Bread & Butter Physics at Muon Colliders

Tao Han

IPPP Topical Meeting on Physics with High-Brightness Stored Muon Beams

- A Higgs factory
- EW physics at high energies
- Precision Higgs physics

Collider benchmark points:

- The Higgs factory:

$E_{cm} = m_H$

$L \sim 1\ fb^{-1}/yr$

$\Delta E_{cm} \sim 5$ MeV

Parameter	Units	Higgs
CoM Energy	TeV	0.126
Avg. Luminosity	$10^{34} cm^{-2}s^{-1}$	0.008
Beam Energy Spread	%	0.004
Higgs Production/10^7 sec		13'500
Circumference	km	0.3

- Multi-TeV colliders:

Lumi-scaling scheme: $\sigma L \sim$ const.

$$L \gtrsim \frac{5\text{ years}}{\text{time}} \left(\frac{\sqrt{s_\mu}}{10\text{ TeV}} \right)^2 2 \cdot 10^{35} \text{cm}^{-2}\text{s}^{-1}$$

1 ab^{-1} /yr

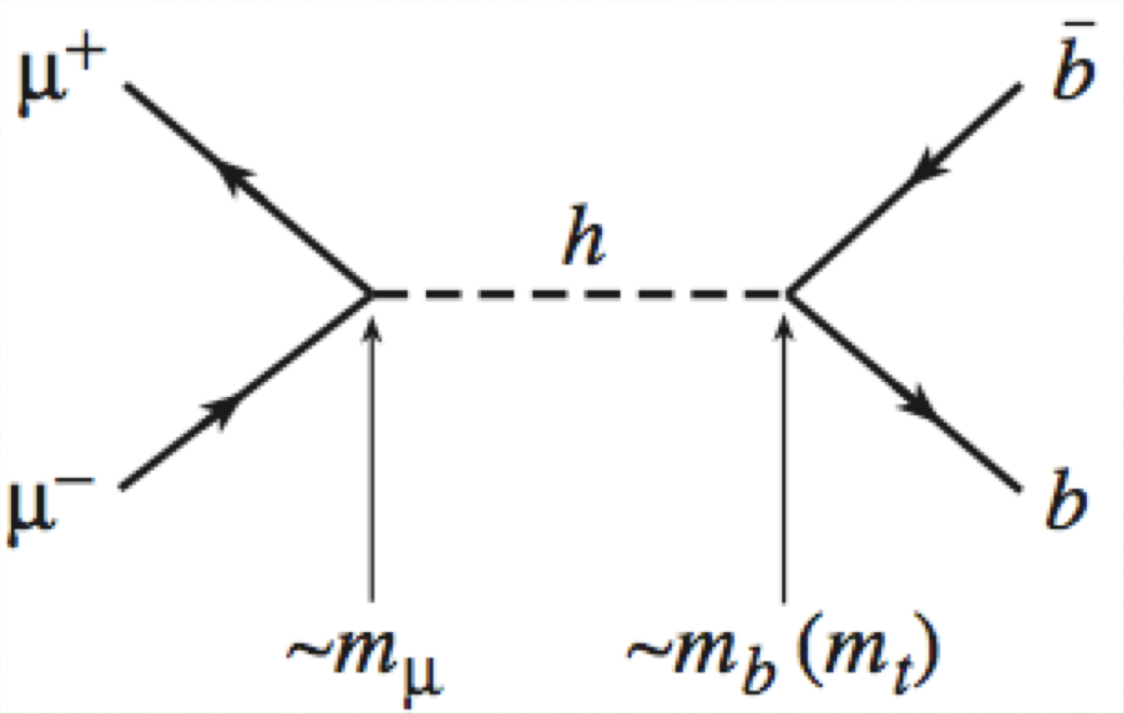
The aggressive choices:

$$\sqrt{s} = 3,\ 6,\ 10,\ 14,\ 30 \text{ and } 100 \text{ TeV}, \quad \mathcal{L} = 1,\ 4,\ 10,\ 20,\ 90, \text{ and } 1000 \text{ ab}^{-1}$$

European Strategy, arXiv:1910.11775; arXiv:1901.06150; arXiv:2007.15684.

1. A Higgs Factory

Resonant Production:



$$\sigma(\mu^+\mu^- \to h \to X) = \frac{4\pi\Gamma_h^2 \mathrm{Br}(h \to \mu^+\mu^-)\mathrm{Br}(h \to X)}{(\hat{s} - m_h^2)^2 + \Gamma_h^2 m_h^2}.$$

$$\begin{aligned}\sigma_{peak}(\mu^+\mu^- \to h) &= \frac{4\pi}{m_h^2} BR(h \to \mu^+\mu^-) \\ &\approx 41 \text{ pb at } m_h = 125 \text{ GeV}.\end{aligned}$$

About O(40k) events produced per fb^{-1}

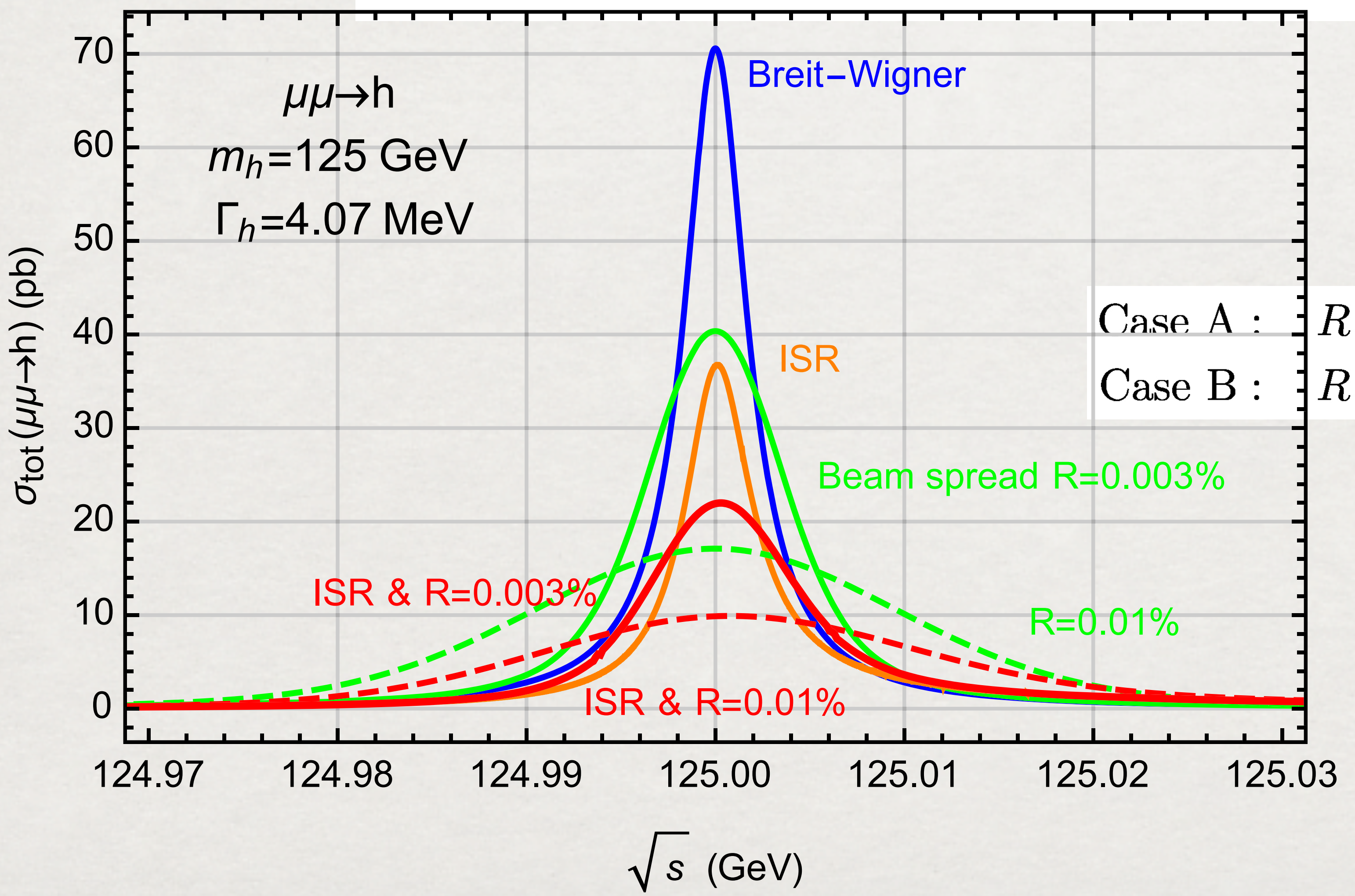
At m_h=125 GeV, Γ_h = 4.2 MeV

$$\frac{\exp[-(\sqrt{\hat{s}}-\sqrt{s})^2/(2\sigma^2_{\sqrt{s}})]}{\sqrt{2\pi}\sigma_{\sqrt{s}}}$$

$$\frac{4\pi\Gamma(h\to\mu\mu)\,\Gamma(h\to X)}{(\hat{s}-m_h^2)^2+m_h^2[\Gamma_h^{\rm tot}]^2}$$

$$\sigma_{\rm eff}(s)=\int d\sqrt{\hat{s}}\,\frac{dL(\sqrt{s})}{d\sqrt{\hat{s}}}\sigma(\mu^+\mu^-\to h\to X)$$

$$\propto\begin{cases}\Gamma_h^2 B/[(s-m_h^2)^2+\Gamma_h^2 m_h^2] & (\Delta\ll\Gamma_h),\\ B\exp[\frac{-(m_h-\sqrt{s})^2}{2\Delta^2}](\frac{\Gamma_h}{\Delta})/m_h^2 & (\Delta\gg\Gamma_h).\end{cases}$$

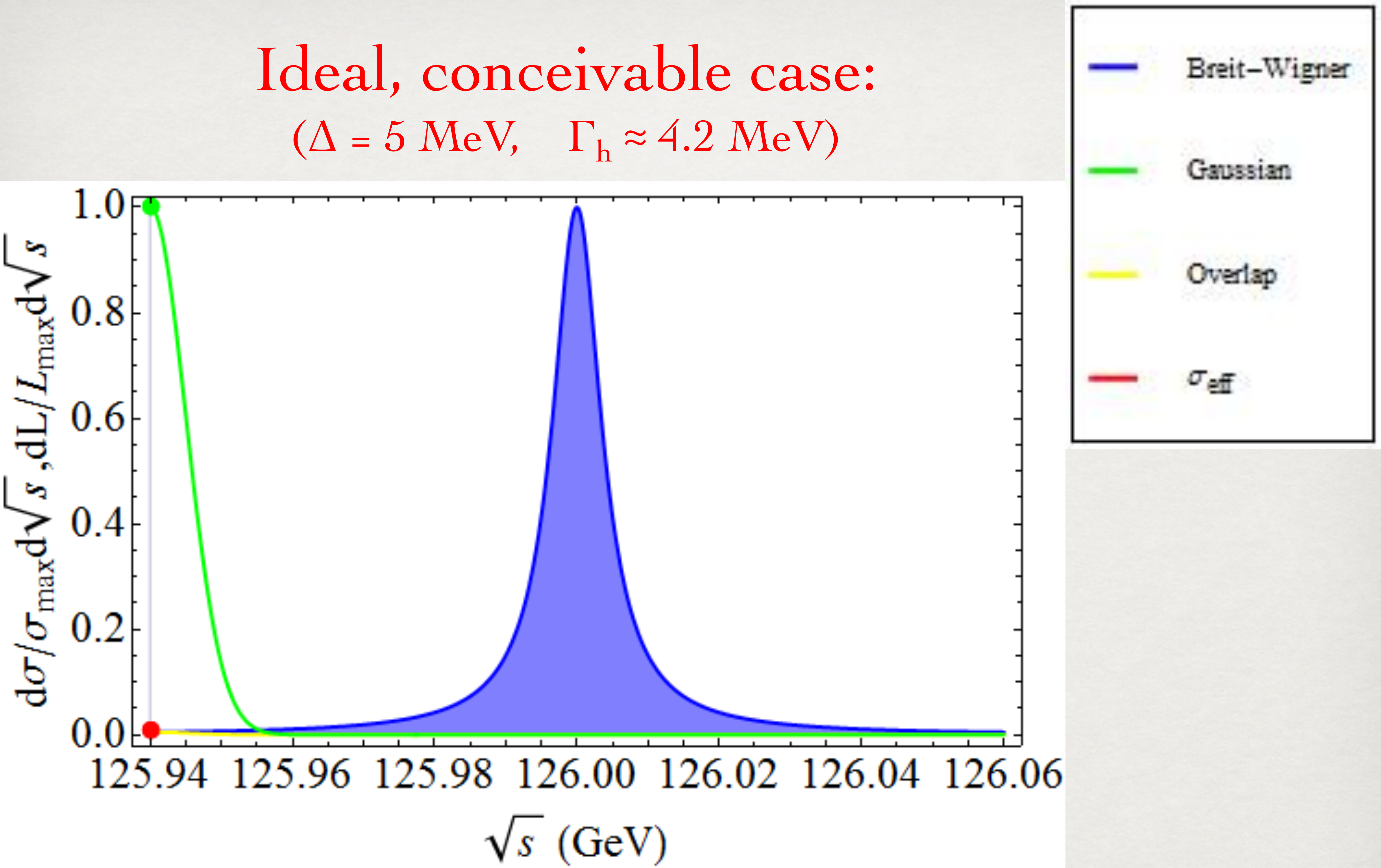


"Muon Collider Quartet":
Barger-Berger-Gunion-Han
PRL & Phys. Report (1995)

Case A : $R=0.01\%$ ($\Delta=8.9$ MeV), $L=0.5$ fb^{-1},
Case B : $R=0.003\%$ ($\Delta=2.7$ MeV), $L=1$ fb^{-1}.

TH, Liu: 1210.7803;
Greco, TH, Liu: 1607.03210

Ideal, conceivable case:
(Δ = 5 MeV, $\Gamma_h \approx 4.2$ MeV)



An optimal fitting would reveal Γ_h

Achievable accuracy at the Higgs factory:

TABLE I. Effective cross sections (in pb) at the resonance $\sqrt{s} = m_h$ for two choices of beam energy resolutions R and two leading decay channels, with the SM branching fractions $\mathrm{Br}_{b\bar{b}} = 56\%$ and $\mathrm{Br}_{WW^*} = 23\%$ [9]. a cone angle cut: $10^o < \theta < 170^o$

	$\mu^+\mu^- \to h$	$h \to b\bar{b}$		$h \to WW^*$	
R (%)	$\sigma_{\rm eff}$ (pb)	$\sigma_{\rm Sig}$	$\sigma_{\rm Bkg}$	$\sigma_{\rm Sig}$	$\sigma_{\rm Bkg}$
0.01	16	7.6		3.7	
0.003	38	18	15	5.5	0.051

Good S/B, S/√B → % accuracies

Table 3

Fitting accuracies for one standard deviation of Γ_h, B and m_h of the SM Higgs with the scanning scheme for two representative luminosities per step and two benchmark beam energy spread parameters.

$\Gamma_h = 4.07$ MeV	L_{step} (fb^{-1})	$\delta\Gamma_h$ (MeV)	δB	δm_h (MeV)
$R = 0.01\%$	0.05	0.79	3.0%	0.36
	0.2	0.39	1.1%	0.18
$R = 0.003\%$	0.05	0.30	2.5%	0.14
	0.2	0.14	0.8%	0.07

~ 3.5%

TH, Liu: 1210.7803;
Greco, TH, Liu: 1607.03210

2. A Multi-TeV Collider

- **EW physics at ultra-high energies:**

$$\frac{v}{E}: \; \frac{v \; (250 \text{ GeV})}{10 \text{ TeV}} \approx \frac{\Lambda_{QCD} \; (300 \text{ MeV})}{10 \text{ GeV}}$$

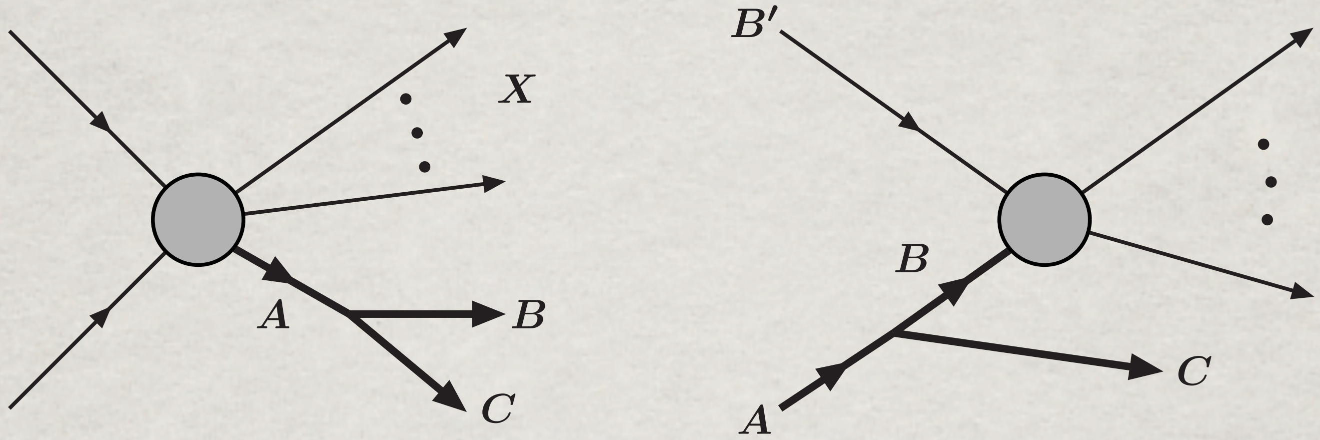
$$v/E, \; m_t/E, \; M_W/E \to 0!$$

- A massless theory:
  → splitting phenomena dominate!
- EW symmetry restored:
  → $SU(2)_L \times U(1)_Y$ unbroken gauge theory
- v/E as power corrections
  → Higher twist effects.

J. Chen, TH, B. Tweedie, arXiv:1611.00788;
G. Cuomo, A. Wulzer, arXiv:1703.08562; 1911.12366.

Ciafaloni et al., hep-ph/0004071; 0007096; A. Manohar et al., 1803.06347.
C. Bauer, Ferland, B. Webber et al., arXiv:1703.08562; 1808.08831.

EW splitting physics: EW PDFs & showering



$$d\sigma_{X,BC} \simeq d\sigma_{X,A} \times d\mathcal{P}_{A\to B+C}$$

$$E_B \approx zE_A, \quad E_C \approx \bar{z}E_A, \quad k_T \approx z\bar{z}E_A\theta_{BC}$$

$$\frac{d\mathcal{P}_{A\to B+C}}{dz\, dk_T^2} \simeq \frac{1}{16\pi^2} \frac{z\bar{z}\,|\mathcal{M}^{(\mathrm{split})}|^2}{(k_T^2 + \bar{z}m_B^2 + zm_C^2 - z\bar{z}m_A^2)^2}$$

- On the dimensional ground: $|\mathcal{M}_{split}|^2 \sim k_T^2$ or m^2
- When SU(2) quantum numbers not summed/averaged, factorized formalism may NOT be valid: → Bloch-Nordsieck theorem violation

Ciafaloni et al., hep-ph/0004071; 0007096
C. Bauer, Ferland, B. Webber et al., arXiv:1703.08562; 1808.08831.
A. Manohar et al., 1803.06347, J. Chen, TH, B. Tweedie, arXiv:1611.00788.

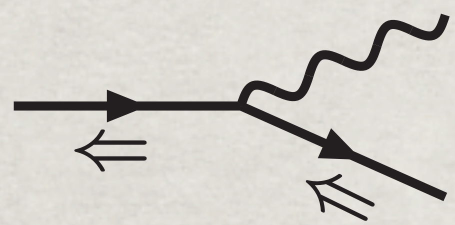
EW splitting functions:

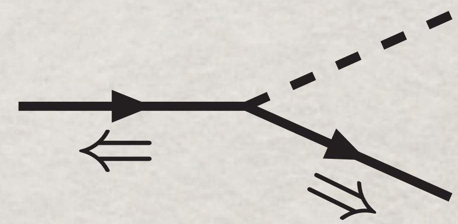
Start from the unbroken phase – all massless.

$$\mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_\phi + \mathcal{L}_f + \mathcal{L}_{Yuk}$$

Chiral fermions: f_s, gauge bosons: $B, W^0, W^\pm$; $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(h - i\phi^0) \end{pmatrix}$

e.g.: fermion splitting:





	$\frac{1}{8\pi^2}\frac{1}{k_T^2}\left(\frac{1+\bar{z}^2}{z}\right)$		$\frac{1}{8\pi^2}\frac{1}{k_T^2}\left(\frac{z}{2}\right)$
	$\to\ V_T\ f_s^{(\prime)}$	$[BW]^0_T\ f_s$	$H^{0(*)}\ f_{\text{-}s}$ or $\phi^\pm\ f'_{\text{-}s}$
$f_{s=L,R}$	$g_V^2(Q^V_{f_s})^2$	$g_1 g_2 Y_{f_s} T^3_{f_s}$	$y^2_{f_R^{(\prime)}}$

Ciafaloni et al., Hep-ph/0505047.

Infrared & collinear singularities (P_{gq})

Collinear singularity, Chirality-flip, Yukawa

EW Symmetry breaking & Goldstone-boson Equivalence Theorem (GET):

Lee, Quigg, Thacker (1977); Chanowitz & Gailard (1984)

At high energies $E >> M_W$, the longitudinally polarized gauge bosons behave like the corresponding Goldstone bosons.
(They remember their origin!)

"Scalarization" to implement the Goldstone-boson Equivalence Theorem (GET):

$$\epsilon(k)_L^\mu = \frac{E}{m_W}(\beta_W, \hat{k}) \approx \frac{k^\mu}{m_W} + O(m_W/E)$$

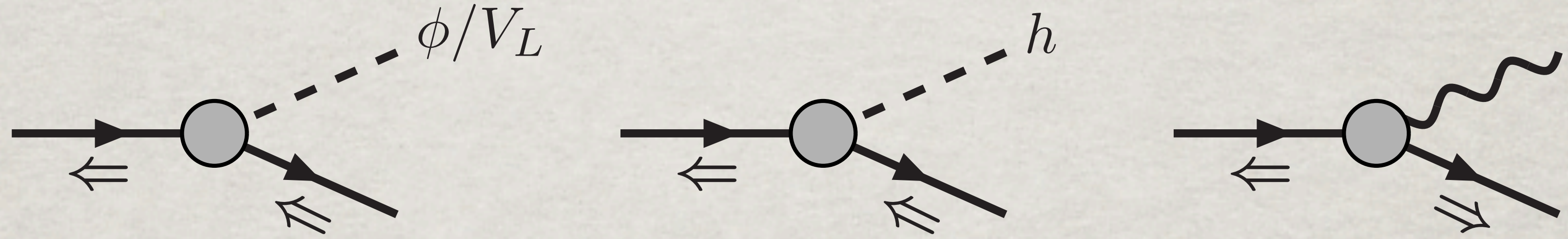
GET violation as power corrections v/E.
Like in QCD: higher-twist effects Λ_{QCD}/E.

J. Chen, TH, B. Tweedie, arXiv:1611.00788;
G. Cuomo, A. Wulzer, arXiv:1703.08562; 1911.12366.

Splitting in a broken gauge theory:

New fermion splitting: $\frac{v^2}{k_T^2}\,\frac{dk_T^2}{k_T^2} \sim (1-\frac{v^2}{Q^2})$

V_L is of IR, h no IR



$$\frac{\frac{1}{16\pi^2}\frac{v^2}{\tilde{k}_T^4}\left(\frac{1}{z}\right)}{\to\ V_L\ f_s^{(\prime)}\ (V\neq\gamma)} \qquad \frac{\frac{1}{16\pi^2}\frac{v^2}{\tilde{k}_T^4}}{h\ f_s} \qquad \frac{\frac{1}{16\pi^2}\frac{v^2}{\tilde{k}_T^4}}{V_T\ f_{\text{-}s}^{(\prime)}}$$

Chirality conserving:
Non-zero for massless f

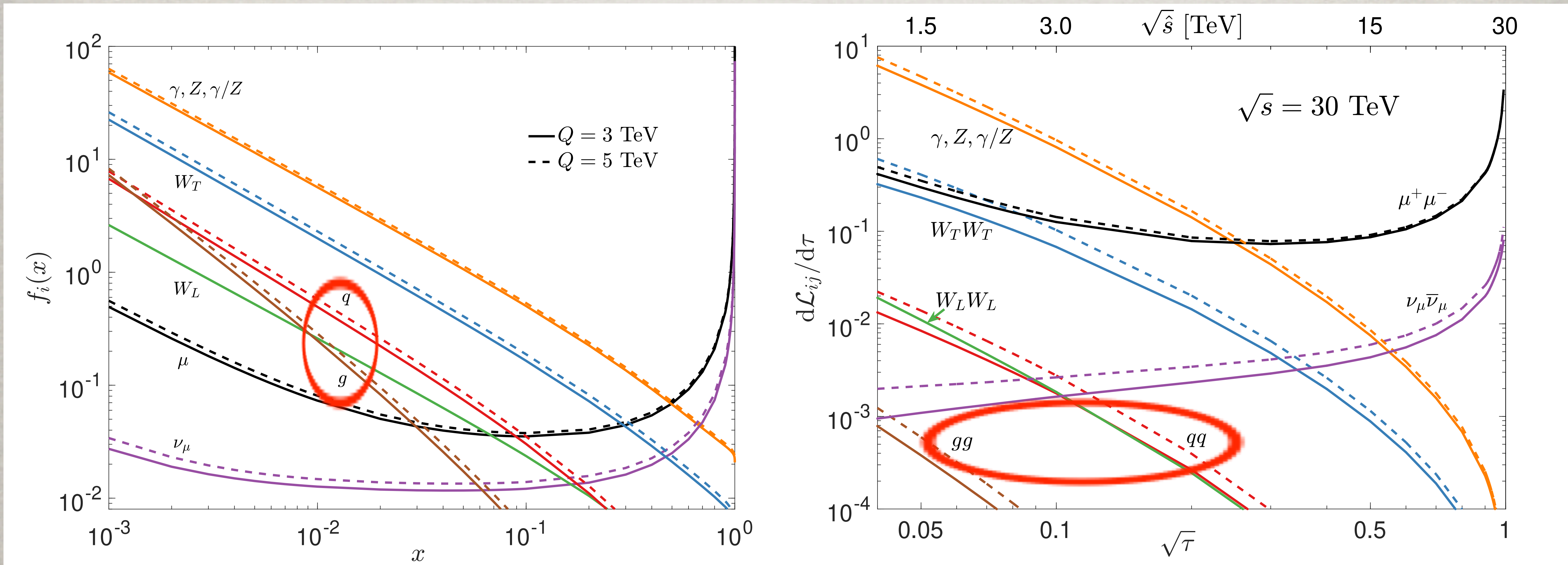
Chirality flipping:
$\sim m_f$

The DPFs for W_L thus don't run at leading log:
"Bjorken scaling" restored (higher-twist effects)!

- **EW PDFs at a muon collider:**

"partons" dynamically generated

$$\frac{\mathrm{d} f_i}{\mathrm{d}\ln Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P^I_{i,j} \otimes f_j$$



$\mu^\pm$: the valance. $\ell_R,\ \ell_L,\ \nu_L$ and $B, W^{\pm,3}$: LO sea.

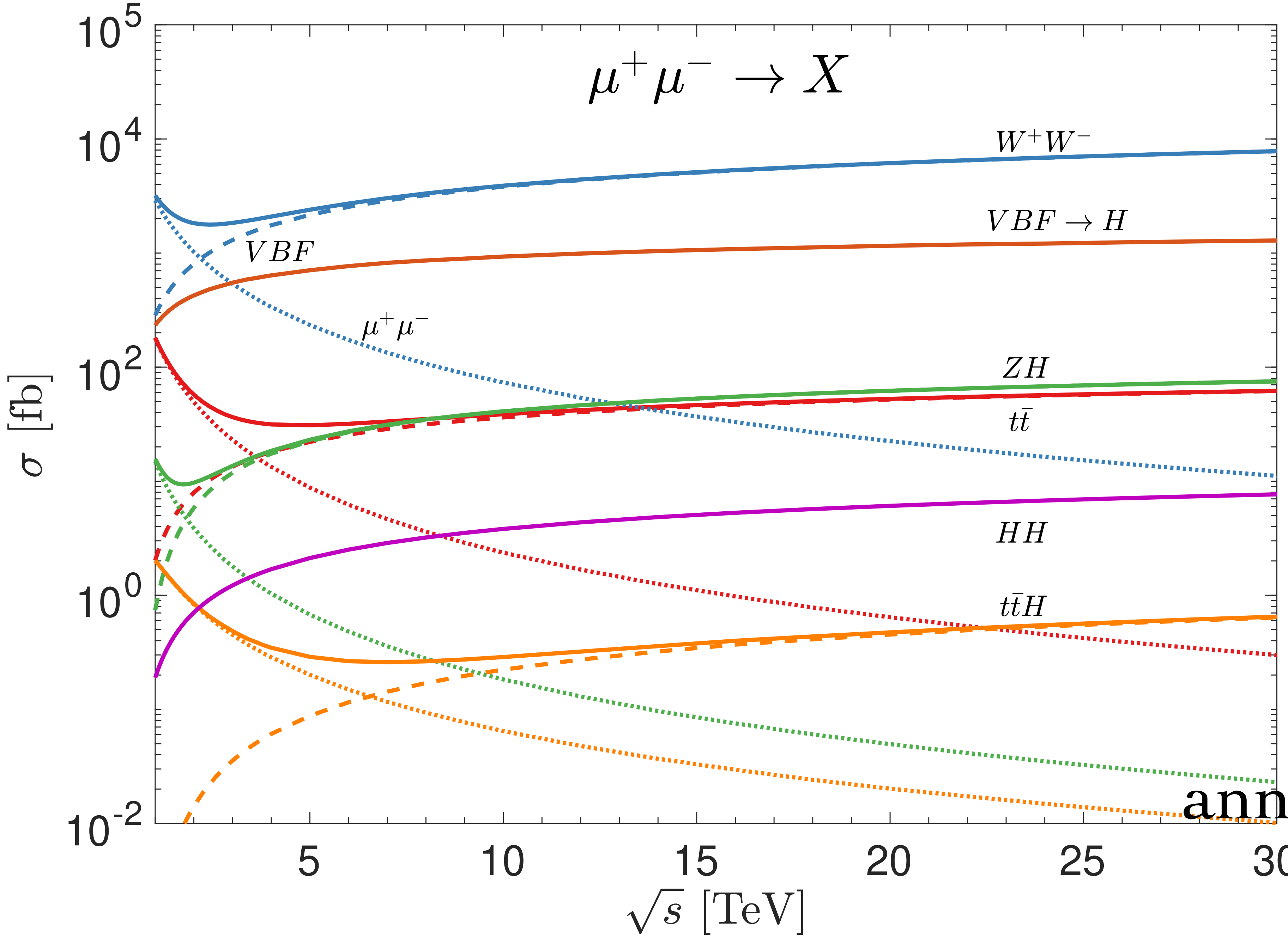
Quarks: NLO; gluons: NNLO.

TH, Yang Ma, Keping Xie, arXiv:2007.14300

• "Semi-inclusive" processes

Just like in hadronic collisions:

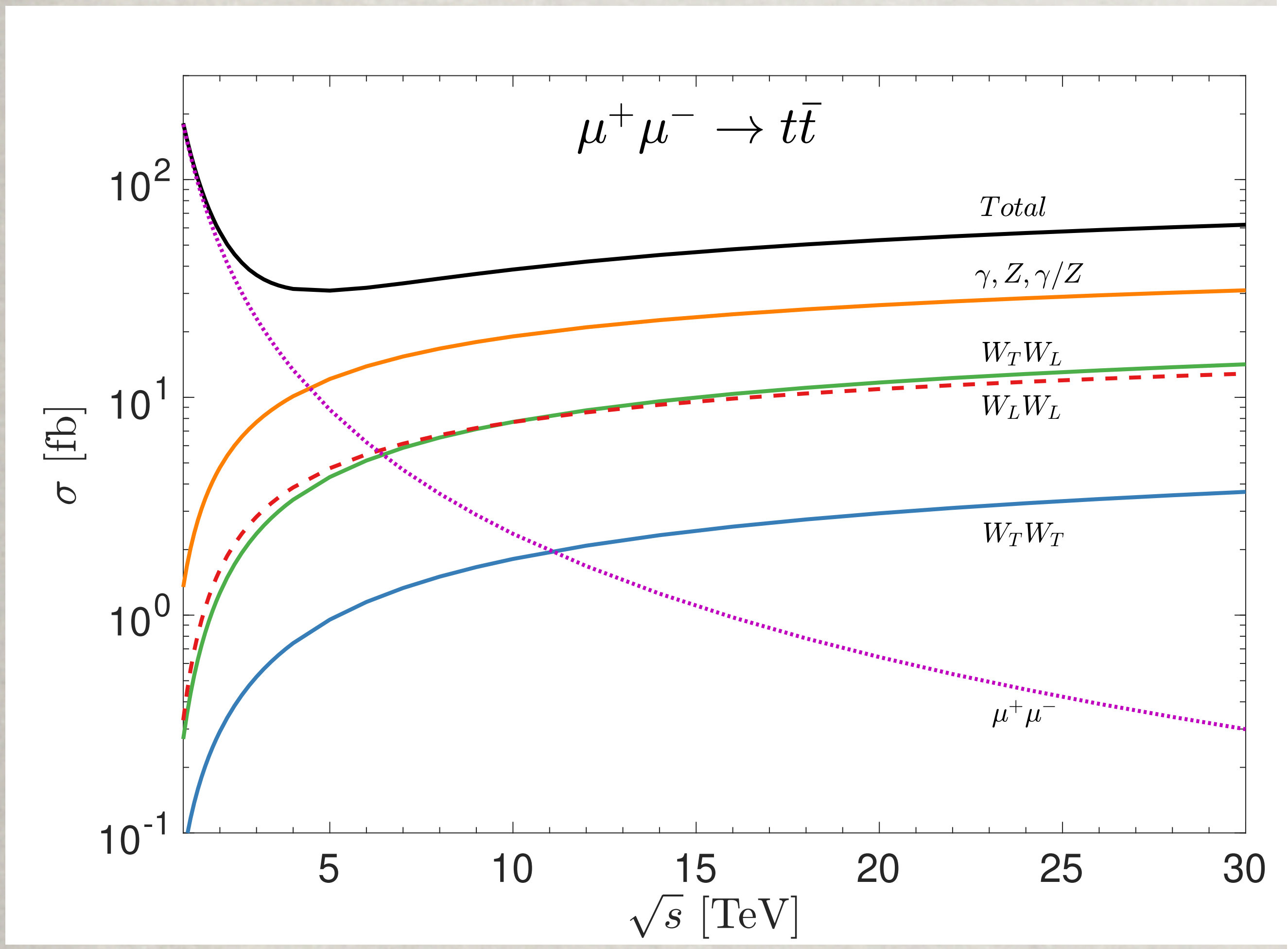
$\mu^+\mu^-$ → exclusive particles + remnants

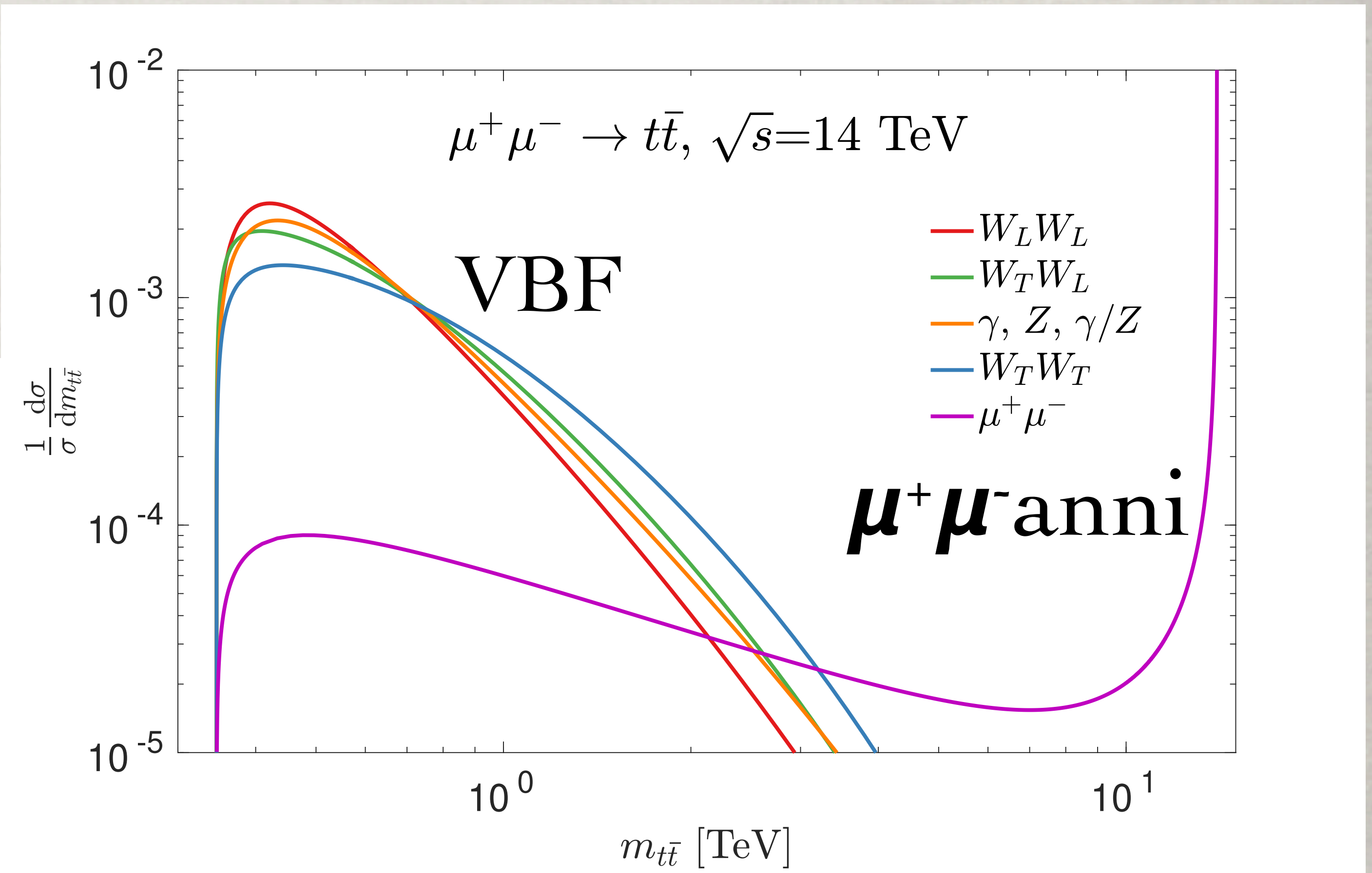


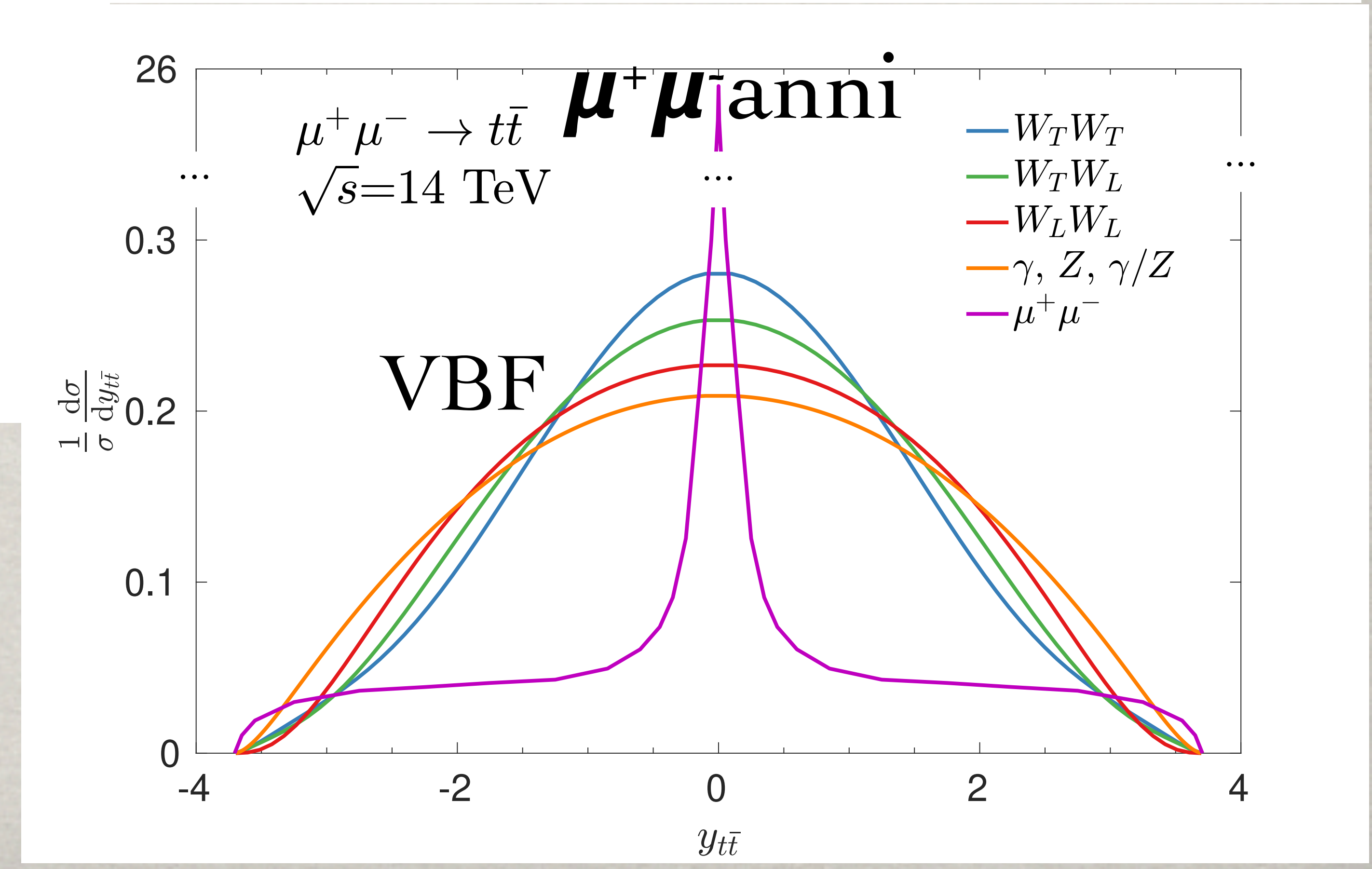
VBF

$\mu^+\mu^-$ annihilations

Underlying sub-processes:





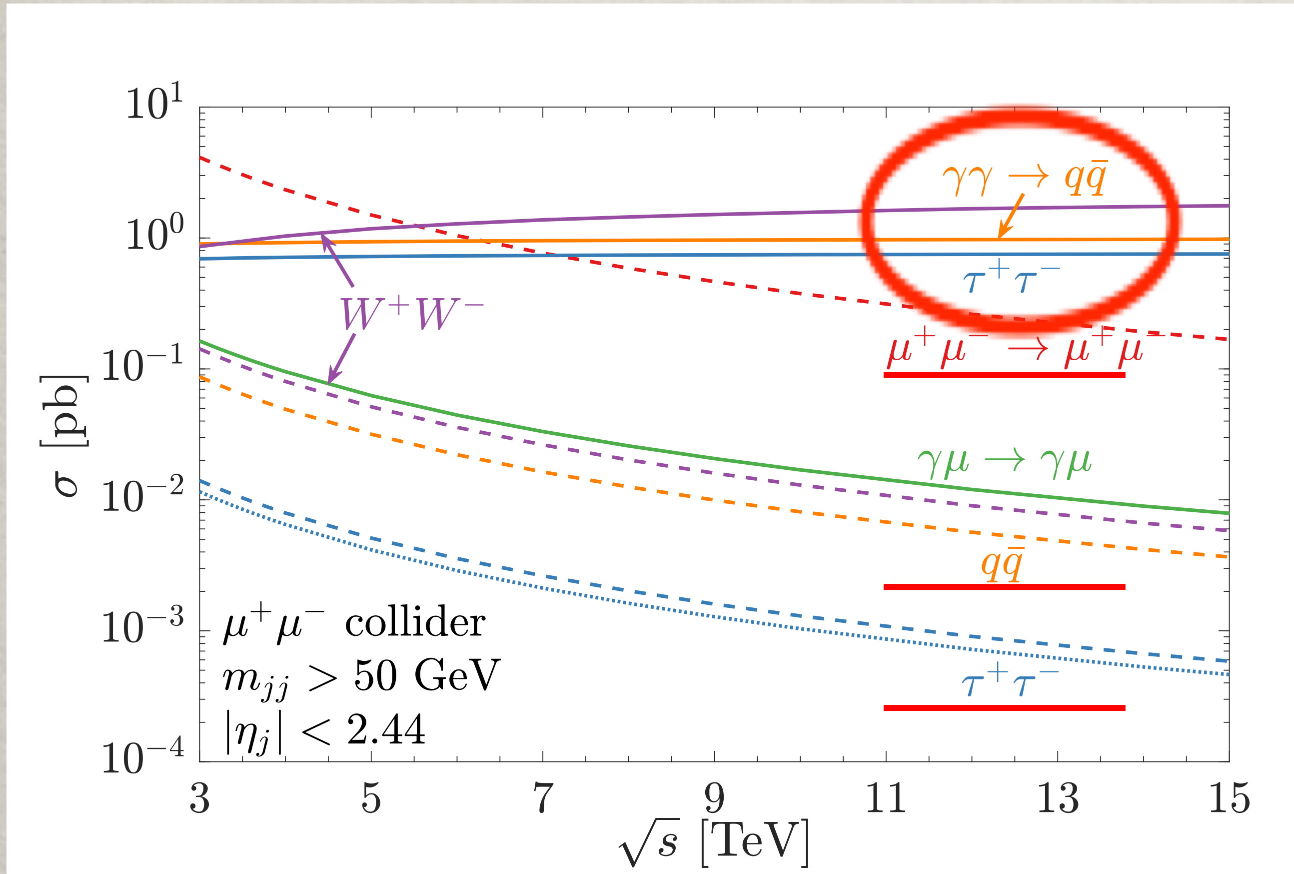


Partonic contributions

• Jets at low energies

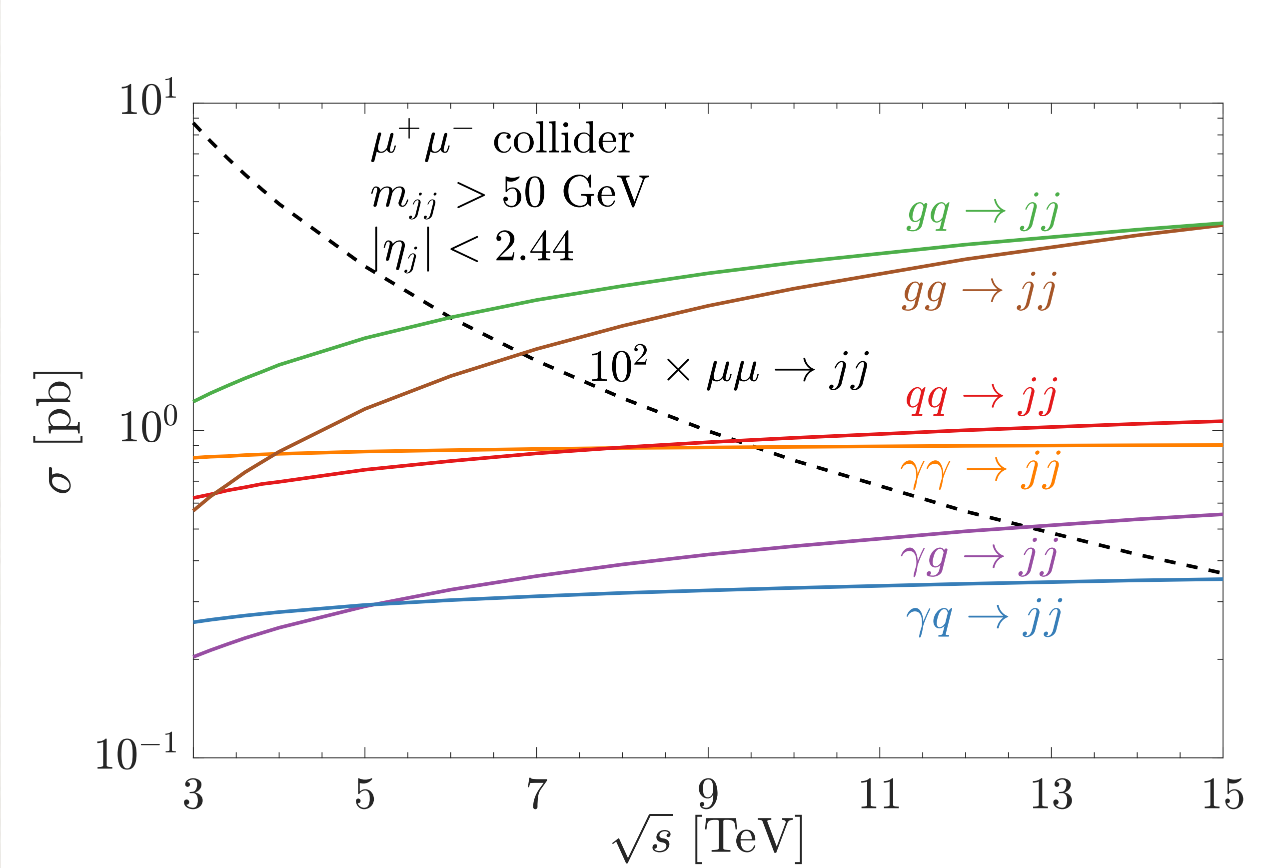
For **μ⁺μ⁻** annihilation: $\sigma_{ann} \sim \frac{\alpha^2}{s}$

For partonic fusion: $\sigma_{fusion} \sim \frac{\alpha^2}{m_{jj}^2}\log^2(\frac{Q^2}{m^2})$



Di-jet production: QCD dominates

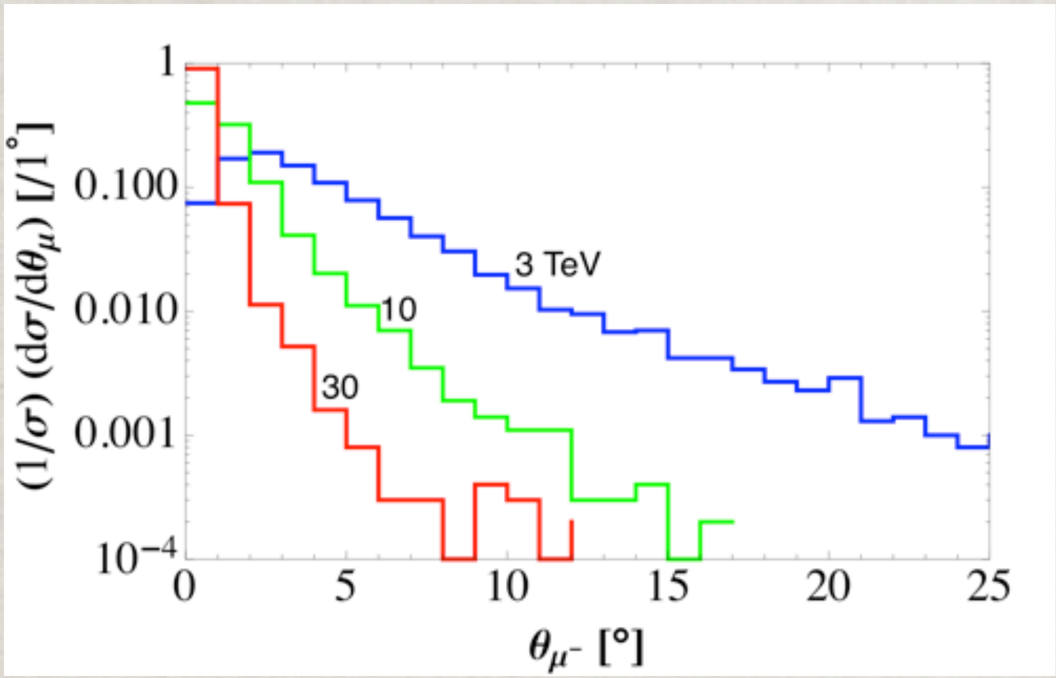
$\gamma\gamma \to q\bar{q},\ \gamma g \to q\bar{q},\ \gamma q \to gq,$
$qq \to qq(gg),\ gq \to gq,$ and $gg \to gg(q\bar{q})$

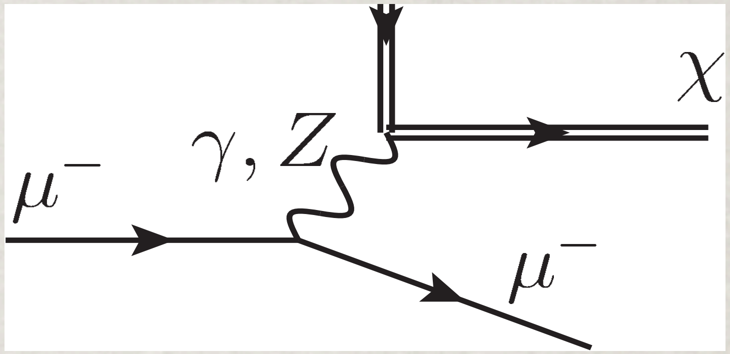


TH, Yang Ma, Keping Xie, to appear soon.

• Unique kinematic features:

- Forward tagging:





$\theta_\mu \approx M_Z/E_\mu$

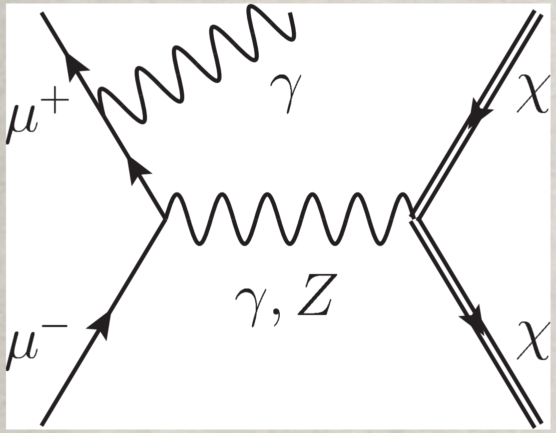
$\theta_\mu \sim 0.02 \approx 1.2^\circ$ at 10 TeV.

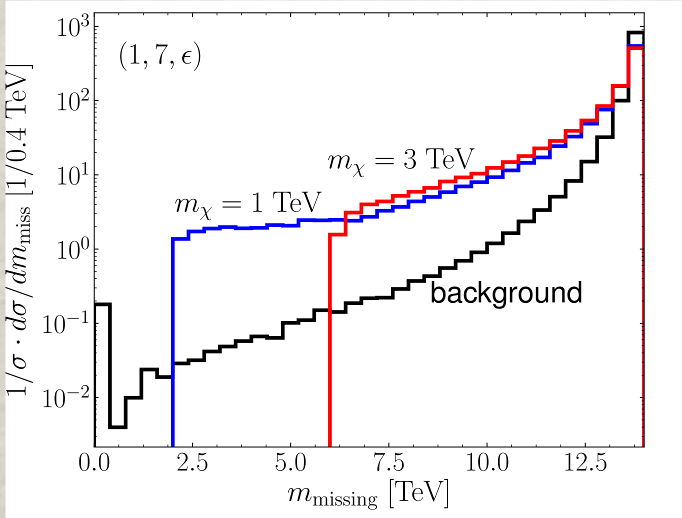
$10^\circ < \theta_{\mu^\pm} < 170^\circ.$

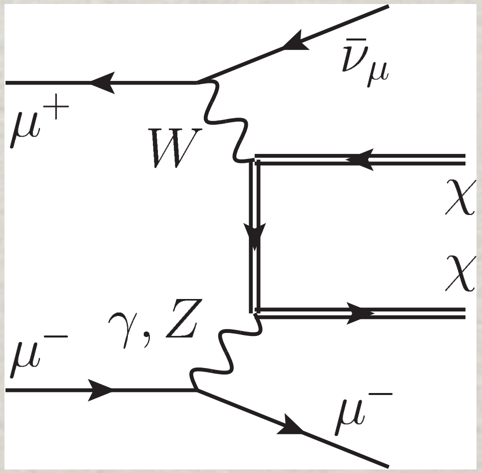
- "Recoil mass" → "missing mass": $m^2_{\text{missing}} \equiv (p_{\mu^+} + p_{\mu^-} - \sum_i p_i^{\text{obs}})^2$

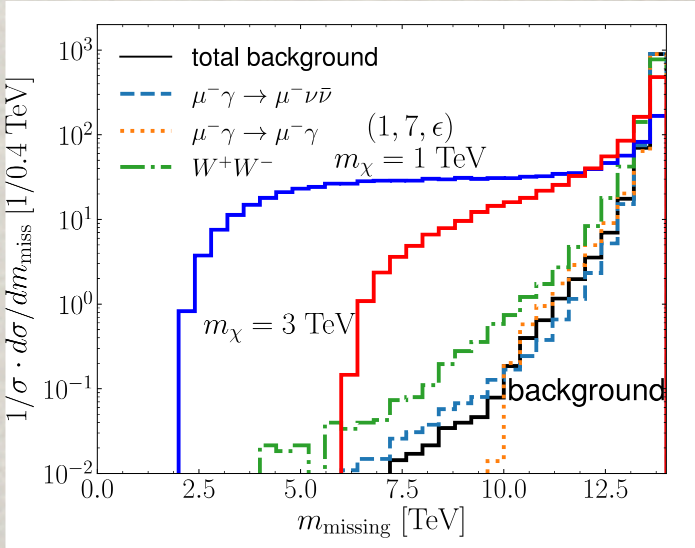
$m^2_{\text{missing}} \equiv (p_{\mu^+} + p_{\mu^-} - p_\gamma)^2 > 4m_\chi^2$

$m^2_{\text{missing}} = (p^{\text{in}}_{\mu^+} + p^{\text{in}}_{\mu^-} - p^{\text{out}}_{\mu^\pm})^2 > 4m_\chi^2.$





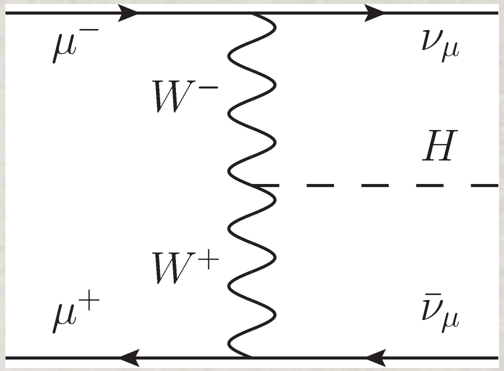




TH, Z. Liu, L.T. Wang, X. Wang: arXiv:2009.11287

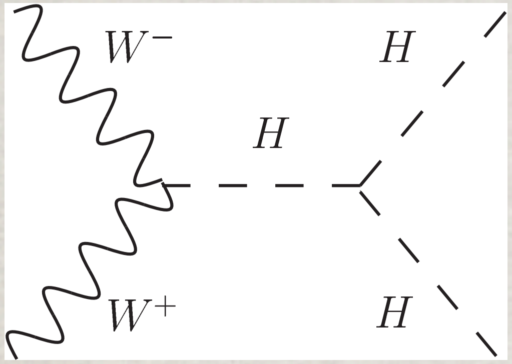
3. Precision Higgs Physics

$$\mu^+\mu^- \to \nu_\mu \bar{\nu}_\mu \, H \qquad (WW \text{ fusion}),$$
$$\mu^+\mu^- \to \mu^+\mu^- \, H \qquad (ZZ \text{ fusion}).$$

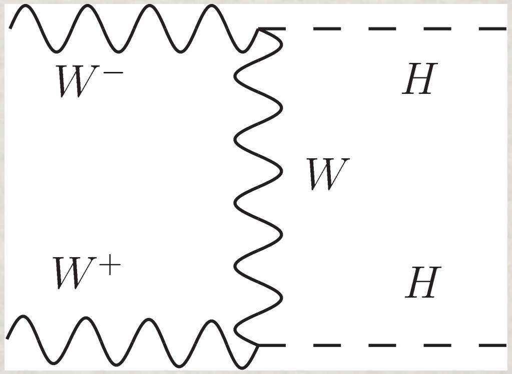


WWH / ZZH couplings

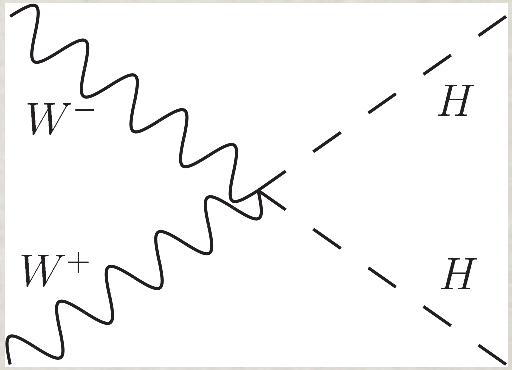
HHH / WWHH couplings:



(a)



(b)



(c)

$\sqrt{s}$ (TeV)	3	6	10	14	30
benchmark lumi (ab^{-1})	1	4	10	20	90
σ (fb): $WW \to H$	490	700	830	950	1200
$ZZ \to H$	51	72	89	96	120
$WW \to HH$	0.80	1.8	3.2	4.3	6.7
$ZZ \to HH$	0.11	0.24	0.43	0.57	0.91
$WW \to ZH$	9.5	22	33	42	67
$WW \to t\bar{t}H$	0.012	0.046	0.090	0.14	0.28
$WW \to Z$	2200	3100	3600	4200	5200
$WW \to ZZ$	57	130	200	260	420

10M H

500k HH

TH, D. Liu, I. Low, X. Wang, arXiv:2008.12204

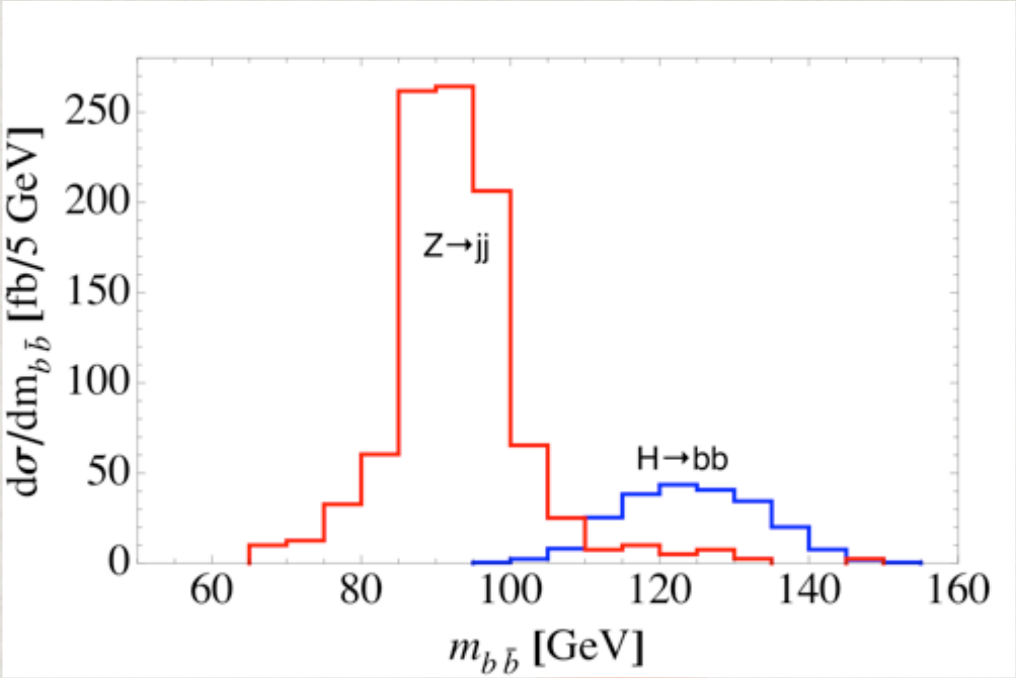
Achievable accuracies

$$\mathcal{L} \supset \left(M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu\right)\left(\kappa_V \frac{2H}{v} + \kappa_{V_2} \frac{H^2}{v^2}\right) - \frac{m_H^2}{2v}\left(\kappa_3 H^3 + \frac{1}{4v}\kappa_4 H^4\right)$$

Leading channel H → bb:

$$\Delta E/E = 10\%.$$

$$10^\circ < \theta_{\mu^\pm} < 170^\circ.$$



$\sqrt{s}$ (lumi.)	3 TeV (1 ab^{-1})	6 (4)	10 (10)	14 (20)	30 (90)	Comparison
WWH ($\Delta\kappa_W$)	0.26%	0.12%	0.073%	0.050%	0.023%	0.1% [41]
$\Lambda/\sqrt{c_i}$ (TeV)	4.7	7.0	9.0	11	16	(68% C.L.)
ZZH ($\Delta\kappa_Z$)	1.4%	0.89%	0.61%	0.46%	0.21%	0.13% [17]
$\Lambda/\sqrt{c_i}$ (TeV)	2.1	2.6	3.2	3.6	5.3	(95% C.L.)
$WWHH$ ($\Delta\kappa_{W_2}$)	5.3%	1.3%	0.62%	0.41%	0.20%	5% [36]
$\Lambda/\sqrt{c_i}$ (TeV)	1.1	2.1	3.1	3.8	5.5	(68% C.L.)
HHH ($\Delta\kappa_3$)	25%	10%	5.6%	3.9%	2.0%	5% [22, 23]
$\Lambda/\sqrt{c_i}$ (TeV)	0.49	0.77	1.0	1.2	1.7	(68% C.L.)

Table 7: Summary table of the expected accuracies at 95% C.L. for the Higgs couplings at a variety of muon collider collider energies and luminosities.

 TH, D. Liu, I. Low, X. Wang, arXiv:2008.12204

Summary

- **s-channel Higgs factory:**
  - Direct measurements on Y_μ & Γ_H
  - Other BRs comparable to e^+e^- Higgs factories
- **Multi-TeV colliders:**
  - Unprecedented accuracies for WWH, WWHH, H^3, H^4
  - Decisive coverage for minimal WIMP DM $M \sim 0.5\ E_{cm}$
  - New particle (H) mass coverage $M_H \sim (0.5 - 1)E_{cm}$
  - Further complementarity: Astro/Cosmo/GW etc.
  - Bread & butter SM EW physics in the new territory:
    EW factorization theorem violation;
    Goldstone boson equivalence violation

An exciting journey ahead!

Please join the efforts at: International muon collider collaboration:
https://simba3.web.cern.ch/simba3/SelfSubscription.aspx?groupName=MUONCOLLIDER-DETECTOR-PHYSICS
Muon Collider Forum: SNOWMASS-MUON-COLLIDER-FORUM@FNAL.GOV
at https://snowmass21.org/energy/start#communications.