Bounce of Nothing

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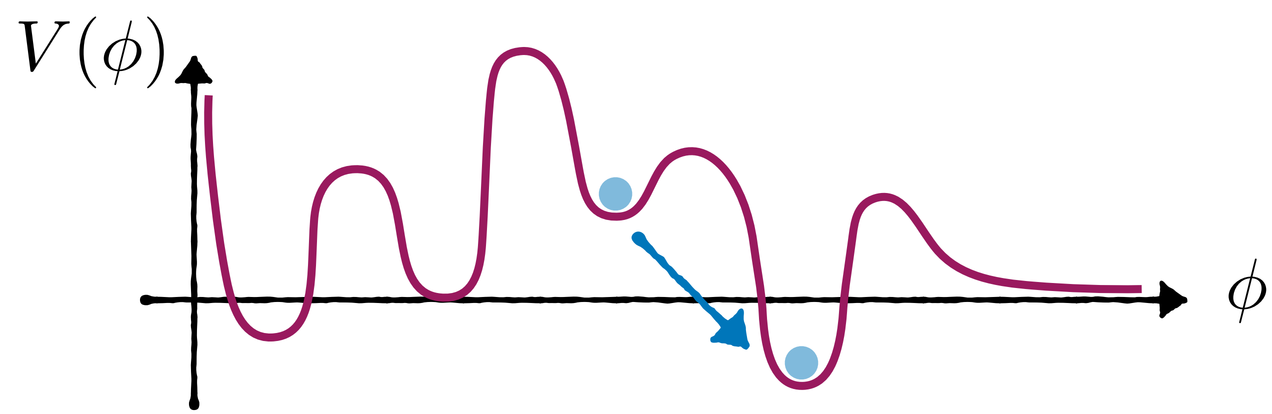
Motivation

Vacuum stability is one of the most important aspects of any theory

The semiclassical formalism of Coleman *et al.* has been the standard tool to study vacuum decay in QFT

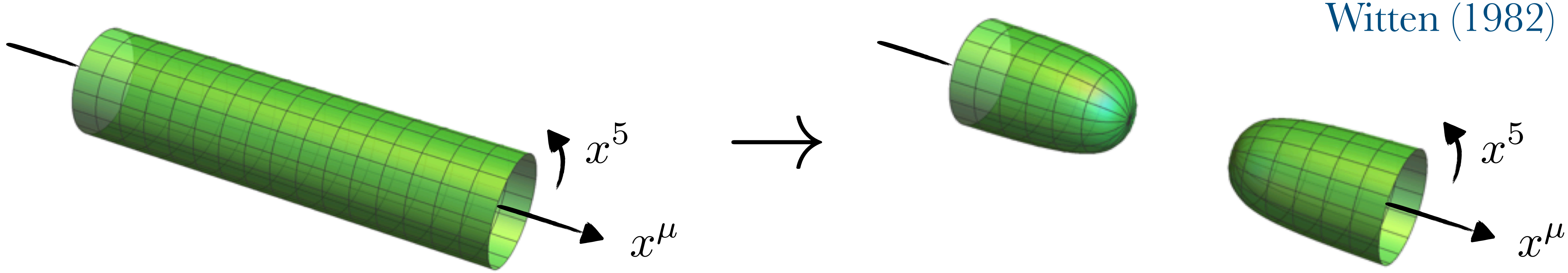
including gravity



Coleman; Callan, Coleman (1977)
Coleman, De Luccia (1980)

Theories with compact extra dimensions can decay via an exotic type of gravitational instability that "destroys the spacetime" — a ***bubble of nothing***

Witten (1982)



Motivation

Why should we care about bubbles of nothing?

- Extra dimensions an ingredient of many well-motivated BSM theories

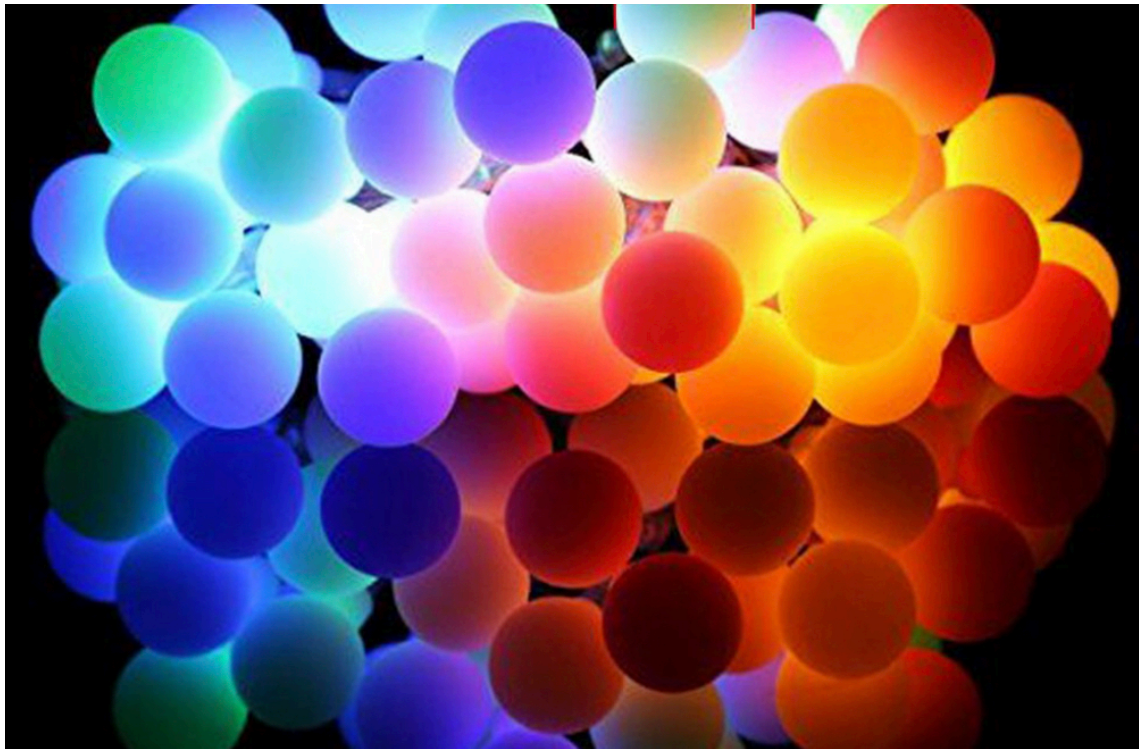
including string theory!

could *our* universe decay into a bubble of nothing?

- Metastability of de Sitter space?!

- Could play role in vacuum selection

e.g. in the string landscape



Motivation

- Implications for supersymmetry breaking?!

e.g. Blanco-Pillado, *et al.*; 1606.03095
Garcia-Etxebarria, *et al.*; 2005.06494

Witten (1982)
Dine, Fox, Gorbatov; hep-th/0405190

- Universe $\to$ "nothing" $\overset{?}{\Rightarrow}$ "nothing" $\to$ Universe

e.g. Vilenkin (1982)
Hawking, Turok (1998)

Blanco-Pillado, *et al.*; 1104.5229
Brown, Dahlen; 1111.0301

- What is "nothing" in a UV-complete theory of quantum gravity?!

e.g. Witten, PLB 206 (1988)
Silverstein, McGreevy; hep-th/0506130
Agrawal *et al.*; 2009.10077

as bad as it sounds, or
another phase of gravity?

In this talk...

What are the necessary conditions for bubbles of nothing to be possible ***realistic theories***?

4-dim de Sitter vacuum

no low energy SUSY

stabilized extra dimensions

- What are the conditions on the radion potential for a BON to exist?

- Intermediate object between Witten's bubble and CDL bounce

- Effect on the decay rate: what happens in the limit $\Lambda_{\mathrm{CC}} \to 0$?

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Vacuum decay in QFT

Coleman; Callan, Coleman (1977)

Semiclassical treatment of vacuum decay centered on finding the ***bounce***

$$\left.\frac{\delta S_E[\phi]}{\delta\phi}\right|_{\phi_{\rm b}} = 0 \qquad \text{with} \qquad \phi_{\rm b}(t_E, \mathbf{x}) \to \phi_{\rm fv} \qquad \text{as} \qquad \begin{cases} t_E \to \pm\infty \\ |\mathbf{x}| \to \infty \end{cases}$$

and a single negative eigenvalue!

The bounce is $\mathcal{O}(4)$-symmetric, i.e. $\phi_{\rm b}(\xi)$ with $\xi = \sqrt{t_E^2 + \mathbf{x}^2}$

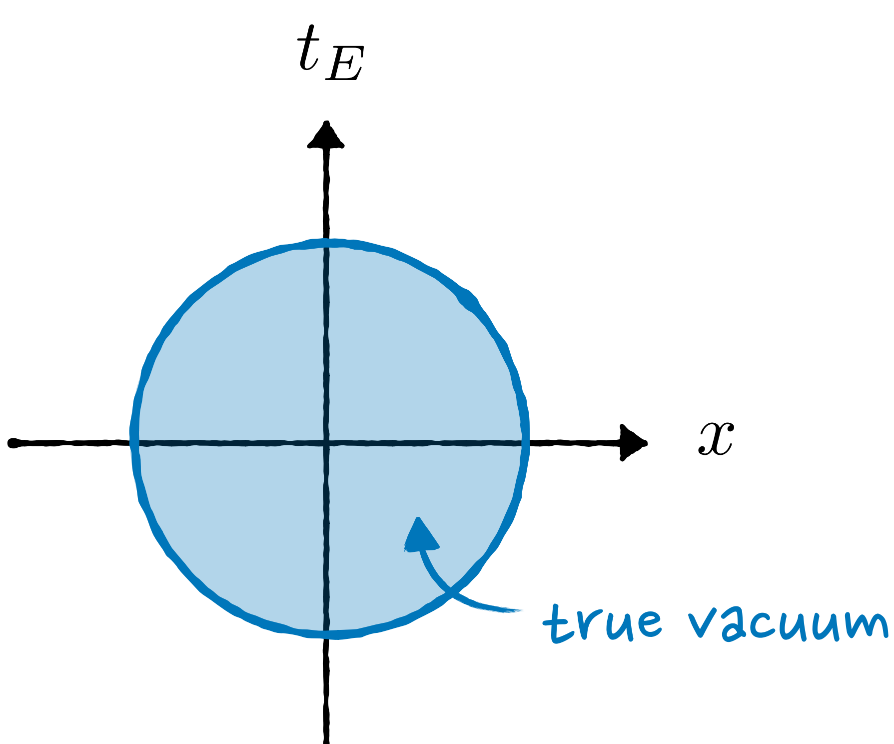
$$\phi'' + \frac{3}{\xi}\phi' = \frac{dV}{d\phi} \qquad \text{with} \qquad \phi \xrightarrow{\xi\to\infty} \phi_{\rm fv} \qquad \underline{\text{and}} \qquad \phi'(0) = 0$$

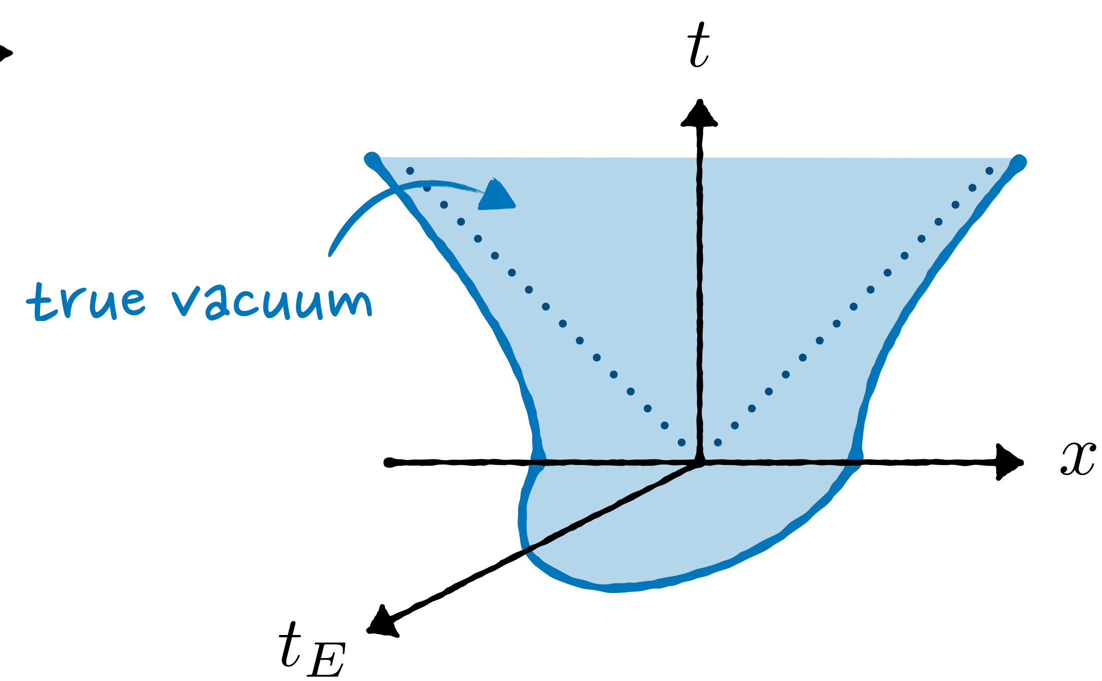
Bounce action determines the decay rate of the false vacuum

$$\Gamma = m^4 e^{-\Delta S_E} \qquad \text{with} \qquad \Delta S_E = S_E[\phi_{\rm b}] - S_E[\phi_{\rm fv}]$$

Analytic continuation of the bounce gives evolution of bubbles post nucleation

$R_b^2 < \mathbf{x}^2 + t_E^2$ $\quad t_E \to it \quad$ $R_b^2 < \mathbf{x}^2 - t^2$





Tunneling in the presence of gravity

Coleman, De Luccia (1980)

Now metric dof: Assumed $\mathcal{O}(4)$-symmetry restricts attention to metrics of form

$$ds = d\xi^2 + \rho(\xi)^2 d\Omega_3$$

curvature radius of the unit 3-sphere

$$d\Omega_3 = d\psi^2 + \sin^2\psi \, d\Omega_2$$

Scalar + Einstein equations

$$\begin{cases} \phi'' + \dfrac{3\rho'}{\rho}\phi' = \dfrac{dV(\phi)}{d\phi} \\ \rho'^2 = 1 + \dfrac{\rho^2}{6m_{\mathrm{Pl}}^2}\left(\phi'^2 - 2V(\phi)\right) \end{cases}$$

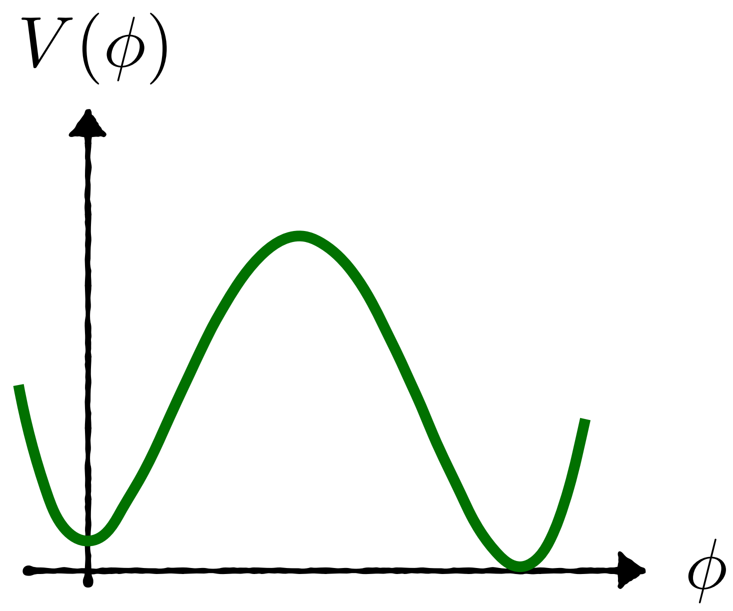
e.g. for a de Sitter false vacuum:

$$\phi_{\mathrm{dS}} = \mathrm{const.} \quad \text{and} \quad \rho_{\mathrm{dS}}(\xi) = l_{\mathrm{dS}} \sin\left(\frac{\xi}{l_{\mathrm{dS}}}\right) \quad \text{with} \quad l_{\mathrm{dS}} = \sqrt{\frac{3m_{\mathrm{Pl}}^2}{V_{\mathrm{fv}}}}$$

dS radius

Coleman-De Luccia

Coleman, De Luccia (1980)

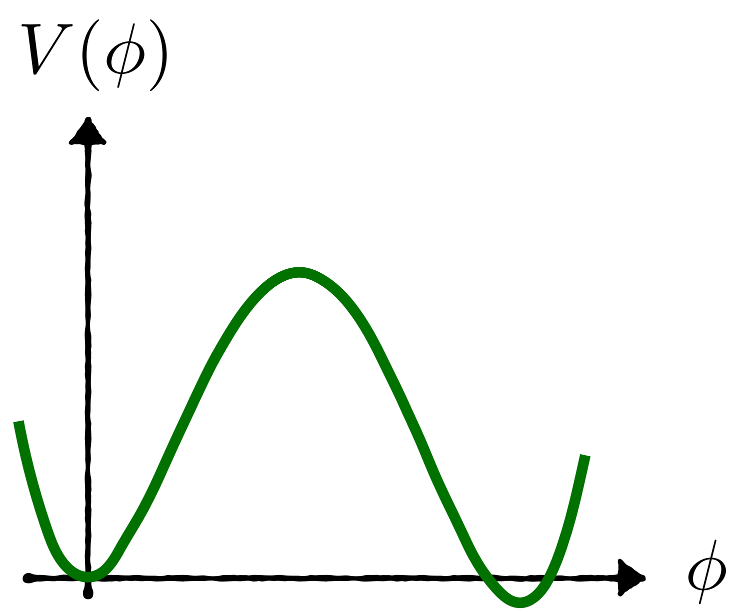


The changes are quantitative…

$$\Delta S_E = \frac{27\pi^2\sigma^4/2\epsilon^3}{\left(1+\frac{3\sigma^2}{4m_{\rm Pl}^2\epsilon}\right)^2}$$

$$\epsilon \equiv V_{\rm tv} - V_{\rm fv} \ll V_{\rm barrier}$$

…and qualitative



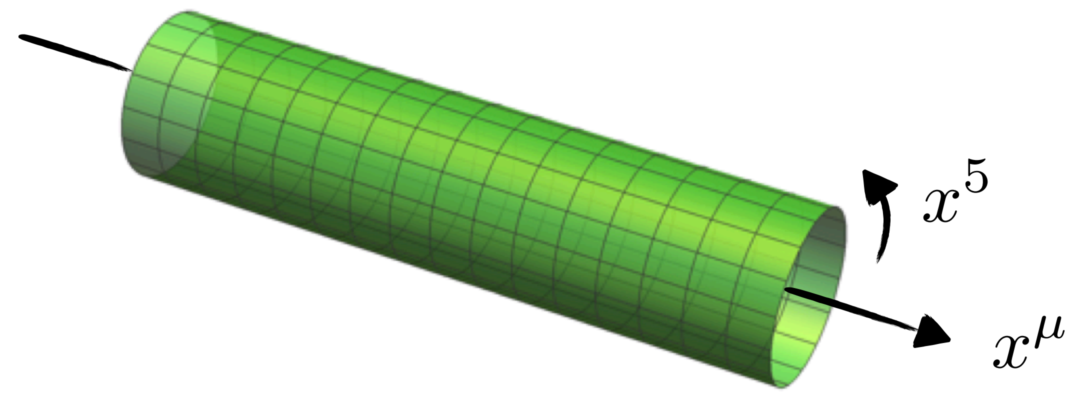
$$\Delta S_E = \frac{27\pi^2\sigma^4/2\epsilon^3}{\left(1-\frac{3\sigma^2}{4m_{\rm Pl}^2\epsilon}\right)^2}$$

$\Rightarrow$ no decay for $\epsilon = V_{\rm tv} - V_{\rm fv} \le \dfrac{3\sigma^2}{4m_{\rm Pl}^2}$!!!

Instability of the Kaluza-Klein vacuum

Witten (1982)

Witten's bubble of nothing is an instability of the (purely gravitational and unstabilized) Kaluza-Klein vacuum $\mathbb{M}_4 \times S^1$



$$ds = \underbrace{-dt^2 + d\mathbf{x}^2}_{\mathbb{M}_4} + (dx^5)^2$$

$\rightarrow$ $x^5 \sim x^5 + 2\pi R$

The relevant instanton is the 5-dimensional Euclidean Schwarzschild solution:

$$ds_E = \left(1 - \frac{R^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_3 + \left(1 - \frac{R^2}{r^2}\right)(dx^5)^2$$

$r \in [R, \infty)$

$x^5 \sim x^5 + 2\pi R$

5D Euclidean Schwarzschild

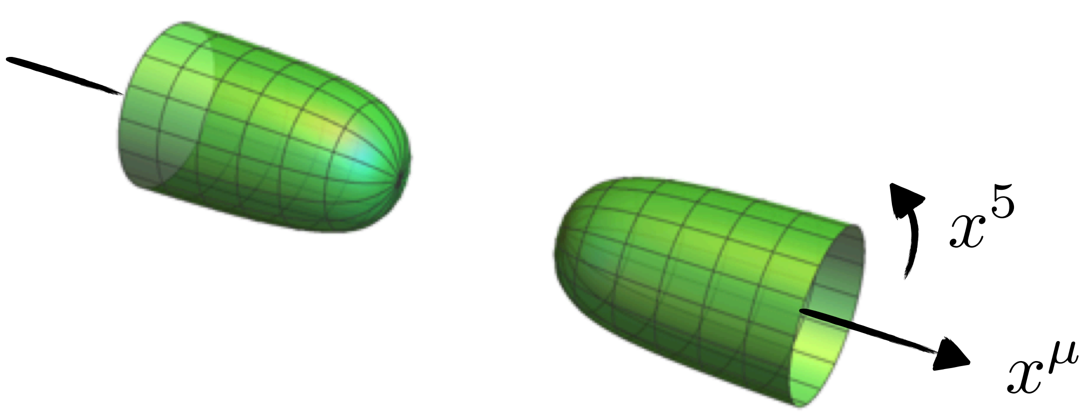
Witten (1982)

$$ds_E = \left(1 - \frac{R^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_3 + \left(1 - \frac{R^2}{r^2}\right)(dx^5)^2$$

Euclidean black holes have neither an interior nor a singularity

$$\begin{cases} \lambda \equiv \sqrt{2R(r-R)} \in [0, \infty) \\ \alpha \equiv x^5/R \in [0, 2\pi) \end{cases} \Rightarrow \quad ds_E \simeq \underbrace{d\lambda^2 + \lambda^2 d\alpha^2}_{\mathbb{R}^2} + \underbrace{R^2 d\Omega_3}_{S^3}$$

near r=R



Smooth geometry: S^1 shrinks to zero, and all curvature invariants near the tip of the "cigar" set by the scale $\sim 1/R$

A bubble of... nothing

Witten (1982)

After analytic continuation, $\psi \to \pi/2 + i\chi$ (equiv. $t_E \to it$):

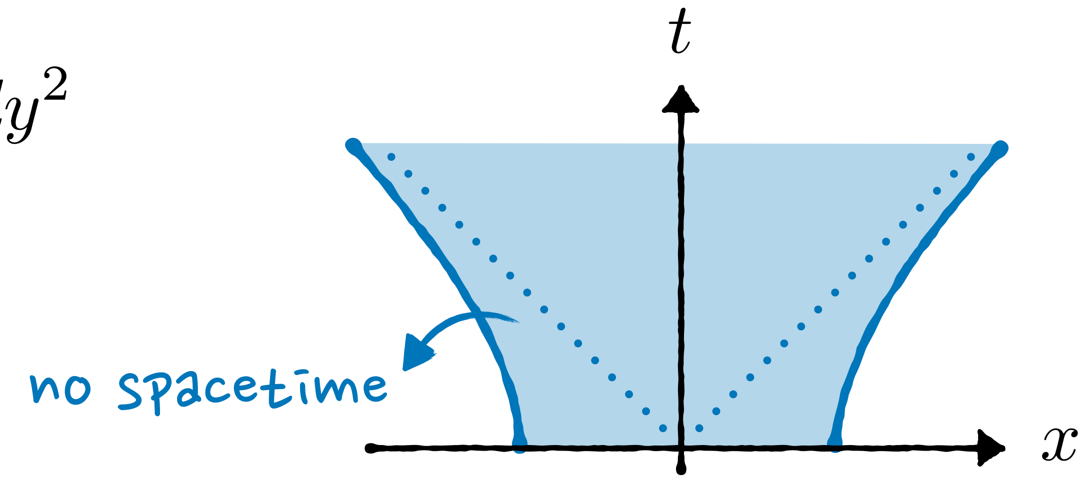
$$ds = \left(1 - \frac{R^2}{r^2}\right)^{-1} dr^2 - r^2 d\chi^2 + r^2 \cosh^2\chi \, d\Omega_2 + \left(1 - \frac{R^2}{r^2}\right) dy^2$$

what is this??

Asymptotically, for $r \gg R$:

$$ds \simeq -r^2 d\chi^2 + dr^2 + r^2 \cosh^2\chi \, d\Omega_2 + dy^2$$

$$= \underbrace{-dt^2 + d\tilde{r}^2 + \tilde{r}^2 d\Omega_2}_{\mathbb{M}_4} + \underbrace{dy^2}_{S^1}$$



however...

$$r^2 = \mathbf{x}^2 - t^2 \geq R^2 \qquad \Rightarrow \qquad |\mathbf{x}_{\text{wall}}| = \pm\sqrt{R^2 + t^2}$$

Decay rate

Witten (1982)

Tunneling exponent given by the action of 5D Euclidean Schwarzschild:

$$\Delta S_E = -\frac{1}{16\pi G_5}\int_{\mathcal{M}} d^5x\,\sqrt{g}\,\mathcal{R} - \frac{1}{8\pi G_5}\int_{\partial\mathcal{M}} d^4x\,\sqrt{h}\,K - S_E^{(\mathrm{flat})}$$

$$\Delta S_E = \pi^2 m_{\mathrm{Pl}}^2 R^2$$

$\gg 1$ provided $R \gg m_{\mathrm{Pl}}^{-1}$

But...

The Kaluza-Klein vacuum $\mathbb{M}_4 \times S^1$ is a solution to Einstein's equations for any radius of the KK circle: extra dimension is not stabilized

* however *

however... extra dimensions must be stabilized in any realistic model!

What is the effect of a stabilizing potential on bubble of nothing instabilities?

this will be the topic of the rest of my talk

Bubble of nothing as a CDL problem

Dine, Fox, Gorbatov (2006)

Dimensional reduction of 5-dimensional Einstein's gravity

$\Rightarrow$ real scalar field minimally coupled to gravity in 4 dimensions

$$ds_E = e^{-\sqrt{\frac{2}{3}}\frac{\phi}{m_{\rm Pl}}}\underbrace{(g_{\mu\nu}dx^\nu dx^\nu)}_{\text{4D metric}} + e^{2\sqrt{\frac{2}{3}}\frac{\phi}{m_{\rm Pl}}}\underbrace{dy^2}_{\text{radion}}$$

radion

4D metric

Witten's bubble can be written as the solution to a 4-dimensional CDL problem in the absence of a potential for the scalar field:

Schwarzschild radial coordinate

$$\xi \equiv \int_R^r \frac{d\hat{r}}{(1-R^2/\hat{r}^2)^{1/4}}$$

CDL O(4)-symmetric coordinate

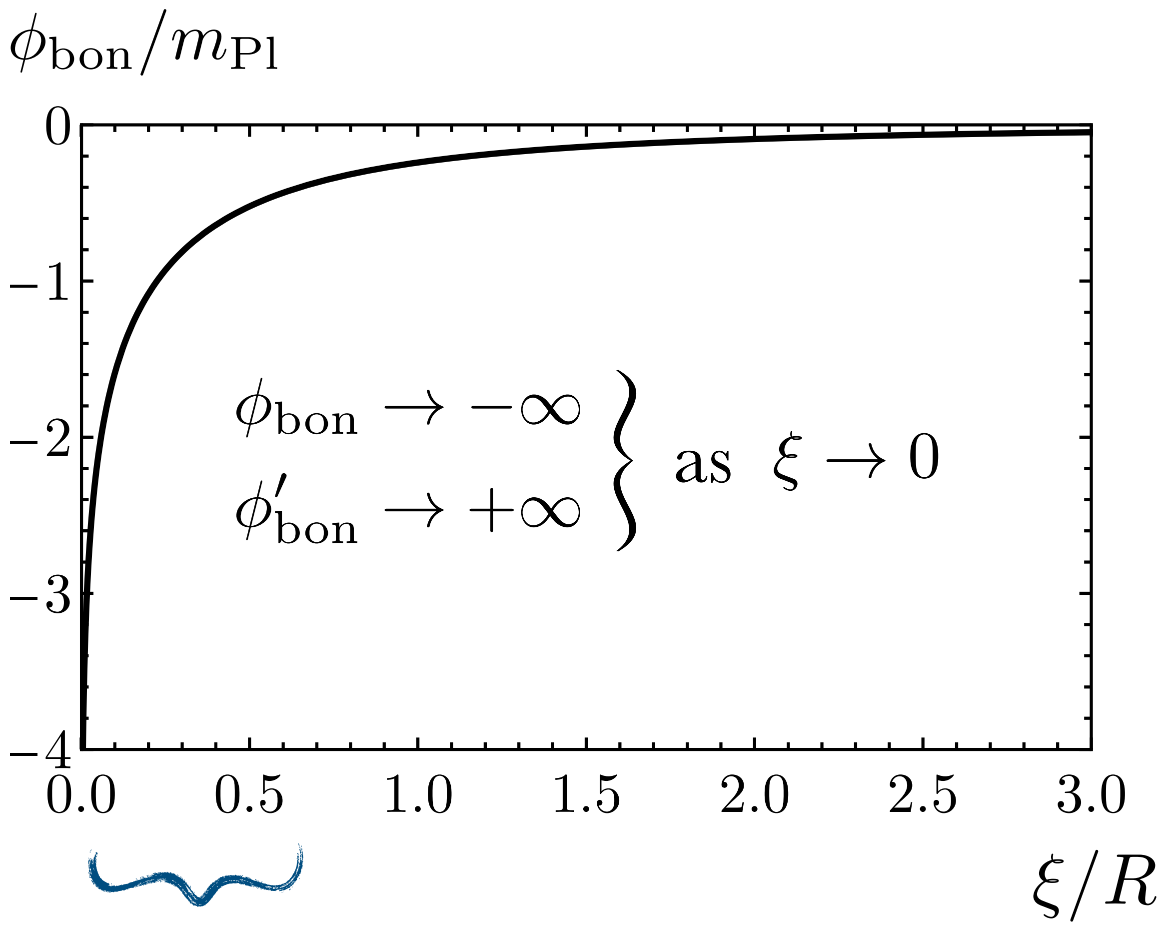
$$\phi'' + \frac{3\rho'}{\rho}\phi' = \cancel{\frac{dV(\phi)}{d\phi}}$$

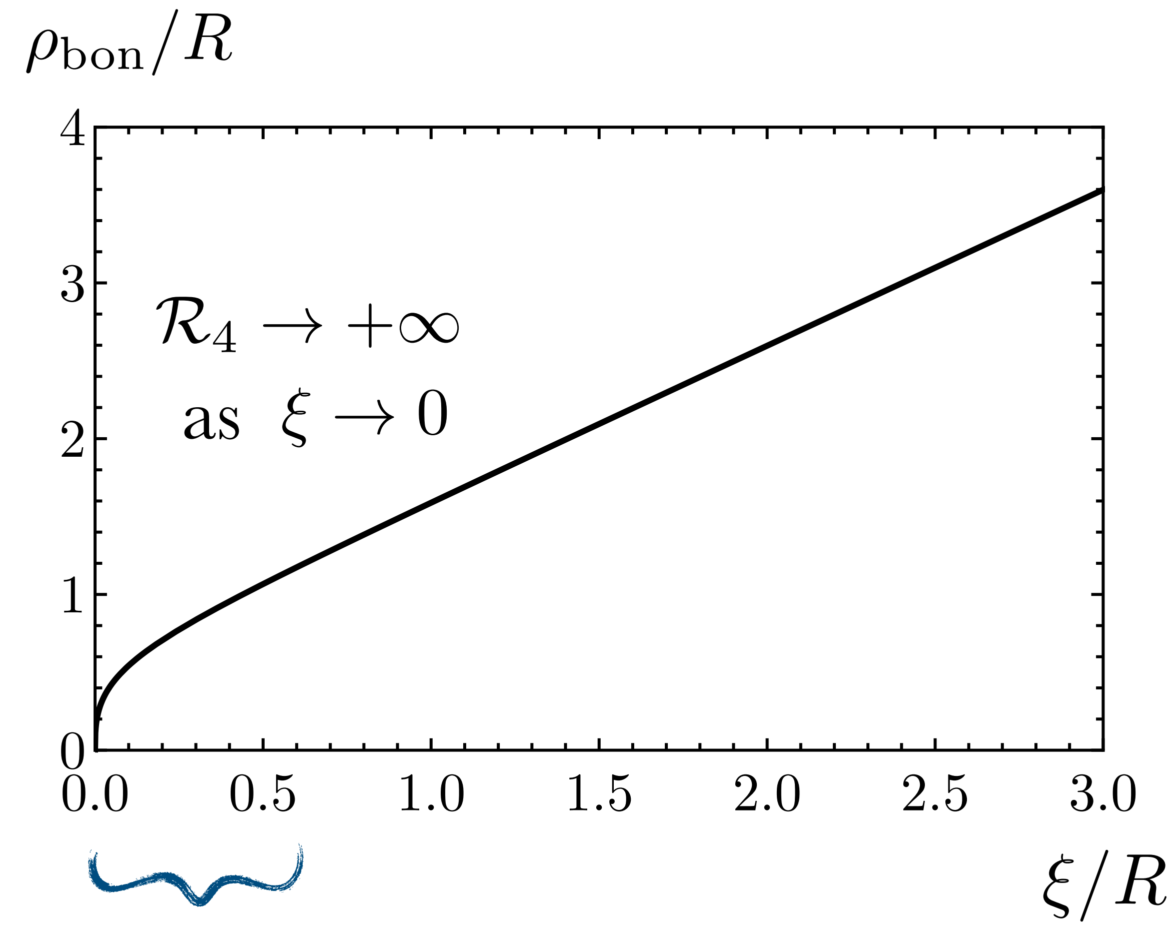
$$\rho'^2 = 1 + \frac{\rho^2}{6m_{\rm Pl}^2}(\phi'^2 - \cancel{2V(\phi)})$$

Bubble of nothing as a CDL problem

Dine, Fox, Gorbatov (2006)

Rather, a special kind of CDL problem… "singular" at the center of the bounce





$$m_{\rm Pl}\sqrt{\frac{2}{3}}\log\left(\frac{3\xi}{2R}\right)$$

for $\xi \ll R$

$$R\left(\frac{3\xi}{2R}\right)^{1/3}$$

for $\xi \ll R$

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Dine, Fox, Gorbatov (2006)

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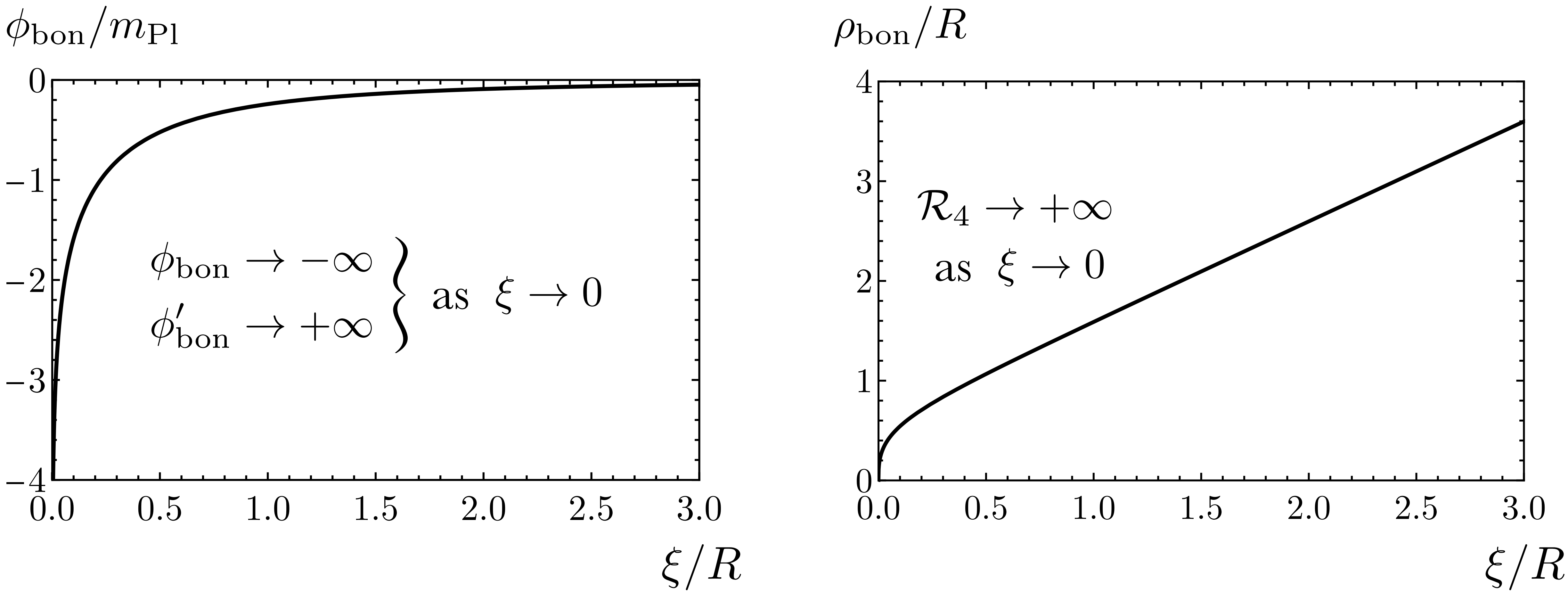


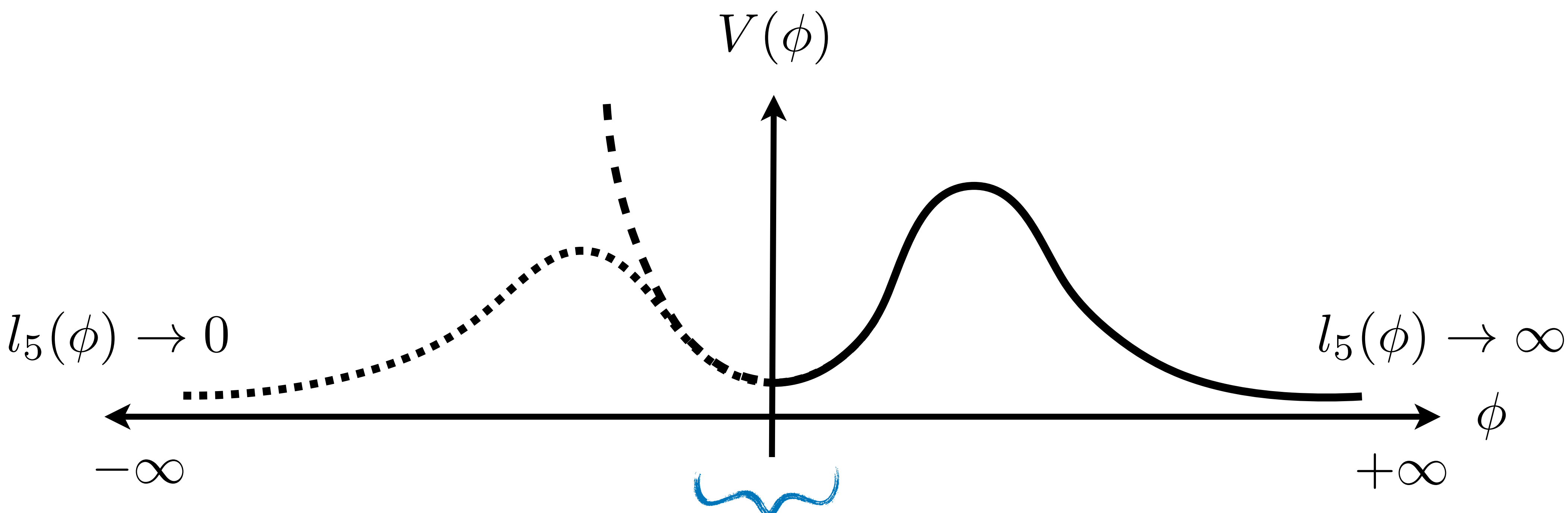
Crucially: $-m_{\rm Pl}^2 \mathcal{R}_4 + (\phi')^2 \equiv 0 \quad \Rightarrow \quad$ finite action

can now turn a potential for the scalar field!

Stabilized extra dimensions

In any realistic construction, extra dimensions *must* be stabilized

Survival of BON in the presence of a potential for the radion
is a necessary requirement if they are to be relevant in 'real life'



$$V(\phi) \simeq V_{\rm fv} + \frac{1}{2} m^2 \phi^2$$

with $\quad V_{\rm fv} \geq 0$

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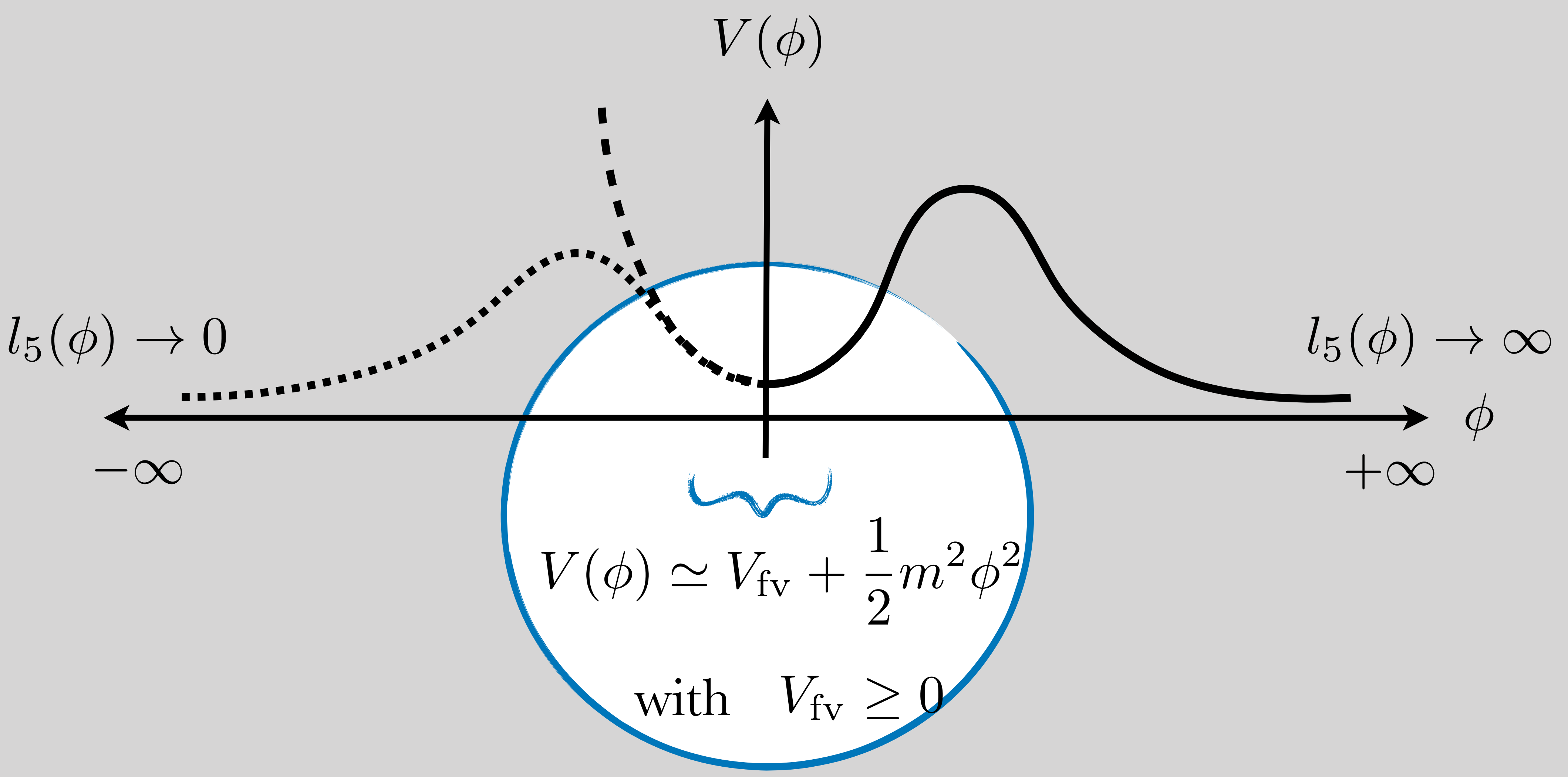

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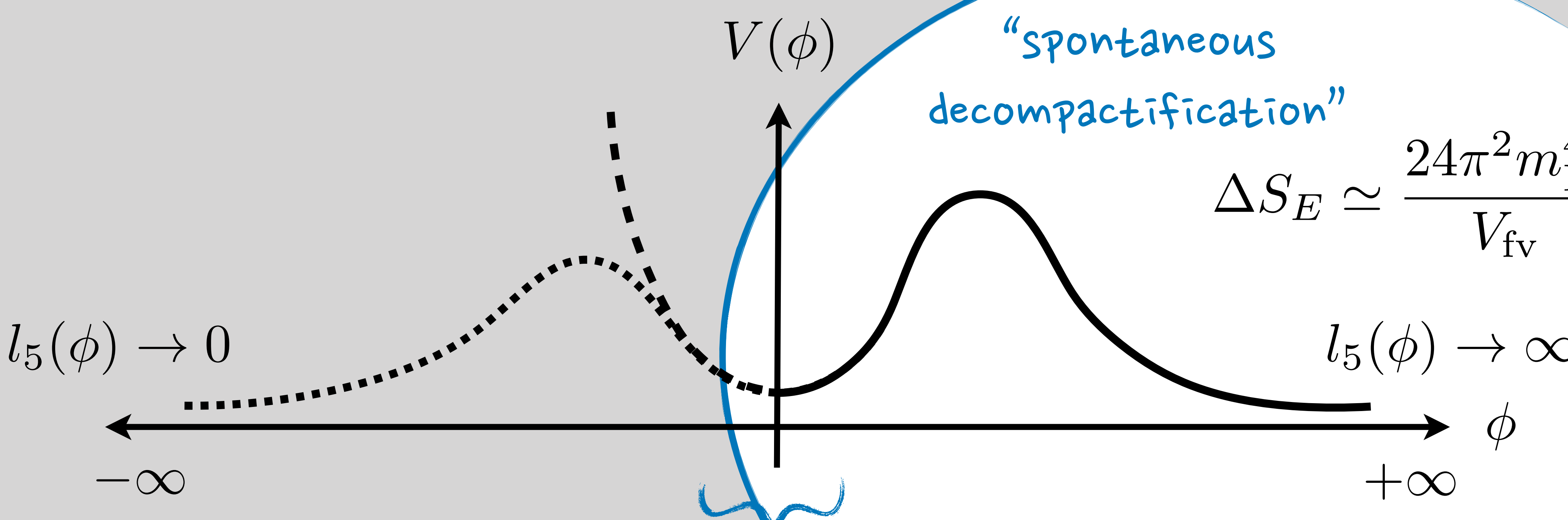

$$\Delta S_E \simeq \frac{24\pi^2 m_{\rm Pl}^4}{V_{\rm fv}}$$

$$V(\phi) \simeq V_{\rm fv} + \frac{1}{2}m^2\phi^2$$

with $V_{\rm fv} \geq 0$

e.g. Giddings (2003)
Giddings, Myers (2004)

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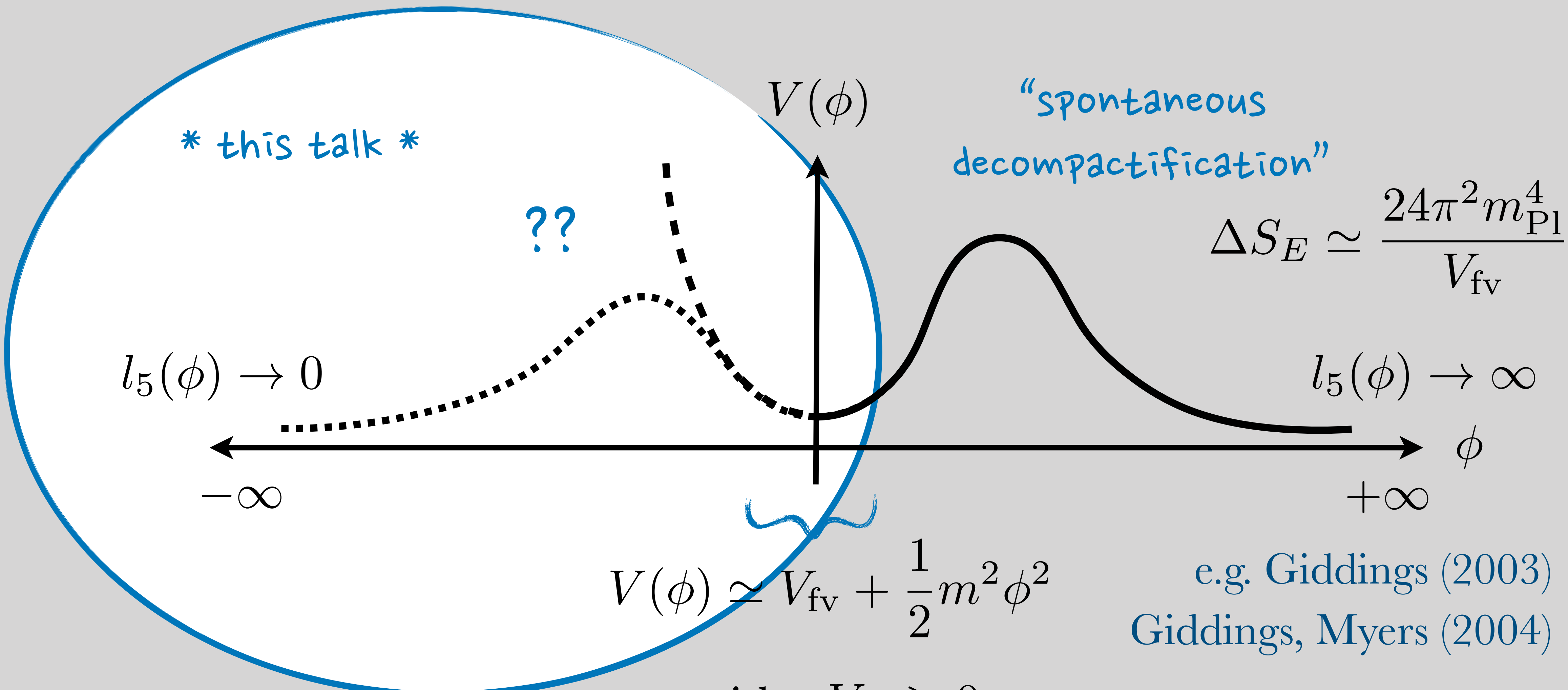

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with $V_{\rm fv} \geq 0$

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Bubble existence conditions

What types of scalar potentials are compatible with the existence of a bubble of nothing?

Consider scalar potentials of the form $V(\phi) \simeq V_0 e^{\gamma\phi/m_{\rm Pl}}$ as $\phi \to -\infty$

* with no assumption regarding the sign of γ *

e.g. $\phi'' + \frac{3\rho'}{\rho}\phi' = \frac{dV(\phi)}{d\phi}$ evaluated on the bubble of nothing:

$\propto \xi^{-2}$

$$\left.\frac{dV(\phi)}{d\phi}\right|_{\rm bon} \simeq \frac{\gamma V_0}{m_{\rm Pl}}\left(\frac{3\xi}{2R}\right)^{\sqrt{\frac{2}{3}}\gamma}$$

$$\sqrt{\frac{2}{3}}\gamma + 2 > 0 \quad \Rightarrow \quad \gamma > -\sqrt{6}$$

we will assume this is true in the rest of this talk

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A generalization of Witten's bubble

One-parameter family of bubble of nothing solutions to the CDL equations for $V(\phi) \equiv 0$:

$$\phi_\eta(\xi) \equiv \sqrt{\frac{3}{2}} m_{\rm Pl} \log \eta + \phi_{\rm bon}(\xi \cdot \eta^{-3/2})$$

$$\rho_\eta(\xi) \equiv \eta^{3/2} \rho_{\rm bon}(\xi \cdot \eta^{-3/2})$$

- When $V(\phi) \equiv 0: \quad G_5 \to \eta^{-1} G_5 \quad \Rightarrow \quad \eta \equiv 1$ WLOG

- When $V(\phi) \neq 0: \qquad ds_E \simeq d\lambda^2 + \lambda^2 d\alpha^2 + \eta^2 R^2 d\Omega_3$

near $\xi = 0$

radius of the 'hole' that nucleates in spacetime

$$\eta = \frac{R_3}{R} \neq 1$$

radius of the KK circle in the false vacuum

radius of the BON (self-consistently) determined by the features of the potential

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Some simplifying assumptions...

- Most interested in the behavior of the decay rate in limit of vanishing vacuum energy, $V_{\rm fv} \to 0$

  We take the de Sitter radius to the largest length scale:

$$l_{\rm dS} \gg R, m^{-1}$$

ignore terms that vanish as $l_{dS} \to \infty$

- Also:

$$m \ll 1/R \qquad\qquad V_0 \lesssim m_{\rm Pl}^2 m^2 \ll m_{\rm Pl}^2/R^2$$

$mR \ll 1$ will be the expansion parameter we use to build the bounce solution

can ignore details of potential to the left of false vacuum

None of these assumptions are necessary (or important!) for either existence or qualitative properties of solutions — but will make our life easier

(assumptions are relaxed in our numerical analysis)

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A bubble of nothing for dS vacua

Under this assumption, bounce solution behaves like the bubble nothing well into the regime where $\rho \gtrsim R$ and $\rho' \simeq 1$:

$$\rho(\xi) \simeq \xi + \alpha\, \eta^{3/2} R \qquad\qquad \phi(\rho) \simeq \sqrt{\frac{3}{2}} m_{\rm Pl} \left\{ \log \eta - \frac{\eta^3}{2} \left(\frac{R}{\rho} \right)^2 \right\}$$

$$\alpha = \sqrt{\pi}\,\Gamma\left(3/4\right)\Gamma\left(1/4\right)^{-1} \simeq 0.6$$

Assuming $\eta - 1 \ll 1$ (TBJ a posteriori), the solution remains BON-like until it approaches the false vacuum:

$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} \simeq m^2 \phi \qquad \Rightarrow \qquad \phi_i(\rho) \simeq C\, \frac{K_1(m\rho)}{m\rho}$$

For $\rho \gg m^{-1}$:

$$\frac{\phi_i(\rho)}{m_{\rm Pl}} \propto -\frac{e^{-m\rho}}{(m\rho)^{3/2}}$$

approaches the false vacuum exponentially fast

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A bubble of nothing for dS vacua

Provided $V_{\rm fv} > 0$ the solution eventually transitions into the de Sitter false vacuum:

$$\rho(\xi) \simeq l_{\rm dS} \sin\left(\frac{\xi + \alpha R}{l_{\rm dS}}\right)$$

Demanding continuity:

$$\eta = \frac{R_3}{R} \simeq 1 + \underbrace{\frac{m^2 R^2}{4} \log(mR)^{-1}}_{\text{small if } mR \ll 1} + \mathcal{O}\left(m^2 R^2\right)$$

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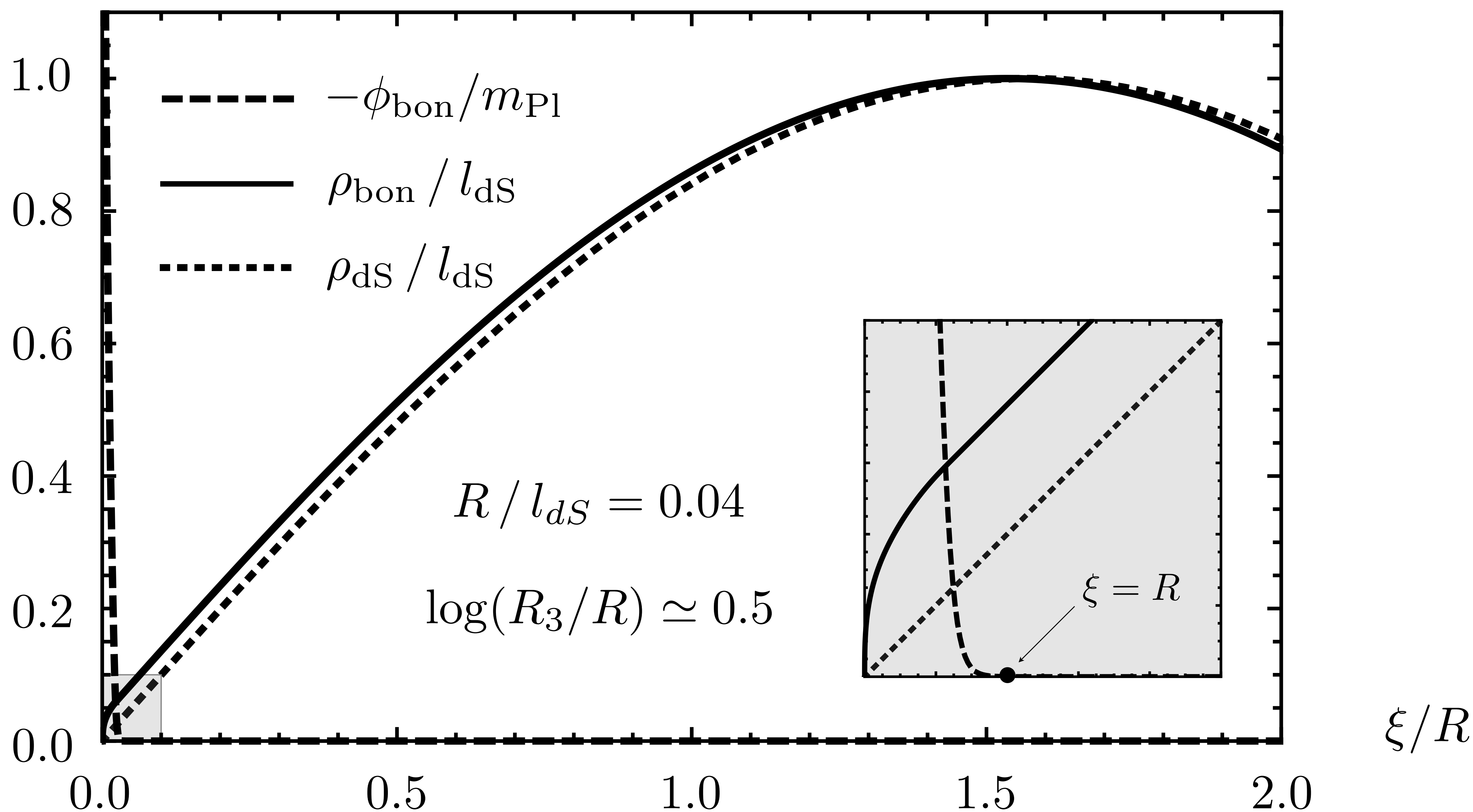
A bubble of nothing for dS vacua

Instanton describing a BON instability of a de Sitter vacuum:



instanton "stitches" together BON and false vacuum (de Sitter) solutions

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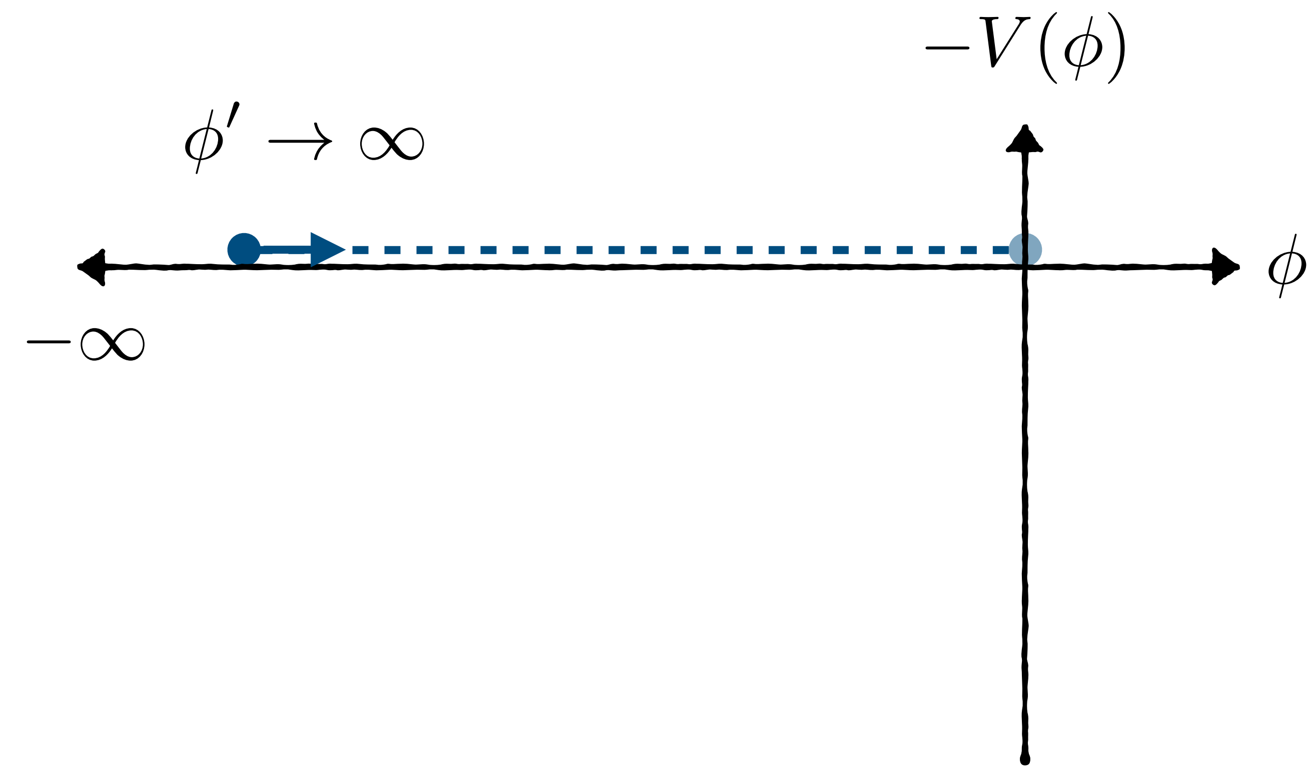
Shooting… with "infinite velocity"

Witten's bubble ($V(\phi) \equiv 0$):

$$\phi'' + \frac{3\rho'}{\rho}\phi' = 0$$

all "friction" (opposite to CDL thin-wall bounce)

The singular bc's of the bubble of nothing correspond to shooting with "infinite velocity"



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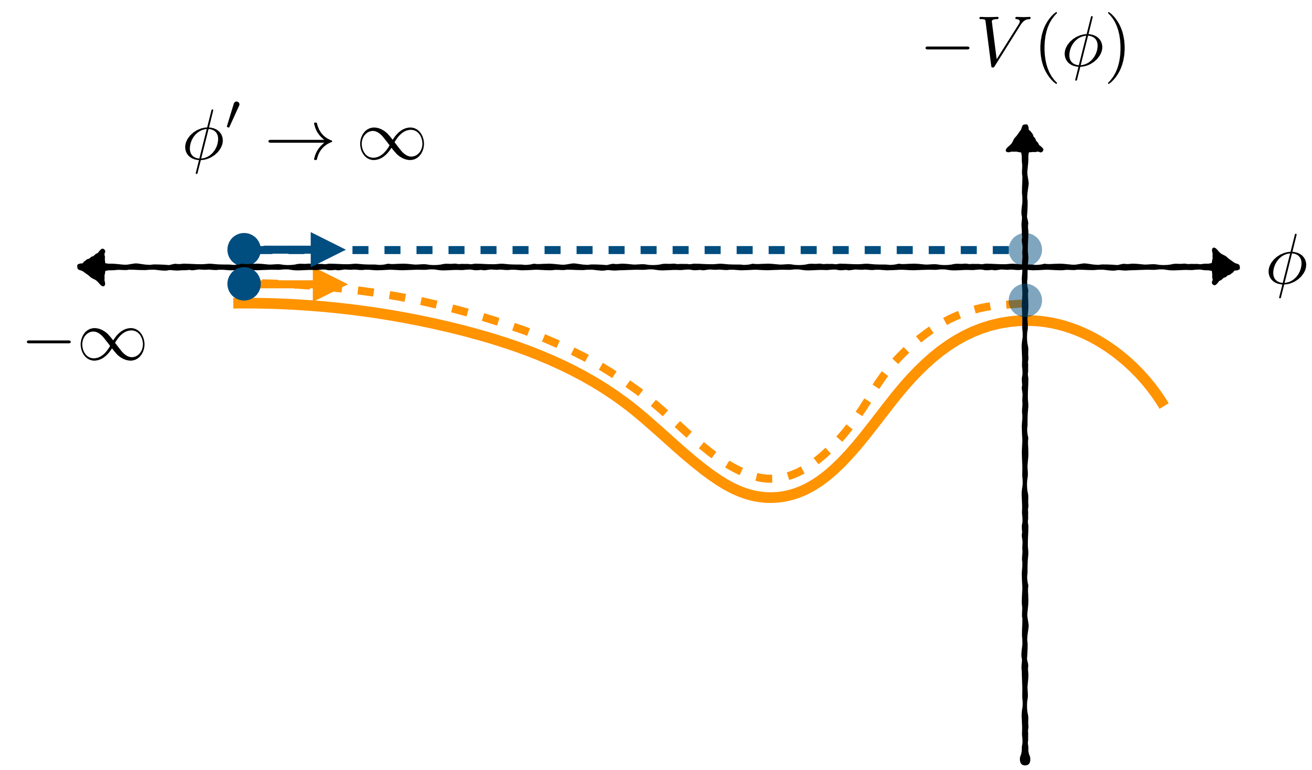

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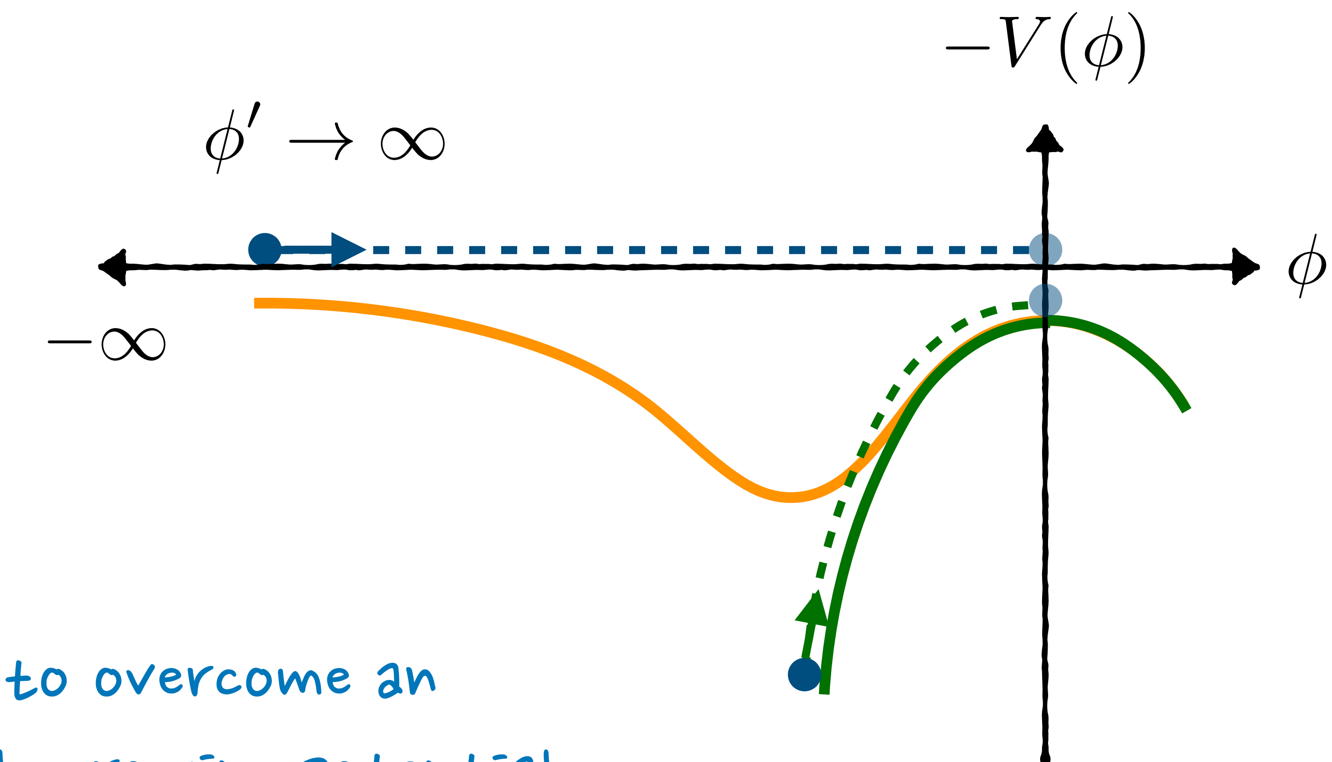

enough to overcome an exponentially growing potential

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Decay rate

$$\Delta S_E \;=\; \pi^2 m_{\rm Pl}\sqrt{\frac{2}{3}}\rho(\xi)^3\phi'(\xi)\Big|_{\xi=0} \;-\; 2\pi^2\int_0^{\xi_{\rm max}} d\xi\rho^3 V \;-\; S_E\Big|_{\rm dS}$$

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$$\Delta S_E\Big|_{\xi=0} = \pi^2 m_{\rm Pl}^2 R^2\eta^3 \simeq \pi^2 m_{\rm Pl}^2 R^2\left\{1+\frac{3}{4}m^2R^2\log(mR)^{-1}\right\}$$

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Decay rate

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$$\Delta S_E\Big|_V \; \simeq \; -2\pi^2\int^{m^{-1}} d\rho\rho^3(V - V_{\rm fv}) \; \simeq \; -\frac{3\pi^2}{8}m_{\rm Pl}^2 m^2 R^4 \log(mR)^{-1}$$

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Decay rate

$$\Delta S_E \;=\; \pi^2 m_{\rm Pl}\sqrt{\frac{2}{3}}\rho(\xi)^3\phi'(\xi)\Big|_{\xi=0} \;-\; 2\pi^2\int_0^{\xi_{\rm max}} d\xi\,\rho^3 V \;-\; S_E\Big|_{\rm dS}$$

In total…

$$\boxed{\Delta S_E \simeq \pi^2 m_{\rm Pl}^2 R^2\left\{1+\frac{3}{8}m^2R^2\log(mR)^{-1}\right\}} \quad + \mathcal{O}(\pi^2 V_0 R^4)$$

small correction if $V_0 \ll m_{\rm Pl}^2/R^2$

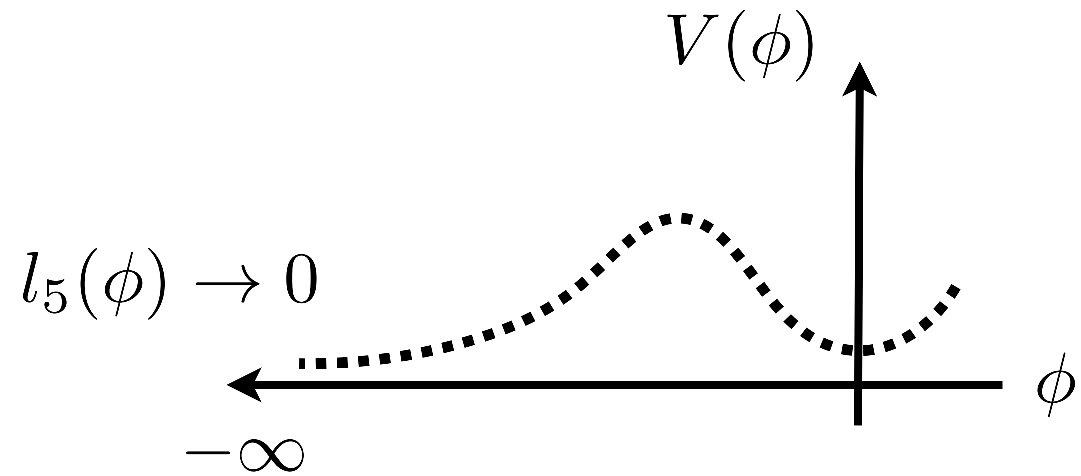
a *finite* correction to the action of Witten's bubble in the limit $V_{fv} \to 0$

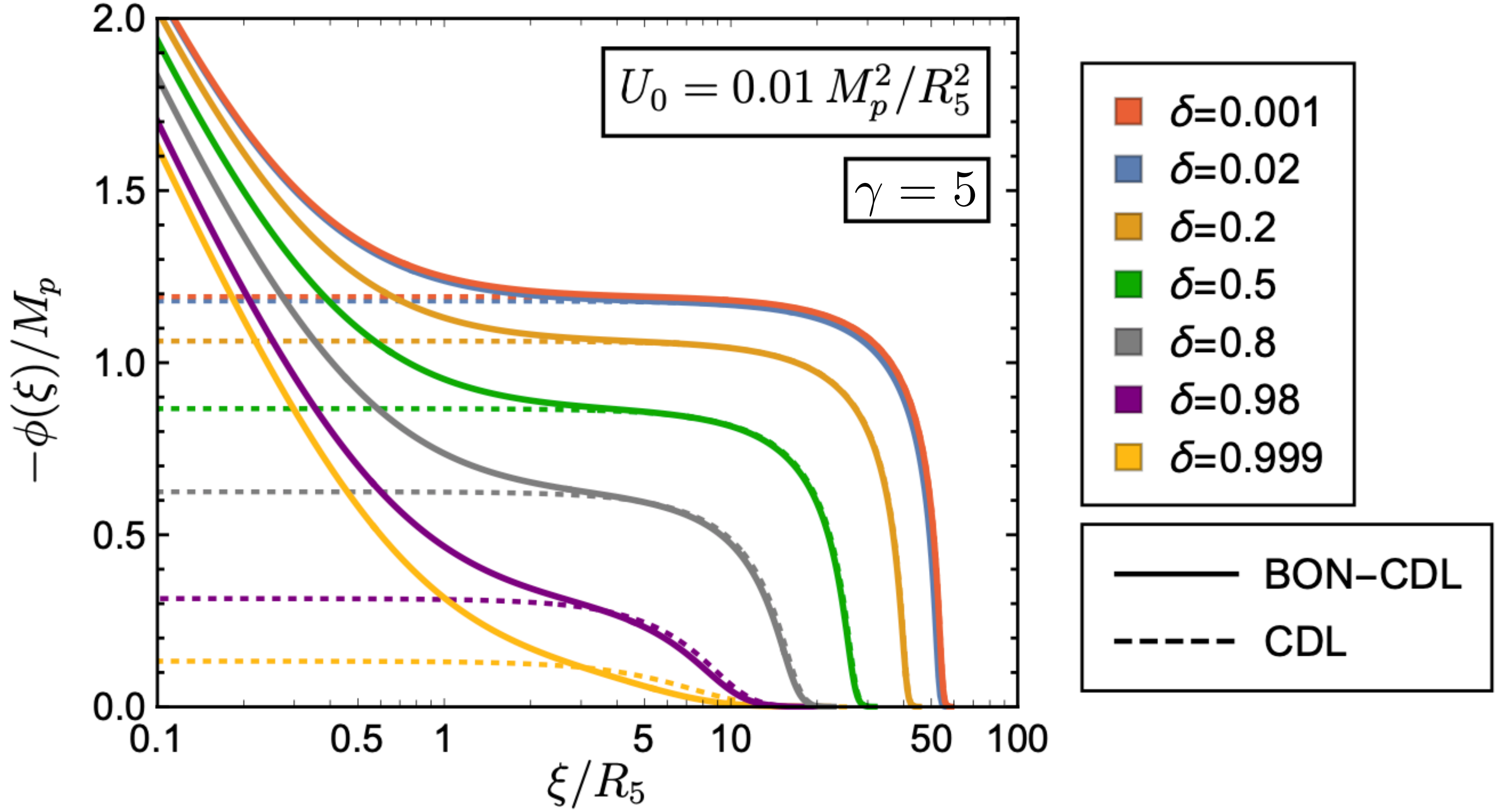
c.f. spontaneous decompactification: $\Delta S_E \simeq \dfrac{24\pi^2 m_{\rm Pl}^4}{V_{\rm fv}}$

* BON-like decay always faster for sufficiently small vacuum energy *

Exotic bounces

Depending on details of the potential, there may be additional instanton solutions with BON-like behavior





although not the bounce of least action!

Draper, **IGG**, Lillard; 2105.10507

Conclusions

- Bubble of nothing instabilities survive even in the presence of a stabilizing potential

even if the potential grows in the compactification limit
(although with limitations)

- Decay rate remains finite in the limit of vanishing vacuum energy

BON will be the dominant decay chanel for small enough cc
(provided supersymmetry is not restored in that limit)

- Potential implications for vacuum selection, e.g.

$$\Gamma_{\rm bon}/H_0^4 \gtrsim 1 \qquad \Rightarrow \qquad 2\pi R \gtrsim 50\, m_{\rm Pl}^{-1}$$

much left to understand about the physics of bubbles of nothing

Thank you!