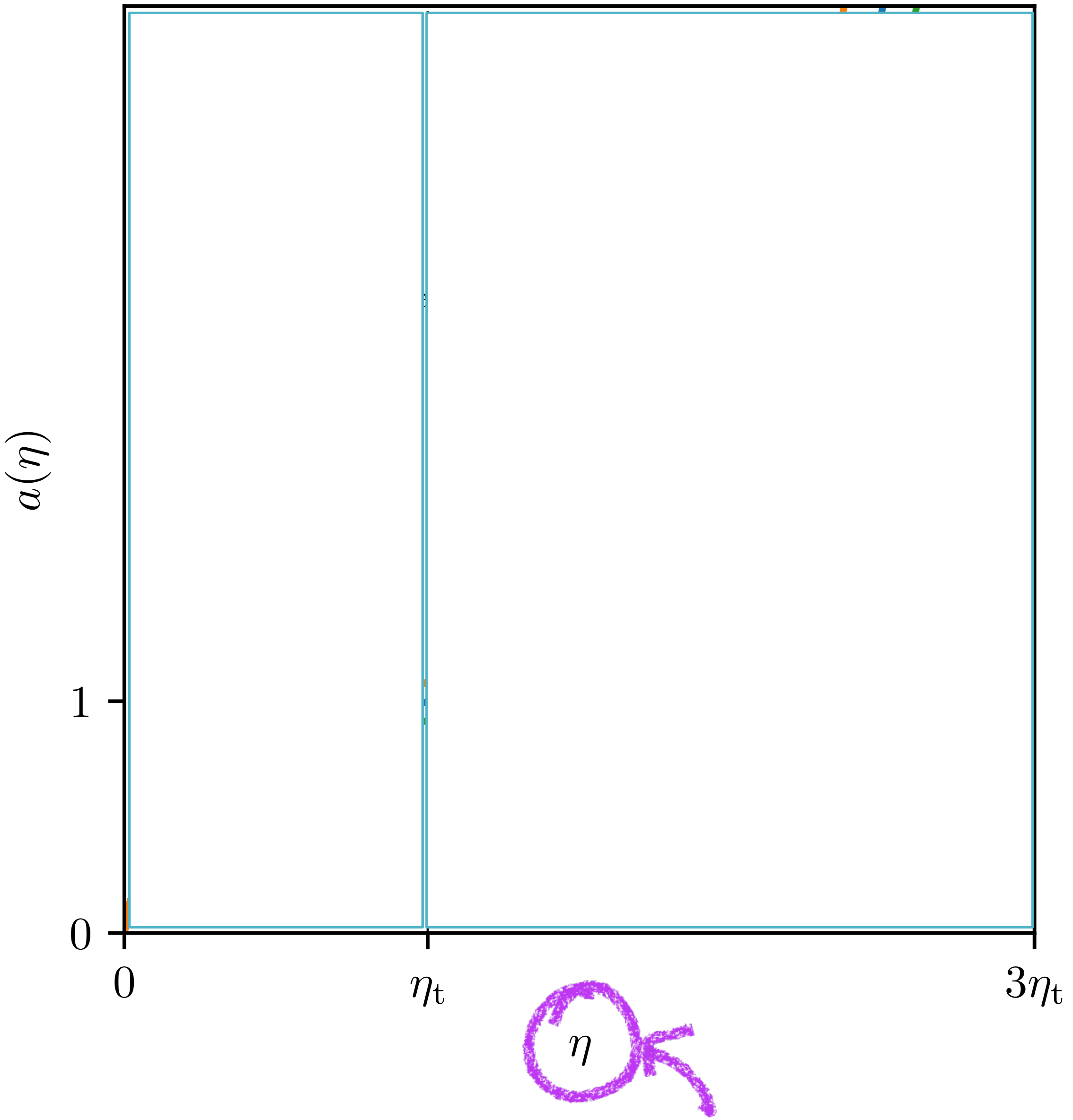


Curved Primordial Power Spectra

Inflationary model



Working with conformal time!

Taken from:

[Thavanesan et al. (arXiv: 2009.05573)]

Background equations

$$(\phi')^2 \gg a^2 V(\phi)$$

(Kinetic Dominance)

$$\mathcal{H}' + 2\mathcal{H}^2 + 2K = a^2 V(\phi), \quad \approx 0$$

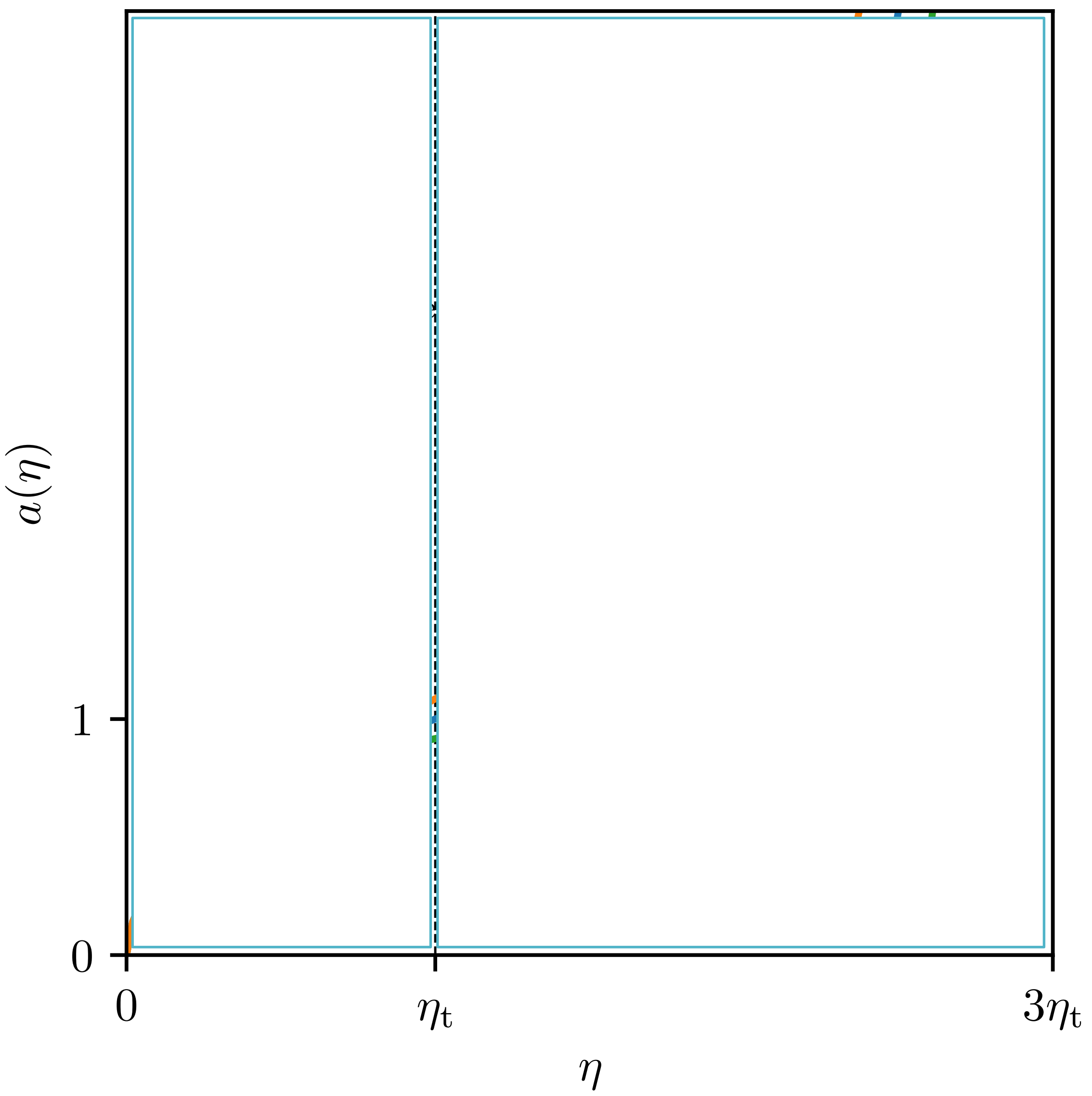
$$a = \sqrt{S_K(2\eta)} : 0 \leq \eta < \eta_t$$

$$S_K(x) = \begin{cases} \sin(x) & K = +1 \\ x & K = 0 \\ \sinh(x) & K = -1 \end{cases}$$

Taken from:

[Thavanesan et al. (arXiv: 2009.05573)]

Inflationary model



Taken from:

[Thavanesan et al. (arXiv: 2009.05573)](https://arxiv.org/abs/2009.05573)

Background equations

$$(\phi')^2 \ll a^2 V(\phi)$$

(Ultra-Slow-Roll)

$$\mathcal{H}' - \mathcal{H}^2 - K = \frac{1}{2}\phi'^2 . \quad \approx 0$$

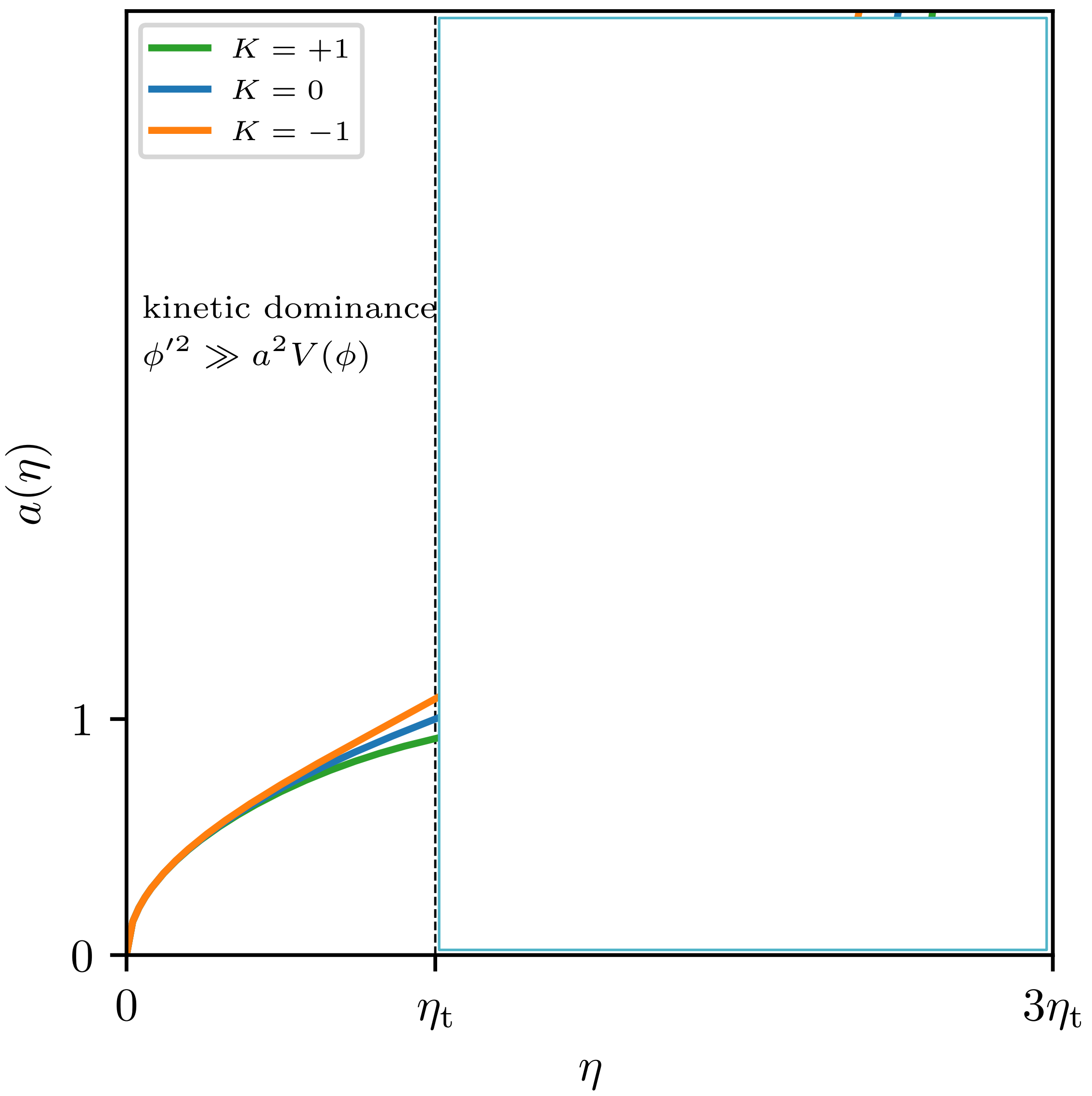
$$a = [S_K(2\eta_t)]^{3/2} / S_K(3\eta_t - \eta) : \eta_t \leq \eta < 3\eta_t$$

$$S_K(x) = \begin{cases} \sin(x) & K = +1 \\ x & K = 0 \\ \sinh(x) & K = -1 \end{cases}$$

Taken from:

Thavanesan et al. (arXiv: 2009.05573)

Inflationary model



Taken from:

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Curved Mukhanov-Sasaki (MS) equation

$$v = \mathcal{Z}\mathcal{R}, \quad \text{and} \quad \mathcal{Z} = \frac{a\phi'}{\mathcal{H}}\sqrt{\frac{\mathcal{D}^2}{\mathcal{D}^2 - K\mathcal{E}}}, \quad \mathcal{E} = \frac{\phi'^2}{2\mathcal{H}^2}.$$

$$v_k'' + \left[\mathcal{K}^2 - \left(\frac{\mathcal{Z}''}{\mathcal{Z}} + 2K + \frac{2K\mathcal{Z}'}{\mathcal{H}\mathcal{Z}}\right)\right] v_k = 0.$$

$$\mathcal{K}^2(k) = \begin{cases} k^2, & k \in \mathbb{R},\, k > 0: \;\; K = 0, -1, \\ k(k+2), & k \in \mathbb{Z},\, k > 2: \;\; K = +1. \end{cases}$$

Taken from:

Thavanesan et al. (arXiv: 2009.05573)

Logolinear series expansions

$$x(\eta) = \sum_{j,k} [x_j^k]\, \eta^j (\log \eta)^k,$$

where $[x_j^k]$ are twice-indexed real constants defining the series, with square brackets used to disambiguate powers from superscripts. We make definitions for upper and lower indexed functions of η via partial summation:

$$x_j(\eta) = \sum_k [x_j^k] (\log \eta)^k, \qquad [x^k](\eta) = \sum_j [x_j^k]\, \eta^j,$$

$$x(\eta) = \sum_j x_j(\eta)\, \eta^j = \sum_k [x^k](\eta)\, (\log \eta)^k.$$

Taken from:

Thavanesan et al. (arXiv: 2009.05573)

Logolinear series expansions

$$(\phi')^2 \gg a^2 V(\phi)$$

(Kinetic Dominance)

Up to the first curvature terms, we have:

$$\phi' = \pm\sqrt{\frac{3}{2}\frac{1}{\eta}} \pm \frac{\sqrt{6}K}{3}\eta + \mathcal{O}(\eta^3),$$

$$\mathcal{H} = N' = \frac{1}{2\eta} - \frac{2K}{3}\eta + \mathcal{O}(\eta^3),$$

$$a = e^N = e^{N_\mathrm{P}}\eta^{1/2} - \frac{e^{N_\mathrm{P}}K}{3}\eta^{5/2} + \mathcal{O}(\eta^{9/2}),$$

Taken from:

[Thavanesan et al. (arXiv: 2009.05573)](https://arxiv.org/abs/2009.05573)

Curved Mukhanov-Sasaki (MS) equation

$$(\phi')^2 \gg a^2 V(\phi)$$

(Kinetic Dominance)

$$\frac{\mathcal{Z}''}{\mathcal{Z}} + 2K + \frac{2K}{\mathcal{H}}\frac{\mathcal{Z}'}{\mathcal{Z}} = -\frac{1}{4\eta^2} + \frac{32K}{3} - \frac{24K^2}{\mathcal{K}^2(k)} + \mathcal{O}(\eta^2).$$

$$v_k'' + \left[\mathcal{K}^2 - \left(\frac{\mathcal{Z}''}{\mathcal{Z}} + 2K + \frac{2K\mathcal{Z}'}{\mathcal{H}\mathcal{Z}}\right)\right] v_k = 0.$$

$$v_k'' + \left[\kappa_-^2 + \frac{1}{4\eta^2}\right] v_k = 0,$$

$$\kappa_-^2(k) = \mathcal{K}^2(k) - \frac{32K}{3} + \frac{24K^2}{\mathcal{K}^2(k)}.$$

Taken from:

Thavanesan et al. (arXiv: 2009.05573)

Curved Mukhanov-Sasaki (MS) equation

$$(\phi')^2 \ll a^2 V(\phi)$$

(Ultra-Slow-Roll)

$$\frac{\mathcal{Z}''}{\mathcal{Z}} + 2K + \frac{2K\mathcal{Z}'}{\mathcal{H}\mathcal{Z}} \simeq \frac{a''}{a} + 4K$$
$$= \frac{2}{(\eta - 3\eta_{\mathrm{t}})^2} - \frac{11K}{3} + \mathcal{O}[(\eta - 3\eta_{\mathrm{t}})^2].$$

$$v_k'' + \left[\mathcal{K}^2 - \left(\frac{\mathcal{Z}''}{\mathcal{Z}} + 2K + \frac{2K\mathcal{Z}'}{\mathcal{H}\mathcal{Z}}\right)\right] v_k = 0.$$

$$v_k'' + \left[\kappa_+^2 - \frac{2}{(\eta - 3\eta_{\mathrm{t}})^2}\right] v_k = 0,$$

$$\kappa_+^2 = \mathcal{K}^2(k) - \frac{11K}{3}.$$

Taken from:

Thavanesan et al. (arXiv: 2009.05573)

Curvature tension in Cosmology

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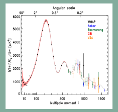
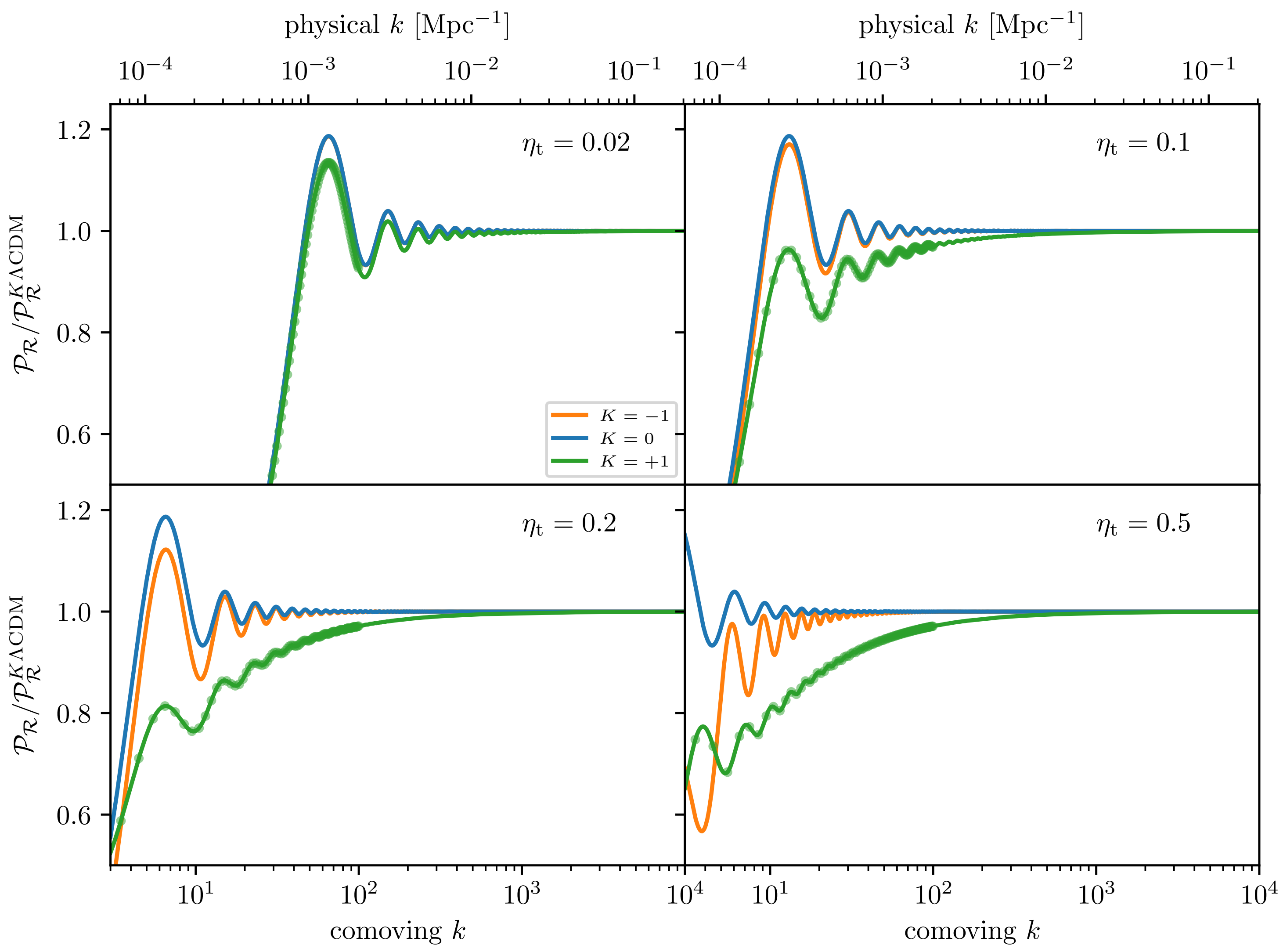


Curved Primordial Power Spectra

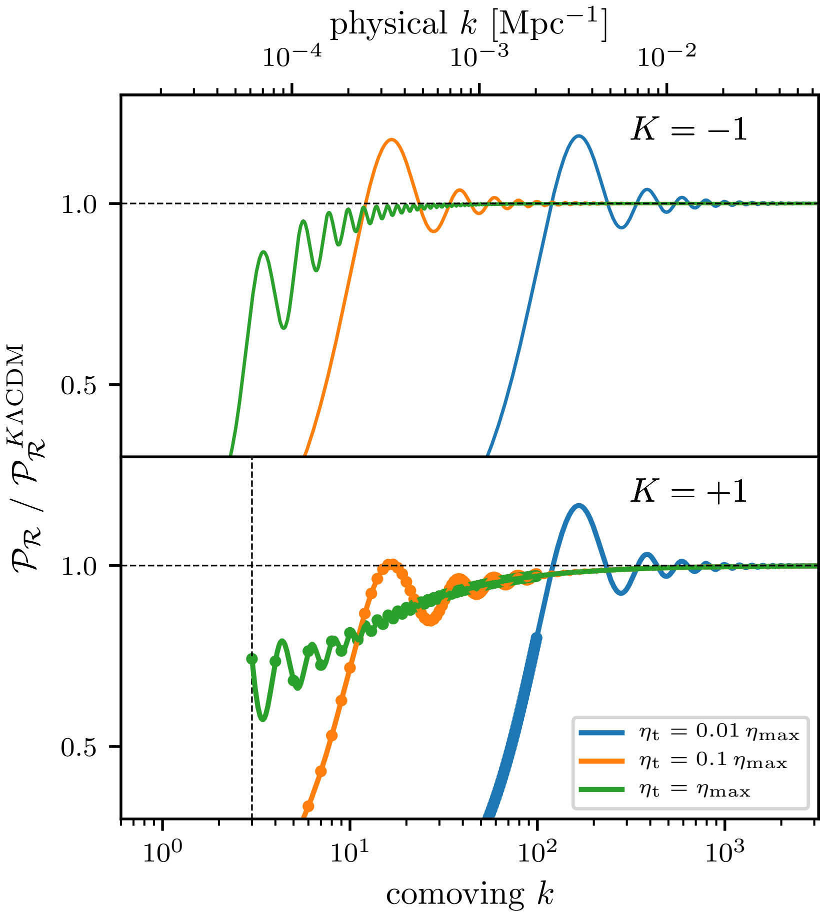


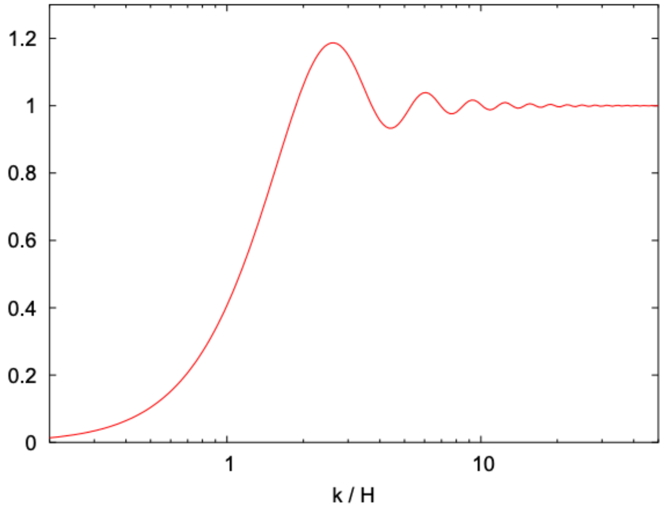
Primordial Power Spectra

Taken from:

Ayngaran Thavanesan. Analytical approximations for curved primordial power spectra (supplementary mate- rial), September 2020. URL https://doi.org/10.5281/zenodo.4024321.

Primordial Power Spectra





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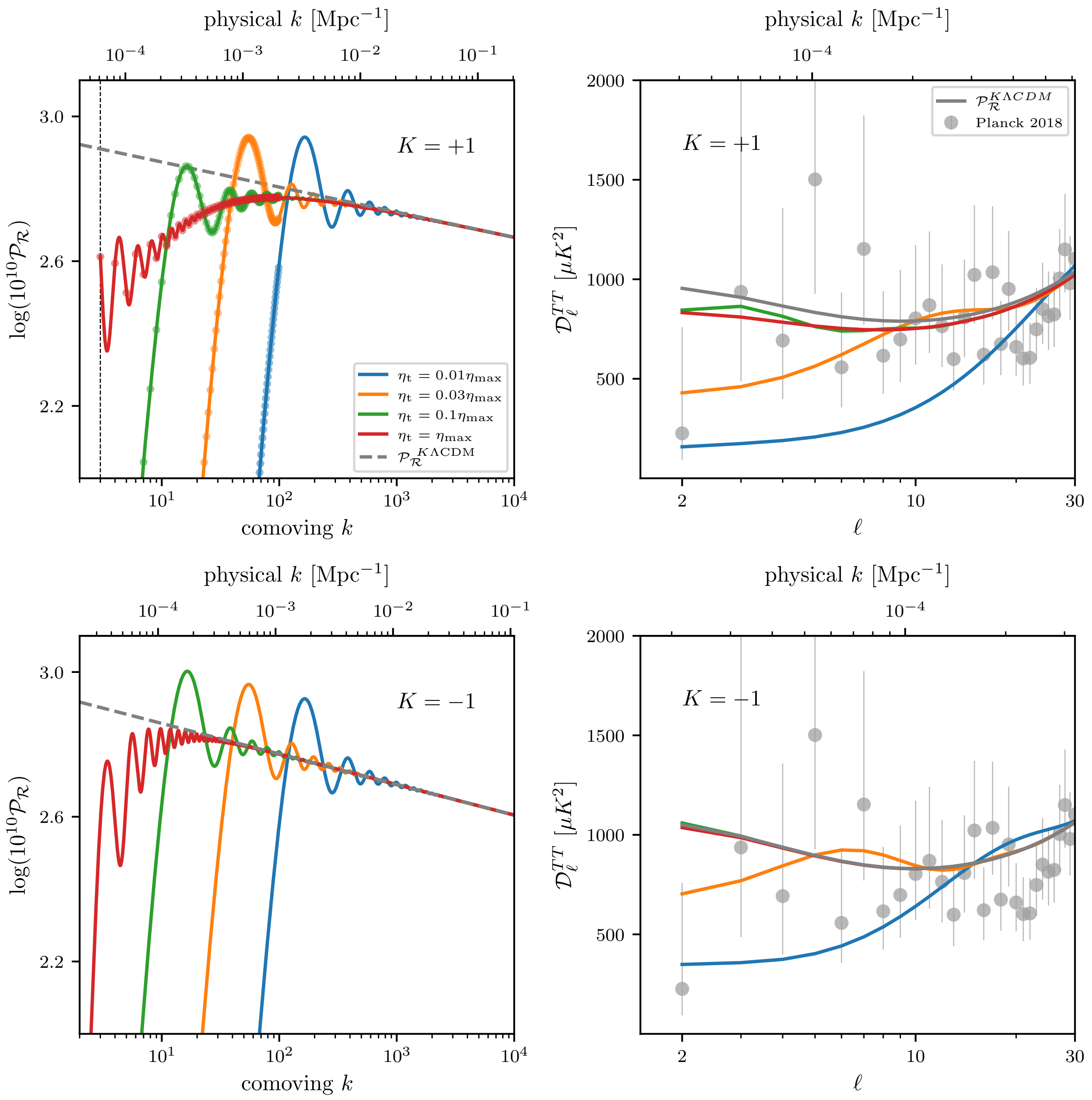
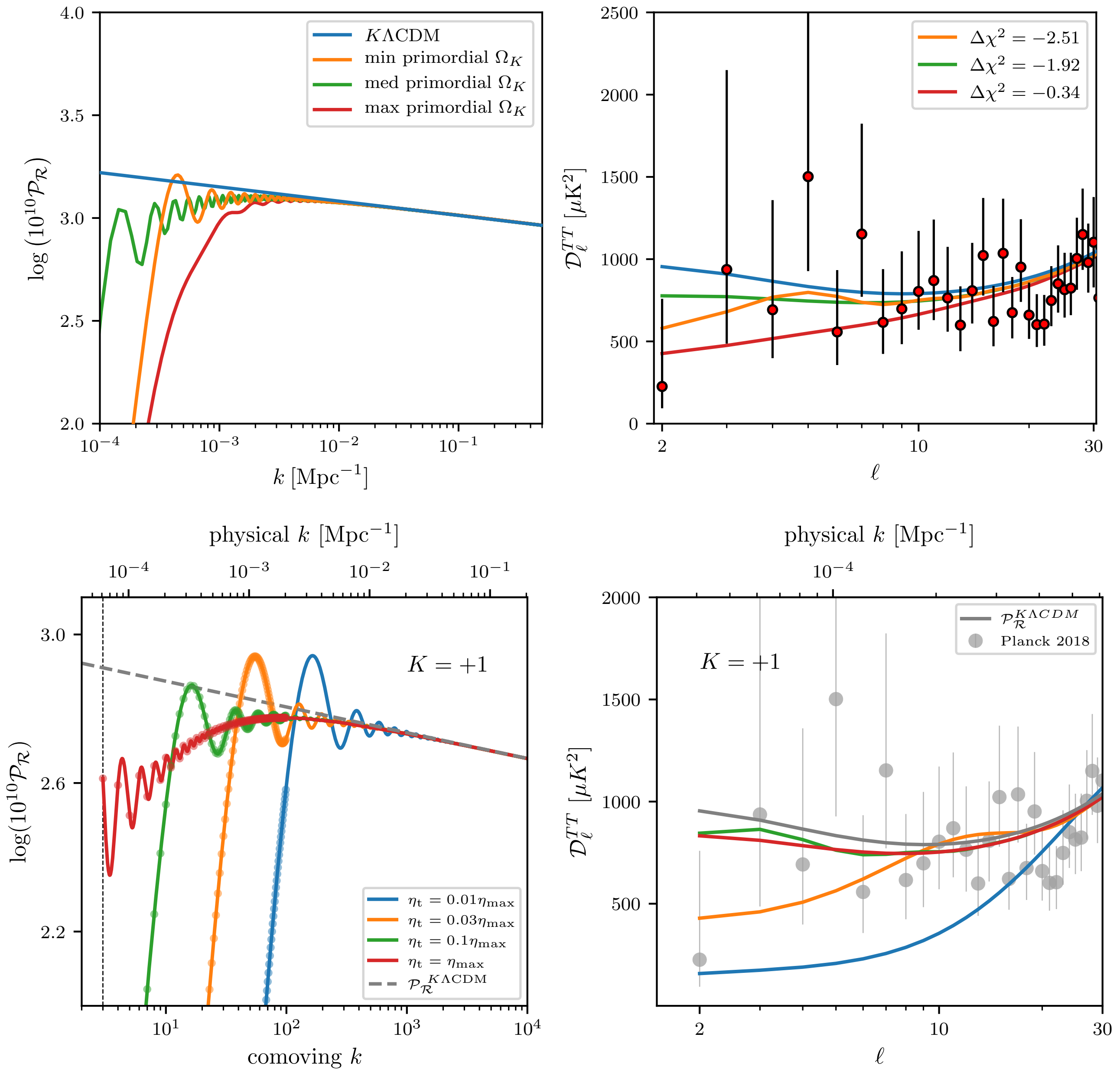
CMB Spectra



Taken from:

[Thavanesan et al. (arXiv: 2009.05573)]

CMB Spectra



Taken from:

[Thavanesan et al. (arXiv: 2009.05573)] and [Handley (arXiv:1907.08524)].

Conclusions + Future

We have now:

- Derived an analytical method to compute curved primordial power spectra.
- Gained better insight into the physics of curved inflating universes.
- A framework, which can be extended to include potential dependance as well as varying initial conditions.

Current and future work includes:

- Combining our work on curvature and upcoming work on initial conditions (Gessey-Jones *et al. in prep*).
- Extending our “general” Contaldi approximation to three-point function (bispectrum) calculations.

References

https://arxiv.org/pdf/1401.2253.pdf

https://arxiv.org/pdf/1901.07540.pdf

https://arxiv.org/pdf/astro-ph/0303636.pdf

https://arxiv.org/pdf/0907.5424.pdf

https://arxiv.org/pdf/1907.08524.pdf